THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENT'S ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA)2020

ADDITIONAL MATHEMATICS

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042 ADDITIONAL MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is pleased to issue this report on Students' Item Response Analysis (SIRA) in Additional Mathematics for FTNA 2020 in order to inform teachers, parents, students, policy makers and other education stakeholders on how the students responded to the assessment items. The report will enable the stakeholders to take appropriate measures to enhance the performance of students and enable them to master the topics which need more emphasis in teaching and learning.

The analysis of students' responses was done in order to identify the areas in which the students did well or poorly. Basically, the report highlights the factors for poor performance in order to understand what the education system managed or failed to offer to learners in their two years of Secondary Education.

The factors noted for poor performance include the students' inability to: solve linear and quadratic equations algebraically; determine the locus of points moving at equal distance from the fixed points and solve simultaneous equations graphically. Extracts from the students' responses are used in this report to illustrate the reasons for poor or good performance that could be used as a practical guide to teachers and students in the teaching and learning process.

The National Examinations Council of Tanzania believes that the education stakeholders will work on the challenges the students faced when attempting the assessment questions and take appropriate measures to improve the performance in this subject.

Finally, the Council would like to thank the examination officers and all others who participated in the preparation of this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Students' Item Response Analysis (SIRA) report in Additional Mathematics subject is prepared to provide feedback to stakeholders about the performance of the students.

The paper comprised 10 compulsory questions. Each question carried 10 marks. The questions were set basing on the 2005 Form I and II Additional Mathematics syllabi.

In 2020, a total of 516 students sat for the Additional Mathematics examination. As compared to the FTNA 2019 results, 527 students sat for the examination. This represents a decrease in number of students in this assessment by 2.09 percent. Table 1 presents the summary of the students' performance in 2019 and 2020.

Table 1: The Performance of the Students in Additional Mathematics FTNA 2019 and 2020

Year	Code	Subject Name	Students Sat	Passed		Grades				
				No.	%	A	B	C	D	F
2019	042	Additional Mathematics	527	318	60.34	15	28	146	129	209
2020			516	392	75.97	70	89	137	96	124

Table 1 shows that the students' performance in Additional Mathematics was good as 75.97 percent of the students who sat for the assessment passed, of which 70 (17.86%) students got grade A, 89 (22.70%) got grade B, 137 (34.95%) got grade C and 124 (31.63%) got grade D. These results show that the students' performance in this subject increased by 7.85 percent in comparison with students' results of 2019. However, 124 (24.03%) students performed poorly as they obtained grade F. The percentages of students who passed the assessments in Additional Mathematics got different grades as shown in Figure 1.

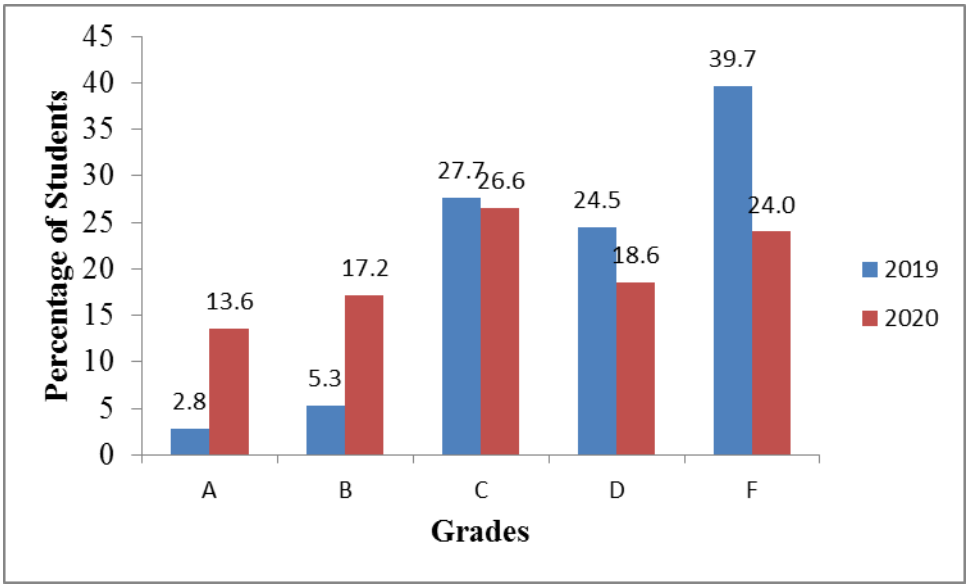


Figure 1: *Distribution of Grades A, B, C, D and F for the 2019 and 2020 Additional Mathematics Assessment*

Figure 1 shows that, the performance in Additional Mathematics was better than that of 2019, since the percentages of students who passed the assessment with grades A and B were 13.6 and 17.2 respectively. In 2019, only 2.8 percent of the students got grade A while 5.3 percent got grade B.

The analysis of students' performance in each question is presented in section 2.0. The section consists of a short description about the requirements of questions and the analysis on how the students responded to the questions. Extracts for both well and poorly performed questions are included in the analysis for each question. The factors that contributed to good or poor performance in each question are illustrated by using samples of students' responses. Therefore, the analysis in each question can be used as a practical guide to teachers and students in order to improve teaching and learning, and eventually students' performance in future Additional Mathematics assessments.

The analysis of students' performance in the topics assessed is also shown in Appendix I whereby green, yellow and red colours represent good, average and poor performance respectively. Likewise, the comparison of the students' performance in each topic for the FTNA 2019 and 2020 in Additional Mathematics is shown in Appendix II. Finally, the recommendations are included at the end of this report to help students, teachers and the government to enhance the students' performance in future Additional Mathematics assessment.

2.0 ANALYSIS OF STUDENTS' PERFORMANCE IN EACH QUESTION

This section gives the analysis of the students' performance in each question. The national assessment results are based on the score intervals of 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 which are equivalent to excellent, very good, good, satisfactory and fail respectively. For the purpose of this report, the performance in each question is considered good, average or weak if the percentage of students who scored 30 percent or more of the marks allocated for the question is in the intervals 65 – 100, 30 – 64 or 0 – 29 respectively.

2.1 Question 1: Numbers

This question had parts (a) and (b). In part (a), the students were required to write down all factors of 30 which are greater than 2. In part (b), they were given the whole numbers 14472 and 91896 and required to identify the number which is divisible by both 8 and 9.

The analysis of data shows that 513 (99.4%) students attempted this question. Among them, 381 (75.0%) students scored marks ranging from 3 to 10. Therefore, the students' performance in this question was good. Figure 2 shows the performance in question 1.

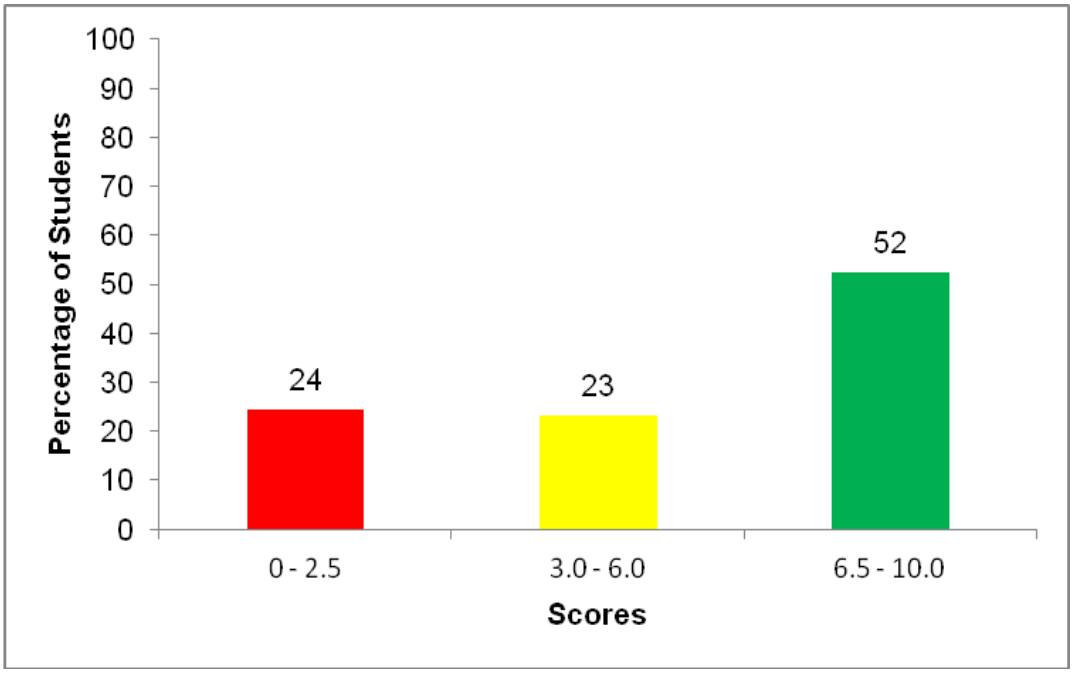


Figure 2: *Students' performance in question 1*

Figure 2 shows that 52 percent of students scored marks ranging from 6.5 to 10 whereby 1.4 percent of them scored 10 marks. The students who performed well in part (a) were able to recall and write down all the factors of 30 as 1, 2, 3, 5, 6, 10, 15 and 30, then selected the factors which are greater than two and listed them correctly as 3, 5 ,6, 10, 15 and 30. These students demonstrated good understanding on the topic of Numbers. In part (b), they managed to identify that 472 is the number which is formed by taking the last three digits of the whole number 14472. Then, they recognized 472 as the number which is divisible by 8 hence 14472 is divisible by 8. The students carried out the computations to verify that 14472 is divisible by 9 because the sum of digits of 14472 results into a number which is divisible by 9. Similarly, the students were able to identify that the number 896 is formed by taking the last three digits of 91896 and is divisible by 8 as well. However, they concluded that 91896 is not divisible by 9 because the sum of all digits in this number gives 33 which is not divisible by 9. Extract 1.1 is a sample response from one of the students who managed to answer this question correctly.

1. (a) Write down all factors of 30 which are greater than 2.
(b) Given the whole numbers 14472 and 91896; which number is divisible by both 8 and 9?

a) Factors of 30 which are greater than 2

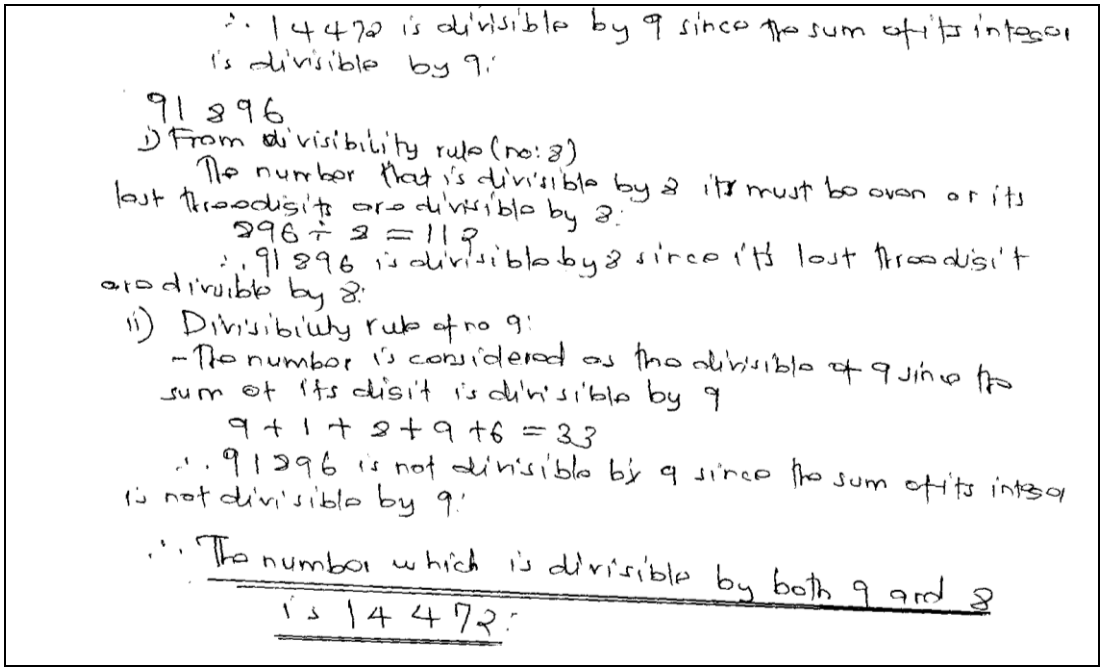
2 | 30
3 | 15
5 | 5
1

Factors are: 3, 5, 6, 10, 15, and 30 itself
∴ Factors which are greater than 2 are 3, 5, 6, 10, 15 and 30.

By visibility rule (8)
The number that is divisible by 8, It's last three digit must be even or divisible by 8
It's last two digit = 472
$472 \div 8 = 59$
∴ 14472 is divisible by 8 since it's last three digits are divisible by 8.

ii) From visibility rule (9)
- The number is said to be divisible by 9 since the sum of its digit is divisible by 9

$1 + 4 + 4 + 7 + 2 = 18$
$18 \div 9 = 2$



Extract 1.1: A correct response from one of the students

In Extract 1.1, the student listed 3, 5, 6, 10 15 and 30 as the factors of 30 which are greater than 2. Also, the student applied the rule of divisibility of 8 and 9 in showing that 14472 is divisible by 8 and 9.

Figure 2 also shows that 24 percent of the students scored below 3 out of 10 marks. The weak performance of the students in this category was due to the following reasons: in part (a), the students confused factors with multiples of numbers. They listed the multiples of 30 as $30\times1=30$, $30\times2=60$, $30\times3=90$, $30\times4=120$ and $30\times5=150$ instead of listing the factors of 30 which are bigger than 2 namely 3, 5, 6, 10, 15, and 30. Other students failed to abide by the demands of the question as they listed various types of numbers. For example, one of the students wrote a set of even numbers greater or equal to 2 such as 2, 4, 6, 8, 10 and 12. In part (b), they failed to recall and apply the divisibility rule to test a number which is divisible by 8 and 9 from the given whole numbers. The analysis of the students' responses shows that they decided to test whether the given numbers are divisible by 8 and 9 by using long division method as follows; $\frac{14472}{8}$, $\frac{91896}{8}$, $\frac{14472}{9}$ and $\frac{91896}{9}$. Thus, they seemed to lack the intended skills as stipulated in the Additional Mathematics syllabus. Extract 1.2 is a sample response from one of the students illustrating how they failed to answer correctly part (a) of this question.

1. (a) Write down all factors of 30 which are greater than 2.
(b) Given the whole numbers 14472 and 91896; which number is divisible by both 8 and 9?

SOLUTION (a)

30 x 1 = 30
30 x 2 = 60
30 x 3 = 90
30 x 5 = 150

∴ The multiples of 30 which are greater than 2 are 3, 5, 7 and.....

Extract 1.2: An incorrect response from one of the students

In Extract 1.2, the student carried out computations to find the multiples of 30, then concluded by recognizing a set of numbers 3, 5 and 7 as the factors 30 greater than 2.

2.2 Question 2: Algebra

This question comprised parts (a) and (b). In part (a), the students were required to simplify the expression $6(x+1)+2(x+2y)-8x+10y-2(3+4y)$. In part (b), they were required to: (i) use elimination method to solve the simultaneous equations $6m=-2n+14$ and $2m+5n=9$ and (ii) solve the linear inequality $7<3y+1\leq 13$.

