THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEMS RESPONSE ANALYSIS REPORT FOR THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2017

041 BASIC MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is pleased to issue the report on Students' Item Response Analysis (SIRA) in Basic Mathematics for the Form Two National Assessment (FTNA) 2017, in order to inform teachers, parents, policy makers and other education stakeholders, on how the students responded to the assessment items. The report will enable the stakeholders to understand topics which need more emphasis in teaching and learning in order to take appropriate measures to improve the performance of the students.

The analysis of the students' responses was done in order to identify the areas in which the students faced problems, did well or averagely. Basically, the report highlights the factors for good or poor performance in order to understand what the education system managed or was unable to offer to the learners in their two years of Secondary Education.

The performance in this assessment was generally poor. The factors noted for the poor performance include lack of knowledge and skills on the assessed topics, inability to use concepts/formula/laws correctly, failure of the students to identify the demands of the questions and lack of skills to comprehend word problems mathematically or diagrammatically. Extracts of the students' responses from the scripts are used in this report to illustrate the reasons behind the poor or good performance. These extracts can be used as a guide by teachers and students.

The National Examinations Council of Tanzania believes that education stakeholders will work on the challenges faced by students while attempting assessment questions in order to take appropriate measures to improve the performance in this subject.

The National Examinations Council of Tanzania will highly appreciate comments and suggestions from teachers, students and the public in general that can highlight any area for improvement in writing future Students' Item Response Analysis reports.

Finally, the Council would like to thank all the Examination Officers and others who participated in the preparation of this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of the students' performance in 041 Basic Mathematics assessment for Form Two National Assessment (FTNA 2017). The analysis highlights the strengths and weaknesses that were observed when the students were answering the questions in order to provide a general overview of the students' performance.

The paper comprised of 10 compulsory questions. Each question carried 10 marks. The questions were set basing on the 2005 Form I and II Basic Mathematics syllabi.

A total of 485,494 students sat for the 041 Basic Mathematics assessment, out of which 155,092 (32.00%) students got different grades as shown in Figure 1.

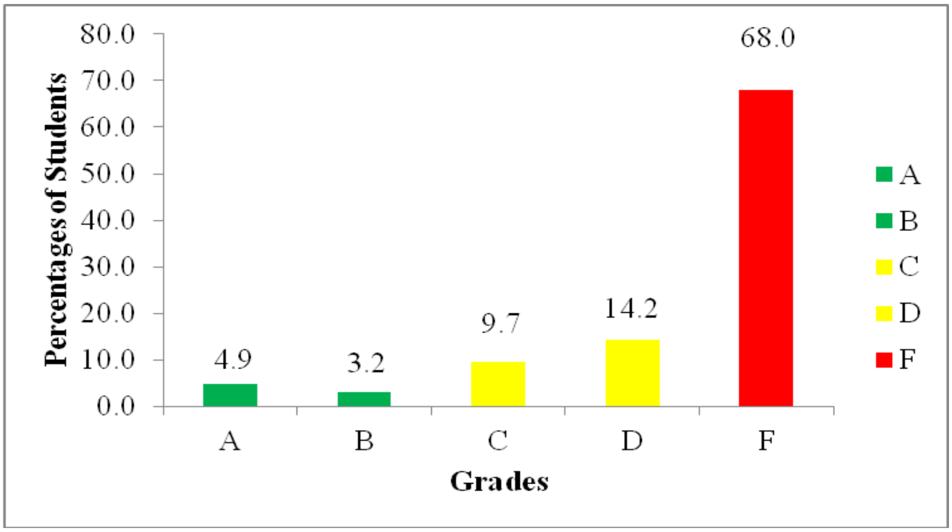


Figure 1: Students' Grades in FTNA 2017

This performance had an increase of 10.45 percent when compared to the FTNA 2016 assessment results, whereby 21.55 percent of 408,191 students who sat for the assessment got different grades, as shown in Figure 2.

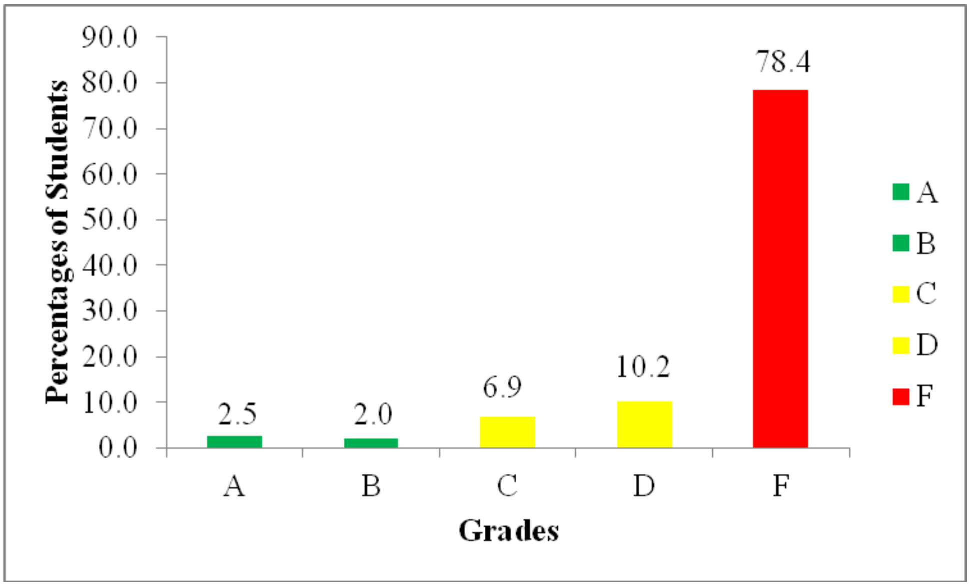


Figure 2: Students' Grades in FTNA 2016

The analysis of the students' performance in each question is presented in section 2.0. It consists of a short description of the requirements of the questions and the analysis on how students responded to questions. Extracts of well performed and poorly done questions are included in the analysis of a particular question. The factors that accounted for good or poor performance in each question have been indicated and illustrated using samples of students' responses. Therefore, the analysis of each question could be used as a practical guide to teachers and students to improve teaching and learning, and eventually students' performance.

