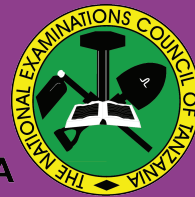




THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**STUDENTS' ITEM RESPONSE ANALYSIS
REPORT ON THE FORM TWO NATIONAL
ASSESSMENT (FTNA) 2024**

ADDITIONAL MATHEMATICS



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042 ADDITIONAL MATHEMATICS

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FOREWORD

This report presents the Students' Item Response Analysis (SIRA) on the Form Two National Assessment, Additional Mathematics subject which was conducted in November 2024. The report highlights the strengths and weaknesses that students demonstrated in their responses to each question. The purpose is to provide feedback that teachers, students and other education stakeholders can use to improve teaching and learning of the subject.

The students' performance in the FTNA 2024 Additional Mathematics assessment was generally good. The report identifies seven (7) topics which students performed well, one (1) topic with average performance, and one (1) with weak performance. The students' good performance was attributed to good competence in the tested concepts, while their average performance was due to moderate understanding of the subject matter. On the other hand, the weak performance was attributed to insufficient knowledge, misinterpretation of questions, use of incorrect or inappropriate formulae, and inability to perform basic operations.

The National Examinations Council of Tanzania expects that the recommendations provided in this report will help students, teachers, and other education stakeholders to develop appropriate strategies for improving students' performance in Additional Mathematics.

The National Examinations Council of Tanzania appreciates the contributions of examination officers and all those who participated in the preparation of this report.



Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The report on Students’ Item Response Analysis (SIRA) in Additional Mathematics paper aims at providing feedback to stakeholders including teachers and students about the students’ performance in the 2024 Form Two National Assessment (FTNA). The paper was set according to 2021 Form Two National Assessment Format which is bases on the 2010 Additional Mathematics syllabus for secondary school, Form I – II. The paper consisted of ten compulsory questions, each worth 10 marks.

In 2024, a total of 392 students sat for the FTNA in Additional Mathematics, which is 49 less than the number of students who sat for the assessment in 2023. Overall performance was good as 84.95 percent of the students passed. When compared to 2023 where 70.29 percent of the students passed the assessment, the 2024 performance has improved by 14.66 percent. Figure 1 compares the percentage of students who achieved each grade in Additional Mathematics 2023 and 2024.

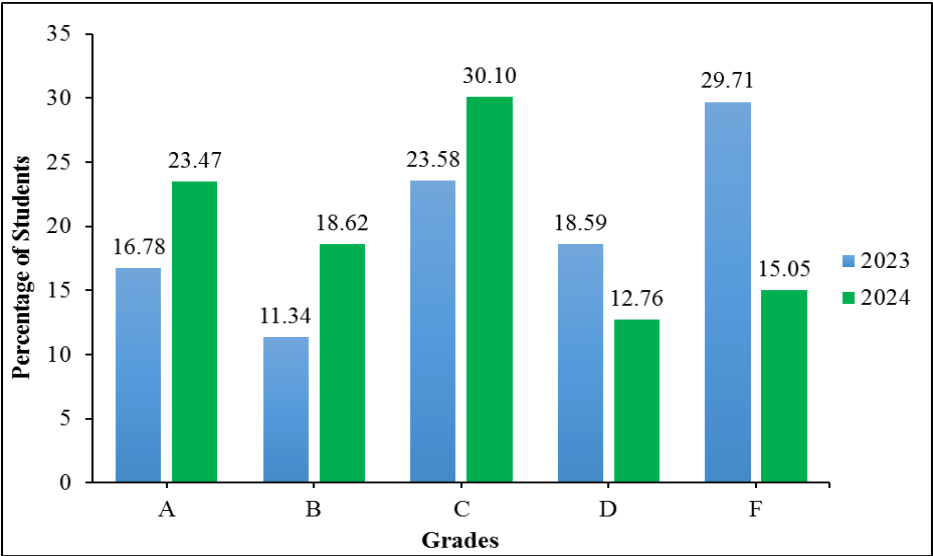


Figure 1: Performance of Students by Grades in 2023 and 2024

Figure 1 shows that, the percentage of students who attained grades A, B and C in 2024 is higher compared to 2023. On the other hand, the percentage of students who attained grades D and F in 2023 is higher compared to 2024.

This report has four (4) sections including introduction as Section 1.0. Section 2.0 of this report provides a detailed analysis of students' performance for each question. It includes brief explanations of the requirements for each question, along with an analysis of students' responses. Extracts of both good and weak responses are included to illustrate the analysis. Additionally, factors influencing performance on each question are highlighted using samples of students' work.

The analysis of the students' performance on each topic assessed is given out in Section 3.0. Finally, Section 4.0 of the report provides conclusions and recommendations for improving performance in future assessments. Also the report includes Appendices I and II. Appendix I presents the performance analysis of the assessed topics, whereby green, yellow, and red colours indicate good, average, and poor performance, respectively. Appendix II compares students' performance in each topic for 2023 and 2024 in Additional Mathematics.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section presents the analysis of students' performance on each question. The national assessment results are based on score intervals of 75–100, 65–74, 45–64, 30–44, and 0–29, which correspond to excellent, very good, good, satisfactory, and fail, respectively. Moreover, the performances' percentage on each question is divided into three categories, which are weak performance (0–29 percent), average performance (30–64 percent), and good performance (65–100 percent).

2.1 Question 1: Numbers

The question was composed of two parts: (a) and (b). In part (a), the students were informed that one Form Two student at a certain school had a birthday party. During the occasion, some drinks were distributed among the six tables in the following manner: Wine = 7, 8, 11, 13, 20, 23, Juice = 4, 8, 12, 20, 32, 52 and Water = 1, 2, 4, 5, 6, 7. The students were required to identify the distribution that follows the Fibonacci sequence. In part (b), the students were required to use the divisibility rule to identify the number of

villages that can acquire the sewing machines distributed by the tailoring mart association. Each village has the following number of people who belong to the association: Village, A = 69388, B = 125362, C = 164369 and D = 378978. Therefore, to qualify for a sewing machine, a minimum of 19 members from the same village must be present.

The analysis of the data revealed that 309 (78.82%) students scored 0 to 2.5 marks, 21 (5.36%) scored 3.0 to 6.0 marks, and 62 (15.82%) students scored 6.5 to 10 marks. Thus, the students' performance on this question was weak, as only 21.18 percent of the students who attempted this question passed. Figure 2 presents the performance summary.

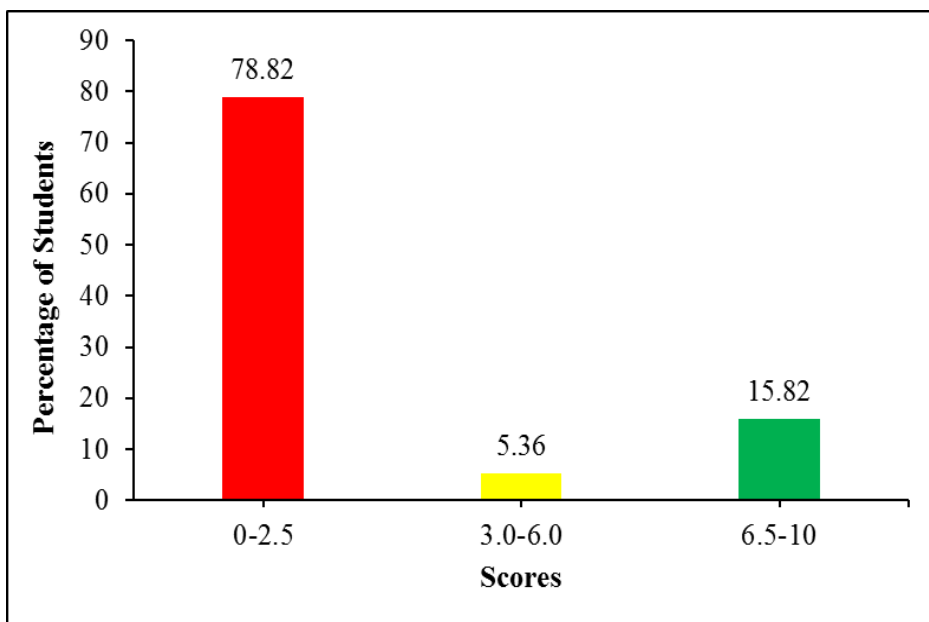
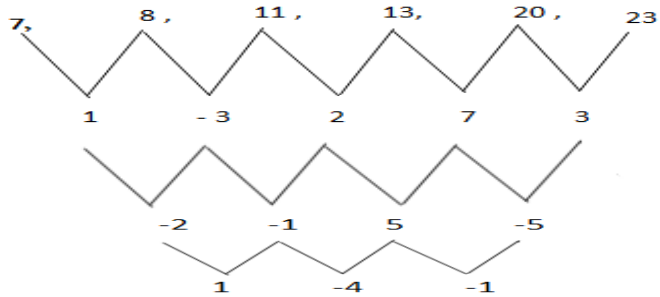


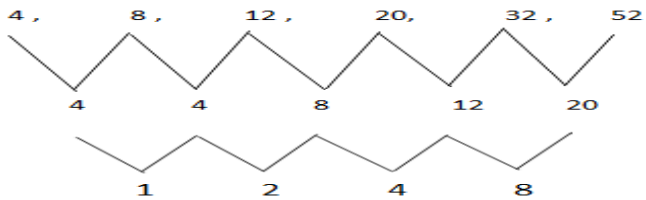
Figure 2: *Students' Performance in Question 1*

Further analysis revealed that 64 (16.3%) students scored zero. Various challenges prevented these students from correctly answering this question. In part (a), some students misinterpreted the Fibonacci sequence by identifying types of numbers from the given sequence. For instance, some students identified prime numbers as {7, 11, 13, 23}, odd numbers as {1, 5, 7} and even numbers as {2, 4, 6, 8, 20, 32, 52}, from the distributions. Moreover, other students responded to the question by simply illustrating the pattern for each drink, as follows:

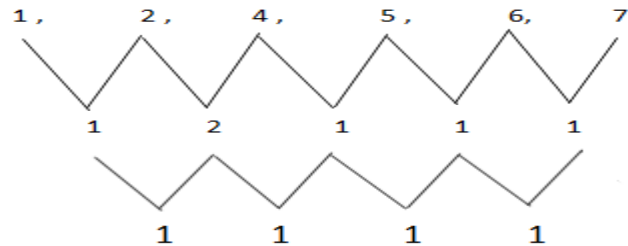
Wine:



Juice:



Water:



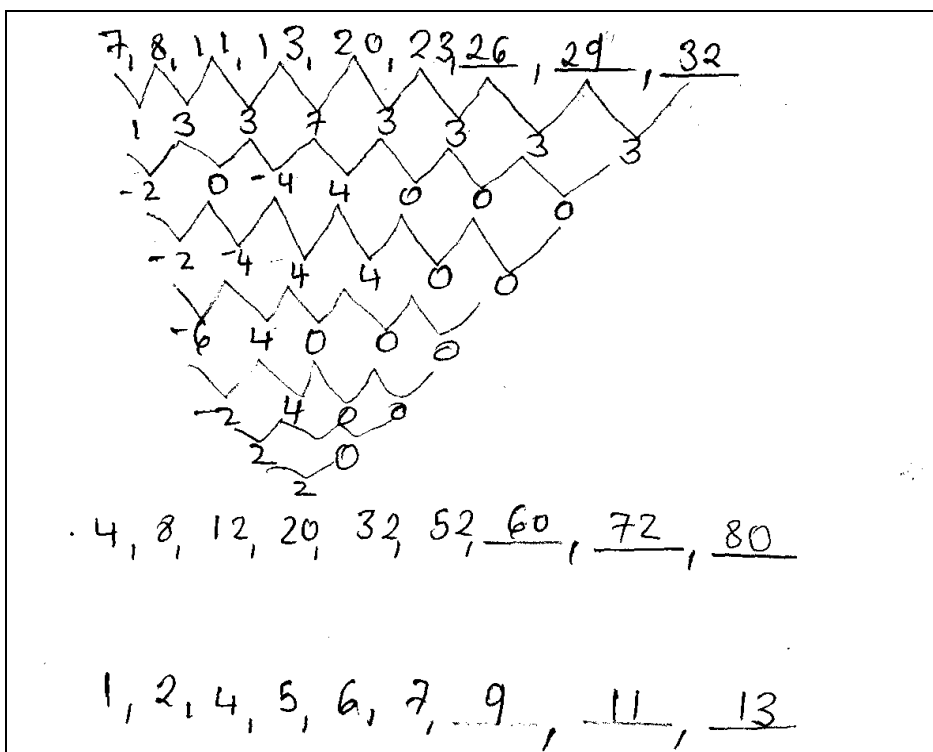
Thereafter, it was commented that all patterns represent Fibonacci sequence.

Additionally, most students did not verify the Fibonacci rule; instead, they inappropriately identified the next terms of the distributions using incorrect patterns such as Pascal's triangle (Extract 1.1).

In part (b), the analysis revealed that some students were not familiar with the divisibility rule of 19. For instance, some students added up all the digits of each village's population, then divided the total by 19. Thus, they obtained Village A = 1.79, Village B = 1, Village C = 1.53 and Village D = 2.21. Then, incorrectly concluded that there was only one machine. Other students added all the population digits for each village, then added the results as well. That is, they obtained Village A = 34, Village B = 19, Village C = 29 and Village D = 42. Later,

worked on sum, which resulted in 124. Then, they concluded that 124 villages could acquire the sewing machine.