The analysis of data shows that out of 516 (100%) students who attempted this question, 75 percent scored from 3 to 10 marks. The analysis also shows that 24 percent of the students scored below 3 out of 10 marks, 23 percent scored from 3 to 6 marks and 52 percent scored from 6.5 to 10 marks. It was also noted that 105 (20.3%) students scored all 10 marks while 52 (10.1%) students scored 0. Generally, this question was performed well as shown in Figure 2.

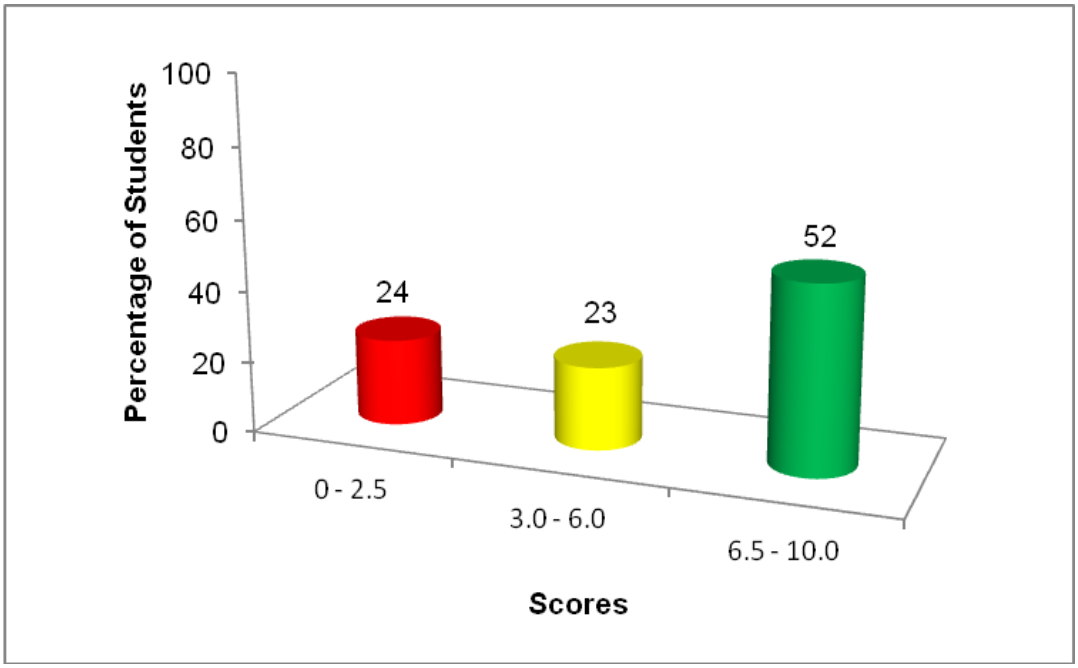


Figure 3: *Students' performance in question 2*

The students who responded to this question correctly demonstrated the following strengths: in part (a), the students had sufficient knowledge on the BODMAS rule. Firstly, they were able to open the brackets of the given expression correctly to get $6x+6+2x+4y-8x+10y-6-8y$. Then, they managed to arrange correctly the like terms as $6x+2x-8x+4y+10y-8y-6+6$. Finally, they got $6y$ as the most simplified form of the given expression. In part (b) (i), the students were familiar with the elimination method of solving simultaneous equations. Most of them rearranged correctly the two equations in the form $ax+by=c$ as $6m+2n=14$ and $2m+5n=9$. Later, they multiplied the second equation by 3 so that the two equations had the same leading coefficients that is $6m+2n=14$ and $6m+15n=27$ of which they solved to get $n=1$ and $m=2$. In part (b) (ii), the students solved correctly the inequality $7<3y+1\leq 13$ in two cases. In the first case, they solved the inequality $7<3y+1$ to get $2<y$. In the second case, the inequality $3y+1\leq 13$ was solved to get $y\leq 4$. Finally, the two solutions were combined to get $2<y\leq 4$ as the required solution of the given inequality. Extract 2.1 is a sample response from one of the students illustrating how they managed to answer the question correctly.

(a) Simplify the following expression $6(x+1)+2(x+2y)-8x+10y-2(3+4y)$

(b) (i) Use elimination method to solve the following simultaneous equations: $6m=-2n+14$ and $2m+5n=9$.

(ii) Solve the following linear inequality: $7<3y+1\leq 13$.

soln:

a) $6(x+1)+2(x+2y)-8x+10y-2(3+4y)$

$6x+6+2x+4y-8x+10y-6-8y$

by $6x+2x-8x+4y+10y-8y+6-6$

$8x-8x+14y-8y+6-6$

$=6y$

$\therefore 6(x+1)+2(x+2y)-8x+10y-2(3+4y)=6y$

b) (i) $6m=-2n+14$

$\begin{cases} 6m+2n=14 \ldots (i) \\ 2m+5n=9 \ldots (ii) \end{cases}$

• Let 'Eliminate 'm'

$2\{6m+2n=14$

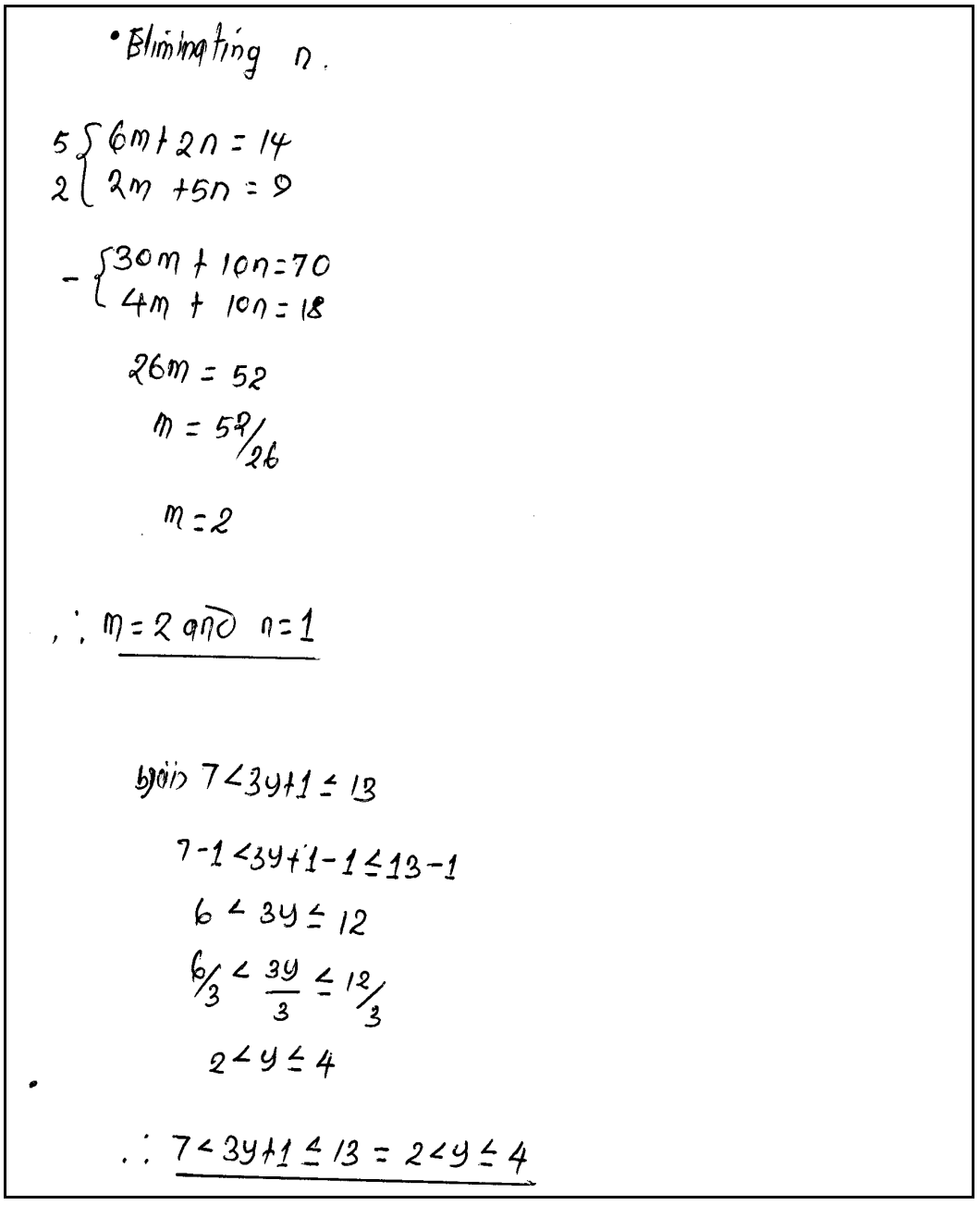
$6\{2m+5n=9$

$-\begin{cases} 12m+4n=28 \\ 12m+30n=54 \end{cases}$

$-26n=-26$

$n=-26/-26$

$n=1$



Extract 2.1: A correct response from one of the students who attempted question 2

In Extract 2.1, the student was able to collect like terms so as to simplify the given expression in part (a). In part (b), the students solved correctly the two linear simultaneous equations in (i) and the given linear inequality in (ii).

Despite the good performance, there were 126 (24%) students who scored low marks. These students encountered the following challenges: in part (a), some

students opened the brackets correctly but could not proceed further with the process of simplifying the resulting expression into $6y$. Other students failed to apply the BODMAS rule to simplify expression they were given. For instance, one student opened the brackets by writing $6x+1+2x+2y-8x+10y-6+4y$ but simplified it incorrectly into $x+22y$ instead of $6y$. The students were supposed to realize that the long form of the BODMAS rule is Open Brackets of Division, Multiplication, Addition and Subtraction and that it is used to explain the order of operation of a mathematical expression. In part (b) (i), the students managed to rearrange the given equations into $6m+2n=14$ and $2m+5n=9$ respectively but could not multiply each of these equation by a suitable number so that the two equations had the same terms in *m* or *n* that could be solved by using elimination method to get the value of *m* and *n*. Further analysis shows that other students could not realize the need to rearrange the given simultaneous equations into $6m+2n=14$ and $2m+5n=9$. For example, one of the students applied the elimination method on $\begin{matrix}1\\6\end{matrix}\begin{cases}6m=-2n+14\\2m+5n=9\end{cases}$ incorrectly and got wrong values of *m* and *n* that is $m=5.2$ and $n=22.6$ instead of $m=2$ and $n=1$. This shows that the students had insufficient knowledge and skills in solving simultaneous equations. In part (b) (ii), some students confused the inequality signs (< and $\leq$) with the equality sign (=) hence were unable to solve the inequality $7<3y+1\leq 13$. For instance, one of the students replaced the given inequality signs in $7<3y+1\leq 13$ with an equal sign as a result he/she solved $7<3y+1=3y+1\leq 13$ to get $2<y=y\leq 4$ instead of $2<y\leq 4$. Other students lacked the skills of solving the inequalities and decided to perform irrelevant calculations. For instance, one student subtracted 1 in the given inequality to get $7<3y\leq 12$, then solved it to get $\frac{7}{3}<y\leq 4$ which is an incorrect. Extract 2.2 is a response of a student who got wrong answer in part (a) of this question.

2. (a) Simplify the following expression $6(x+1)+2(x+2y)-8x+10y-2(3+4y)$

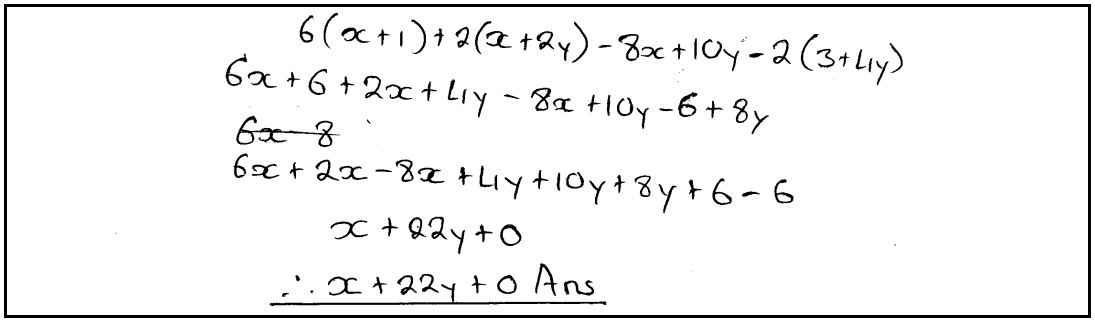
(b) (i) Use elimination method to solve the following simultaneous equations: $6m=-2n+14$ and $2m+5n=9$.

(ii) Solve the following linear inequality: $7<3y+1\leq 13$.

Solution

(a)

To simplify



Extract 2.2: An incorrect response from one of the students

In Extract 2.2, the student failed to apply the BODMAS rule to simplify the given expression.

2.3 Question 3: Geometrical Constructions

The question required the students to: (a) draw the line segment XY, then divide it into two equal parts and (b) construct the Hexagon with sides of length 4cm each.

The analysis shows that the question was attempted by 509 (98.6%) students, out of which 76.81 percent of the students scored marks ranging from 3 to 10. Further analysis shows that 23.18 percent of the students scored from 0 to 3 marks, 28.68 percent scored from 3.0 to 6.0 marks and 48.13 percent scored above 6 out of 10 marks. The analysis also shows that 7 (1.4%) students scored all 10 marks while 80 (15.7%) students scored 0. Therefore, the students' performance in this question was good. These statistics are summarized in Figure 4.