The analysis of the students' performance in the topics assessed is shown in appendix I whereby green, yellow and red colours represent good, average and poor performance respectively. Finally, the recommendations are included at the end of this report to help students, teachers and the government to improve students' performance in future Basic Mathematics assessment.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE IN EACH QUESTION

This section provides the analysis of the students' performance in each question. The performance in each question was considered as good; average or weak if the percentage of students who scored 30 percent or more of the marks assigned for the question was in the intervals 65 – 100, 30 – 64 and 0 – 29 respectively.

2.1 Question 1: Numbers

In this question, the students were required to: (a) find the L.C.M and G.C.F of 13, 52 and 104 and (b) round off the number 568,356 to the nearest thousands and ten thousands.

The question was attempted by all students of which 68.71 percent scored from 3 to 10 marks and among them 8.96 per cent scored all the 10 marks. The data shows that question 1 was the best performed question in this assessment. Figure 3 represents the data graphically.

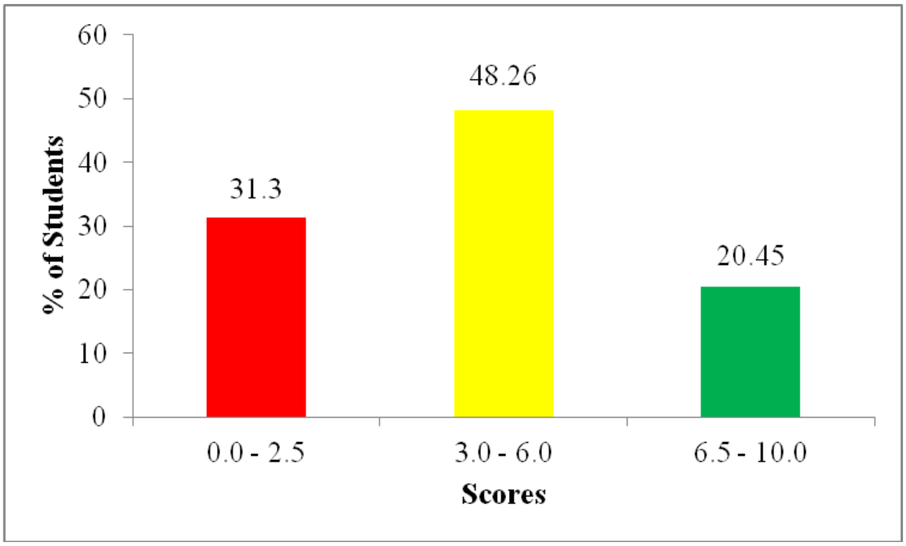


Figure 3: Students' Performance

As indicated in Figure 3, the majority of students had average performance. A few students scored full marks. Most of them employed the prime factorization method or the listing method to find the great common factor and least common multiple of 13, 52 and 104 in part (a). In finding the

GCF, they managed to identify the largest number that could divide 13, 52 and 104 which was 13. Also, in finding the LCM, most students realised that they had to use the fact that LCM is the smallest number that 13, 52 and 104 will divide into. Thus, 104 was the smallest number that contains 13, 52 and 104 as factors.

In part (b), they were able to recall and use correctly the round off procedures of numbers. Such round off procedures were identifying the digit to be rounded; leaving the digit the same if the next digit to it is less than 5 and increasing the digit by 1 if the next digit to it is greater or equal to 5. Therefore to round off 568,356 to the nearest thousands they kept 8 as it is in the 1000s position. Then, they did not change it because the next digit is 3 which is less than 5 and hence they got 568,000. It was also noted that the students were able to round off the same number to the nearest ten thousands as 570,000 because the digit next to 6 is 8 and is greater than 5, see Extract 1.1.

Extract 1.1

1. (a) Find the LCM and GCF of 13, 52 and 104.

Solution.

2	13	52	104
2	13	26	52
2	13	13	26
13	13	13	13
	1	1	1

LCM = 2 x 2 x 2 x 13

= 104.

GCF = 13.

∴ The LCM = 104 and GCF = 13.

(b) Round off the number 568,356 to the nearest thousands and ten thousands.

Solution.

568356

6 — ones

5 — tens

3 — hundreds

8 — thousands

6 — ten thousands

5 — hundred thousands

Round off to thousands

568356 = 568,000

Round off to ten thousands.

568356 = 570,000

∴ Round off the number to thousands is 568,000 and ten thousands is 570,000.

Extract 1.1 shows how well the student answered part (a) showing good understanding of the concepts of the great common factor, least common multiple and rounding off techniques.

In spite of the good performance, several students scored low marks. These students confused the Lowest Common Multiple (LCM) with the Greatest Common Factor (GCF) and vice versa in part (a). Such confusion resulted into failure to find the correct values of the LCM and GCF. They ended up getting the LCM as 13 and GCF as 104 instead of 104 and 13 respectively. Others lacked skills on how to find the prime factorisation for each of the numbers that were given to them because they were unable to break down the given numbers to the primes by the division method.