Furthermore, some students applied inappropriate rules. For example, they multiplied the last digit of each village's population by 5 and subtracted the product from the remaining numbers, which is the rule of divisibility for 17. That is, Village A = 69388 \Rightarrow 6938 - (8 \times 5) = 6898. Likewise, for Village B = 12526, Village C = 16391, and Village D = 37857. They then concluded that Village A and Village B, each with one sewing machine, qualified for the acquisition of many machines. Likewise, others applied the divisibility rule for 4, such as for Village A, they took 69388 \Rightarrow 88 \div 4 = 22 because the number of villages is 4. Furthermore, some students responded to the question by dividing the given populations directly by 19. Extract 1.1 presents a sample response from a student who failed to correctly attempt the question.



$$A = 69388, B = 125362, C = 164369, \text{ and } D = 378978$$

Soln

$$A = 69388$$

$$A = 6 + 9 + 3 + 8 + 8$$

$$A = 34 \text{ Divisibility by } 2$$

$$B = 1 + 2 + 5 + 3 + 6 + 2$$

$$B = 19 \text{ Divisibility by } 3$$

$$C = 1 + 6 + 4 + 3 + 6 + 9$$

$$C = 29 \text{ Divisibility by } 3$$

$$D = 3 + 7 + 8 + 9 + 7 + 8$$

$$D = 42 \text{ Divisibility by } 2$$

∴ The villages can acquire the sewing machines is $34 + 19 + 29 + 42 = 124$
124 villages

Extract 1.1: A sample of incorrect responses to question 1.

In Extract 1.1, part (a), the student calculated the next three terms of each distribution contrary to the requirement of the question. In part (b), the student added all the digits of the populations for each village and then summed up the answers resulting in an incorrect answer.

Despite the weak performance, 17 (4.3%) students responded correctly to this question and scored all 10 marks. In part (a), the students demonstrated their competency with the Fibonacci sequence by accurately understanding its definition, which states, "If the sum of two preceding numbers is equal to the next number, then the sequence is a

Fibonacci.” Then, they accurately applied the Fibonacci rule to determine the next number. They discovered that the distribution of juice follows a Fibonacci sequence; that is, Juice: $4 + 8 = 12$, $8 + 12 = 20$, $12 + 20 = 32$, $20 + 32 = 52$. They also verified that distributions for wine and water are not stipulating Fibonacci because, Wine: $7 + 8 \neq 11$ and Water: $1 + 2 \neq 4$. Finally, the students came to the conclusion that only Juice = 4, 8, 12, 20, 32, 52 is the Fibonacci sequence.

In part (b), the students demonstrated familiarity with the rule of divisibility for 19, which stipulates that a number becomes divisible by 19 when it adds two times the last digit to the remaining number. Thus, the students performed basic operations based on the rule and identified only three (3) villages, namely, Village A, B, and C with populations of 69388, 125362, and 164369, respectively, qualified to acquire the sewing machines. Extract 1.2 presents a sample response from a student who correctly answered this question.

Wine = 7, 8, 11, 13, 20, 23

$$7 + 8 = 15$$

It is not representing Fibonacci sequence.

Juice = 4, 8, 12, 20, 32, 52

$$4 + 8 = 12$$

$$12 + 8 = 20$$

$$20 + 12 = 32$$

$$32 + 20 = 52$$

It represents Fibonacci sequence.

Water = 1, 2, 4, 5, 6, 7

$$1 + 2 = 3$$

It is not representing Fibonacci sequence.

Juice which is 4, 8, 12, 20, 32, 52 represents Fibonacci sequence.

A number is divisible by 19 by taking the last digit multiply it by 2 then add the result from the rest until you get the small number which is divisible by 19.

$$\begin{array}{r}
 A = 69388 \\
 \begin{array}{r}
 69\overset{1}{3}8(8) \times 2 \\
 + \quad 16 \\
 \hline
 69\overset{2}{5}(4) \times 2 \\
 + \quad 8 \\
 \hline
 70\overset{3}{3}(3) \times 2 \\
 + \quad 6 \\
 \hline
 7\overset{4}{6}(6) \times 2 \\
 + \quad 12 \\
 \hline
 19 \\
 \hline
 19 \Rightarrow 1
 \end{array}
 \end{array}$$

Village A can acquire the sewing machine.

$$\begin{array}{r}
 \text{Village B} = 125362 \\
 \begin{array}{r}
 125\overset{1}{3}6(2) \times 2 \\
 + \quad 4 \\
 \hline
 1254\overset{2}{0}(0) \times 2 \\
 + \quad 0 \\
 \hline
 125\overset{3}{4}(4) \times 2 \\
 + \quad 8 \\
 \hline
 13\overset{4}{3}(3) \times 2 \\
 + \quad 6 \\
 \hline
 19 \\
 \hline
 19 \Rightarrow 1
 \end{array}
 \end{array}$$

Village B can acquire sewing machine

$$\begin{array}{r}
 C = 164369 \\
 \begin{array}{r}
 164\overset{1}{3}6(9) \times 2 \\
 + \quad 18 \\
 \hline
 164\overset{2}{5}(4) \times 2 \\
 + \quad 8 \\
 \hline
 165\overset{3}{3}(3) \times 2 \\
 + \quad 6 \\
 \hline
 17\overset{4}{7}(1) \times 2 \\
 + \quad 2 \\
 \hline
 19 \\
 \hline
 19 \Rightarrow 1
 \end{array}
 \end{array}$$

Village C can acquire sewing machine

$$\begin{array}{r}
 D = 378978 \\
 \begin{array}{r}
 37\overset{1}{8}97(8) \times 2 \\
 + \quad 16 \\
 \hline
 379\overset{2}{1}(3) \times 2 \\
 + \quad 6 \\
 \hline
 379\overset{3}{9}(7) \times 2 \\
 + \quad 14 \\
 \hline
 39\overset{4}{9}(3) \times 2 \\
 + \quad 6 \\
 \hline
 4\overset{5}{0}(5) \times 2 \\
 + \quad 10 \\
 \hline
 14 \\
 \hline
 19
 \end{array}
 \end{array}$$

Village D cannot acquire sewing machine

∴ Three villages can acquire sewing machine.

Extract 1.2: A sample of the correct responses to question 1.

In Extract 1.2, part (a), the student demonstrated competence with the Fibonacci sequence by accurately using its definition. In part (b), the students successfully applied the rule of divisibility for the number 19.

2.2 Question 2: Algebra

The question consisted of parts (a) and (b). In part (a), the students were required to find the solution set of the inequality $-18 < 6x + 6 \leq 36$ and show it on a number line. In part (b), the students were required to calculate the value of “g” if $K = 12$ and $T = 6$, given that $K^2 = \frac{T^2 g}{T + g}$.

The data depicts that 101 (25.76%) students scored marks ranging from 0 to 2.5, 119 (30.36%) scored marks ranging from 3.0 to 6.0, and 172 (43.88%) students scored marks ranging from 6.5 to 10. Figure 3 presents the percentage summary of the students' performance on this question.

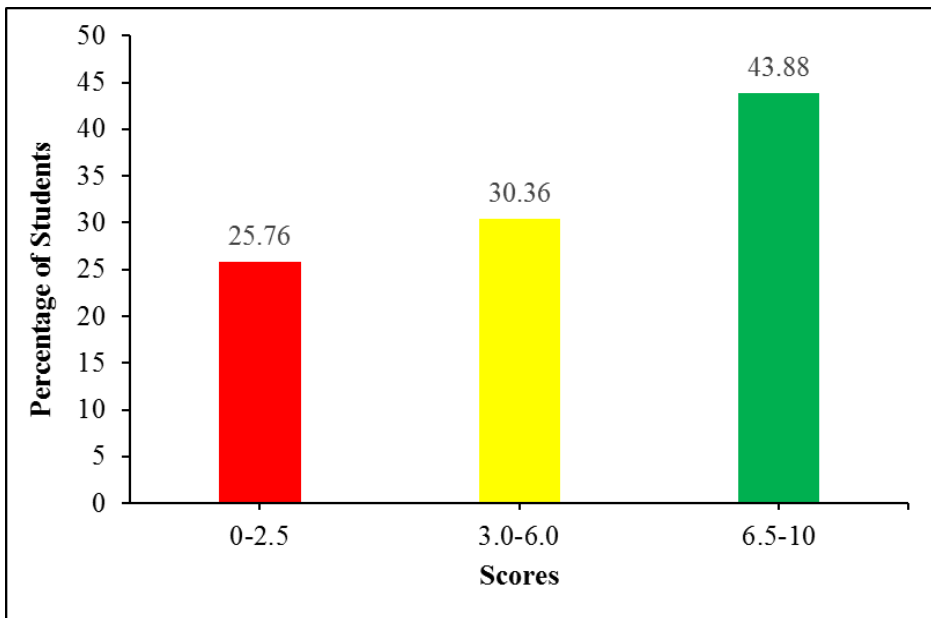
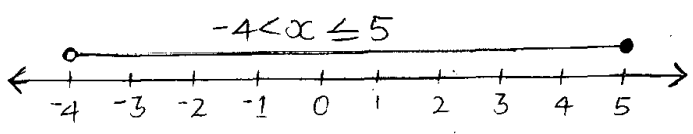


Figure 3: *Students' Performance in Question 2*

The performance of students on this question was generally good, as 74.24 percent of the students who attempted this question passed. Furthermore, the data reveals that 19.1 percent of students who attempted the question correctly responded to all items. In part (a), the students applied their algebraic knowledge, correctly got $-4 < x \leq 5$ and displayed it on a number line (Extract 2.1). In part (b), the students

obtained $g = \frac{K^2 T}{T^2 - K^2}$ from $K^2 = \frac{T^2 g}{T + g}$ then, substituted the values of $K=12$ and $T=6$ to the equation and got $g=-8$. Alternatively, students substituted the given values of K and T into the equation $K^2 = \frac{T^2 g}{T + g}$, and obtained $g=-8$. Extract 2.1 is a sample of the correct response from one of the students who attempted this question.

$$\begin{aligned}
 & -18 < 6x + 6 \leq 36 \\
 & -18 < 6x + 6 \leq 36 \\
 & -18 < 6x + 6 \\
 & -18 - 6 < 6x \\
 & \frac{-24}{6} < \frac{6x}{6} \\
 & -4 < x \\
 & 6x + 6 \leq 36 \\
 & 6x \leq 36 - 6 \\
 & \frac{6x}{6} \leq \frac{30}{6} \\
 & x \leq 5 \\
 & -4 < x \leq 5
 \end{aligned}$$


$$\begin{aligned}
\frac{K^2}{1} &= \frac{T^2 g}{T+g} \\
\frac{K^2}{1} &= \frac{T^2 g}{T+g} \\
K^2(T+g) &= T^2 g \\
K^2 T + K^2 g &= T^2 g \\
K^2 T &= T^2 g - K^2 g \\
\frac{K^2 T}{T^2 - K^2} &= \frac{g(T^2 - K^2)}{T^2 - K^2} \\
g &= \frac{K^2 T}{T^2 - K^2} \\
K &= 12 \\
T &= 6 \\
g &= \frac{12^2 \times 6}{12^2 - 6^2} \\
g &= \frac{144 \times 6}{36 - 144} \\
g &= \frac{864}{-108} \\
g &= \frac{-864}{108} \\
g &= -8 \qquad \therefore g = -8
\end{aligned}$$

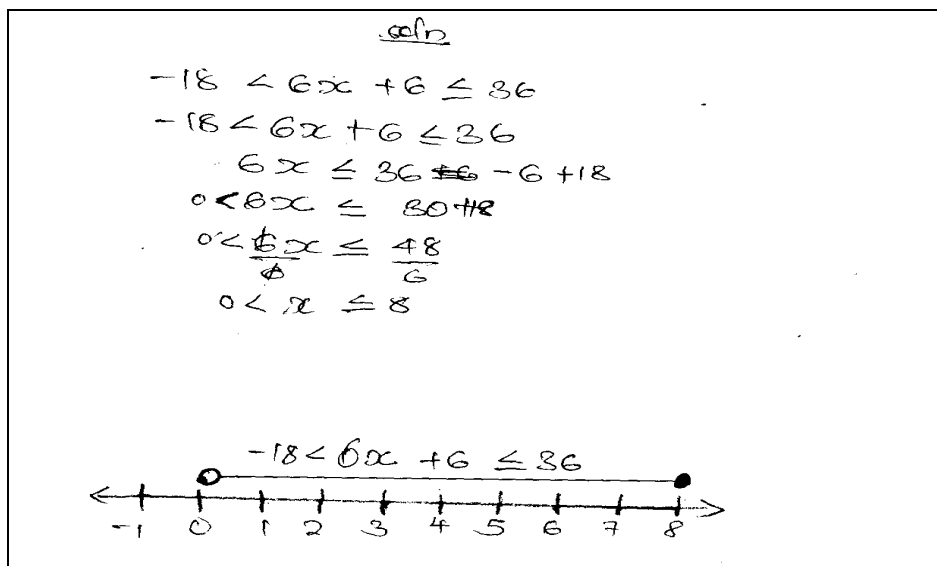
Extract 2.1: A sample of the correct responses to question 2

In Extract 2.1, part (a), the student correctly manipulated the equation, performed basic algebraic operations, and finally represented the solution on the number line. In part (b), the student correctly made g the subject and performed substitution to obtain its value.