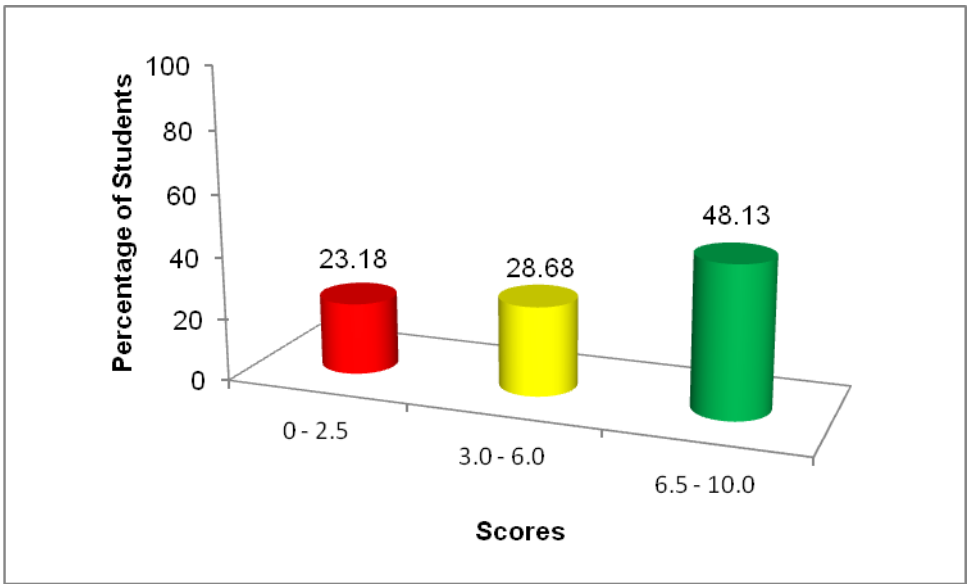
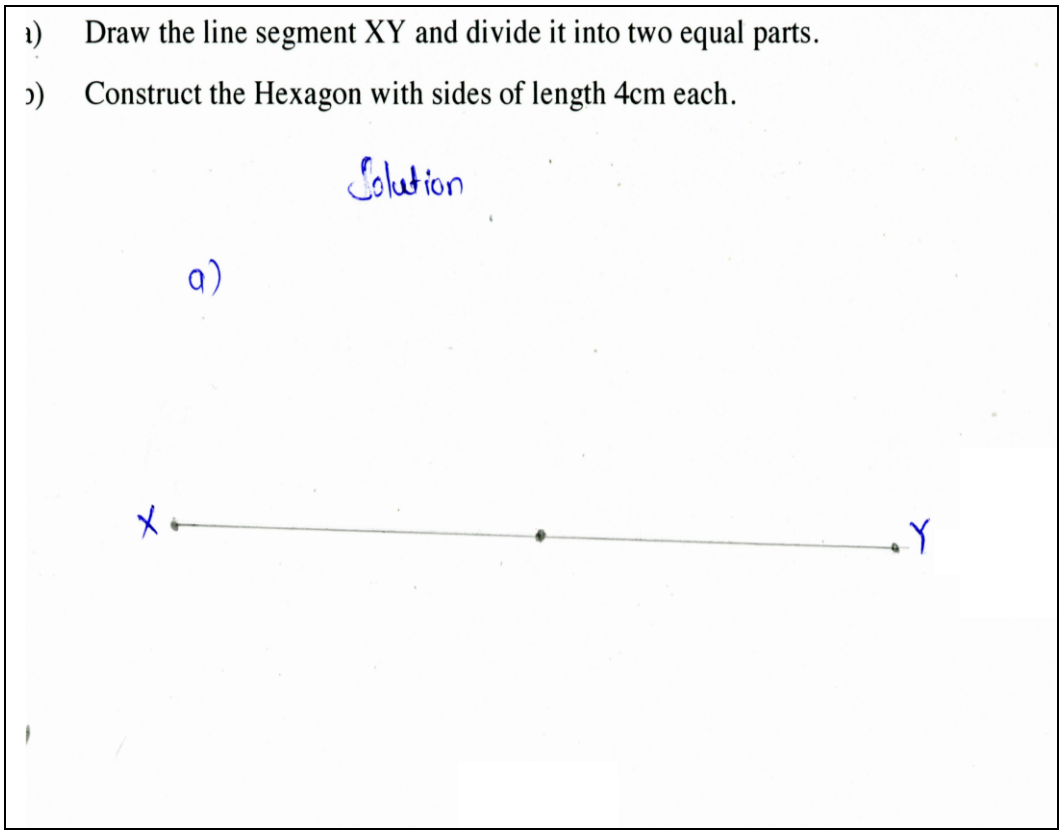
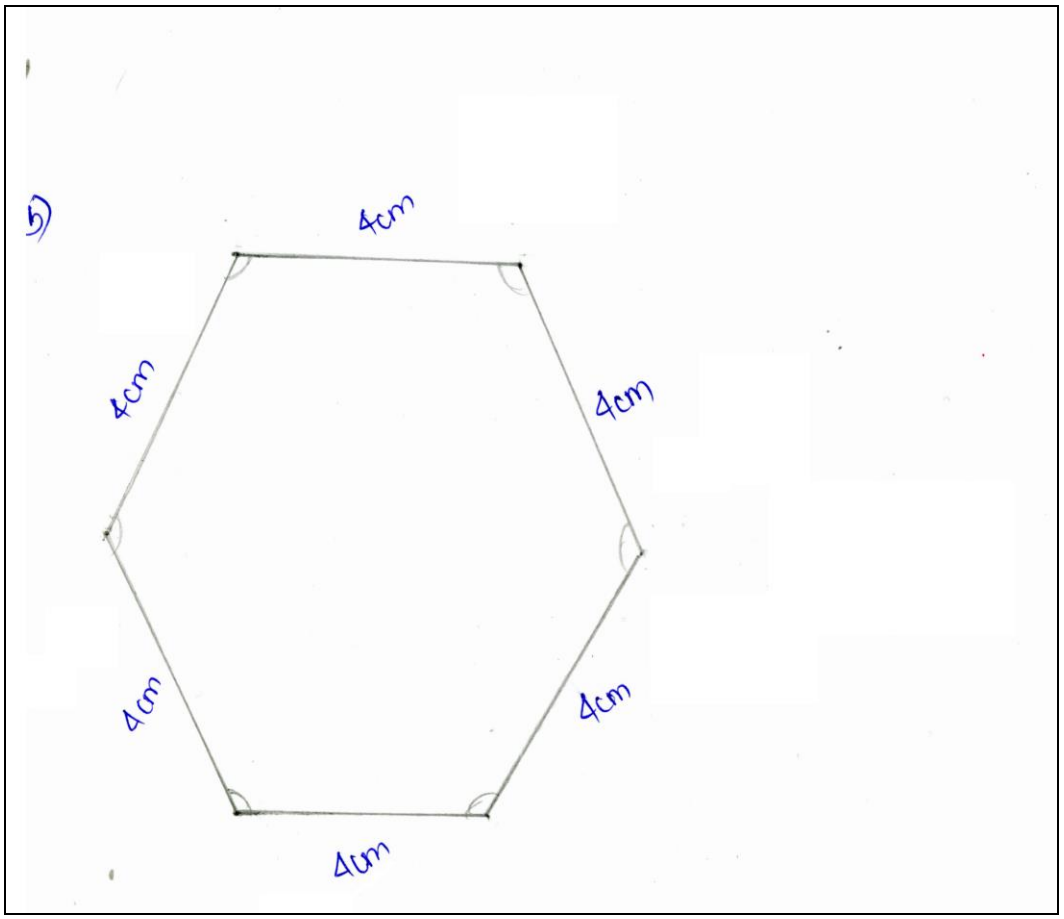


Figure 4: *Students' performance in question 3*

The students who attempted question 3 correctly demonstrated the following competences: in part (a), they were able to draw the line segment XY by labelling it with two end points X and Y and used a pair of dividers to divide it into two equal parts. In part (b), the students who drew the correct hexagon were able to: draw a circle of radius 4 cm by using a compass. They managed to maintain the angle of contact they set for the compass so as to make six small dots on the edge of the circle by using a pencil; connected six dots with a ruler and erased the original circle and the dots along the edges so as to remain with the Hexagon with sides of length 4 cm each. Other students recognized the hexagon as a six sided polygon. Therefore, they were able to compute the value of the interior angle for a six-sided polygon. For this reason, they used the formula $\frac{n-2\ \times 180^{0}}{n}$ to determine the value of each interior angle in the polygon as 120°. Thereafter, they constructed correctly the required polygon using a protractor and a ruler keeping the length of each side equal to 4 cm and the size of each interior angle equal to 120°. Extract 3.1 is a sample solution from one of the students who performed well.



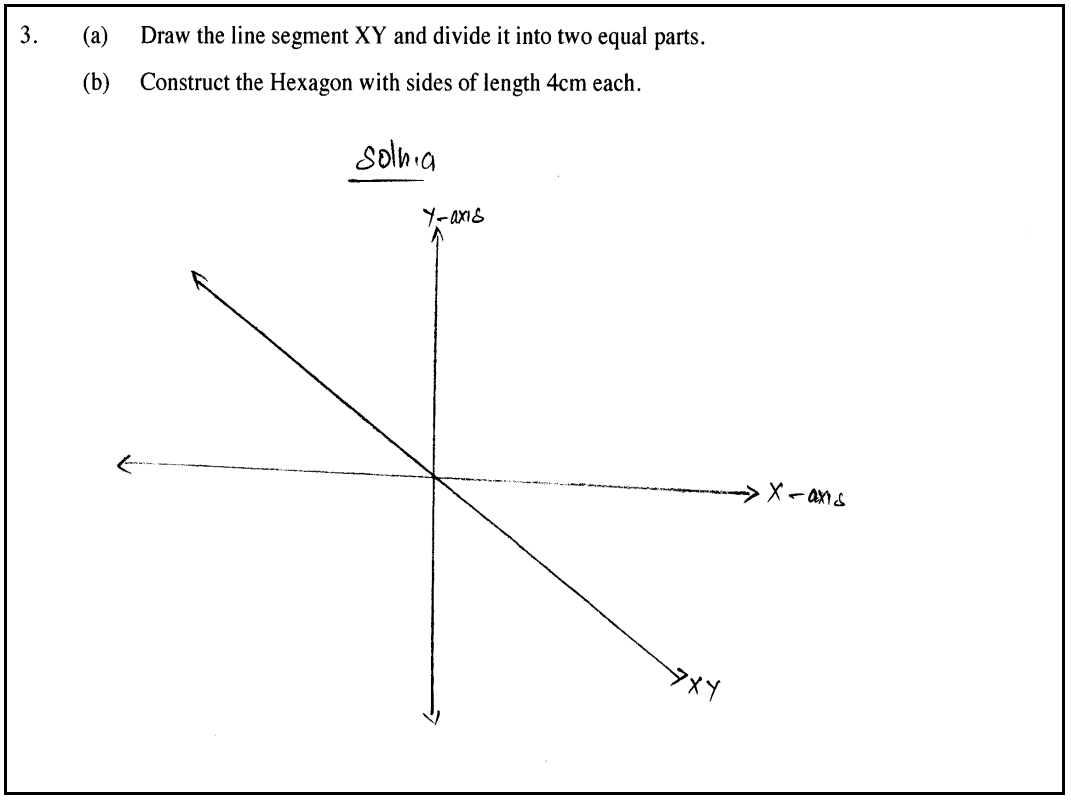


Extract 3.1: A correct response from one of the students

In Extract 3.1, the student demonstrated the ability to divide the given line into segments of equal parts in part (a). The student also constructed correctly a diagram showing a six sided polygon in part (b).

The students who did not respond to the question correctly encountered the following challenges: In part (a), some students recognized the term line segment as a line with arrows on both sides hence drew the straight line $\overleftrightarrow{XY}$ instead of line segment $\overline{XY}$. The analysis shows further that several students perceived the line segment $\overline{XY}$ as an xy-plane. Thus, found themselves drawing the x-axis and y-axis contrary to the requirements of the question. In part (b), the students failed to realize that the hexagon is a six-sided polygon, as a result they constructed different polygons with more or less number of sides than six sides such as octagon, pentagon, square and heptagon. However, it was surprising to see how several

students used the formula $\frac{(n-2)\times 180^0}{n}$ to calculate the value for the interior angle but could not draw the required polygon. Extract 3.2 is a sample response from one of the students who responded part (a) of the question incorrectly.



Extract 3.2: An incorrect response from one of the students

As Extract 3.2 shows, the student bisected the x - axis and y – axis using the line $y = -x$.

2.4 Question 4: Locus

In this question the students were required to find the locus of a point which is equidistant from points (0, 2) and (0, -3).

The analysis shows that 488 (94.6%) students attempted this question. Among them, 237 (48.6%) students scored from 3 to 10 marks. Therefore, the students' performance in this question was average. Figure 5 shows the students' performance in this question.

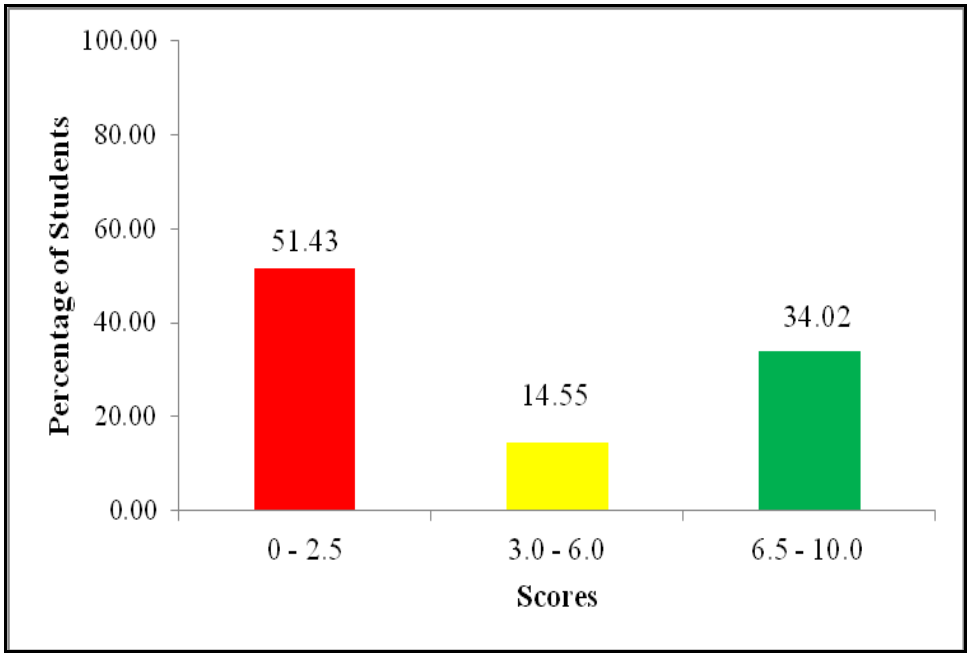
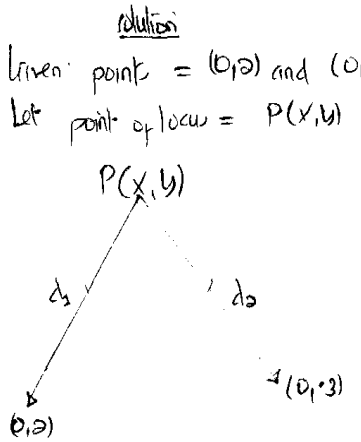
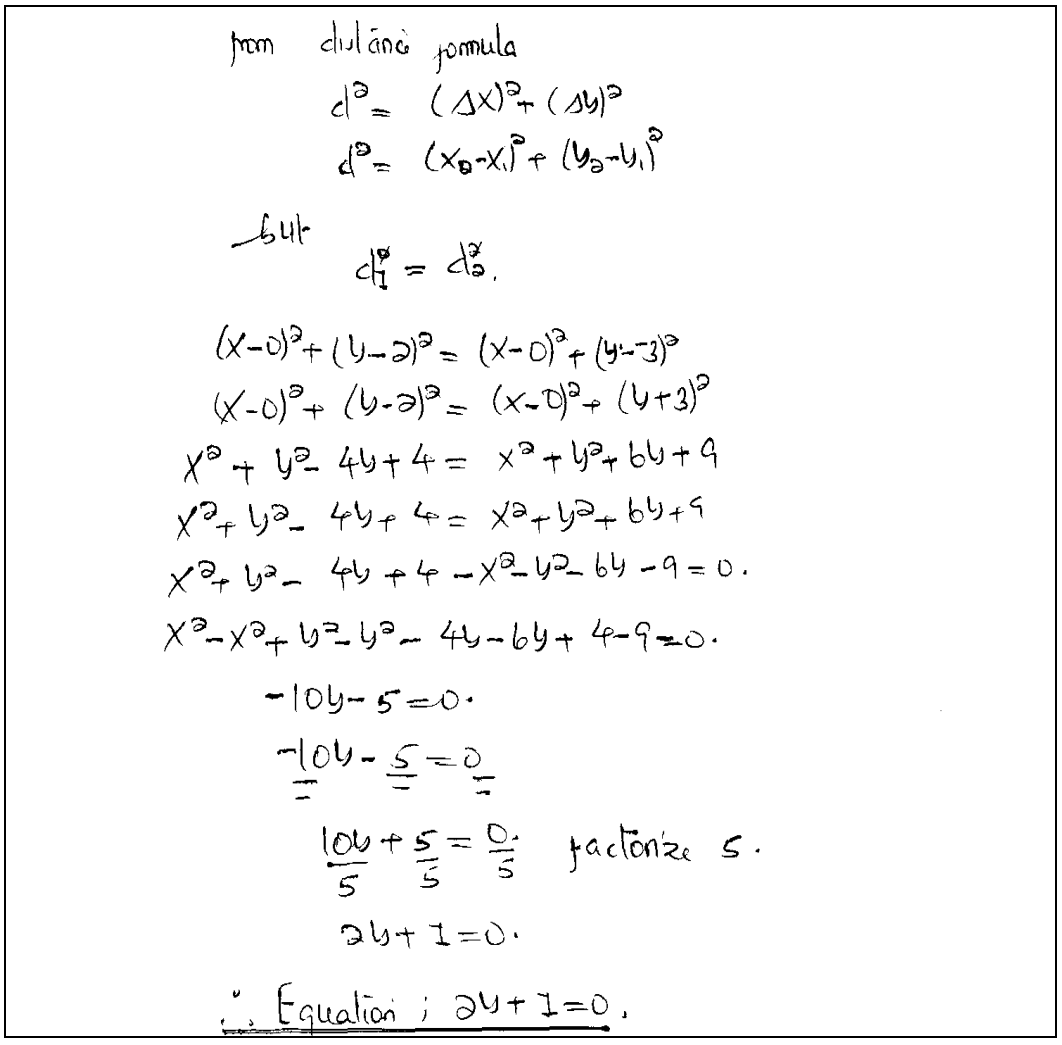


Figure 5: *Students' performance in question 4*

A total of 166 (34.02%) students scored high marks whereby 141 (28.9%) students scored 10 marks. The students who scored all marks were able to: identify that the distance from point x, y to (0, 2) is equal to its distance from (0, -3). In addition, they applied correctly the distance formula and used the points (0, 2), x, y and (0, -3) to determine the locus $2y+1=0$. Extract 4.1 is a sample response from one of the students showing how they managed to answer this question correctly.