In part (b), several students failed to round off 568,356 to the nearest thousands and ten thousands. The analysis of the students' responses revealed that there were students who simply indicated the place value. Thus, answers such as thousands is a place value of 8, ten thousands is a place value of 6 were frequently seen in their work. Others completely lacked the round off techniques of numbers. For example, some students rounded off 568,356 to the nearest thousands as 5,683 while others rounded it as 9,000 contrary to the requirements of the question. Extract 1.2 is a sample answer from the script of one of the students showing how they failed to answer this question.

Extract 1.2

568,356 to thousands

= 9,000

568,356 to Ten thousands

= 70,000

Extract 1.2 shows how the student failed to round-off the given number into thousands and ten thousands as per question demand.

2.2 Question 2: Fractions and Decimals

The question comprised of parts (a) and (b) as follows:

(a) Determine the improper fraction of $\frac{3}{5}\times 4\frac{1}{5}\div\frac{18}{25}$.

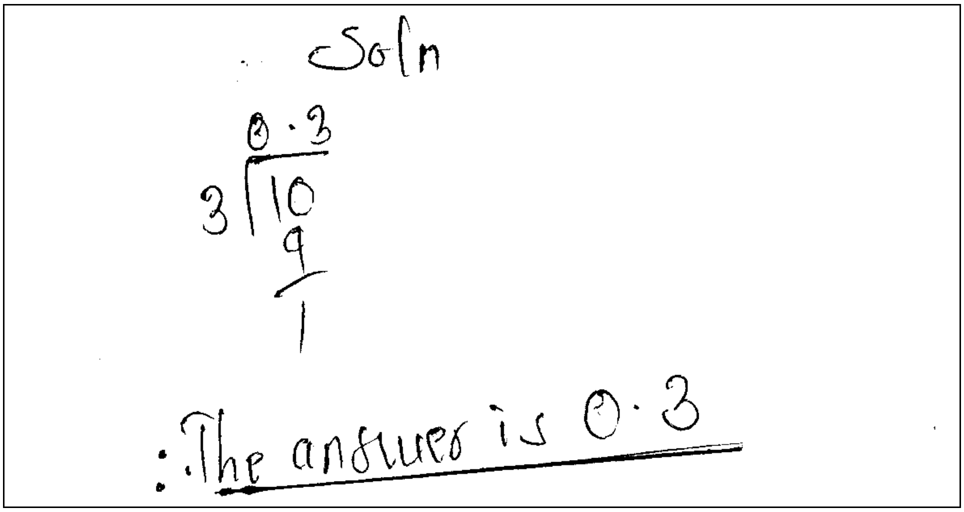
(b) Convert $\frac{1}{3}$ into a repeating decimal.

Question 2 was attempted by 486,083 students of which half of them (50.07%) scored from 3 to 10 marks, indicating an average performance. In this question, 44.2 percent of the students who attempted it scored a 0 mark. These students were unable to multiply and divide the given fractions in part (a). It was observed that some students could not find the required fraction often due to computational errors such as $4\frac{1}{5}=\frac{26}{5}$ instead of $4\frac{1}{5}=\frac{21}{5}$ while others wrote the given expression incorrectly for example one student wrote it as $\frac{3}{5}\times 4\frac{1}{4}\div\frac{18}{25}$ instead of $\frac{3}{5}\times 4\frac{1}{5}\div\frac{18}{25}$. It was also observed that several students did not remember to invert the fraction $\frac{18}{25}$ to $\frac{25}{18}$ so they changed $\frac{3}{5}\times 4\frac{1}{4}\div\frac{18}{25}$ into $\frac{3}{5}\times\frac{21}{5}\times\frac{18}{25}$ instead of $\frac{3}{5}\times\frac{21}{5}\times\frac{25}{18}$

indicating that they had inadequate knowledge on how to divide fractions. Further analysis of the responses indicated that some students gave a final answer as a mixed fraction i.e. $3\frac{1}{2}$ instead of an improper fraction i.e. $\frac{7}{2}$ which meant the full requirements of the question were not observed.

In part (b), they did not realise that when 1 is divided by 3 yields a repeating decimal 0.333…. which can be written as $0.\dot{3}$. Extract 2.1 shows the sample solution of a student who failed to perform part (b) of this question correctly.

Extract 2.1



Extract 2.1 shows the student who divided the numerator by the denominator to get 0.3 which was wrong.

On the other hand, the percentage of students who scored full marks was 13.62. The students were able to correctly perform the operations such as addition, multiplication and division on the given expression and they were aware that a fraction whose denominator is smaller than the numerator is called an improper fraction. Thus, they represented correctly the required answer as $\frac{7}{2}$ in part (a). In part (b), the students divided 1 by 3 to get 0.333.... and then managed to represent the part that repeats itself by placing a dot over the recurring pattern as shown in Extract 2.2.

Extract 2.2

Solution.

$\frac{3}{5} \times 4\frac{1}{5} \div \frac{18}{25}$

$\frac{3}{5} \times \frac{21}{5} \div \frac{18}{25}$

BODMAS

$\frac{3}{5} \times \frac{21}{5} \times \frac{25}{18} = \frac{3}{\cancel{5}_1} \times \frac{21}{\cancel{5}_1} \times \frac{\cancel{25}^{5}}{18}$

$= \frac{\cancel{3} \times 21}{\cancel{18}\ 6}$

$= \frac{21}{6} = \frac{7}{2}$

$\therefore = \frac{7}{2}$.