However, 25.7 percent of students scored low marks due to various difficulties. In part (a), a number of students performed incorrect operations when simplifying the inequality $-18 < 6x + 6 \leq 36$. For

instance, some students incorrectly shifted -18 to the right-hand side of the inequality, leading to an incorrect result $6x + 6 \leq 36 + 18$. Thereafter, they incorrectly applied the absolute value technique, resulting in $\pm|6x + 6 \leq 54|$, leading to an incorrect solution $-10 \geq x \leq 8$ which they then presented on a number line. Furthermore, the analysis revealed that a large number of students committed computational errors. For instance, some students correctly divided $-18 < 6x + 6 \leq 36$ by 6 and got $-3 < x + 1 \leq 6$. Thereafter, they simplified wrongly to obtain $-3 < x \leq 5$ which resulted to an incorrect solution of a sketch presented on a number line. Similarly, some students simplified $-18 < 6x + 6 \leq 36$ by collecting the constant terms to the right-hand side of the inequality, resulting in $0 < 6x \leq 48$ and hence $0 < x \leq 8$. Finally, they presented an incorrect solution on a number line.

In part (b), most students made some conceptual errors. For instance, some students wrongly expressed g in terms of K and T and obtained an incorrect equation $g = \frac{KT}{T - K}$, which after substituting $K = 12$ and $T = 6$ resulted in $g = -12$. Extract 2.2 is a sample of response from one of the students who faced difficulties when attempting this question.



$$\begin{aligned}
 k^2 &= \frac{T^2 g}{T + g} \\
 \sqrt{T^2 g} &= \sqrt{k^2 T} + \sqrt{k^2 g} \\
 Tg &= kT + kg \\
 kT &= Tg - kg \\
 kT &= g(T - k) \\
 \frac{kT}{T - k} &= \frac{g(T - k)}{T - k} \\
 g &= \frac{kT}{T - k}
 \end{aligned}$$

Given that $k = 12$ and $T = 6$.

$$\begin{aligned}
 g &= \frac{12 \times 6}{6 - 12} = \frac{72}{-6} \\
 &= -12
 \end{aligned}$$

$$g = -12$$

∴ The value of g is -12

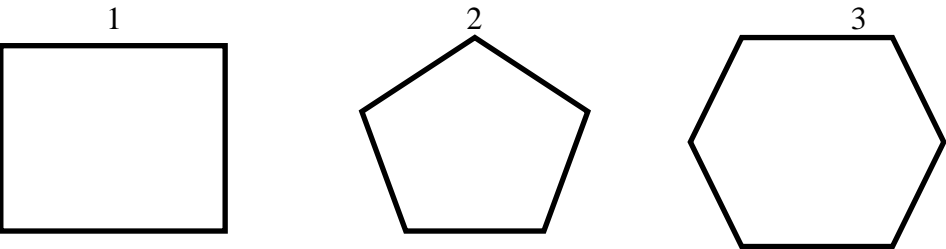
Extract 2.2: A sample of incorrect responses to question 2.

In Extract 2.2, part (a), the student presented an incorrect solution on the number line. In part (b), the student incorrectly applied the square root, leading to a wrong value of g .

2.3 Question 3: Geometrical Constructions

The question comprised parts (a) and (b), which stated as follows:

Study the regular polygons 1, 2, and 3 and answer the questions that follow.



- (a) Complete the following table by writing the number of triangles and sum of interior angles for each polygon.

Polygon	Number of triangles	Sum of all interior angles
1		
2		
3		

- (b) Write down the formula for calculating the sum of all interior angles of a polygon with n sides.

All 392 students attempted this question, of whom 44 (11.22%) scored below 3.0 marks while 348 (88.78%) scored 3.0 to 10 marks. Thus, the students' performance on this question was generally good, as 88.78 percent of the students who attempted the question passed. Figure 4 presents the summary of students' performance in this question.

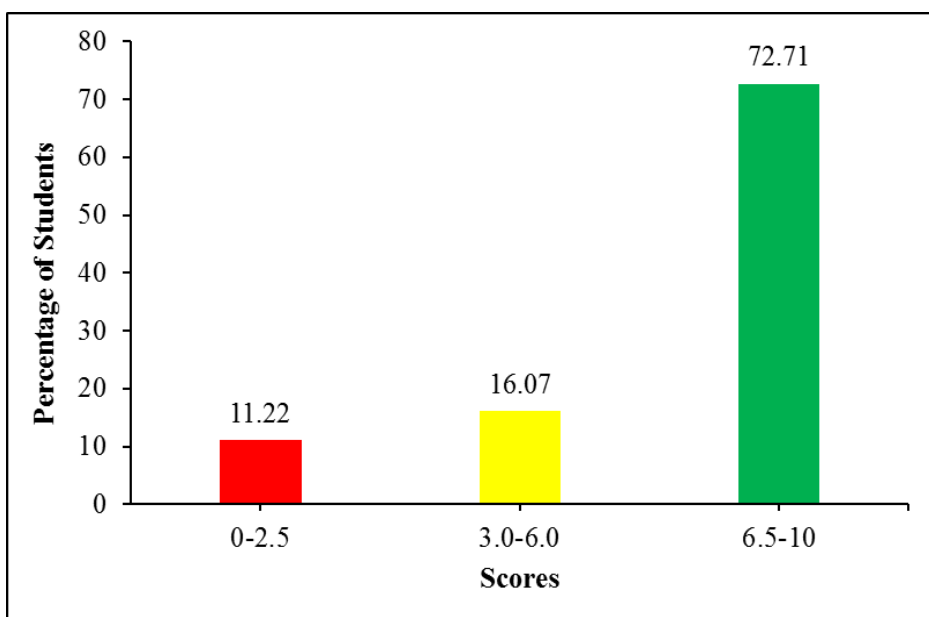


Figure 4: *Students' Performance in Question 3*

The students who correctly answered part (a) applied the formula for calculating the number of triangles (N) for each polygon with n sides as $N = n - 2$. These students correctly identified the number of sides of polygons 1, 2, and 3 as 4, 5, and 6 and consequently obtained the number of triangles for polygons as 2, 3 and 4, respectively. The students also deduced the formula to determine the sum of the interior angles (S) of polygons 1, 2 and 3 by considering the number of triangles as $S = \text{number of triangles} \times 180^\circ$ and consequently obtained 360° , 540° and 720° respectively. In part (b), students correctly recalled the formula for calculating the sum of all interior angles of a regular polygon with n sides as $S = (n - 2) \times 180^\circ$. Extract 3.1 is a sample of a correct responses from one of the students who attempted this question.

Polygon	Number of triangles	Sum of all interior angles
1	2	360°
2	3	540°
3	4	720°

The formula for calculating the sum of all interior angles of a polygon is $(n - 2)180^\circ$.

Extract 3.1: A sample of the correct responses to question 3.

In Extract 3.1, part (a), the student managed to find the number of triangles and the sum of all interior angles. In Part (b), the student recalled correctly the formula for the sum of all interior angles.

Despite the majority of students demonstrating their strengths in this question, 24 students (6.1%) scored a zero. For instance, in part (a), some students counted the number of sides of the polygons and got 4, 5 and 6 triangles instead of 2, 3, and 4, respectively. Similarly, in determining the sum of all interior angles of the polygon, some students applied incorrect formulae, commonly $\left(\frac{n-2}{n}\right) \times 180^\circ$ and obtained incorrect answers 90° , 108° and 120° . In responding to part (b), some students wrote the incorrect formula, particularly the formula for calculating the sum of interior and exterior angles of a polygon, that is $e + i = 180^\circ$. Likewise, others wrote $n = \frac{1+n^2}{n+n}$. Extract 3.2 presents a sample response from a student who was unable to provide the correct answer.

Polygon	Number of triangles	Sum of all interior angles
1	4	90°
2	5	108°
3	6	120°

Sum of all interior angle = $\frac{(n-2)}{n} \times 180$

Extract 3.2: A sample of incorrect responses to question 3.

In Extract 3.2, part (a), the student encountered challenges in finding the number of triangles and the sum of all angles. In part (b), the student recalled incorrect formula for the sum of all interior angles of a regular polygon.

2.4 Question 4: Coordinate Geometry

The question had two parts (a) and (b). In part (a), the students were given equations of two lines $5x + 6y = 5$ and $kx - 3y = 10$, then required to calculate the values of k if they are (i) parallel and (ii) perpendicular. In part (b), they were required to show whether the points (i) $(-1, -4)$, $(1, 2)$ and $(3, 8)$, and (ii) $(0, 1)$, $(1, 3)$ and $(2, 5)$ are collinear.

The data analysis shows that 392 (100%) students attempted this question, with 90 (22.96%) scoring between 0 and 2.5 marks, 135 (34.44%) scoring between 3.0 and 6.0 marks, and 167 (42.60%) scoring 6.5 marks or above. Generally, students' performance on this question was good, as 77.04 percent of students passed. Figure 5 provides a summary of students' performance on this question.

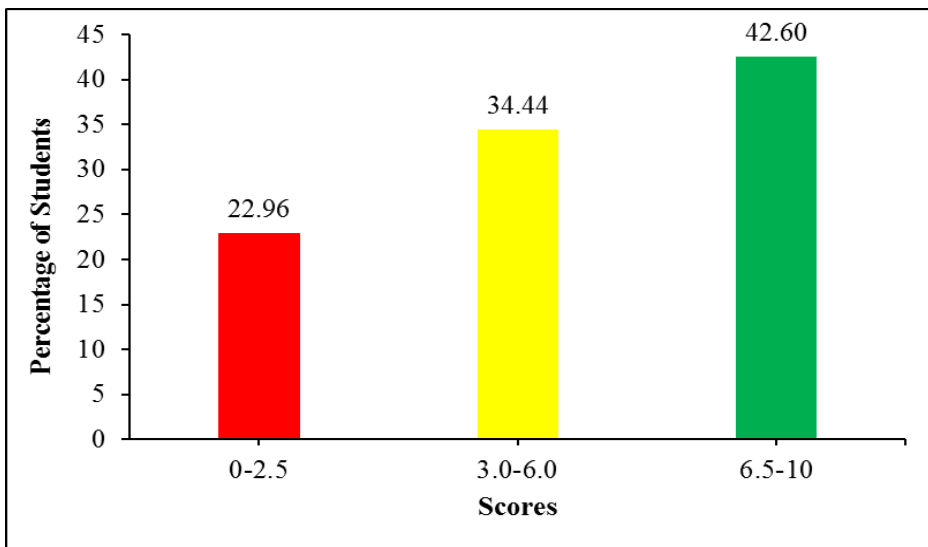


Figure 5: *Students' Performance in Question 4*

The analysis of data shows that 13.3 percent scored full marks. In part (a) the students rewrote the equations $5x + 6y = 5$ and $kx - 3y = 10$ in standard form as $y = -\frac{5}{6}x + \frac{5}{6}$ and $y = \frac{1}{3}kx - \frac{10}{3}$, and identified that the slopes of the two lines are $M_1 = -\frac{5}{6}$ and $M_2 = \frac{1}{3}k$. In responding to part (a) (i), the students were conversant with the fact that, the parallel

lines have the same slope, therefore, they equated M_1 and M_2 resulting in the equation $-\frac{5}{6} = \frac{1}{3}k$ and consequently $k = -\frac{5}{2}$. In attempting (a) (ii), these students were conversant with the fact that the product of the slopes of the two perpendicular lines is -1 , hence they formulated the equation $-\frac{5}{6} \times \frac{1}{3}k = -1$ and solved it to get $k = \frac{18}{5}$.

In part (b) (i), the students correctly recalled the condition for collinear points, that is, three or more points are collinear if they lie on the same line. Then, they determined the slope of the line using the formula, $\text{slope}(M) = \frac{y_2 - y_1}{x_2 - x_1}$ using the points $(-1, -4)$, $(1, 2)$ and $(3, 8)$ to get $M_1 = 3$, $M_2 = 3$ and $M_3 = 3$. Finally, they concluded that since $M_1 = M_2 = M_3$ then, the given points are collinear. Similarly, in part (b) (ii), the students correctly determined the slopes using the points $(0, 1)$, $(1, 3)$ and $(2, 5)$, and managed to get $M_1 = M_2 = M_3 = 2$. Therefore, they commented that the given points are collinear. Extract 4.1 provides a sample of a response from one of the students who correctly attempted the question.

Soln

① $m_1 = m_2$

$$5x + 6y = 5$$

$$\frac{6y}{6} = \frac{-5x + 5}{6}$$

$$y = \frac{-5x}{6} + \frac{5}{6}$$

$$m_1 = \frac{-5}{6} = m_2$$

$$kx - 3y = 10$$

$$\frac{-3y}{-3} = \frac{-kx + 10}{-3}$$

$$y = \frac{-kx}{3} + \frac{10}{3}$$

$$m_2 = \frac{+k}{3}$$

$$\frac{-5}{6} \times \frac{+k}{3}$$

$$-5 \times 3 = +k \times 6$$

$$\frac{+6k}{+6} = \frac{-15}{+6}$$

$$\therefore k = \frac{-5}{2}$$

② $m_1 m_2 = -1$

$$5x + 6y = 5$$

$$\frac{6y}{6} = \frac{-5x + 5}{6}$$

$$y = \frac{-5x}{6} + \frac{5}{6}$$

$$m_1 = \frac{-5}{6}$$

$$kx - 3y = 10$$

$$\frac{-3y}{-3} = \frac{-kx + 10}{-3}$$

$$y = \frac{+kx}{3} + \frac{10}{3}$$

$$m_2 = \frac{k}{3}$$

$$\frac{-5}{6} \times \frac{k}{3} = -1 \quad \frac{-5k}{18} \times \frac{-1}{1}$$

$$\frac{-5k}{-5} = \frac{-18}{-5}$$

$$k = \frac{18}{5}$$

Soln

① $m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \frac{2+4}{1+1} = \frac{8-2}{3-1}$

$m = \frac{6}{2} = \frac{6}{2} \quad 3 = 3$

\therefore It is collinear

② $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-1}{1-0} = \frac{5-3}{2-1}$

$\frac{2}{1} = \frac{2}{1}$

$m = 2$

\therefore It is collinear

Extract 4.1: A sample of the correct responses to question 4.