4. Find the locus of a point which is equidistant from points (0, 2) and (0,-3).



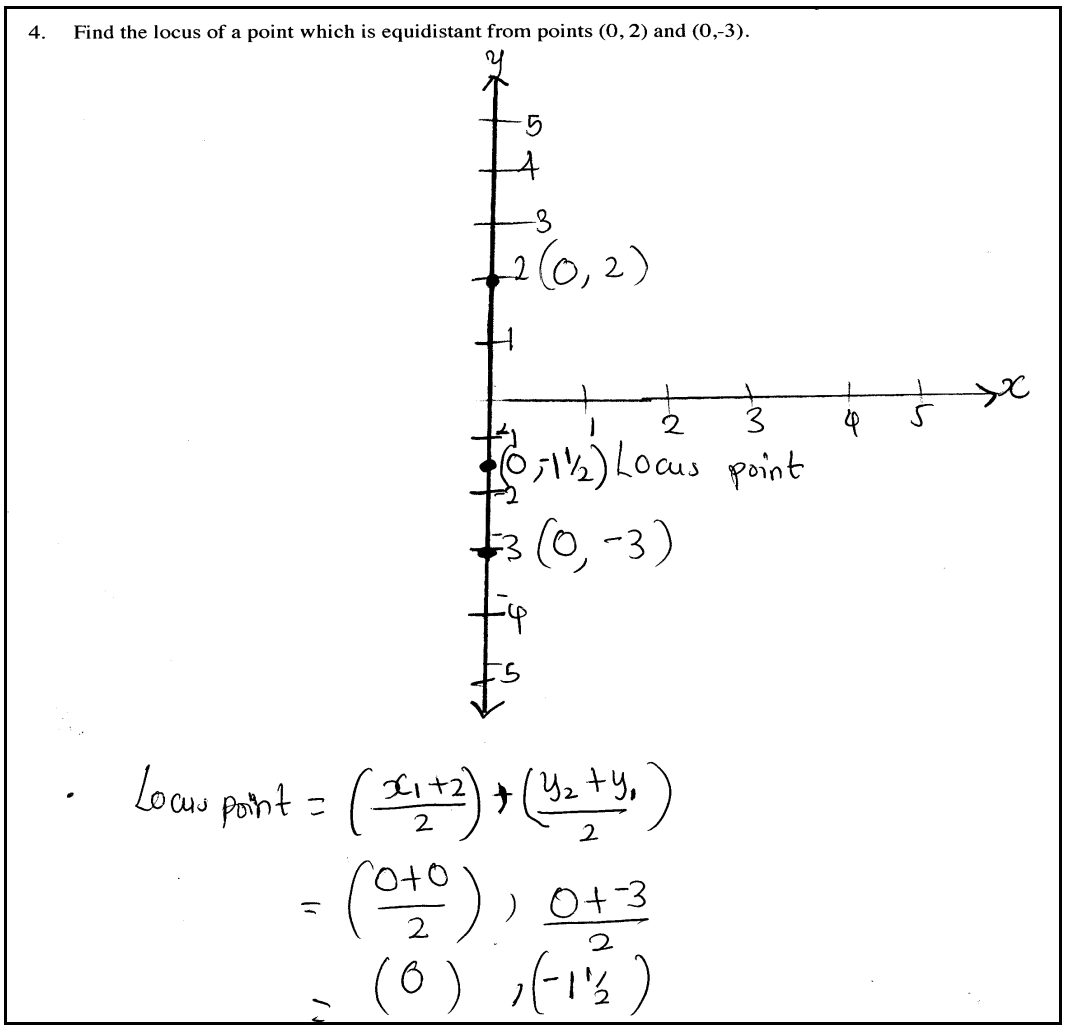


Extract 4.1: A correct response from one of the students

In Extract 4.1, the student managed to determine the locus of a point moving at equal distance from a fixed point.

On the other hand, a total of 251 students equivalent to 51.43 percent scored low marks of which 207 (42.4%) students scored 0. These students failed to apply the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the required equation of a locus. Some of the students used incorrect formulae such as $\sqrt{y-a^{\,2} + y-b^{\,2} = r^2}$. Thus, they ended up with the incorrect answer for the locus such as $2y + 13 = 0$ instead of $2y + 1 = 0$. Further analysis shows that other students evaluated the midpoint of a line joining the points (0, 2) and (0, -3) to get

$x, y = 0, 0.5$. This was wastage of time because the given question did not require them to find the midpoint. Extract 4.2 is a sample solution from one of the students who got the question wrong.



Extract 4.2: An incorrect response from one of the students

In Extract 4.2, the student evaluated the mid-point contrary to the requirements of the question.

2.5 Question 5: Coordinate Geometry

The question required students to find the coordinates of the points of intersection of the graphs of $y = x^2 - x - 3$ and $y = x$.

The analysis shows that 44.5 percent of the students scored from 3 to 10 marks, among them, 25.61 percent scored from 6.5 to 10 marks. Therefore, the students' performance in this question was of average. Figure 6 summarizes the students' performance in this question.

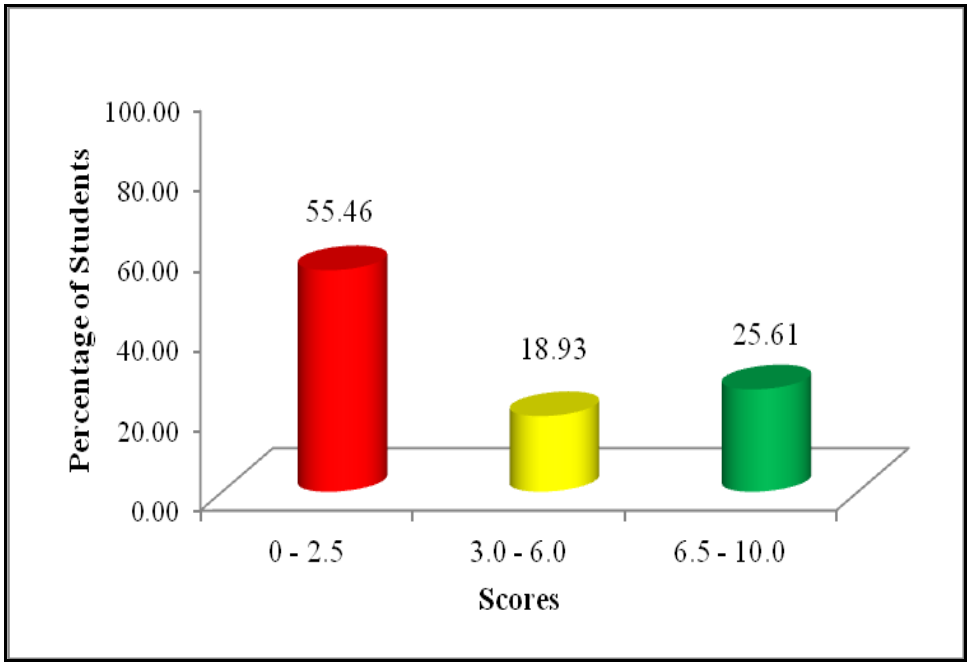
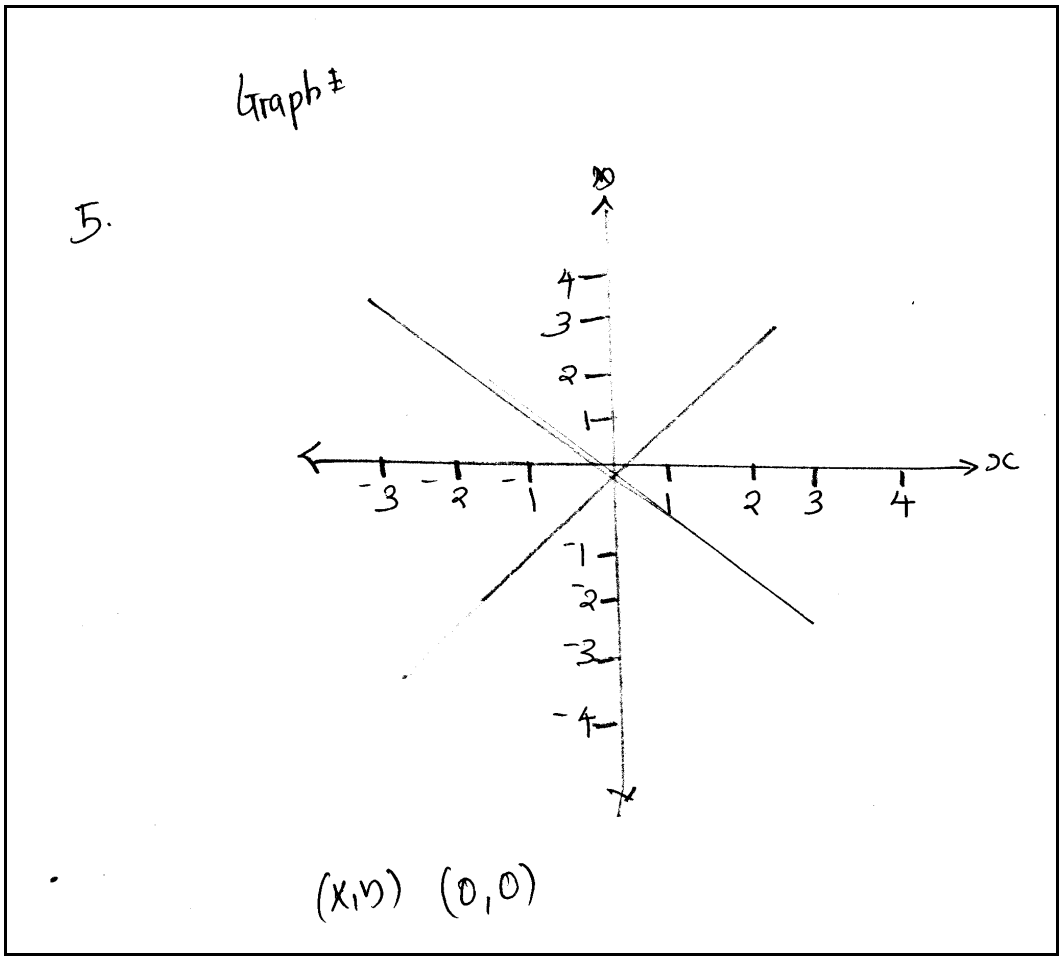


Figure 6: *Students' performance in question 5*

As seen in Figure 6, more than half of the students (55.46%) scored low marks. Some of these students failed to equate the given equations to get the points of intersections $3,3$ and $-1,-1$. Other students prepared a table of values for $y=x$ and $y=x^2-x-3$ with wrong entries for *x* and *y*. Thus, they drew incorrect graphs which produced wrong points of intersection. Extract 5.1 is a sample of poor responses showing some mistakes done by one of the students who responded incorrectly to this question.

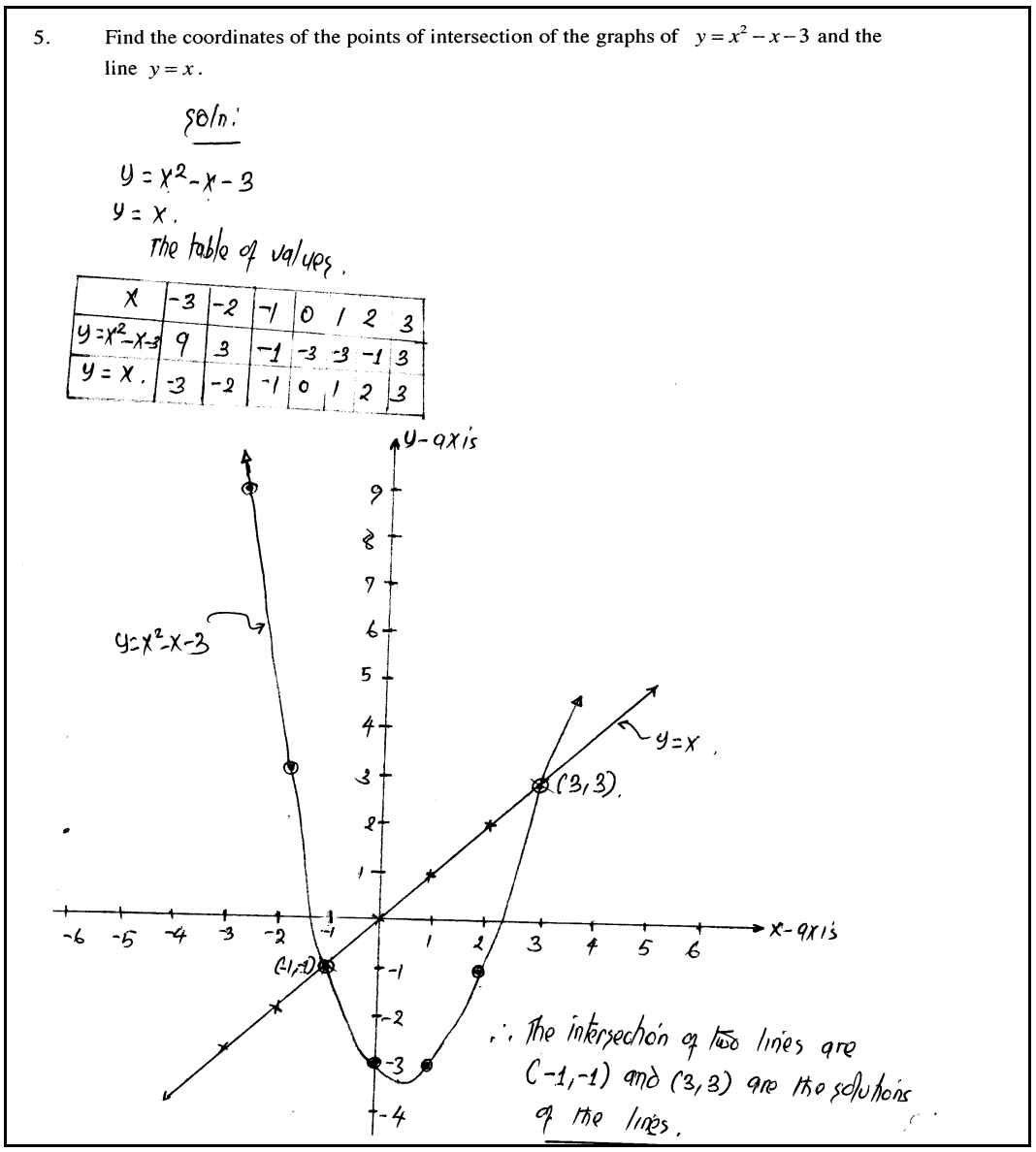


Extract 5.1: An incorrect response from one of the students

In Extract 5.1, the student drew irrelevant graphs indicating lack of knowledge on the assessed concept.