(b) Convert $\frac{1}{3}$ into a repeating decimal.

Solution

To convert $\frac{1}{3}$ into a repeating decimal.

Divide $\frac{1}{3}$

$$
\begin{array}{r}
0.33333\ldots \\
3\,\overline{)\,10} \\
-9 \\
\hline
10 \\
-9 \\
\hline
10 \\
-9 \\
\hline
10 \\
-9 \\
\hline
10 \\
-9 \\
\hline
1
\end{array}
$$

$0.33333\ldots$ into repeating decimal $= 0.\dot{3}$

Extract 2.2 shows the student who managed to multiply and divide correctly the fractions involved in part (a). In part (b), the student used correctly the long division technique to change the given fraction into a recurring decimal.

2.3 Question 3: Units, Loss and Profit

The question had parts (a) and (b). In part (a), the students were asked to change 15km into centimetres. In part (b), they were required to find the time in which sh. 200, 000 will earn sh. 48,000 at the rate of 4% interest per annum.

The analysis of data shows that 187,591 students equivalent to 38.56 percent scored from 3 to 10 marks, and among them, 45,675 students equivalent to 9.4 percent scored full marks. Based on this analysis, question 3 is among the four questions with an average performance.

The students who scored high marks were able to identify the location of km and cm in the metric system of units i.e. km, hm, dam, m, dm, cm and mm; multiply 15 by 10^5 and got 1,500,000 cm as required, since there are 5 intermediate units. Such units are hm, dam, m, dm, and cm. The students also recalled the formula $I = \frac{PRT}{100}$ correctly and realized that the value of T in part (b) could be obtained by inserting $P = 200{,}000/=$, $I = 48{,}000/=$ and $R = 4\%$ into the formula $T = \frac{100I}{PR}$. Extract 3.1 shows the work of one of such students who attempted the question correctly in both parts.

Extract 3.1

3. (a) Change 15 km into centimeters.

K	H	D	M	d	c	m
1	0	0	0	0	0	0
	1	0	0	0	0	0
		1	0	0	0	0
			1	0	0	0
				1	0	0
					1	0

1 km = 100,000 cm
15 km = ?

$\frac{15\,km \times 100{,}000\,cm}{1\,km} = 1500{,}000\,cm$

$\therefore = \underline{\underline{1{,}500{,}000\,cm}}$

(b) Find the time in which sh. 200,000 will earn sh. 48,000 at the rate of 4% inte
per annum.

Solution:

Principal = 200,000/=

Interest = 48,000/=

Rate = 4%

$\times 100 \quad I = \frac{PRT}{100} \times 100$

$\frac{100I}{PR} = \frac{PRT}{PR}$

$\boxed{\therefore T = \frac{100I}{PR}}$

Time = $\frac{100I}{PR}$

Time = $\frac{100 \times 48{,}000}{200{,}000 \times 4}$

$$\text{Time} = \frac{\cancel{4800000}\ 6}{\cancel{800000}\ 1}$$

$$\text{Time} = 6/1 = 6$$

$$\therefore \underline{\text{Time}} = \underline{6 \text{ years}}$$

Extract 3.1 shows how the student was able to correctly change the units from km to cm in part (a). In part (b), he/she was able to apply the appropriate formula to find the time in years, over which the principal would earn the given interest.

Despite this average performance, there were students who scored low marks. Such students were unable to perform arithmetic calculations on metric units of length that is changing 15 km into centimetres in part (a). For instance, some students regarded 1 kilometre to be equivalent to 10^{-5} centimetre as a result they ended up multiplying 15 by 10^{-5} to get the incorrect answer 0.00015 cm.

In part (b), some students recalled the formula $I = \frac{PRT}{100}$ correctly but could not rearrange it into $T = \frac{100I}{PR}$ to find the required number of years while others could not identify the data to be entered into the formula i.e. P=shs 200,000, Rate=4%, I=shs 48,000. Such students got only the marks for the formula. It was observed that, there were students who performed calculations which were not related to the demands of the question. Such students were given no marks. Extract 3.2 is a sample answer from one of the students illustrating one of these cases.

Extract 3.2

3. (a) Change 15 km into centimeters.

1cm = 100000km

? 15

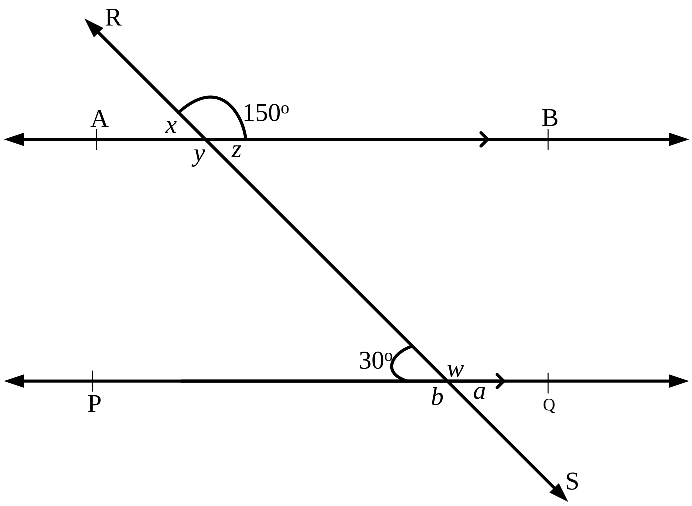
$$\frac{1\text{cm} \times 15\cancel{\text{km}}}{100\,000\cancel{\text{km}}} = \frac{15\text{cm}}{100{,}000}$$

0.0015cm

Extract 3.2 shows the student who performed wrong conversion on the units of length that was given showing lack of knowledge on the tested concept.