In Extract 4.1, part (a), the student correctly applied the condition for parallel and perpendicular lines to get the value of k . In part (b), the student applied correctly the condition for collinear points.

In spite of the good performance, 86 (21.9%) students scored 2.5 marks or less. In part (a) (i), some students added both equations, as $5x + 6y = 5 + kx - 3y = 10$, then wrongly simplified to obtain

$$k = -\frac{15-3y}{6x}.$$

Furthermore, other students equated the terms containing x variable instead of slopes; that is, $5x = kx$, consequently they got $k = 5$. Also, some students responded to the question by using an inappropriate approach. For example, they solved the two equations simultaneously; that is, they made y the subject, and got $y = \frac{5-5x}{6}$.

Later, they substituted it into the equation $kx - 3y = 10$, and obtained

$$k = \frac{45}{14}, \text{ rather than } k = -\frac{5}{2}.$$

In part (a) (ii), some students perceived the word perpendicular as the intersection of sets; hence they wrote $5x+6y=5 \cap kx-3y=10$ and then wrongly simplified to obtain $k=4yx$. Furthermore, the analysis shows that other students committed with arithmetic error. For example, they managed to identify the gradient of the line $5x+6y=5$ as $m_1 = -\frac{5}{6}$, similarly for the line $kx-3y=10$, they got the second slope $m_2 = k$ before putting it into standard form. Thereafter, they correctly recalled the condition for perpendicular lines and applied it to get $k = \frac{6}{5}$. Likewise, other students did not understand the demand of the question; they solved the two equations simultaneously and incorrectly manipulated some operations, resulting in $k=19$ rather than $k = \frac{18}{5}$.

In part (b), some of the students responded by plotting points on x, y -plane without finding the slopes of the lines joining the points. Moreover, some students applied an incorrect formula; they wrote $m_1 = (x_2 + x_1, y_2 + y_1)$. Thus in responding to part (b) (i), they wrote $(3+-1, -4+8)$, and $(0+1, 1+3)$, consequently obtained points $(2, 4)$ and $(1, 4)$. Additionally, other students responded to this question by equating the sum of abscissa and ordinates, for instance, in (b) (i) $-1+1+3 = -4+2+8$ and obtained $3=10$ while in (b) (ii) $0+1+2 = 1+3+5$ and obtained $3=9$. Extract 4.2 provides a sample of a response from one of the students who faced difficulties when responding to this question.

Solution:

9) i) Parallel

$$5x + 6y = 5 \cap kx - 3y = 10$$

$$5x + 6y = 5 \cap kx - 3y = 10$$

$$5 - 10 - 5 + 5x - x \cup 6y - 3y = k$$

$$5 + 5x - x \cup 6y - 3y = k$$

$$5 + 4x \cup 6y - 3y = k$$

$$5 + 4x \cup 4y = k$$

$$x \cup 4y = k$$

$$4yx = k$$

$$\underline{\underline{\text{Parallel} = 4yx = k}}$$

ii) Perpendicular

Soln

$$(5x + 6y = 5) \cap kx - 3y = 10$$

$$5x + 6y = 5 \text{ or } kx - 3y = 10$$

$$10 - 5 - 6y + 3y + k \times x$$

$$5 - 3y + k \times x$$

$$2y + k \times x$$

$$2y + k \times x$$

$$2y + x = k$$

$$k = 2xy.$$

ln perpendicular $k = 2xy$.

Soln.

$$-1 + 1 + 3 = -4 + 2 + 8$$

$$3 = 2 + 8$$

$$+3 = 10$$

$$10 + 3 = \underline{13}$$

ii) $(0, 1)$, $(3, 2)$ and $(2, 5)$

$$0 + 1 + 2 = 1 + 3 + 5$$

$$\underline{3} = \underline{9}$$

$$\underline{3} = \underline{9}$$

$$\underline{\underline{3}}$$

Extract 4.2: A sample of incorrect responses to question 4.

In Extract 4.2, the student was not familiar with the conditions for parallel lines, perpendicular lines, and collinear points, which led to incorrect responses.

2.5 Question 5: Locus

The question consisted of two parts (a) and (b). In part (a), the students were given a statement that a cow was tied up by a rope while the other end of the rope was tied to a pole that was fixed at point B. The cow moved around the pole such that the rope is always equidistant from point B. With the aid of a sketch diagram, the students were required to describe the locus of the cow while moving around the point B. Part (b) was as follows: If a point T moving in a plane is equidistant between the points $M(5, 2)$ and $N(-1, 1)$, determine the locus of T and hence describe its nature.

All 392 students attempted the question, of whom 135 (34.44%) scored 0 to 2.5 marks, 123 (31.38%) scored from 3.0 to 6.0 marks, and 134 (34.18%) students scored from 6.5 to 10 marks. Therefore, the overall performance of the students in this question was good because 65.56 percent of students passed. Figure 6 represents the summary of students' performance in percentages.

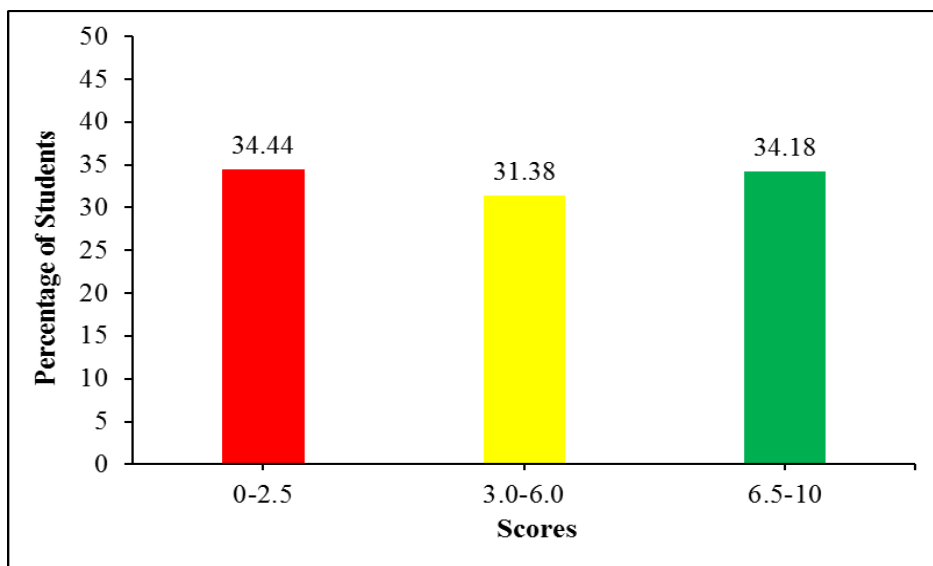


Figure 6: *Students' Performance in Question 5*

In part (a), the students who managed to provide the correct responses interpreted and described correctly that the locus formed by a cow is a circle with centre B and the radius being the length of the rope as illustrated in Extract 5.1.

In part (b), the students correctly applied the formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for calculating the distance between two points. Later, they assumed that point T is (x, y) and thus they substituted $T(x, y)$ and $M(5, 2)$ into the formula, resulting in $TM = \sqrt{(x-5)^2 + (y-2)^2}$ and $TN = \sqrt{(x+1)^2 + (y-1)^2}$. Thereafter, they equated TM and TN to obtain the locus of T that is, $12x + 12y = 27$ which describes a straight line. Extract 5.1 illustrates the correct response to this question from one of the students.

Soln

Diagram

A = cow
B = fixed point

∴ locus of circular objects

solution

$T(x, y)$

$M(5, 2)$

$N(-1, 1)$

$\sqrt{TM} = \sqrt{TN}$

$$(\sqrt{(x-5)^2 + (y-2)^2})^2 = (\sqrt{(x+1)^2 + (y-1)^2})^2$$

$$(x-5)^2 + (y-2)^2 = (x+1)^2 + (y-1)^2$$

$$(x-5)(x-5) + (y-2)(y-2) = (x+1)(x+1) + (y-1)(y-1)$$

$$x^2 - 5x - 5x + 25 + y^2 - 2y - 2y + 4 = x^2 + x + x + 1 + y^2 - y - y + 1$$

$$x^2 - 10x + 25 + y^2 - 4y + 4 = x^2 + 2x + 1 + y^2 - 2y + 1$$

$$x^2 + y^2 - 10x + 25 + 4 - 4y = x^2 + y^2 + 2x - 2y + 1 + 1$$

$$x^2 + y^2 - 10x - 4y + 29 = x^2 + y^2 + 2x - 2y + 2$$

$$0 = 2x + 10x - 2y + 4y + 2 - 29$$

$$0 = 12x + 2y - 27$$

$$12x + 2y - 27 = 0$$

\therefore The locus of T is $12x + 2y - 27 = 0$

and its nature is a locus of a straight line.

Extract 5.1: A sample of the correct responses to question 5.

In Extract 5.1, part (a), the student correctly described the locus formed by a cow. In part (b), the student correctly applied the formula for calculating the distance between two points.

However, the analysis of data revealed that 55 (14%) students got zero. In part (a), some students presented sketch diagrams indicating incorrect loci, such as a straight line, trapezium, and isosceles triangle instead of a circle. Also, when responding to part (b), the students applied an incorrect formula, including the gradient formula $M = \frac{y_2 - y_1}{x_2 - x_1}$ then,

substituted (5, 2) and (-1, 1) to get $m = \frac{1}{6}$ and consequently the locus

$y = \frac{x + 7}{6}$. Likewise, other students applied the midpoint formula

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ then replaced (x_1, y_1) and (x_2, y_2) with (5, 2) and (-1, 1) respectively, to obtain (2, 1.5) which is contrary to the requirement of the question. On the other hand, some students calculated the distance between two points, $M(5, 2)$ and $N(-1, 1)$, using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and obtained $d = \sqrt{37}$.

Extract 5.2 is a sample response from one of the students who encountered challenges in this question.

<p><u>Solution</u></p> <p>Point T</p> <p>Points</p> <p>$M(5, 2)$ & $(-1, 1)$</p> <p>$x_1 \quad y_1 \quad \text{slope} \quad x_2 \quad y_2$</p> <p>$M = \frac{y_2 - y_1}{x_2 - x_1}$</p> <p>$M = \frac{1 - 2}{-1 - 5}$</p> <p>$M = \frac{+1}{+6}$</p> <p>$M = \frac{1}{6}$</p>	
<p>$M = (-1, 1) (x_1, y_1)$</p> <p>$x_1 \quad y_1 \quad x_2 \quad y_2$</p> <p>$\frac{1}{6} = \frac{y - 1}{x + 1}$</p> <p>$x + 1 = 6y - 6$</p> <p>$6y = x + 1 + 6$</p> <p>$\frac{6y}{6} = \frac{x + 7}{6}$</p> <p>$y = \frac{x + 7}{6}$</p>	<p>Locus T is</p> <p>$y = \frac{x + 7}{6}$</p> <p>nature = $\frac{1}{6}$</p>

Extract 5.2: A sample of incorrect responses to question 5.

In Extract 5.2 the student applied an inappropriate formula to find the locus.

2.6 Question 6: Symmetry

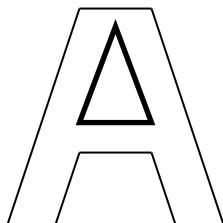
The question comprised three parts (a), (b), and (c), which state that:

(a) The students were required to determine the number of lines of symmetry in the following objects:

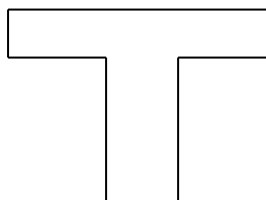
- (i) A ball
- (ii) A pencil

(b) Use a dotted line(s) to verify the symmetry of the following letters;

(i)



(ii)



(c) Sketch the following polygons and examine all lines of symmetry:

- (i) Square
- (ii) Equilateral triangle
- (iii) Rhombus

The analysis of data revealed that 335 (85.46%) students scored marks ranging from 6.5 to 10, 37 (9.44%) scored marks ranging from 3.0 to 6.0, and 20 (5.10%) students scored 0 to 2.5 marks. The students' performance summary is presented in Figure 7.