In Figure 6, it is also shown that 25.61 percent of the students scored above 6 out of 10 marks allocated for this question. The students who responded correctly this question replaced y in the equation $y = x^2 - x - 3$ with x to get the equation $x^2 - 2x - 3 = 0$; solved the equation $x^2 - 2x - 3 = 0$ using either the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or method of completing the square or factorization by splitting the middle term or by inspection to get $x = 3$ and $x = 1$. Then, by substituting the x values into the equation $y = x$, they gave the correct values of $y = 3$ and $y = -1$ and hence the required points of intersection. Other students

attempted the question graphically by drawing the graphs of $y = x^2 - x - 3$ and $y = x$ on the same axes and determined the points of intersection for the two graphs as (-1, -1) and (3, 3). Extract 5.2 is a sample response from one of the students who responded to this question correctly.



Extract 5.2: A correct response from one of the students

In Extract 5.2, the student prepared a table of values with correct entries and presented a correct graph for $y = x^2 - x - 3$ and $y = x$. Finally, the points of intersection were identified correctly.

2.6 Question 6: Symmetry

This question had parts (a) and (b). In part (a), the students were required to: (i) draw all lines of symmetry in an equilateral triangle and (ii) determine the number of lines of symmetry in an equilateral triangle. In part (b), the students were required to state whether (i) a circle and (ii) rhombus are symmetrical or not.

The analysis of data shows that 510 (98.8%) students attempted this question. Among them, 454 (76.8%) students scored from 3 to 10 marks. Therefore, the students' performance in this question was good. Figure 7 shows the percentage of students who got weak, average and good performance.

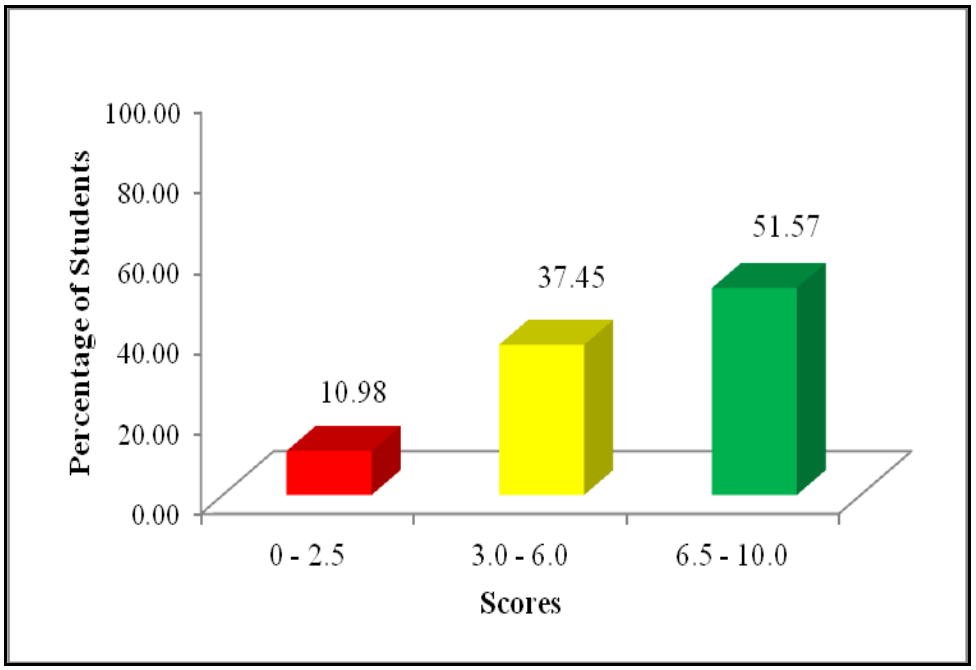
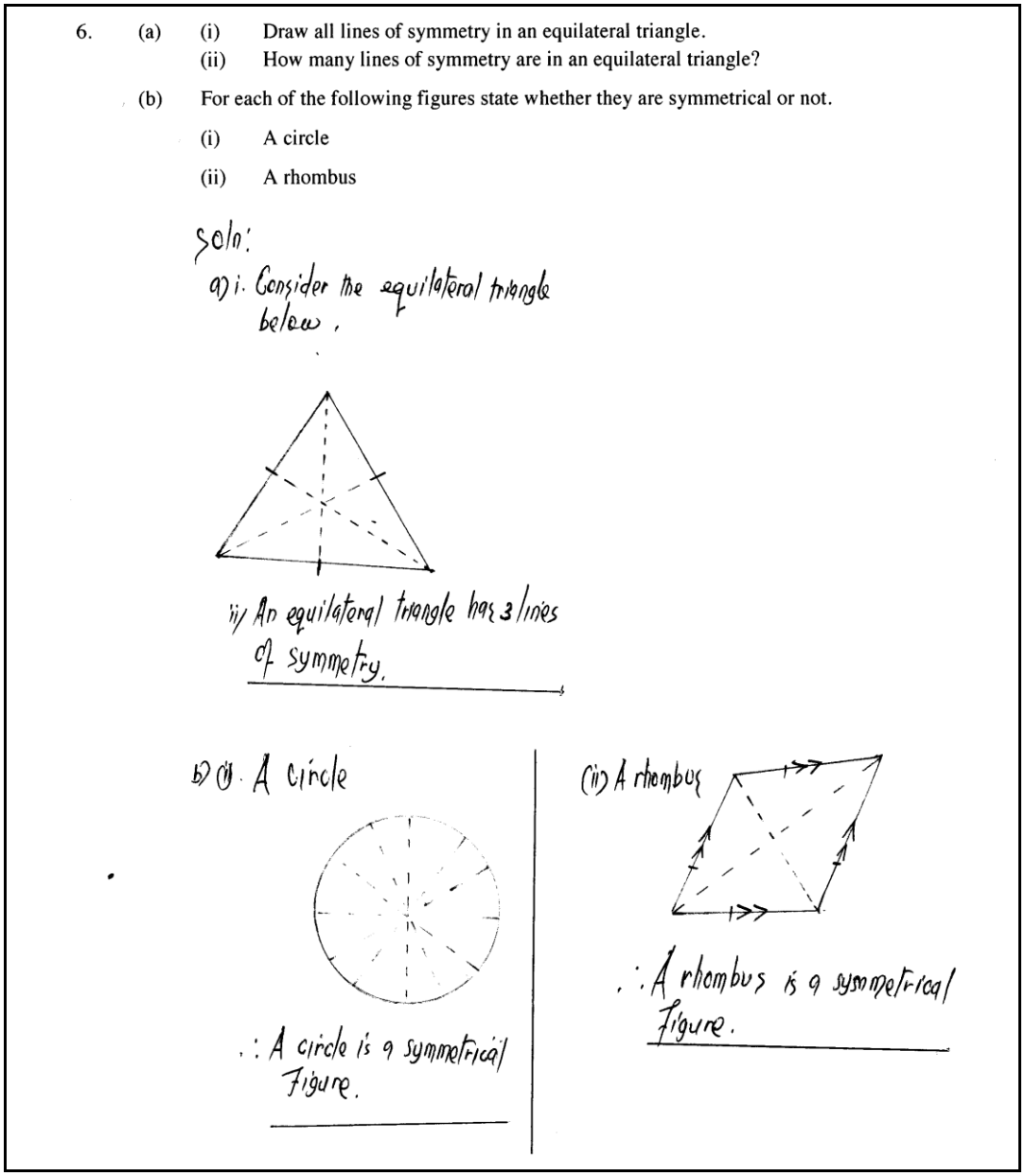


Figure 7: *Students' performance in question 6*

Figure 7 shows that 51.57 percent of students scored above 6 out of 10 marks whereby 83 (16.3%) students scored all 10 marks. The students who responded correctly to part (a) (i) of this question managed to recall the properties of an equilateral triangle and drew it as they recognized that it is the three equally sided polygon whose interior angles are 60° each. Thereafter, the students were able to draw three lines of symmetry that bisect the three angles of the equilateral triangle. In part (a) (ii), the students managed to state the number of lines of symmetry of the equilateral triangle as they were drawn in part (a) (i) and concluded that the equilateral triangle has three lines of symmetry. In part (b), the students demonstrated knowledge and skills of symmetrical figures as they were able to state

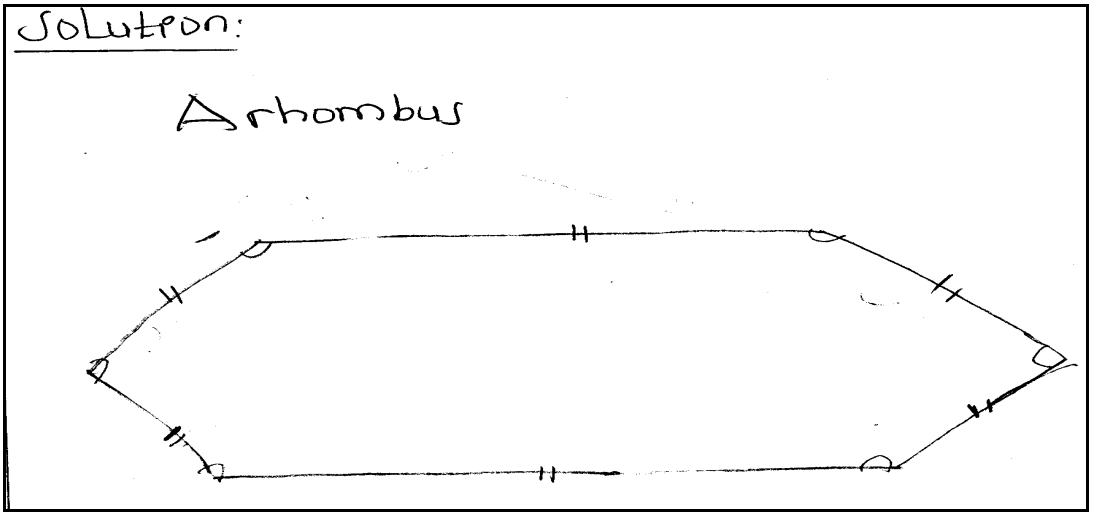
that a circle and rhombus are symmetrical figures with infinity and two lines of symmetry respectively. Extract 6.1 is a sample response from one of the students who answered correctly this question.



Extract 6.1: A correct response from one of the students

In Extract 6.1, the student was able to draw all lines of symmetry of an equilateral in part (a). In part (b), the student identified a circle and rhombus as symmetrical figures.

On the other hand, 10.98 percent of the students scored below 3 out of 10 marks allotted to this question. The students encountered the following challenges: in part (a), a number of students failed to abide by the demand of the question. For instance, one of the students drew only one line of symmetry from the single corner bisecting an angle and sides of an isosceles triangle instead of drawing the line of symmetry from either corner of an equilateral triangle bisecting it. There were also several students who failed to recall the properties of the equilateral triangle which resulted into their failure to draw the lines of symmetry. This implies that the students had inadequate knowledge on symmetrical figures. Moreover, a few students confused the equilateral triangle with other polygons such as the quadrilateral and isosceles triangle. In part (b) (i), some students failed to recall an approach that drawing lines from either sides bisecting 360^0 angle of a circle into two halves with 180^0 each could offer apparent reason that the circle is symmetrical with infinity numbers of lines of symmetry. As a result, they ended up stating that a circle is not symmetrical which is not correct. Likewise, in part (b) (ii), the students failed to recall and apply the approach that drawing lines from either side bisecting a rhombus into two equal parts could make a strong reason that a rhombus is symmetrical with two lines of symmetry. In addition, several students came out with an incorrect statement that a rhombus is symmetrical with four lines of symmetry. Extract 6.2 is a sample response from one of the students who sketched a wrong polygon in part (b) (ii) of this question.



Extract 6.2: An incorrect response from one of the students

In Extract 3.2, the student confused a rhombus with a six sided polygon. The student was supposed to understand that the rhombus is a parallelogram with four equal sides and sometimes one with no right angle.

2.7 Question 7: Logic

The question consisted of parts (a), (b) and (c). In part (a), the students were given the premises such that P stands for *Anna is the tallest girl in form two* and Q stands for *Anna is an intelligent girl in form two*. The students were required to write verbal statements for (i) $\sim P \wedge \sim Q$ and (ii) $P \leftrightarrow \sim P$. In part (b), the students were required to draw an electrical circuit for the statement $(p \wedge q) \vee r$. In part (c), the students were required to test the validity of $\sim p \rightarrow \sim q$ by using a truth table.

The analysis shows that 31.36 percent of the students who attempted this question scored below 3 out of 10 marks, 27.42 percent from 3.0 to 6.0 and 41.22 percent scored above 6 out of 10 marks. Generally, the students' performance in this question was good as 68.6 percent of the students got more than 3 marks. Figure 8 illustrates the students' performance in question 7.