2.4 Question 4: Geometry, Perimeter and Areas

This question had parts (a) and (b). In part (a), the students were required to find the size of angles *a, b, w, x, y* and *z* in the following figure. In part (b), they were asked to find perimeter of a square which has an area of 25cm^2.



The analysis of data shows that about one third of the students (33.68%) scored from 3 to 10 marks and 56,892 (11.71%) students scored 6.5 to 10 marks. Thus question 4 was averagely performed.

Despite the fact that the question was averagely performed, more than half of the students (56.43%) scored zero. Such students were unable to use the properties of angles formed when the transversal line RS crosses the parallel lines AB and PQ in part (a). Many of them did not realise that the given drawing had equal vertically opposite angles, pairs of alternate interior angles, pairs of alternate exterior angles and equal corresponding angles. Furthermore, they did not know that the angles on the same side of a transversal and between parallel lines add up to 180°.

In part (b), the most common error noted was using inappropriate formulas such as $A = \frac{\pi D^2}{4}$, $p = l^2$, $p = \frac{Area}{4}$ etc. The students were not aware that the perimeter is the distance around the edge of the square while area is the space covered within the edges of the square. In answering this part, they were required to use both formulas for area and perimeter of a square i.e. $A = l^2$ or length times length, while the perimeter is $p = 4l$ or 4 times length. Extract 4.1 is a sample answer illustrating how the students failed to answer part (b).

Extract 4.1

(b) Find the perimeter of a square, if its area is $25\ cm^2$.

area($\frac{1}{2}$ x 2)

area = $\frac{1}{2}$ x 25cm^2

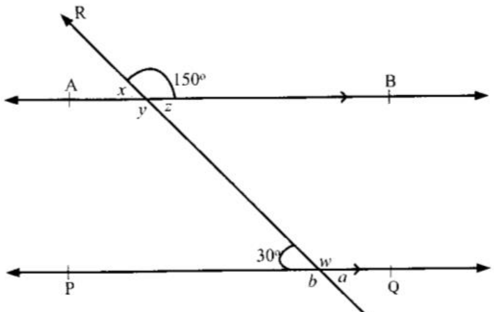
area = 50

area of perimeter = 50cm^2

Extract 4.1 shows that the student lacked knowledge and skills on the topic of perimeters and areas, as a result performed calculations which were not related to the demands of part 4 (b).

Only a few students (7.78%) answered this question correctly and scored full marks. These students were able to find the sizes of different angles formed by the parallel lines and a transversal that is *a, b, w, x, y* and *z* in part (a). In part (b), they were able to correctly apply the formulas to find the area and perimeter of the square. Extract 4.2 shows a sample answer from one among the students who answered the question correctly.

Extract 4.2



$x + 150° = 180°$ (By straight line rule)

$x = 180° - 150°$

$\underline{x = 30°}$

$\underline{w = 150°}$ (corresponding angles)

$w = b$ (vertical opposite angles)

$\underline{b = 150°}$

$30° = a$ (vertical opposite angles)

$\underline{a = 30°}$

$x = z$ (vertical opposite angles)

$\underline{z = 30°}$

$150° = y$ (vertical opposite angles)

$\underline{y = 150°}$

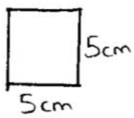
(b) Find the perimeter of a square, if its area is $25\ cm^2$.

$A = S \times S$

$\sqrt{25cm^2} = \sqrt{S^2}$

$5cm = S$

One side = 5cm



$P = S \times 4$

$P = 5cm \times 4$

$\underline{P = 20cm}$

The responses in Extract 4.2 show that the student had adequate knowledge and skills on the tested concepts.

2.5 Question 5: Algebra and Exponents

Question 5 had parts (a) and (b). In part (a), the students were required to find the value of *x* in the equation $9 \times 3^{4x} = 27^{(x-1)}$. In part (b), they were required to factorize the quadratic expression $6x^2 - 11x + 4$ by splitting the middle term.

In this question, most students (87.44%) scored below 3 marks and among them 83.69 percent scored 0 indicating that the topics of Algebra and Exponents were not clear to the majority of students. Hence, the general performance of students in these topics was poor.

The students who scored low marks in part (a) failed to write $9 \times 3^{4x} = 27^{(x-1)}$ in power as $3^2 \times 3^{4x} = 3^{3(x-1)}$ which was an important step in obtaining the required value of *x*. For instance, several students were expressing the given exponential equation as $27^{4x} = 27^{(x-1)}$ see Extract 5.1, thus ended up getting the incorrect answer $x = -\frac{1}{3}$.

In part (b), a number of students demonstrated a lack of understanding about the factorization method by splitting the middle term. For example there were students who equated the given expression to 0 and then solved it for *x* contrary to the requirements of the question. In factorizing the given expression by using the method of splitting the middle term, the students were required to identify two numbers which add up to -11 and when multiplied its product is 24. Using this condition, the two numbers were supposed to be -8 and -3 and the factorized expression would be $(3x-4)(2x-1)$. Further analysis on the students' responses revealed that, some students picked numbers that did not match the condition stated and as a result ended up with incorrect factors. Extract 5.2 is a sample answer from one of the students' script illustrating this case.