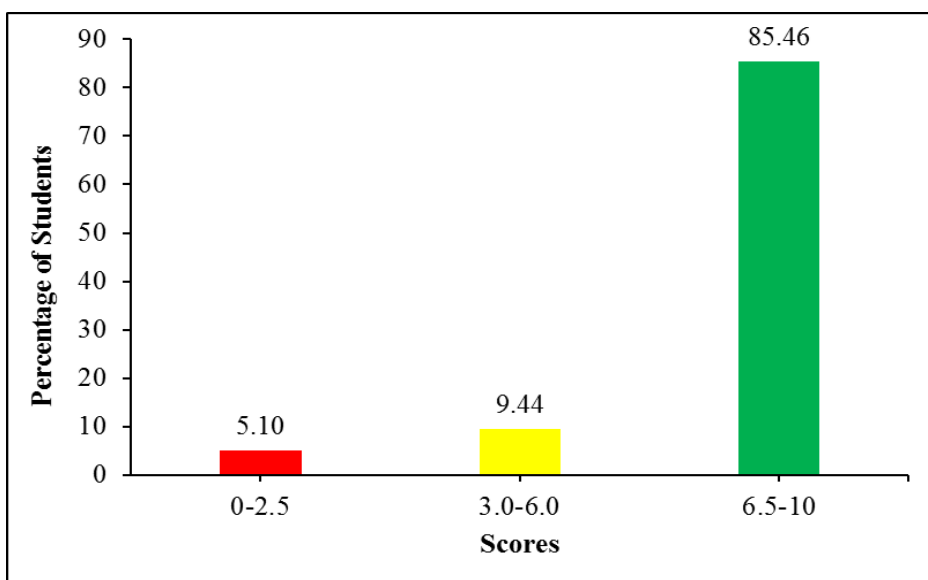
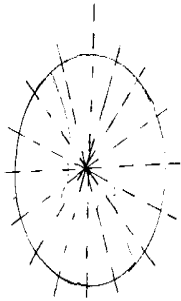


Figure 7: *Students' Performance in Question 6*

As Figure 7 shows, the students' performance was generally good since 94.40 percent of students who attempted this question passed. The students who were able to respond to this question correctly showed adequate knowledge and skills on symmetry. In part (a), the students wrote that “a ball has an infinite number of lines of symmetry” while “a pencil has one line of symmetry”. Likewise, in part (b), the students demonstrated their competence as they managed to use dotted line(s) to verify the symmetry of the given letters as illustrated in Extract 6.1. In part (c), the analysis shows that the students were conversant with shapes of different polygons, as they correctly sketched the polygons and identified all lines of symmetry (Extract 6.1).

(i) A ball



(ii) A pencil

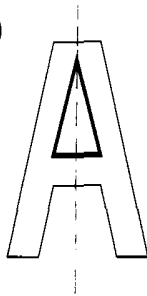


A pencil has one line of symmetry.

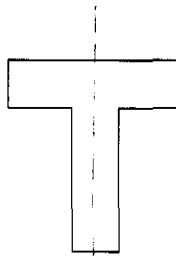
A ball has infinity lines of symmetry

(b) Use a dotted line(s) to verify the symmetry of the following letters;

(i)

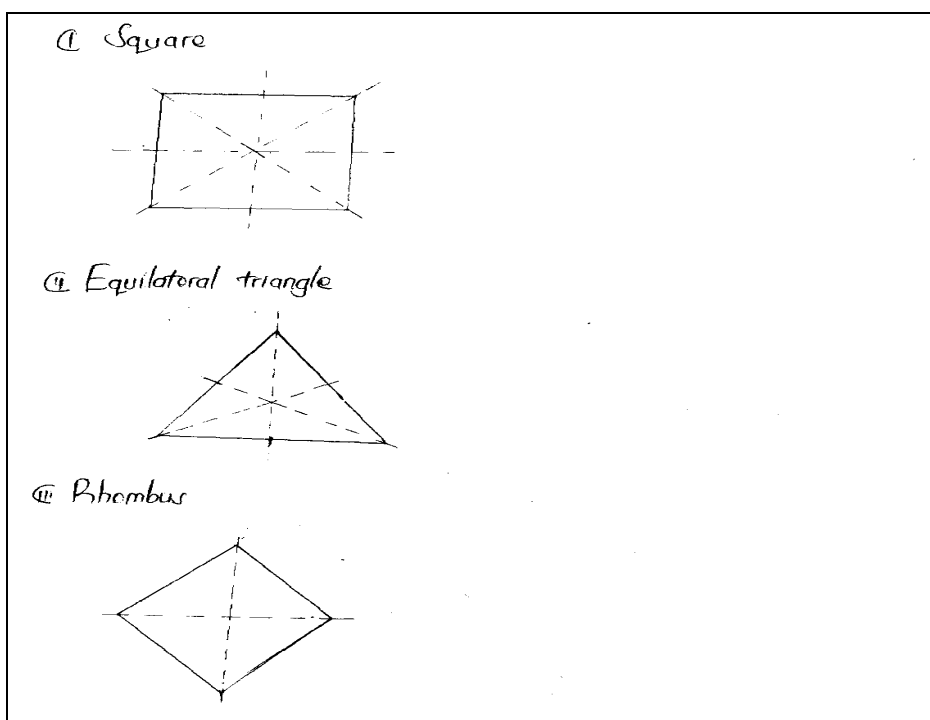


(ii)



(i) Has one line of symmetry

(ii) Has one line of symmetry.

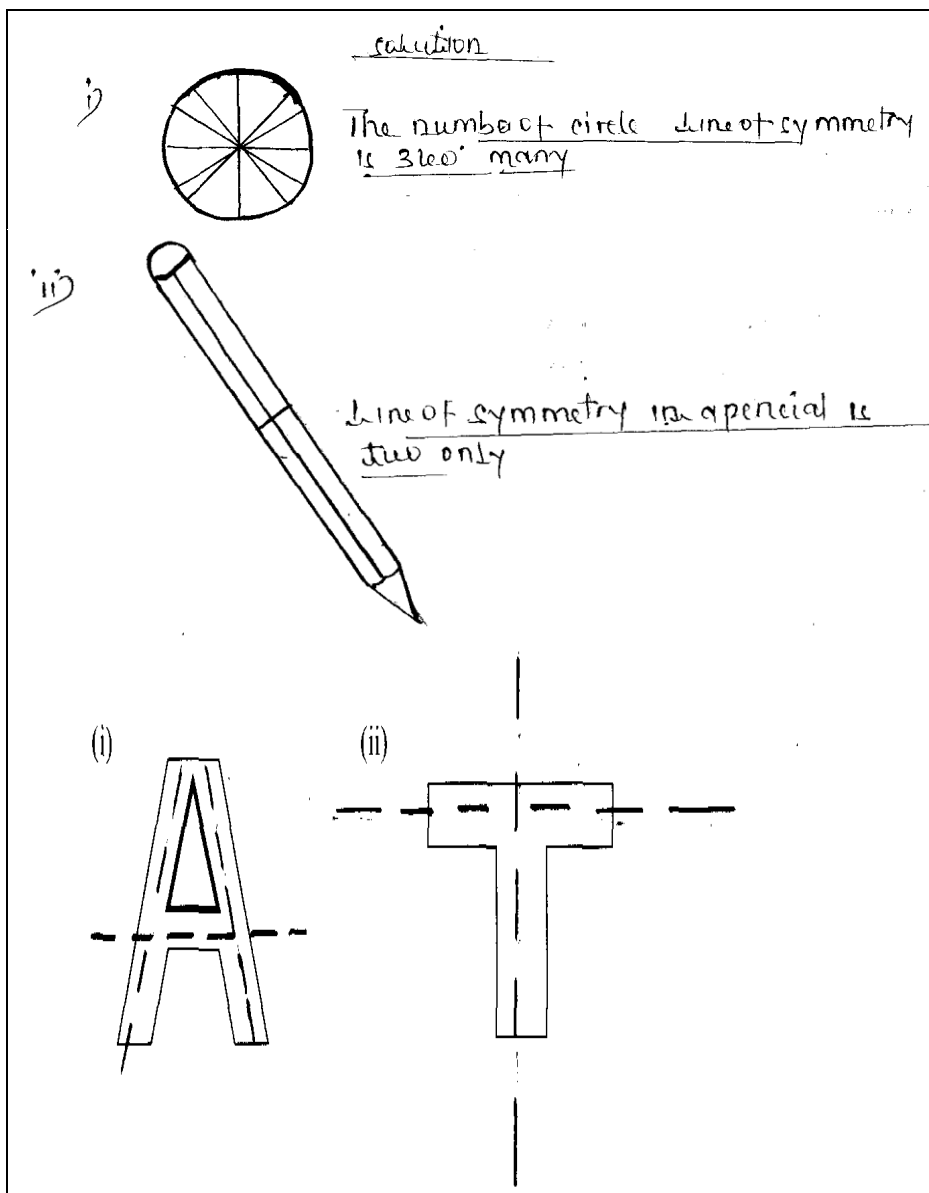


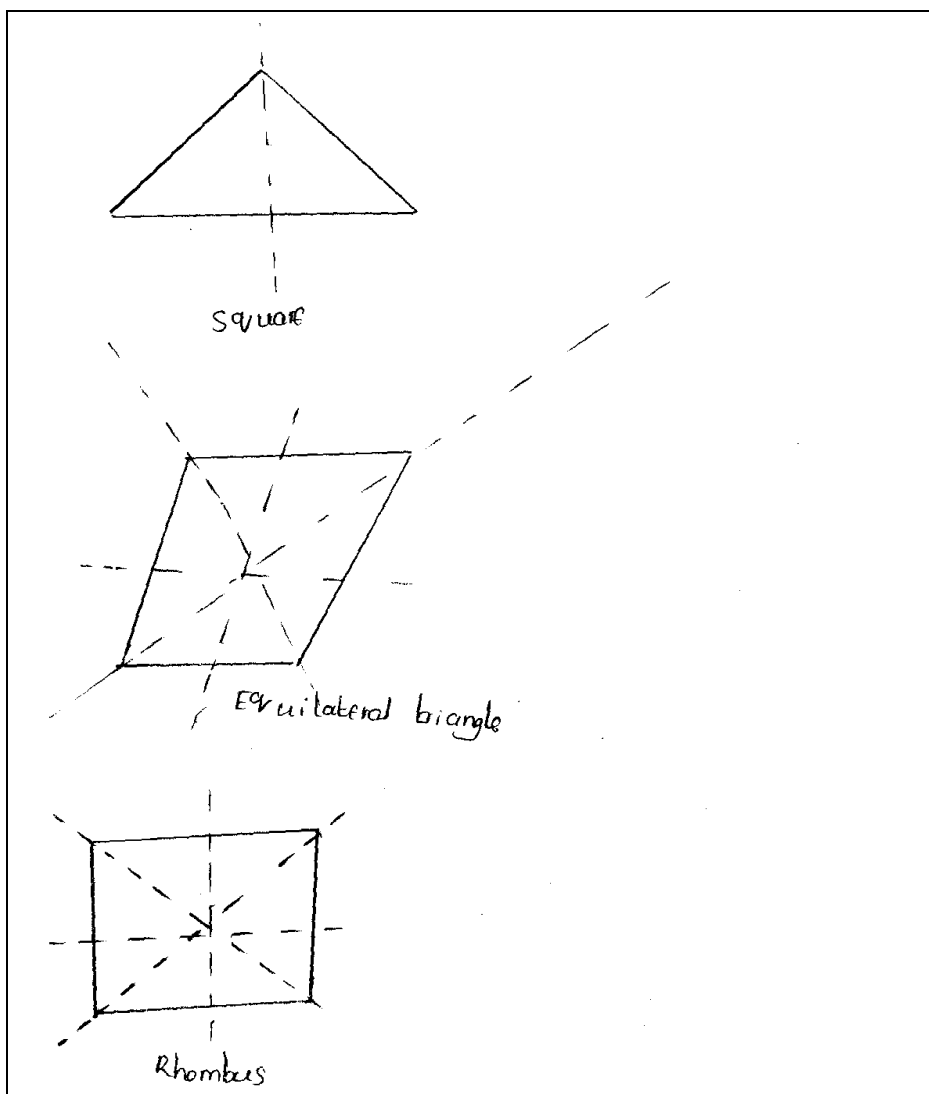
Extract 6.1: A sample of the correct responses to question 6.

In Extract 6.1, part (a), the student correctly drew a ball and pencil and also indicated all their lines of symmetry. In part (b), the student correctly drew all dotted lines of symmetry. In part (c), the student correctly sketched the polygons and identified all lines of symmetry.

On the other hand, the analysis of the data revealed that 5.1 percent of students who scored 2.5 marks or less, were encountered with different challenges. In part (a), most of these students were not competent on the concept of symmetry, as a result, they stated the incorrect number of lines of symmetry. For instance, in part (a) (i), some students stated that a ball has two lines of symmetry while others stated it that has 360 lines. In part (a) (ii), some students stated that a pencil has 0 lines of symmetry, and others stated that it had two lines of symmetry. Also, in part (b), the analysis shows that some students misinterpreted the question; instead of using a dotted line(s) to verify the symmetry, they redrew the given letters using dotted line(s) as illustrated in Extract 6.2. Similarly, other students responded to this part by showing more than one dotted line(s), instead of one. Likewise, in part (c) (i), some students

drew triangles, and circles while in part (c) (ii), they drew parallelograms and rectangles and in part (c) (iii), most of the responses were cylinders, hexagons and crosses. A sample response from one of the students who faced challenges while attempting the question is illustrated in Extract 6.2.





Extract 6.2: A sample of incorrect responses to question 6.

In Extract 6.2, part (a), the student failed to determine the number of lines of symmetry. In part (b), the student showed more than one dotted line of symmetry, and in part (c), he/she did not recognise the shapes of polygons as well as identify lines of symmetry.