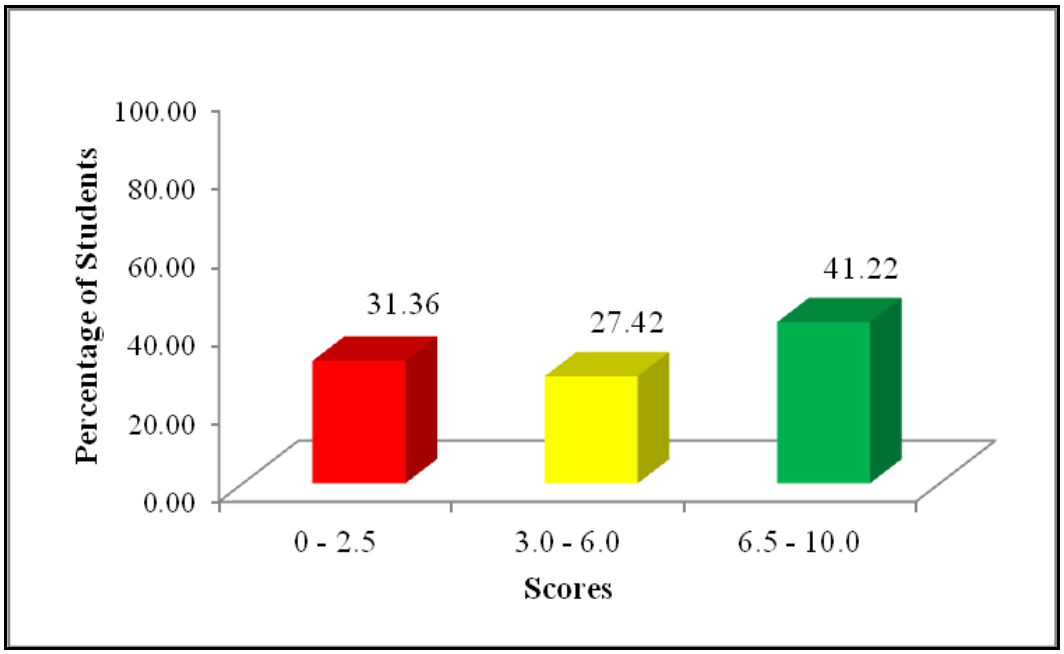


Figure 8: *Students' Performance in question 7*

The students, who answered the question correctly, had adequate knowledge on the concept tested. In part (a), the students made the correct interpretation of the logical connectives $\sim$, $\wedge$ and $\leftrightarrow$ which meant; *not, and* as well as *if and only if* respectively. Hence, they described the verbal statements for $\sim P \wedge \sim Q$ and $P \leftrightarrow \sim Q$ as (i) Anna is not the tallest and not an intelligent girl in form two/ Anna is neither the tallest nor intelligent girl in form two and (ii) Anna is the tallest girl if

and only if she is not an intelligent girl in form two. In part (b), the students were able to draw the series connection of the statements p and q in parallel connection with the statement r to form an electrical circuit representing the compound statement $(p \wedge q) \vee r$. In part (c), the students constructed a truth table for $[(p \rightarrow q) \wedge (r \vee \sim q) \wedge \sim r] \rightarrow \sim p$ with columns p, q $\sim p$, $\sim q$ and $\sim p \rightarrow \sim q$ and were able to conclude that the statement $\sim p \rightarrow \sim q$ is not valid since the last column on the right contains the truth values T and F. Extract 7.1 is a sample of a good response from one of the students.

7. (a) If P stands for "Anna is the tallest girl in Form Two." and Q stands for "Anna is an intelligent girl in Form Two."; write a verbal statement for each of the following :

(i) $\sim P \wedge \sim Q$.

(ii) $P \leftrightarrow \sim Q$.

(b) Draw an electrical circuit for the statement $(p \wedge q) \vee r$.

(c) Use truth table to test the validity of $\sim p \rightarrow \sim q$.

Solution. 7(a)

P - Anna is the tallest girl in form two.

Q - Anna is an intelligent girl in form two.

(i) $\sim P \wedge \sim Q$

Anna is neither the tallest nor intelligent girl in form two

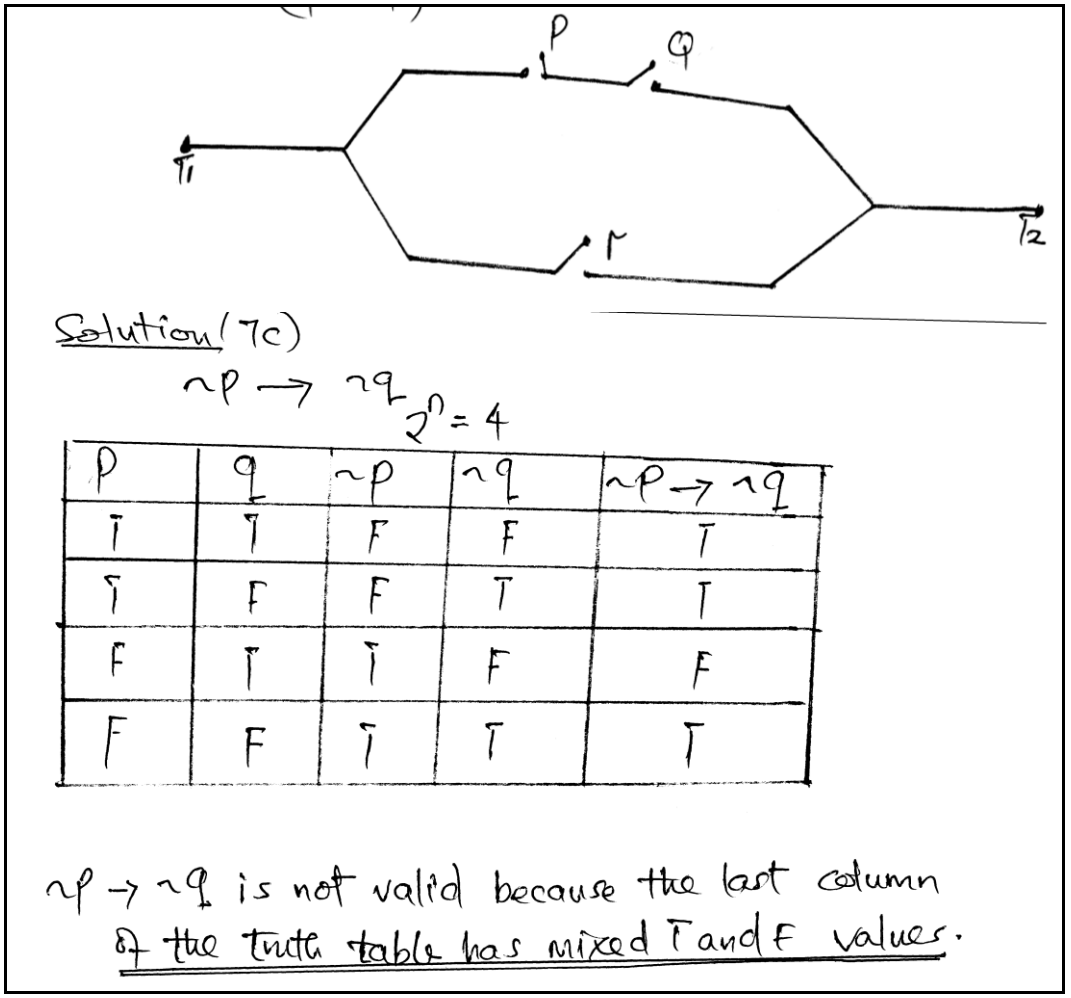
(ii) $P \leftrightarrow \sim Q$

Anna is the tallest girl if only she is an intelligent girl in form two.

b

(b) Solution.

$(P \wedge Q) \vee r$

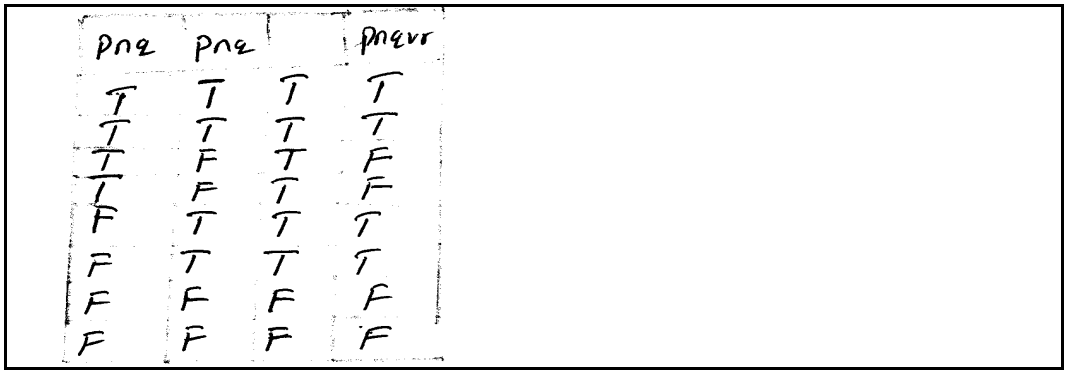


Extract 7.1: A correct response from one of the students

In Extract 7.1, the students wrote correctly the verbal representation of the given statements in part (a). The student was able to draw an electrical circuit containing three propositions in series and parallel connection in part (b). The student was able to test the validity of the given statement in part (c).

Despite the good performance, nearly one third of the students (31.36%) failed to answer the question accordingly. In part (a), the students confused writing the verbal statement with testing the validity of a logical statement. For instance, one of the students tested the validity of the given statements as both $\sim P \wedge \sim Q$ and $P \leftrightarrow \sim Q$ were valid. The analysis also reveals that some students who answered part (b) incorrectly failed to connect the switches *p*, *q* and r in order to construct a network circuit. The analysis shows further that a number of students drew the electrical circuits correctly but failed to label them accordingly. In part (c), several students

failed to use truth tables for testing the validity of $\sim p \rightarrow \sim q$. Such students could not identify the number of rows in the truth table for a compound statement involving two statements. The students should have known that the number of rows in the truth table depends on the number of different components in the statement and can be written as 2^n, where *n* is the number of the components forming the statement. In this case, the number of rows was 4. Extract 7.2 is a sample of responses from the student who got part (c) of this question wrong.



Extract 7.2: An incorrect response from one of the students

In Extract 7.2, the student constructed a truth table with 8 instead of 2 rows.

2.8 Question 8: Variations

The question had parts (a) and (b). In part (a), the students were given the following statement: The variable x and y are directly proportional to each other. If $x = 3$ and $y = 12$, thereafter they were asked to find the equation relating x and y. In part (b), the students were given that the variable r is directly proportional to t, and r is 6 when t is 18, and were required to find r when t is 24.

The question was attempted by 507 (98.3%) students, out of whom 61.45 percent scored from 6.5 to 10 marks and 0.4 percent scored all 10 marks. The analysis shows further that 17.87 percent of the students had average performance; their scores ranged from 3.0 to 6.0 marks and 20.68 percent scored from 0 to 2.5 marks. Generally, the students' performance in this question was good, as Figure 9 shows.

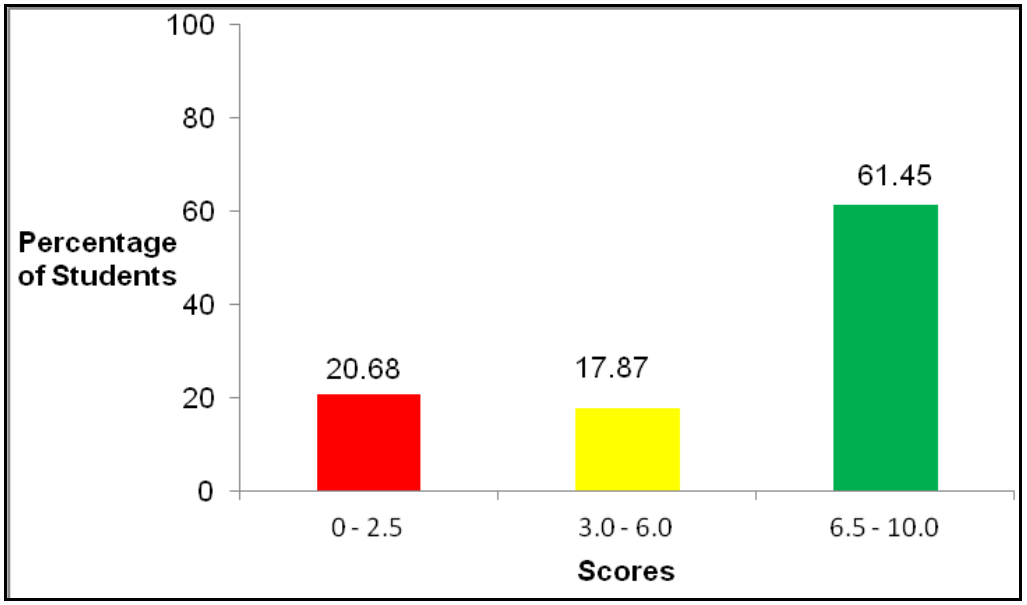


Figure 9: *Students' performance in question 8*

The students who answered part (a) correctly were able to: express the statement *the variable* x *and* y *are directly proportional to each other* as a mathematical equation $x = ky$; substitute $x = 3$ and $y = 12$ in the equation $x = ky$ to get $k = \frac{1}{4}$ and hence substitute the value of k in equation $x = ky$ to get $x = \frac{1}{4}y$. Finally, they got $y = 4x$. Alternatively, other students transformed the given statement as $y = kx$; substituted $x = 3$ and $y = 12$ into $y = kx$ to get $k = 4$ and substituted the value of k into $y = kx$ to get $y = 4x$ which was the required equation.

In part (b), the students expressed the statement; *the variable* r *is directly proportion to* t as $r = kt$ and substituted $r = 6$ and $t = 18$ to obtain the value of k *as* $\frac{1}{3}$. Thereafter, they substituted the value of k into $r = kt$ in order to get an expression $r = \frac{t}{3}$ whereby $t = 24$. Finally they simplified $r = \frac{24}{3}$ to get $r = 8$. This shows that, they had sufficient knowledge on the concept of variation. Extract 8.1 is a sample response from one of the students who answered correctly this question.

8. (a) The variables x and y are direct proportional to each other. If $x = 3$ when $y = 12$, find the equation relating x and y.

(b) If r is directly proportional to t, and r is 6 when t is 18, find r when t is 24.

Soln:

a) $x \propto y$

$x = ky$

∴ The equation is $x = ky$ where $x = 3, y = 12$

b) $x = ky$

$3 = k \times 12$

$k = 3/12$

$k = 1/4$

From

$x = ky$

$x = (1/4)y$

$(x = y/4)4$

$4x = y$

∴ The equation is $4x - y = 0$.

8. b). $r \propto t$, $r = 6, t = 18$.

$r = kt$

$6 = 18k$

$k = 6/18$

$k = 1/3$,

$r = ?, k = 1/3$ and $t = 24$

from

$r = kt$

$r = 1/3 \times 24$

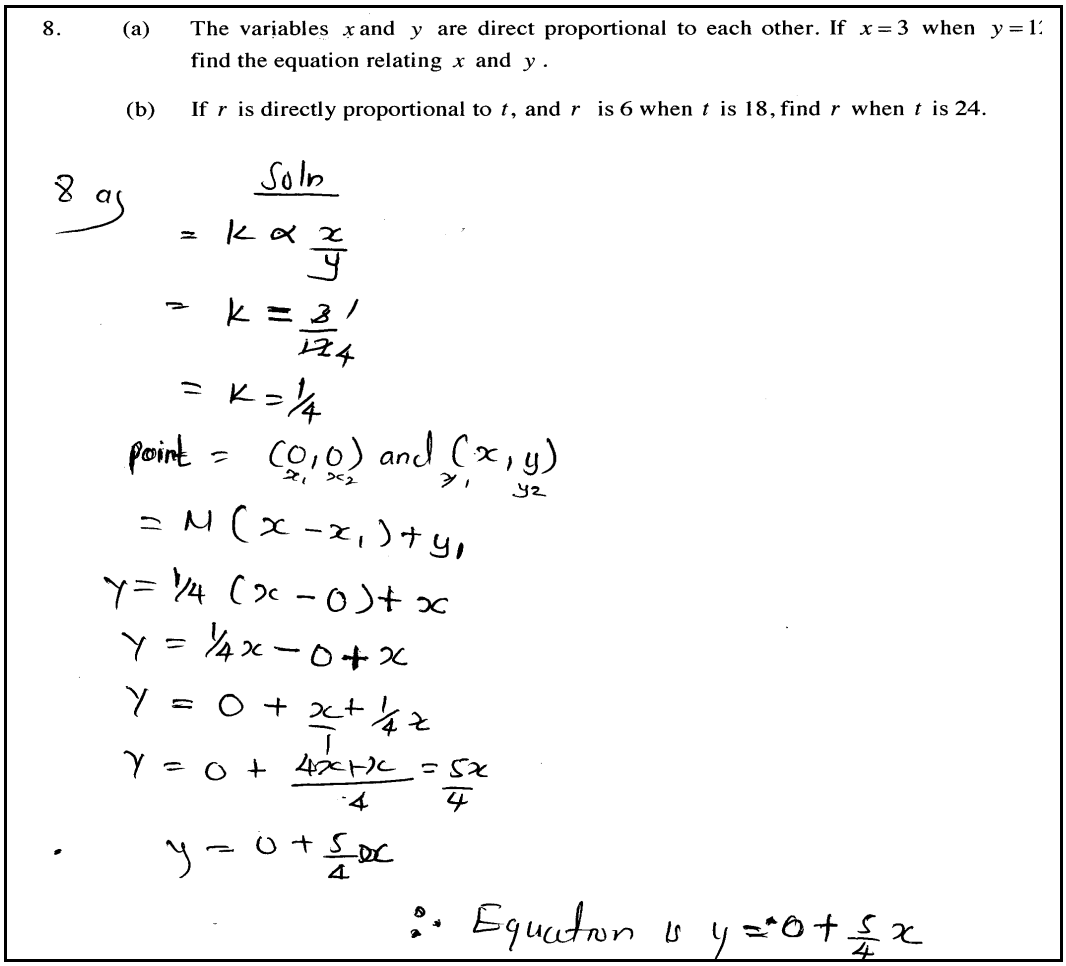
$r = 8$.