Extract 5.1

Soln

$9 \times 3^{4x} = 27^{(x-1)}$

$27^{4x} = 27^{(x-1)}$

Since the bases are equal we equate the exponents

$= 4x = x - 1$

$= 4x - x = -1$

$= \frac{\cancel{3}x}{\cancel{3}} = \frac{-1}{3}$

$$x = \frac{-1}{3}$$

$\therefore$ Value of x is $\frac{-1}{3}$

Extract 5.1 shows how the student failed to apply the laws of exponents in answering part (a).

Extract 5.2

(b) Factorize the expression $6x^2 - 11x + 4$ by splitting the middle term.

Soln:

$6x^2 - 11x + 4$

$= 6x^2 - x - 10x + 4$

$(6x^2 - x) - (10x + 4)$

$2x(3x - 1) - 2(5x - 2)$

$\frac{6x^2}{2} - \frac{11x}{2} + \frac{4}{2} =$

$x^2 - \frac{11x}{2} + 2 = \frac{11}{2}x$

$x^2 - \left(\frac{11}{2}\right)^2 + \frac{1}{2} + 2 = \left(\frac{11}{2}\right)^2 x + \frac{1}{2}$

$x^2 - \frac{121}{4} + \frac{1}{2} = \frac{121}{4} + \frac{1}{2}$

$x^2 - \frac{\cancel{121}+2}{4} + 2 = \frac{121+2}{4}$

$x^2 - \frac{123}{4}x + 2 = \frac{123}{4}x$

$4x^2 - 123x + 2 = 123x$

$4x^2 + 2 = 123x + 123x$

$\frac{4x^2}{4} + \frac{2}{4} = \frac{246x}{4}$

$x^2 + \frac{1}{2} = \frac{246}{4}$

$x = \frac{b \pm \sqrt{-b^2 + 4ac}}{2a}$

$\therefore (x+3)(x-5)$

In Extract 5.2, the student was unable to split the middle term -11x into -3x and -8x and consequently ended up with the factorized expression $(x+3)(x-5)$ instead of $(3x-4)(2x-1)$.

However, there were 17,113 students equivalent to 3.52 percent who did well and scored all the 10 marks. In part (a), the students were able to express the terms in the right hand side and left hand side of $9 \times 3^{4x} = 27^{(x-1)}$ by using a common base to obtain $3^{2+4x} = 3^{3x-3}$ and thereafter equated the exponents of both sides in order to find the required value of *x*. In part (b), the students factorized the expression $6x^2 - 11x + 4$

successfully by splitting the middle term, hence scored full marks. (See Extract 5.3)

Extract 5.3

5. (a) Find the value of x in the equation $9\times3^{4x}=27^{(x-1)}$.

Soln.

$9\times3^{4x}=27^{(x-1)}$

$3^2\times3^{4x}=27^{(x-1)}$

$3^2\times3^{4x}=3^{3(x-1)}$

$3^2\times3^{4x}=3^{3x-3}$

$3^{(2+4x)}=3^{(3x-3)}$

$2+4x=3x-3$

$2+3=3x-4x$

$5=-x.$

$\frac{5}{-1}=\frac{-x}{-1}$

$x=-5$

$\therefore\ x=-5.$

(b) Factorize the expression $6x^2-11x+4$ by splitting the middle term.

Soln.

Required to factorize $6x^2-11x+4$.

$6x^2-11x+4$, $6\times4=24$

Factors of $24=1,2,3,4,6,8,12,24$.

Appropriate pair is 3 and 8.

$6x^2-3x-8x+4$

$(6x^2-3x)(-8x+4)$

$3x(2x-1)-4(2x-1)$

$(3x-4)(2x-1)$.

$\therefore 6x^2-11x+4=(3x-4)(2x-1)$.

Extract 5.3 shows that the student had adequate knowledge on the laws of exponents and applied them correctly in part (a). In part (b), he/she was able to factorize the given quadratic expression as required.

2.6 Question 6: Coordinate Geometry and Transformations

The question consisted of two parts. In part (a), the students were asked to find the equation of a straight line passing through the points (3, 5) and (7, 9) in the form of $y = mx + c$. In part (b), the students were given that a triangle with vertices A(2, 2), B(3, 4) and C(4, 3) is reflected in the *y*-axis and were asked to write down the image of points A, B and C.

In this question, the majority of students (80.07%) scored from 0 to 2.5 out of 10 marks and among them 60.42 percent scored a zero mark, indicating a poor performance in this question.

Only a few students were able to answer part (a) correctly. These students were able to find the slope of the straight line joining points (3, 5) and (7, 9) by finding the ratio of the change in y over the change in x. The students had also an idea that the slope of the line must be the same between any two points e.g. (x, y) and (3, 5). Therefore, they were able to use this slope by equating 1 to either $\frac{y-5}{x-3}$ or $\frac{9-y}{7-x}$ so as to get the equation $y = x + 2$ as required. However, it was disappointing to see how a huge number of students applied the incorrect formula such as $m = \frac{y_1 - x_2}{y_2 - x_1}$, $m = \frac{y_1 - y_2}{x_1 + x_2}$, $m = \frac{x_2 - x_1}{y_2 - y_1}$ and so on to find the slope, a situation that led into an incorrect equation for the straight line. It was noted that, many students were able to find the slope but not all of them were able to use this slope to find the required equation.