2.7 Question 7: Logic

The question had three parts: (a), (b), and (c). In part (a), the students were required to use the truth table to show whether the statement $\sim(p \wedge q) \vee (p \vee q)$ is a tautology or not. In part (b), they were instructed that P stands for “clouds are heavy” and Q stands for “It is raining” and were required to express each of the following in symbolic form; (i) Clouds are heavy, and it is not raining, (ii) If it is not true that clouds are heavy, then it is raining, (iii) It rains if and only if clouds are heavy. Whereas in part (c), they were required to make a truth table for the argument $p \rightarrow (\sim q \wedge \sim p)$.

The analysis of data depicts that, out of 392 students who attempted this question, 56 (14.28%) students scored 0 to 2.5 marks, 76 (19.39%) scored 3.0 to 6.0 marks, and 260 (66.33%) students scored 6.5 to 10 marks. Since 85.72 percent of students who answered the question passed, then the students’ performance in this question was generally good. The summary of the students’ performance in this question is shown in Figure 8.

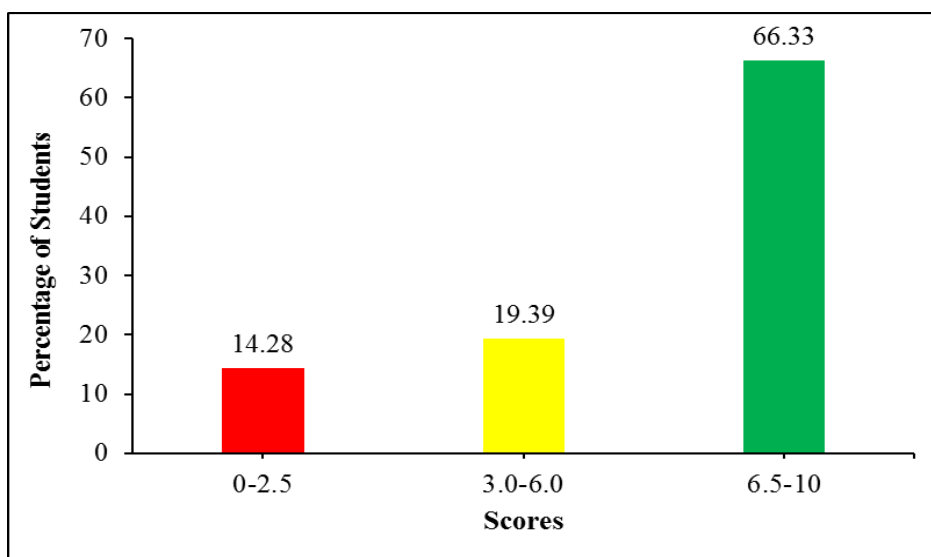


Figure 8: *Students' Performance in Question 7*

The data reveals that 91 (23.21%) students scored full marks. In part (a), these students managed to construct the truth table for the given

proposition, $\sim(p \wedge q) \vee (p \vee q)$ and commented that the proposition is a tautology as seen in Extract 7.1. In part (b), the students were able to identify the appropriate logical connectives and managed to write the correct symbolic statement of the given arguments, that is (i) $P \wedge \sim Q$, (ii) $\sim P \rightarrow Q$ and (iii) $Q \leftrightarrow P$. Likewise, in part (c), the students correctly constructed the truth table with appropriate columns and rows. Thereafter, they assigned the correct entries (truth values) as illustrated in Extract 7.1, which provides a sample of one of the correct solutions to this question.

$\sim(p \wedge q) \vee (p \vee q) \equiv (\sim p \wedge \sim q) \vee (p \vee q)$						
p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$p \vee q$	$\sim(p \wedge q) \vee (p \vee q)$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	T
<u>The statement is tautology.</u>						
<p>i <u>$P \wedge \sim Q$</u></p> <p>ii <u>$\sim P \rightarrow Q$</u></p> <p>iii <u>$Q \leftrightarrow P$</u></p>						
$P \rightarrow (\sim q \wedge \sim p)$						
p	q	$\sim p$	$\sim q$	$\sim q \wedge \sim p$	$P \rightarrow (\sim q \wedge \sim p)$	
T	T	F	F	F	F	
T	F	F	T	F	F	
F	T	T	F	F	T	
F	F	T	T	T	T	

Extract 7.1: A sample of the correct responses to question 7.

In Extract 7.1, part (a), the student correctly assigned the truth values and found all entries in the last column containing T. In part (b), the student correctly identified the appropriate logical connectives and managed to write correct symbolic statements of the given arguments. In part (c), the student constructed a truth table with appropriate columns and rows.

Despite the strengths demonstrated by most of the students, the analysis depicts that some students were unable to respond to the question correctly and scored low marks due to various reasons. In part (a) (i), some students failed to recognise the number of columns and rows for the truth table. Moreover, other students were unable to perform logical operations, resulting in the incorrect entries for the truth table as seen in Extract 7.2. Furthermore, other students failed to determine the correct number of rows for the table, and hence they presented the incorrect truth table. For example,

$\sim (p \wedge q)$	\vee	$p \vee q$
T	T	T
T	F	F
T	F	F
F	T	T
F	F	T
F	T	F

Likewise, some students responded to this part without including important columns for p , q , $p \wedge q$ and $\sim (p \wedge q) \vee (p \vee q)$.

In part (b), some students could not recognise the appropriate logical connectives that led to misinterpretation of the statements. For example, in part (b) (i), most of the incorrect responses were $p \wedge q$, $p \vee q$, TF rather than $P \wedge \sim Q$, in part (b) (ii), they wrote $P \rightarrow Q$, $p \wedge q$, $P + Q$ instead of $\sim P \rightarrow Q$ and in part (b) (iii) some incorrect responses were

$Q + P$, $\sim p$ it is F instead of $Q \leftrightarrow P$. Likewise, in part (c), some students failed to recognise the number of columns and rows for the truth table. For example, some students did not realise that the required truth table had 6 columns and 5 rows; instead, they constructed the truth table with more or less than 6 columns and 5 rows. Moreover, other students responded to the question without adhering to the number of possible combinations of the truth values. Extract 7.2 is a sample of an incorrect response from one of the students who were not able to correctly respond to the question.

$p \wedge q$	$p \vee q$	$(p \vee q)$	$(p \vee q)$	$\sim(p \wedge q)$	$\sim(p \vee q)$
T	f	T	f	f	T
T	f	T	f	T	f
F	T	T	f	T	f
F	T	T	f	f	T

Solution

i/ $p \vee q$ it is T
ii/ $p \wedge q$ it is f
iii/ $p \vee$ it is f

P	q	$P \rightarrow$	$(\sim P)$	$(\sim q)$	$(\sim q \wedge \sim P)$	$P \rightarrow (\sim q \wedge \sim P)$
F	T	F	F	T	T	F
F	T	F	T	F	T	F
F	T	T	F	T	F	T
F	T	T	T	F	T	F
F	T	F	F	T	T	T

Extract 7.2: A sample of incorrect responses to question 7.

In Extract 7.2, parts (a) and (c), the student failed to recognise the required columns and rows, which led to a response to the question without adhering to the number of possible combinations of the truth table. In part (b), the student did not realise the appropriate logic connectives that define the statement.

2.8 Question 8: Algebra

The question stated that in a certain seminar, the storekeeper was required to buy some stationaries. He decided to purchase 2 pens and 4 notebooks, which cost Tsh. 1400/= from the first shop, and then he bought 6 pens and 8 notebooks from the second shop, which cost Tsh. 3600/=. Suppose 1000 participants are expected to attend the seminar; evaluate the total cost that the storekeeper will incur by using the elimination method.

The data analysis depicts that 392 students responded to the question; among them, 89 (22.70%) students scored 0 to 2.5 marks, 76 (19.39%) scored 3.0 to 6.0 marks, and 227 (57.91%) students scored 6.5 to 10 marks. Therefore, the overall performance of the students was good because 77.30 percent of the students who attempted the question passed. Figure 9 provides a summary of the students' performance in this question.

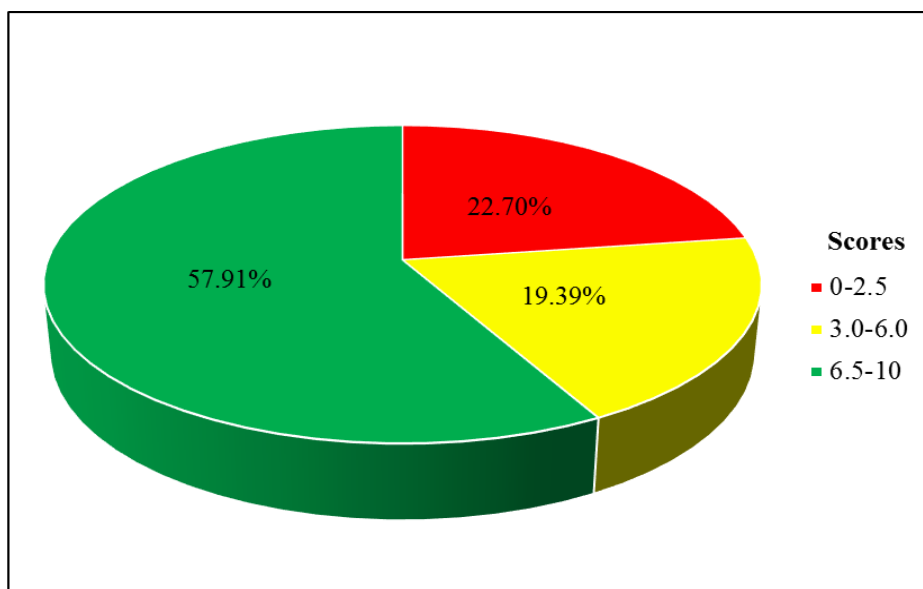


Figure 9: *Students' Performance in Question 8*

The students who attempted this question and managed to score all ten (10) marks assumed that, x and y were the cost of buying one pen and one notebook, respectively. Thereafter, they correctly formulated equations $2x + 4y = 1400$ and $6x + 8y = 3600$ from the word problems. They also correctly solved for x and y by applying the elimination method and obtained $x = 400$ and $y = 150$. Furthermore, they correctly calculated the total cost for buying all the stationaries and obtained Tsh. 550,000. A sample of a correct response from one of the students who attempted the question correctly is provided in Extract 8.1.

<p>pens = x notebooks = y</p> $\begin{array}{r} 2(2x + 4y = 1400) \\ 2(6x + 8y = 3600) \\ \hline 4x + 8y = 2800 \\ - (12x + 24y = 8400) \\ \hline -12x + 16y = -7200 \\ \hline 8y = 1200 \\ \hline y = 150 \end{array}$ $\begin{array}{r} 8(2x + 4y = 1400) \\ 4(6x + 8y = 3600) \\ \hline 16x + 32y = 11200 \\ - (24x + 32y = 14400) \\ \hline -8x = -3200 \\ \hline x = 400 \end{array}$	<p>Soln</p> $\begin{array}{l} x + y \\ 400 + 150 = 550 \\ 550 \times 1000 = 550,000 \\ \therefore \text{The shop keeper will incur} \\ \text{Tsh. 550,000/-} \end{array}$
--	---

Extract 8.1: A sample of the correct responses to question 8.

In Extract 8.1, the student correctly formulated a pair of simultaneous equations from the word problem and correctly applied the elimination method to solve the question.

In spite of good performance, data depicts that 60 (15.3%) students scored zero. These students formulate equations from the word problem, but they failed to apply the elimination method correctly as they committed computational errors. For instance, a certain student

developed $\begin{cases} 2y+4x=1400 \\ 6y+8x=3600 \end{cases}$, then incorrectly eliminated variable x by

multiplying the coefficient of x and y only to the left side of the equations, then solved it to get $x=275$ and $y=220$ (Extract 8.2).

Thereafter, calculated the total cost using $x+y$ and obtained Tsh 495.

Similarly, other students committed computational errors on subtracting the coefficient of variables. For example, the student transformed

correctly the given word problem as $\begin{cases} 2p+4b=1400 \\ 6p+8b=3600 \end{cases}$, thereafter,

obtained $6b=1200$ on subtracting the coefficients of variable b then got $b=200$. Consequently, when calculated, the value of p the student

applied substitution techniques and got $p=300$. Finally, the total cost

equals Tsh 500,000/=. Extract 8.2 is a sample response from one of the students who failed to answer this question correctly.

Solution

Let pens be y
Let notebooks be x

$$\begin{cases} 2y + 4x = 1400 & \text{--- (equation 1)} \\ 6y + 8x = 3600 & \text{--- (equation 2)} \end{cases}$$

$$\begin{array}{r} - \begin{cases} 12y + 24x = 1400 \\ 12y + 32x = 3600 \end{cases} \\ \hline \quad \quad \quad +8x = +2200 \\ \quad \quad \quad +8 \quad \quad \quad +8 \\ \hline \quad \quad \quad x = 275 \end{array}$$

From equation (1) find y

$$\begin{array}{r} 2y + 4 \times 275 = 1400 \\ 2y + 1100 = 1400 \\ 2y = 1400 - 1100 \\ 2y = 300 \\ \frac{2y}{2} = \frac{300}{2} \\ y = 150 \end{array}$$

$x = 275$ is the cost of notebooks and $y = 150$ is the cost of pens

total cost = $y + x$

$x = 275$ is the cost of notebooks and
 $y = 220$ is the cost pens.

$$\begin{aligned}\text{The total cost} &= \text{cost of notebooks} + \text{cost of pens} \\ &= 275 + 220 \\ &= 495\end{aligned}$$

The total cost that the store keeper will incur
is Tsh 495 /=-

Extract 8.2: A sample of incorrect responses to question 8.