∴ The value of r is 8

Extract 8.1: A correct response from one of the students

In Extract 8.1, the student demonstrated understanding on the concepts of variation. The student's work was also well presented and this made it easier for the assessor to mark.

However, 20.68 percent of the students failed to respond to the question accordingly. In part (a), the analysis shows that some of the students were able to transform the statement; *the variables* x *and* y *are direct proportional to each other* mathematically as $x = ky$, but could not go further to find the value of k. Other students formulated incorrect mathematical equations such as $x = y$, $y = 3x$ and $12y = 3$. This shows that they had insufficient knowledge on the concept of variation. In part (b), the analysis shows that several students could not write the equation representing the statement; *the variable* r *is directly proportional to* t. For instance, one of the students wrote the meaningless equation $t + r + t = 6$ instead of $r = kt$. Extract 8.2 is a sample of a response from one of the students who got the question wrong


8. (a) The variables x and y are direct proportional to each other. If $x = 3$ when $y = 1$ find the equation relating x and y.

(b) If r is directly proportional to t, and r is 6 when t is 18, find r when t is 24.

8 a) Soln

$= k \propto \frac{x}{y}$

$= k = \frac{3}{12}/4$

$= k = 1/4$

point = $(0,0)$ and (x,y), $x_1\ x_2$, $y_1\ y_2$

$= M(x - x_1) + y_1$

$y = 1/4\,(x - 0) + x$

$y = 1/4x - 0 + x$

$y = 0 + \frac{x}{1} + 1/4x$

$y = 0 + \frac{4x + x}{4} = \frac{5x}{4}$

$y = 0 + \frac{5}{4}x$

∴ Equation is $y = 0 + \frac{5}{4}x$

Extract 8.2*:* An incorrect response from one of the students

In Extract 8.2, the students failed to interpret the direct variation problem into the required equation.

2.9 Question 9: Sets

The question was as follows; In a class of 105 students, 10 study English and Geography, 8 study History and Geography, 20 study English and History and 5 study all the three subjects. If the number of students studying English only, Geography only and History only are 23, 17 and 27 respectively, (a) show this information on a Venn diagram and (b) determine the number of students who are taking neither of the three subjects.

The analysis of data shows that 19.33 percent of the students who attempted this question scored 2.5 marks or less, 51.48 percent from 3.0 to 6.0 marks and 29.19 percent from 6.5 to 10 marks. Generally, the students' performance in this question was good, as 80.67 percent of the students got 3 marks and above. Figure 10 illustrates the students' performance in this question.

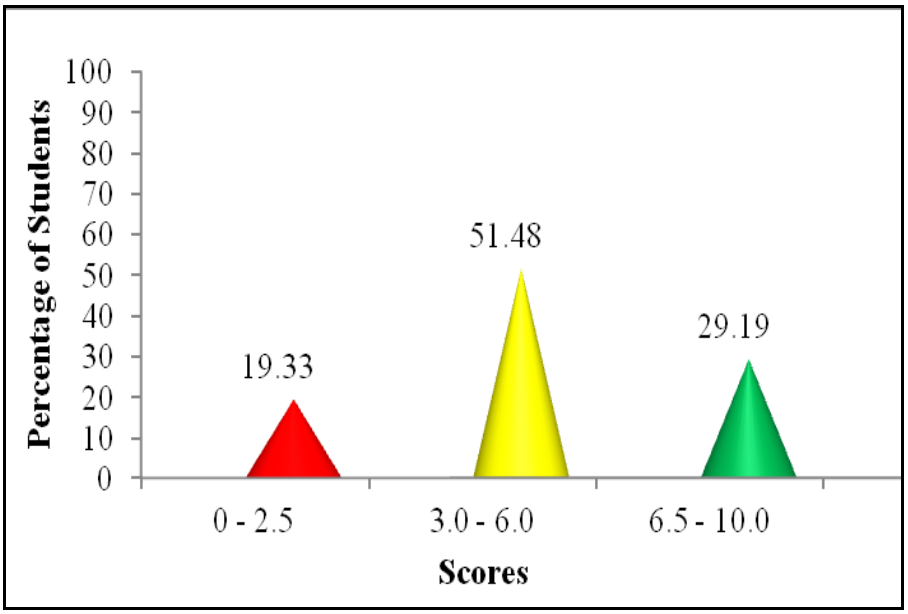


Figure 10: *Students' performance in question 9*

The students who responded to question 9 correctly demonstrated the following competences: in part (a), they were able to represent the given word problem by using Venn diagram as follows: indicating the number of students taking all three as 5, indicating the number of students taking only English, Geography and History as 23, 17 and 27 respectively; indicating the number of students taking English and Geography only, History and Geography only, English and History only by subtracting the number of students taking all subjects from 10, 8 and 20 to get 5, 3 and 15 respectively. By using Venn diagram in part (a), the students were able to

find $105-(27+17+23+3+5+15+5)$ giving 10 as the number of students who are taking neither of the three subjects in part (b). Extract 9.1 is a sample response from one of the students illustrating how the students managed to answer the question correctly.

9. In a class of 105 students, 10 study English and Geography, 8 study History and Geography, 20 study English and History and 5 study all the three subjects. If the number of students studying English only, Geography only and History only are 23, 17 and 27 respectively.

(a) Show this information on a Venn diagram.

(b) How many students are studying neither of the three subjects.

Soln:

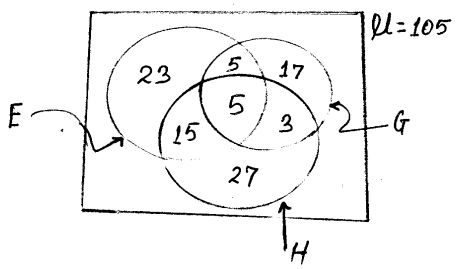
let E - English, G - Geography, H - History

$n(\mathcal{U}) = 105$, $n(E \cap G) = 10$, $n(H \cap G) = 8$

$n(E \cap H) = 20$, $n(E \cap G \cap H) = 5$,

$n(E)_{only} = 23$, $n(G)_{only} = 17$, $n(H)_{only} = 27$

a) The Venn diagram.



b) $n(E \cup G \cup H)' = n(\mathcal{U}) - n(E \cup G \cup H)$

$= 105 - (23+5+17+3+5+15+27)$

$= 105 - 95$

$= 105 - 95$

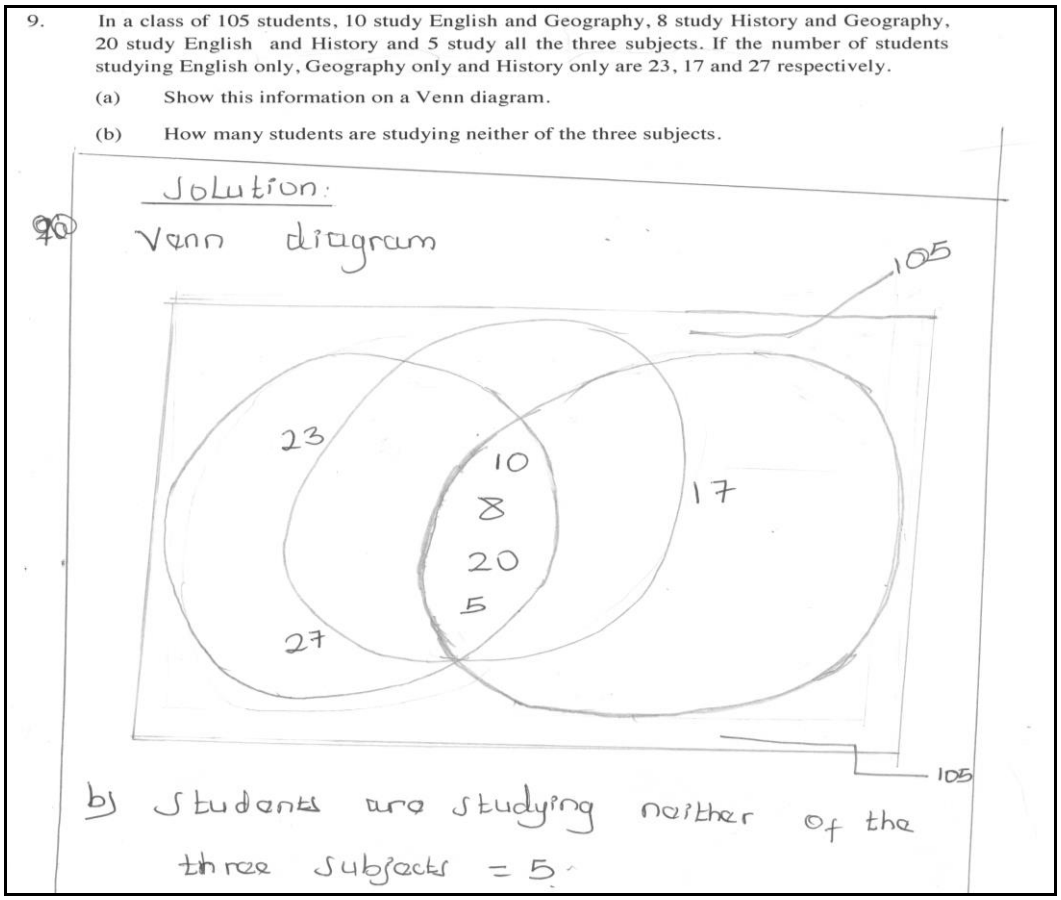
$n(E \cup G \cup H)' = 10$

∴ 10 students study neither of the three subjects

Extract 9.1: A correct response from one of the students

In Extract 9.1, the students represented the given word by using a Venn diagram in part (a). In part (b), the student subtracted the number of students taking at least one subject from the number of students in a class to get the number of students taking none of the three subjects.

In spite of the students' good performance, 98 (19.33%) students scored low marks. In part (a), some of them managed to draw the Venn diagram correctly, but failed to represent the given information by using a Venn diagram. Other students considered the number of students taking all subjects to be x hence indicated $10-x$, $8-x$ and $20-x$ on the Venn diagram as the number of students taking English and Geography only, History and Geography only, English and History only respectively. In addition, there were several students who completely lacked the skills of presenting the given information on Venn diagram. Such students drew incomprehensible diagrams like the one shown in Extract 9.2. In part (b), the analysis of the students' responses shows that the mistakes which were committed in part (a) resulted into incorrect calculations of the number of students who are taking neither of the three subjects. The analysis also shows that there were students who employed inappropriate formulae in their calculations such as $n(E \cup G \cup H)' = n(E \cup G \cup H) - n(\varepsilon)$.



Extract 9.2: An incorrect response from one of the students

In Extract 9.2, the student lacked knowledge and skills on the topic of Sets. As a result, the student drew the diagram which was not related to the demands of the question.

2.10 Question 10: Algebra

This question required the students to: (a) simplify the expression $\frac{a+1}{3}-\frac{(2a+1)}{4}$; (b), solve for c if $\frac{2}{c-1}+\frac{3}{c+1}=\frac{5}{c}$ and (c) solve the simultaneous equations $\begin{cases} c^2+d=9 \\ d+6=2c \end{cases}$.

The analysis shows that almost half (48.93%) of the students scored 3.0 marks and above. Among them, 26.60 percent scored from 3.0 to 6.0 marks. This indicates that the students' performance was average. Figure 11 is a summary of the students' performance in this question.

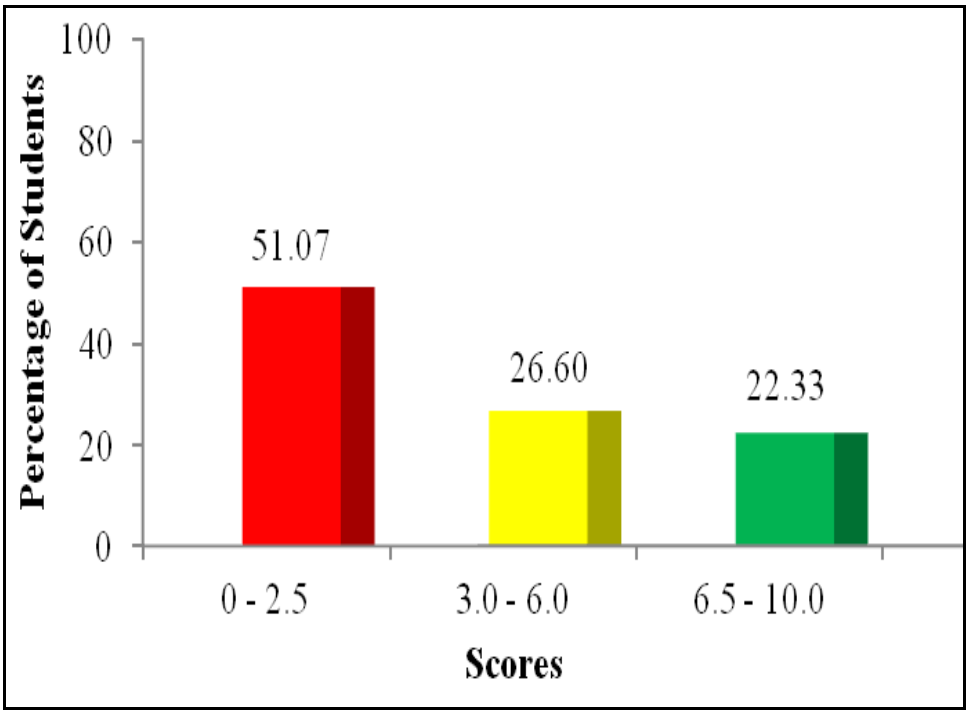
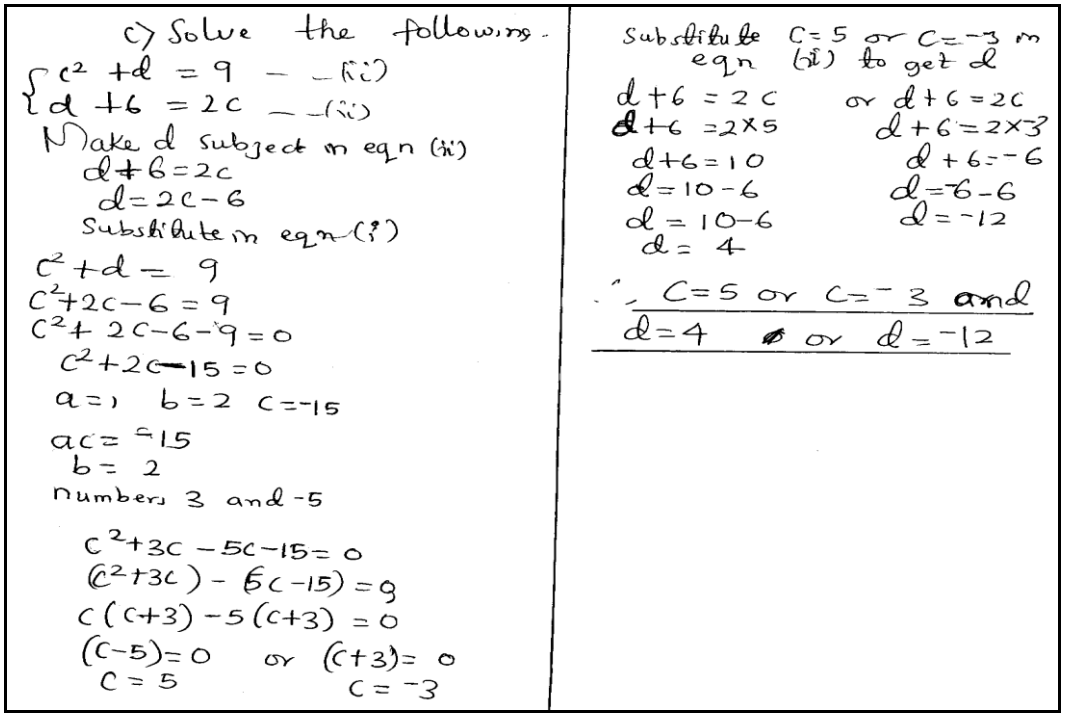