In part (b), only a few students were able to provide correct solutions. Many students were unable to represent the reflections by drawings or use the properties of reflections in order to find the image for the vertices of the triangle. Some students managed to locate the points A, B and C on the x-y plane but failed to write down the image of these points. It was also observed that several students reflected the points in the x – axis contrary to the requirements of the question. These students ended up getting (2, -2), (3, -4), (4, -3), instead of (-2, 2), (-3, 4) and (-4, 3) respectively. This indicates that the students lacked basic knowledge on the characteristics of reflections in a plane. Extract 6.1 is a sample answer showing some of the difficulties the students faced while answering this question.

Extract 6.1

The vertices of a triangle are A (2, 2), B (3, 4) and C (4, 3). If the triangle is reflected in the y-axis, write down the coordinates of the image of points A, B and C.

solution.

From the formula y = ais (means x axis)

A (2, 2) $\xrightarrow{\text{X-axis}}$ A'(2, -2)

B (3, 4) $\xrightarrow{\text{x-axis}}$ B' (3, -4)

C (4, 3) $\xrightarrow{\text{x-axis}}$ C' (4, -3)

In Extract 6.1, the student reflected the given points in the x-axis instead of the y-axis as instructed.

Despite the weaknesses shown, there were 16,861 students (3.47%) who managed to answer this question correctly and scored all the 10 marks. Extract 6.2 is an example of good solution from one of such students.

Extract 6.2

6. (a) Find the equation of the straight line passing through the points $(3, 5)$ and $(7, 9)$. (Express your answer in the form $y = mx + c$).

Soln

gradient (m) = $\frac{\text{change in y}}{\text{Change in x}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Given points, (3, 5) and (7, 9)

$m = \frac{9-5}{7-3} = \frac{4}{4}$

$m = 1$

but equation of the line

$= m = \frac{y - y_1}{x - x_1}$ using point (3, 5)

$\frac{1}{1} \times \frac{y-5}{x-3}$

$y - 5 = x - 3$

$y = x - 3 + 5$

$y = x + 2$

$\therefore$ The equation of the line $= y = x + 2$

(b) The vertices of a triangle are A (2, 2), B (3, 4) and C (4, 3). If the triangle is reflected in the y-axis, write down the coordinates of the image of points A, B and C.

Soln

$A(x,y) = A'(-x,y)$
$B(x,y) = B'(-x,y)$
$C(x,y) = C'(-x,y)$
$\therefore A' = (-2,2)$, $B' = (-3,4)$ and $C' = (-4,3)$

NOTE:- Reflection on the y-axis, $x = 0$.

$\therefore$ Coordinates of the image of points A, B, C
$= A'(-2,2)$, $B'(-3,4)$ and $C'(-4,3)$.

In Extract 6.2, the student attempted the question correctly and managed to use the knowledge and skills learnt in the topic of coordinate geometry to answer part (a). In part (b), he/she recognized that the reflection of (x,y) in the y-axis is (-x,y) and then proceeded to final correct answer.

2.7 Question 7: Radicals and Logarithms

Question 7 had parts (a) and (b). In part (a), the students were asked to rationalize the denominator of $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$. In part (b), they were asked to find the value of $3\log_{10}5 + 5\log_{10}2 - 2\log_{10}2$ without using mathematical tables.

In this question 78.09 percent of the students who attempted it scored below 3 marks and among them 65.08 percent scored zero. Based on these data, this question was poorly performed.

In part (a), it was disappointing to note that some students were able to introduce the rationalizing factor $\left(\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} \times \frac{\sqrt{10}+\sqrt{2}}{\sqrt{10}+\sqrt{2}}\right)$ as their first step but could not continue to get the correct answer because they could not work with operations involving radicals, see Extract 7.1. Such students were unable to multiply the terms in both the numerator and denominator to obtain $\frac{2\sqrt{5}+2}{8}$ or $\frac{\sqrt{5}+1}{4}$ as the simplified form after rationalizing. It was further noted that most students made mechanical errors while manipulating the fraction. Incorrect attempts to rationalise denominators included such thing as $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{10}+\sqrt{2}}$, $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} \times \frac{\sqrt{10}-\sqrt{2}}{\sqrt{10}+\sqrt{2}}$ etc. In part (b), the majority of students were unable to apply the laws of logarithms such as the logarithm of product, logarithm of a quotient and the logarithm of a power in transposing $3\log_{10} 5 + 5\log_{10} 2 - 2\log_{10} 2$ leading to an answer of 3. A sample answer from the script of one of the students who did not do well in part (b) of this question is provided in Extract 7.2.

Extract 7.1

7. (a) Rationalize the denominator of $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$.

soln

$$\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} \times \frac{\sqrt{10}+\sqrt{2}}{\sqrt{10}+\sqrt{2}} = \frac{\sqrt{20}+2}{10+2}$$

$$= \frac{\sqrt{20}+2}{10+2} = \frac{\sqrt{20}+2}{12}$$

$$\therefore \frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} = \frac{\sqrt{20}+2}{12}$$

In Extract 7.1, the student made algebraic errors in multiplying the terms in the numerator. He/she multiplied $\sqrt{10}-\sqrt{2}$ by $\sqrt{10}+\sqrt{2}$ to get 12 instead of 8.

Extract 7.2

(b) Without using mathematical tables, find the value of $3\log_{10} 5 + 5\log_{10} 2 - 2\log_{10} 2$.