In Extract 8.2, the student managed to formulate a pair of simultaneous equations from the word problem but failed to apply the elimination method to solve for x and y which resulted in an incorrect answer.

2.9 Question 9: Sets

The question had two parts: (a) and (b). In part (a), students were required to calculate $n(K \cup L \cup M)$ when given three sets K , L and M such that $n(K) = 18$, $n(L) = 22$, $n(M) = 25$, $n(K \cap L) = 11$, $n(K \cap M) = 15$, $n(L \cap M) = 14$ and $n(K \cap L \cap M) = 9$.

In part (b), the question was as follows: A sample of family surveys composed of 80 families showed that all participants kept at least a goat, a cow, or a dog. It was found that 49 families kept goats, 61 kept cows and 50 families kept dogs. Moreover, 33 families kept goats and cows, 35 kept cows and dogs, and 27 kept goats and dogs. Without using a Venn diagram, the students were asked to calculate the number of families that kept (i) all types of animals, (ii) goats or cows if families

kept dogs only were not supposed to be surveyed, and (iii) dogs or cows if families kept goats only were not supposed to be surveyed.

The data shows that 392 students attempted the question, of whom 175 (44.64%) scored 0 to 2.5 marks and 217 (55.36%) students scored 3.0 to 10 marks, implying average performance. Figure 10 provided a summary of the students' performance on this question.

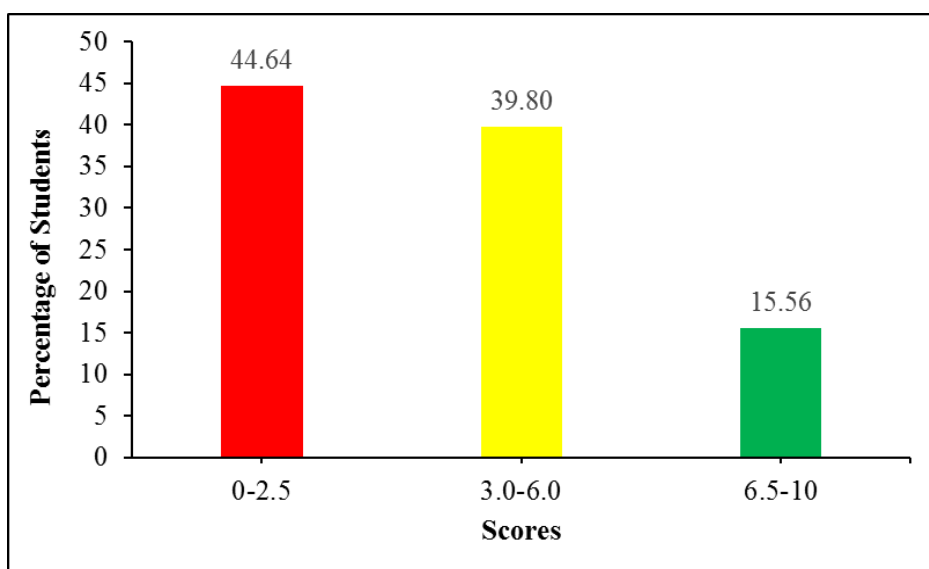


Figure 10: *Students' Performance in Question 9*

Furthermore, the data depicts that 61 (15.56%) students scored more than 6.0 marks. Most students correctly responded to part (a) as they recalled and applied the formula for finding the number of elements in the union of three sets,

$n(K \cup L \cup M) = n(K) + n(L) + n(M) - n(K \cap L) - n(K \cap M) - n(L \cap M) + n(K \cap L \cap M)$ thereafter, they correctly substituted $n(K) = 18$, $n(L) = 22$, $n(M) = 25$, $n(K \cap L) = 11$, $n(K \cap M) = 15$, $n(L \cap M) = 14$ and $n(K \cap L \cap M) = 9$ and performed operations to obtain $n(K \cup L \cup M) = 34$. Other students correctly presented the information using a Venn diagram and correctly interpreted the entries in the respective regions, resulting in $n(K \cup L \cup M) = 34$. Likewise, in part (b) (i), the students correctly realised that the number of families that kept all types of animals means the intersection of all three sets. Then, they correctly recalled the formula involving the three sets as

$n(G \cup C \cup D) = n(G) + n(C) + n(D) - n(G \cap C) - n(G \cap D) - n(C \cap D) + n(G \cap C \cap D)$
 , where G stand for goats, C stand for cows, and D stand for dogs.
 Later, they substituted all the elements into the formula and obtained $n(G \cup C \cup D) = 15$. Also, in part (b) (ii), the students correctly recognised that, the sets to be considered are G and C in a survey. Then, they correctly identified $n(G) = 49$, $n(C) = 61$ and $n(G \cap C) = 3$, and applied the formula, $n(G \cup C) = n(G) + n(C) - n(G \cap C)$ then substituted all the elements and got $n(G \cup C) = 77$. Thus, they concluded that 77 families kept goats or cows.

Similarly, in part (b) (iii), the students correctly identified the families that kept dogs or cows as $n(C) = 61$, $n(D) = 50$, $n(D \cap C) = 35$. Then, they correctly recalled the formula, $n(C \cup D) = n(C) + n(D) - n(C \cap D)$, hence substituted all the elements, and obtained $n(C \cup D) = 76$. Therefore, they concluded that 76 families kept dogs or cows. Extract 9.1 illustrates a sample of one of the students who responded correctly to this question.

<p style="text-align: center;">Soln</p> $n(K \cup L \cup M) = n(K) + n(L) + n(M) - n(K \cap L) - n(K \cap M) - n(L \cap M) + n(K \cap L \cap M)$ $n(K \cup L \cup M) = 18 + 22 + 25 - 11 - 15 - 14 + 9$ $\therefore n(K \cup L \cup M) = 34.$

Soln

$$\textcircled{i} \quad n(G \cup C \cup D) = n(G) + n(C) + n(D) - n(G \cap C) - n(G \cap D) - n(C \cap D) + n(G \cap C \cap D)$$

$$80 = 49 + 61 + 50 - 33 - 27 - 35 + n(G \cap C \cap D)$$

$$80 = 65 + n(G \cap C \cap D)$$

$$80 - 65 = n(G \cap C \cap D)$$

$$n(G \cap C \cap D) = 15$$

\therefore Families that take all types of ^{animals} ~~families~~ are 15 families

$$\textcircled{ii} \quad n(G \cup C) = n(G) + n(C) - n(G \cap C)$$

$$n(G \cup C) = 49 + 61 - 33$$

$$n(G \cup C) = 77$$

\therefore Families that kept goats or cows are 77 families

$$\textcircled{iii} \quad n(D \cup C) = n(D) + n(C) - n(D \cap C)$$

$$n(D \cup C) = 50 + 61 - 35$$

$$n(D \cup C) = 111 - 35$$

$$n(D \cup C) = 76$$

\therefore Families that kept dogs or cows are 76 families

Extract 9.1: A sample of the correct responses to question 9.

In Extract 9.1, part (a), the student correctly recalled and applied the set operation formula and managed to get $n(K \cup L \cup M)$. In part (b), the student applied the correct formula, thereafter substituted all elements, and managed to get the correct answers.

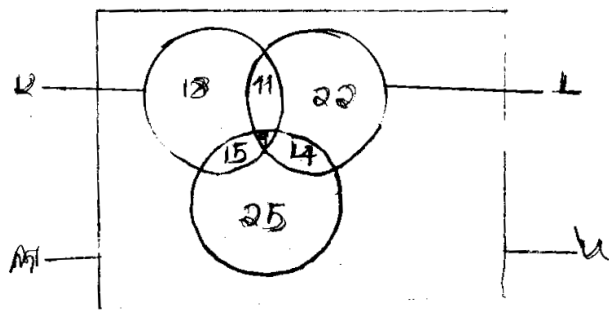
Nevertheless, 27.55 percent of students who attempted the question scored zero due to various challenges. In part (a), some students applied incorrect formulae. For instance, some students wrote,

$n(K \cup L \cup M) = n(K \cup L) + n(K \cup M) + n(L \cap M) + n(K \cap L \cap M)$, resulting in $n(K \cup L \cup M) = 49$, instead of 34. Similarly, other students applied the formula

$n(K \cup L \cup M) = n(K) + n(L) + n(M) + n(K \cap L) + n(K \cap M) - n(L \cap M) - n(K \cap L \cap M)$ which led to $n(K \cup L \cup M) = 68$. Furthermore, some students who applied the Venn diagram failed to insert the entries of the regions correctly (see Extract 9.2).

In part (b) (i), some students misinterpreted the tested concept, resulting in providing irrelevant answers. For example, they wrote, “There are three types of animals,” others wrote, “Animals are goats, cows, and dogs,” while others wrote, “No family that kept all three animals”. Moreover, some students applied the incorrect formula, $n(C) + n(G) + n(D)$, so they identified $n(C) = 61$, $n(G) = 49$ and $n(D) = 50$. Thereafter, substituted into the formula to obtain 160.

In part (b) (ii) and (iii), the majority of students provided responses that did not align with the requirements of the question. For example, in part (b) (ii), some provided that, $n(G \cap C) \text{ only} + n(C \cap D) \text{ only} + n(C \cap D \cap G)$ resulting in $18 + 20 + 15 = 53$. Others applied $n(G \cup C) = n(C) + n(G)$ to get $61 + 49 = 110$. Similarly, others applied the incorrect formula, such as $n(G \cup C \cap D) = n(G) + n(D) - n(G \cap D)$, which finally gave $n(G \cap D) = 14$. Likewise, in part (b) (iii), the responses provided were $n(D \cup C) = n(D) + n(C)$, leading to obtaining $50 + 61 = 111$. Others wrote $n(C) \text{ only} + n(D) \text{ only} + n(C \cap D) \text{ only}$ which equals $8 + 3 + 20 = 31$. Extract 9.2 provides an example of a student who encountered difficulties when answering the question.



$$n(KULUM)$$

$$= 29 +$$

$$53 + 44 + 63 = 160$$

$$\therefore n(KULUM) = 160$$

(i) ^{soluion} let $G = \{\text{families kept goats}\}$
 $C = \{\text{families kept cows}\}$
 $D = \{\text{families kept dogs}\}$

$$n(G \cup C \cup D) = n(G) + n(C) + n(D) - n(G \cap C) - n(G \cap D) - n(C \cap D) + n(G \cap C \cap D)$$

$$80 = 49 + 61 + 50 - 33 - 27 - 35 + n(G \cap C \cap D)$$

$$80 = 49 + 61 + 50 - 33 - 27 - 35 + n(G \cap C \cap D)$$

$$80 = 160 - 85 - n(G \cap C \cap D)$$

$$80 = 75 - n(G \cap C \cap D)$$

$$n(G \cap C \cap D) = 80 - 75$$

$$n(G \cap C \cap D) = 5$$

\therefore 5 families kept all types of animals.

$$\begin{aligned}
 \text{(ii)} \quad n(G \cup C \cup D) &= n(G) + n(D) + n(C) - n(G \cap D) - n(G \cap C) - n(C \cap D) + n(G \cap C \cap D) \\
 80 &= 49 + 50 - n(G \cap D) - 5 \\
 80 &= 99 - 5 - n(G \cap D) \\
 80 &= 94 - n(G \cap D) \\
 80 - 94 &= -n(G \cap D) \\
 -14 &= -n(G \cap D) \\
 \hline
 n(G \cap D) &= 14
 \end{aligned}$$

$$\begin{aligned}
 n(G \cup C \cup D) &= n(D) + n(C) + n(G) - n(C \cap D) - n(G \cap C) - n(G \cap D) + n(G \cap C \cap D) \\
 80 &= 50 + 61 - 35 - 5 \\
 80 &= 111 - 5 \\
 80 &= 106 - 50 \\
 &= n(C \cap D) = 40
 \end{aligned}$$

Extract 9.2: A sample of incorrect responses to question 9.

In Extract 9.2, part (a), the student failed to use the Venn diagram, leading to an incorrect answer. In part (b), the student applied the incorrect formula to all parts, leading to an incorrect answer.

2.10 Question 10: Variations

The question consisted of parts (a), (b), and (c). Part (a) stated that “variable a is directly proportional to the square root of b . If $a = 24$ when $b = 36$, then they were asked to calculate the value of a when $b = 6\frac{1}{4}$ ”. Part (b) was as follows: 16 students can cultivate the school farm for 12 days consecutively. Calculate the number of days for which 8 students can cultivate the same piece of land. Part (c) stated that “ p varies directly as the square of q and inversely as r . If $p = 48$ when $q = 4$ and $r = 16$, solve for p when $q = 8$ and $r = 12$ ”.

The data reveals that 392 students attempted this question, of whom 84 (21.43%) scored from 0 to 2.5 marks, 115 (29.34%) scored from 3.0 to 6.0 marks, and 193 (49.23%) students scored from 6.5 to 10 marks. The overall performance was good because 78.57 percent of students who

attempted the question passed. Figure 11 presents a summary of the students' performance on this question.