Figure 11: *Students' performance in question 10*

As observed in Figure 11, most of the students (51.07%) got less than 3 out of 10 marks. In part (a), most of these students transformed correctly the given expression into $\frac{4\ a+1\ -3\ 2a+1}{12}$ but failed to apply the BODMAS rule on $4\ a+1\ -3\ 2a+1$ to obtain the simplified expression $\frac{1-2a}{12}$. In part (b), the

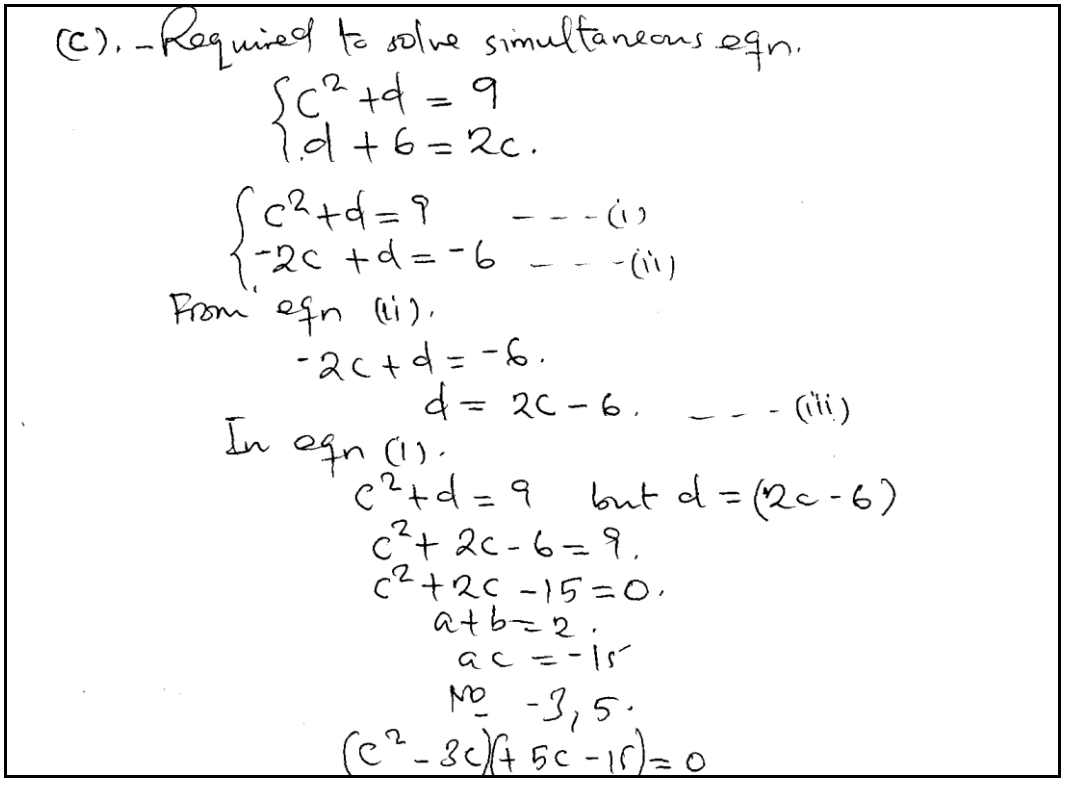
students got the incorrect values of c because they had insufficient skills in performing basic operations on fractions. For examples, one of the students multiplied the numerators 2, 3 and 5 of the fractions in the equation $\frac{2}{c-1}+\frac{3}{c+1}=\frac{5}{c}$ that is $2\times3\times5$ to get 30. Then, they multiplied by 30 on both sides of the given equation to get $c=\frac{-5}{31}$ instead of $c=-1$. Other students solved the given equation by equating the sum of the reciprocals of the fractions $\frac{2}{c-1}$ and $\frac{3}{c+1}$ to the reciprocal of the fraction $\frac{5}{c}$ to get $c=\frac{5}{19}$. In part (c), the students failed to solve the given pair of equations using the method of substitution or elimination to get the values of c and d. However, several students manipulated the given simultaneous equations correctly to get the quadratic equation but failed to solve it to get the required values of c and hence were unable to get the values of d. Extract 10.1, is a sample of a poor response from one of the students showing how the students failed to answer part (c) of this question correctly.



Extract 10.1: An incorrect response from one of the students

In Extract 10.1, the student failed to identify that when 3 is added to -5 the result is -2 and not 2. He/she got the incorrect values of c.

Despite the poor performance of the students as shown in Figure 11, 22.33 percent of the students scored above 6 out 10 marks. In part (a), they were able to: find the LCM of the denominators 3 and 4 to get 12; thereafter, they manipulated the given expression to obtain $\frac{4(a+1)-3(2a+1)}{12}$ which was finally simplified into $\frac{1-2a}{12}$. In part (b), the students had adequate knowledge and skills on algebra as they were able to; find the LCM $c\ c-1\ \ c+1$ of the denominators in the given equation. Then, they multiplied the LCM $c\ c-1\ \ c+1$ on both sides of the given equation to get $\frac{2(c+1)+3(c-1)}{(c-1)(c+1)}=\frac{5}{c}$ and lastly simplified the resulting equation to get $c=5$. In part (c), the analysis of the students' responses shows that a number of students applied correctly the substitution method to manoeuvre the given pair of simultaneous equations into the quadratic equation $c^2+2c-15=0$ which was solved to get $c=3$ or $c=-5$. Further analysis shows that, some students resorted to eliminate the variables in the equations $c^2+d=9$ and $d+6=2c$ to get $c^2+2c-15=0$ which was solved to get the values of *c*. They finally substituted the values of *c* into one of the given equations to find $d=0$ or $d=-16$. Extract 10.2 is a sample of a good response from one of the students.


(c). - Required to solve simultaneous eqn.

$\begin{cases} c^2+d=9 \\ d+6=2c. \end{cases}$

$\begin{cases} c^2+d=9 & ---(i) \\ -2c+d=-6 & ---(ii) \end{cases}$

From eqn (ii).

$-2c+d=-6.$

$d=2c-6. \quad ---(iii)$

In eqn (i).

$c^2+d=9 \quad$ but $d=(2c-6)$

$c^2+2c-6=9.$

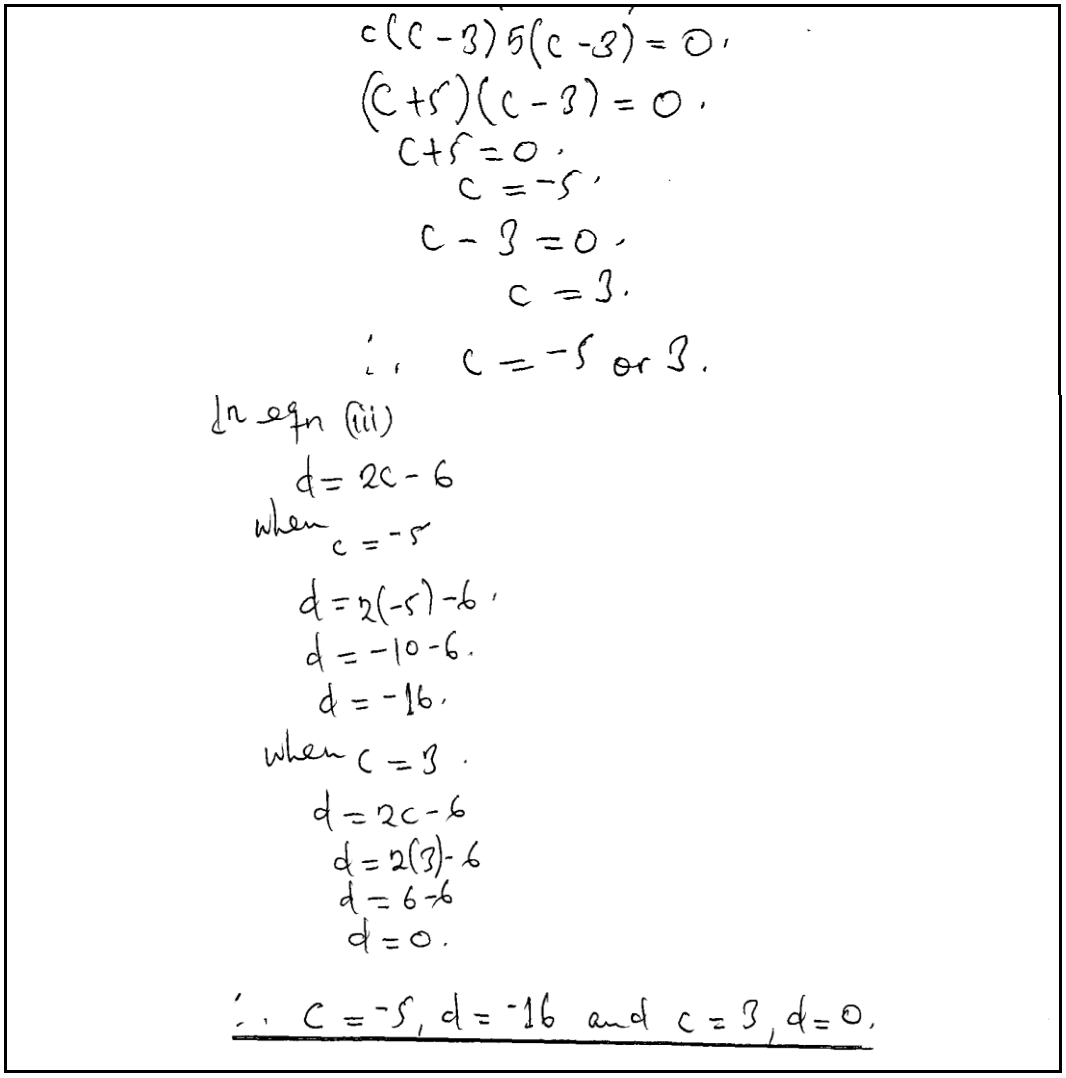
$c^2+2c-15=0.$

$a+b=2.$

$ac=-15$

No $-3, 5.$

$(c^2-3c)(+5c-15)=0$



Extract 10.2: A correct response from one of the students

In Extract 10.2, the student was able to solve a linear equation and a quadratic equation simultaneously.

3.0 ANALYSIS OF STUDENTS' PERFORMANCE IN EACH TOPIC

The assessed topics in FTNA 2020 were *Symmetry, Plan and Elevations, Sets, Variations, Geometrical Constructions, Numbers, Logic, Algebra, Locus and Coordinate Geometry*.

The analysis indicates that the students' performance was good in six (06) topics. These topics are: *Symmetry* (89.0%), *Sets* (80.7%), *Variations* (79.3%), *Geometrical Constructions* (76.8%), *Numbers* (74.3%) and *Logic* (68.6%). The students' good

performance in these topics was due to students' ability to: draw lines of symmetry; identify various types of polygons as symmetrical figures; solve word problems involving three sets using Venn diagrams and solve problems involving direct variations.

On the other hand, the students' performance in three (3) topics was average. The topics under this category are: *Algebra* (62.3%), *Locus* (48.6%) and *Coordinate Geometry* (44.5%). These topics were averagely performed because of the presence of many students who got less than 30 percent of the marks which were allocated to the questions. Other reasons include the students' inability to: solve a linear and a quadratic equation simultaneously; determine the locus of a point moving at equal distance from the fixed points and solve simultaneous equations graphically. The analysis of the students' performance for each topic is presented in Appendix I.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The Students' Items Response Analysis (SIRA) report has been specifically designed to provide the awareness to stakeholders about the students' responses in FTNA 2020. The focus of the analysis was therefore to identify the strengths and weaknesses of the students' responses in various items. The SIRA report in Additional Mathematics 2020 shows that 75.97 percent of the students passed the assessment, in comparison with 60.34 percent who passed the assessment in 2019.

The analysis shows that the students' performance was good in 6 topics and average in 3 topics. The analysis of data for 2019 reveals that, 3 topics had good performance, 5 topics had average performance and 1 topic was poorly performed. The comparison of the students' performance in each topic for two consecutive years is shown in appendix II.

Primarily, the report has revealed the main areas where the students had good, average or poor performance. It is expected that the stakeholders will use the recommendations suggested in this report to improve the performance of students in future Additional Mathematics assessments.

4.2 Recommendations

In order to improve the students' performance in this subject it is recommended that the teachers should;

(a) lead students to discuss on how to find the locus of a point from the fixed points.

(b) lead students to discuss on how to solve a pair of simultaneous equations involving linear and quadratic equations.

(c) encourage students to establish subject clubs with the aim of discussing the topics which were averagely performed.

(d) provide adequate number of exercises to enable the students to have skills of solving simultaneous equations graphically.

Appendix I

Analysis of Students' Performance Per Topic in Additional Mathematics

S/N	Topic	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1	Symmetry	6	89.0	Good
2	Sets	9	80.7	Good
3	Variations	8	79.3	Good
4	Geometrical Constructions	3	76.8	Good
5	Numbers	1	74.3	Good
6	Logic	7	68.6	Good
7	Algebra	2 and 10	62.3	Average
8	Locus	4	48.6	Average
9	Coordinate Geometry	5	44.5	Average

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Appendix II

Comparison of the students' performance in each topic for the 2019 and 2020 Additional Mathematics assessments

S/N	Topic	2019			2020		
		Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1	Symmetry	6	88.8	Good	6	89.0	Good
2	Sets	9	64.7	Good	9	80.7	Good
3	Variations	8	66.6	Good	8	79.3	Good
4	Geometrical Constructions	3	37.1	Average	3	76.8	Good
5	Numbers	1	58.2	Average	1	74.3	Good
6	Logic	7	63.1	Average	7	68.6	Good
7	Algebra	2	53.5	Average	2 & 10	62.3	Average
8	Locus	4	24.7	Weak	4	48.6	Average
9	Coordinate Geometry	5 & 10	54.9	Average	5	44.5	Average