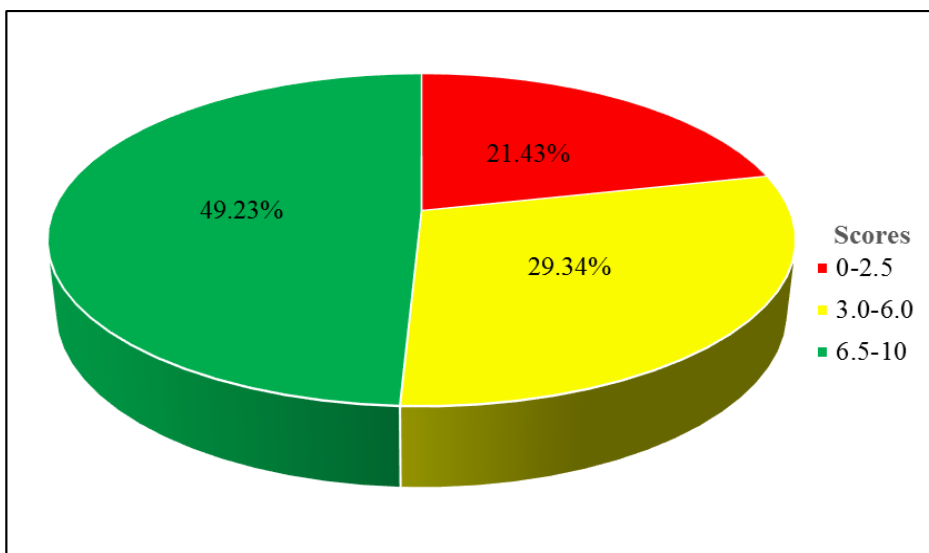


Figure 11: *Students' Performance in Question 10*

In part (a), many students correctly interpreted and rewrote the given information in the mathematical form $a \propto \sqrt{b}$. Thereafter, they introduce the proportionality constant (k) to obtain $a = k\sqrt{b}$ and consequently the equation, $\frac{a_1}{\sqrt{b_1}} = \frac{a_2}{\sqrt{b_2}}$. Thus, they substituted $a_1 = 24$, $b_1 = 36$ and $b_2 = \frac{25}{4}$ into the equation and obtained $a_2 = 10$. Similarly, in part (b), the students rewrote the given word problem into symbolic form, $S \propto \frac{1}{d}$, where S stands for the number of students and d stands for the number of days. Then, they introduced the proportionality constant (k) to obtain $S = \frac{k}{d}$, equivalent to $S_1d_1 = S_2d_2$. Therefore, they substituted $S_1 = 16$, $d_1 = 12$ and $S_2 = 8$ into $S_1d_1 = S_2d_2$ and correctly worked out to obtain $d_2 = 24$ days. Likewise, in part (c), the students correctly transformed the information into a mathematical

model, $p \propto \frac{q^2}{r}$, and introduced a constant of proportionality (k) resulting in $p = \frac{kq^2}{r}$ and consequently $\frac{p_1 r_1}{q_1^2} = \frac{p_2 r_2}{q_2^2}$. Hence, they substituted $p_1 = 48$, $q_1 = 4$, $r_1 = 16$, $q_2 = 8$ and $r_2 = 12$ into $\frac{p_1 r_1}{q_1^2} = \frac{p_2 r_2}{q_2^2}$ and got $p_2 = 256$. Extract 10.1 provides a sample of students' correct responses to this question.

Soln

$$a \propto \sqrt{b}$$

$$\frac{a}{\sqrt{b}} = \frac{k\sqrt{b}}{\sqrt{b}}$$

$$k = \frac{a}{\sqrt{b}}$$

$$k = \frac{24}{\sqrt{36}}$$

$$k = \frac{24}{6}$$

$$k = 4$$

$$a = k\sqrt{b}$$

$$a = 4\sqrt{\frac{25}{4}}$$

$$a = 4 \times \frac{5}{2}$$

$$a = 10$$

∴ Value of $a = 10$

x Students	y days
16	12
8	?

Soln

$$x \propto \frac{1}{y}$$

$$\frac{x}{1} = \frac{k}{y}$$

$$k = xy$$

$$k = 16 \times 12$$

$$k = 192$$

$$\frac{k}{x} = \frac{192}{8}$$

$$y = \frac{k}{x}$$

$$y = \frac{192}{8}$$

$$y = 24 \text{ days}$$

∴ They can cultivate the same piece of land for 24 days

$$\begin{aligned}
 & p \propto \frac{q^2}{r} \quad \text{Sol'n} \\
 & p_1 = \frac{kq^2}{r} \\
 & \frac{kq^2}{q^2} = \frac{p_1 r}{q^2} \\
 & k = \frac{p_1 r}{q^2} \\
 & k = \frac{48 \times 16}{4^2} \\
 & k = \frac{48 \times 16}{16} \\
 & k = 48 \\
 & p = \frac{kq^2}{r} \\
 & p = \frac{48 \times 8^2}{12} \\
 & p = \frac{48 \times 64}{12} \\
 & p = 4 \times 64 \\
 & \therefore p = 256
 \end{aligned}$$

Extract 10.1: A sample of the correct responses to question 10.

In Extract 10.1, the student correctly rewrote the given information into a mathematical model and then solved the equations to get the correct answer. In part (a), the student determined the constant of proportionality (k) and consequently the equation, likewise in part (b) and part (c).

Conversely, 62 (15.82%) students scored 1.0 mark or less. These students faced the following challenges. In part (a), a number of students interchangeably considered the concept of inversely proportional as direct proportional. For instance, some students wrote

$$a \propto \frac{1}{b^2} \text{ and consequently } a = \frac{k}{b^2} \text{ then, they realised that, } a_1 b_1^2 = a_2 b_2^2.$$

Thereafter, substituted the values of $a_1 = 24$, $b_1 = 36$ and $b_2 = 6\frac{1}{4}$ to

obtain $a = 4976\frac{16}{25}$. Also, others considered the concept of square root as cube root; for instance the students formulated a mathematical model as $a \propto \frac{1}{b^3}$. Thereafter, obtained $a = \frac{k}{b^3}$, then they worked on it and

ended up with $a = 13574 \times \frac{4}{25} = 2171\frac{21}{25}$. Likewise, in part (b), a

number of students responded to the question as directly proportional and wrote $S \propto d$ instead of $S \propto \frac{1}{d}$. Therefore, they got $k = \frac{4}{3}$, $d = 6$ instead of $k = 192$ and $d = 24$. Similarly, others equated $\begin{cases} 16 \text{ students} = 12 \\ 8 \text{ students} = ? \end{cases}$, thereafter, applied the cross multiplication techniques, simplifying the result to 6 days instead of 24 days.

In part (c), the students confused the term square and square root, they wrote $p \propto \frac{\sqrt{q}}{r}$, then developed the equation to obtain $p = \frac{k\sqrt{q}}{r}$. Later they recognized that $\frac{p_1 r_1}{\sqrt{q_1}} = \frac{p_2 r_2}{\sqrt{q_2}}$, then substituted $p_1 = 48$, $q_1 = 4$, $r_1 = 16$, $q_2 = 8$ and $r_2 = 12$ to obtain $p = 128$. Moreover, some students committed with an input error after correctly interpreting the given word problem $p \propto q^2 \propto \frac{1}{r}$ and rewrote $k = \frac{pr}{q^2}$. Then, they replaced p , q and r by 48, 4 and 16, respectively, and solved it to obtain the wrong value of $k = 192$. Thereafter, incorrectly solved for p by substituting $k = 192$, $r = 12$ and $q = 8$ to obtain $p = 128$. Extract 10.2 is a sample of the incorrect responses from one of the students who answered this question.

Soln.

$$Q \propto \frac{1}{b^2}$$

$$Q = \frac{k}{b^2}$$

$$k = a b^2$$

$$24 \times 36^2$$

$$k = 31104$$

$$a = \frac{k}{b^2}$$

$$= \frac{31104}{6^2}$$

$$31104 \div 6^2$$

$$31104 \times \frac{4}{25}$$

$$\frac{124416}{25}$$

$$Q = 4976 \frac{16}{25}$$

Let number of students be "s" and number of days be "d"

Soln
Introduce constant "k"
 $S \propto d$
 $S = k d$

$$\frac{16}{12} = \frac{k \times 12}{12}$$

$$k = \frac{16}{12}$$

$$k = \frac{4}{3}$$

Find d when s = 8

$$8 = \frac{4}{3} d$$

$$\text{From } S = k d$$

$$8 \times \frac{4}{3} = \frac{4}{3} \times d \times 3$$

$$\frac{24}{4} = \frac{4 d}{4}$$

$$d = 6$$

\therefore The number of days is 6

soln

$$P \propto \frac{\sqrt{q}}{r}$$

1st find the constant k

$$P = k \frac{\sqrt{q}}{r}$$

but $P = 48$, $q = 4$ and $r = 16$

$$48 = k \frac{\sqrt{4}}{16}$$

$$\cancel{48} = \frac{2k}{16}$$

$$\frac{2k}{2} = \frac{48 \times 16}{2}$$

$$k = 384$$

2nd Find P when $q = 8$ and $r = 12$

From $P = k \frac{\sqrt{q}}{r}$

$$P = \frac{384 \times \sqrt{8}}{12}$$

$$P = 32 \times \sqrt{8}$$

$$P = 32 \times 8^{\frac{1}{2}}$$

$$P = 32 \times 4^{2 \times \frac{1}{2}}$$

$$P = 32 \times 4$$

$$P = 128$$

\therefore The value of P is 128

Extract 10.2: A sample of incorrect responses to question 10.

In Extract 10.2, the student misinterpreted and failed to write the given word problem into a mathematical model, which led to incorrect responses.

3.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH TOPIC

The Additional Mathematics paper on the 2024 Form Two National Assessment was set from nine (9) topics and consisted of 10 compulsory questions. The topics tested were *Numbers*, *Symmetry*, *Algebra*, *Geometrical Constructions*, *Coordinate Geometry*, *Variations*, *Logic*, *Locus*, and *Sets*. Students performed well on seven (7) topics which are *Symmetry* (94.90%), *Geometrical Constructions* (88.78%), *Logic* (85.72%), *Variations* (78.57%), *Coordinate Geometry* (77.04%), *Algebra* (75.77%), and *Locus* (65.56%). The students' good performance in these topics was attributed to their competence in the tested concepts, accurate interpretation of word problems, and their ability to recall and apply appropriate formulas.

However, the performance of students in *Sets* was average (55.36%). The students' average performance on this topic was attributed to their moderate understanding of the subject matter, which was caused by the use of incorrect set formulas, failure to use Venn diagrams to solve set problems, and inability to understand the demands of the questions. Moreover, the analysis shows that students performed poorly on *Numbers* (21.18%). The poor performance was due to incorrect interpretation, inadequate knowledge of the Fibonacci sequence, use of inappropriate methods to find the next numbers in the sequence, failure to recall and apply the rules of divisibility, and inability to perform operations. Appendix I presents the students' performance on each topic 2024.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis revealed that overall performance on the Additional Mathematics Assessment in 2024 improved by 14.66% compared to 2023. Out of the nine (9) topics evaluated, students performed poorly on one (1). Compared to 2023, the performance had significant improvement in *Sets*, *Locus*, *Logic*, and *Geometrical Constructions* and it declined in *Numbers* (see Appendix II). Thus, the key areas where

students demonstrated strengths and weaknesses provide valuable insights for education stakeholders.

4.2 Recommendations

In order to improve more the performance of students in future assessments, the following measures are recommended.

- (a) Due to a lack of students' competence in some areas of *Numbers*, teachers are advised to guide students in generating and applying number patterns using various resources, such as the Fibonacci sequence, Pascal's triangle, and number charts.
- (b) Due to a lack of students' competence in divisibility rules, teachers are advised to guide students in stating the rules for natural numbers using various resources, such as number charts, and applying these rules to simplify computations.
- (c) Given students' difficulties in some areas of Sets, teachers are advised to provide sufficient exercises for learners and emphasise drawing Venn diagrams and solving related problems involving three sets.
- (d) Students to form Mathematics clubs that will enhance sharing of mathematical knowledge through discussions.
- (e) Teachers should guide and encourage students to carefully read and understand the requirements of the questions. This will help them provide accurate and relevant solutions during the National Assessment.

APPENDIX I: Students' Performance on Each Topic in 2024

S/N	Topic	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Symmetry	6	94.90	Good
2.	Geometrical Constructions	3	88.78	Good
3.	Logic	7	85.72	Good
4.	Variations	10	78.57	Good
5.	Coordinate Geometry	4	77.04	Good
6.	Algebra	2 & 8	75.77	Good
7.	Locus	5	65.56	Good
8.	Sets	9	55.36	Average
9.	Numbers	1	21.18	Weak

**APPENDIX II: Comparison of Students' Performance in 2024 and
2023**

S/N	Topic	2023			2024		
		Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks	Question Number	Students who Scored an Average of 30% or Above	Remarks
1.	Symmetry	6	77.78	Good	6	94.90	Good
2.	Geometrical Constructions	3	43.08	Average	3	88.78	Good
3.	Logic	7	56.24	Average	7	85.72	Good
4.	Variations	8	67.80	Good	10	78.57	Good
5.	Coordinate Geometry	5	71.20	Good	4	77.04	Good
6.	Algebra	2 & 9	66.90	Good	2 & 8	75.77	Good
7.	Locus	4	43.99	Average	5	65.56	Good
8.	Sets	10	34.69	Average	9	55.36	Average
9.	Numbers	1	88.66	Good	1	21.18	Weak