$= 3\log_{10} 5 + 5\log_{10} 2 - 2\log_{10} 2$

$= \log_{10} 5^3 \times \log_{10} 2^5 \div \log_{10} 2^2$

$= \log_{10} 125 \times \log_{10} 32 \div \log_{10} 4$

$= 125 \times 32 \div 4$

$= 125 \times 8$

$= 125 \times 8$

$= \underline{1000}$

$\therefore = 1000$

In Extract 7.2, the student applied correctly the power rule $c \log A = \log A^c$ but had a wrong understanding on the use of the quotient and product rule.

Very few students (3.73%) were able to answer this question correctly. A sample answer from one of those students is shown in Extract 7.3

Extract 7.3

. (a) Rationalize the denominator of $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$.

Solution

$\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$

$= \frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} \frac{\times \sqrt{10}+\sqrt{2}}{\times \sqrt{10}+\sqrt{2}}$

$= \frac{\sqrt{20}+\sqrt{4}}{\sqrt{100}+\cancel{\sqrt{20}}-\cancel{\sqrt{20}}-\sqrt{4}}$

$= \frac{\sqrt{20}+2}{10-2}$

$= \frac{\sqrt{20}+2}{8}$

(b) Without using mathematical tables, find the value of $3\log_{10} 5 + 5\log_{10} 2 - 2\log_{10} 2$.

Soln

$3\log_{10}5 + 5\log_{10}2 - 2\log_{10}2$

$= \log_{10}5^3 + \log_{10}2^5 - \log_{10}2^2$

$= \log_{10}125 + \log_{10}32 - \log_{10}4$

$= \log_{10}125 + \log_{10}32/4$

$= \log_{10}125 + \log_{10}8$

$= \log_{10}125 \times 8$

$= \log_{10}1000$

$= \log_{10}10^3$

$= 3\log_{10} 10.$

$= 3$

In Extract 7.2, the student applied correctly the laws of logarithms in simplifying the given logarithmic equation and also managed to rationalize the given fraction in part (a).

2.8 Question 8: Congruence and Similarity

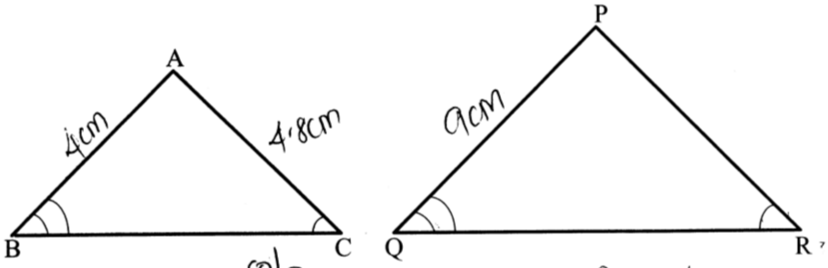
This question consisted of parts (a) and (b). In part (a), the students were required to prove that $\Delta PQS \equiv \Delta PRS$ given that ΔPQR is an isosceles triangle, $\overline{PQ} = \overline{PR}$, $\overline{QS} = \overline{SR}$ and S is a point between Q and R. In part (b), they were given that ΔABC ~ ΔPQR, $\overline{AC} =$ 4.8cm, $\overline{\text{AB}} = 4\text{cm}$, $\overline{\text{PQ}} = 9\text{cm}$ and they were required to find $\overline{PR}$.

This question was attempted by 486,108 students, of whom the majority (85.4%) scored below 3 out of 10 marks and among them 69.55 percent scored a zero mark, indicating an overall poor performance in this question.

The analysis of students' responses revealed that, some of the students who scored low marks in part (a) were unable to use the fact that if $PQ = PR$, $QS = SR$ and PS is shared by triangles PQS and PRS, then the three sides of triangle PQS are congruent to the three sides of triangle PRS and hence the two triangles are congruent by the SSS postulate. These students applied inappropriate congruence theorems such as SAS, AAS etc. Other students completely lacked knowledge on the topic of congruence and hence opted to write anything they thought in their heads. It was also noted that there were students who could not represent the given information on a drawing which would have helped them understand the question.

In part (b), the students were unable to realize that given that triangle ABC is similar to triangle PQR, the corresponding sides are in the ratio $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$, so that $\frac{4}{9} = \frac{4.8}{PR}$. This result was needed in computing the required length of PR. Extract 8.1 is a sample answer illustrating how these students failed to answer part (b) of this question.

Extract 8.1



soln

Since the corresponding sides of similar triangles are equal therefore

$\overline{AC} = \overline{PR}$

$AC = 4.8cm$

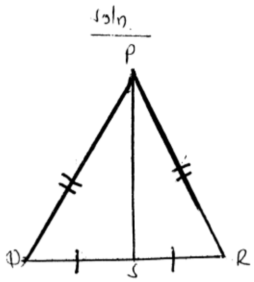
$\therefore \overline{PR} = 4.8cm$

In Extract 8.1, the student had no idea about the conditions for similarity of triangles. He/she treated the length of PR as equal to the length of AC.

Despite the general poor performance in this question, a total of 5,209 students, equivalent to 1.07 percent scored all the ten marks. The students managed to provide correct proof in part (a). They were also able to calculate the correct length PR in part (b). Extract 8.2 illustrates how these students performed in this question.

Extract 8.2

8. (a) PQR is an isosceles triangle whereby $\overline{PQ} = \overline{PR}$ and $\overline{QS} = \overline{SR}$. If S is a point between Q and R prove that $\Delta PQS \equiv \Delta PRS$.



Given: $\overline{PQ} = \overline{PR}$, $\overline{QS} = \overline{SR}$

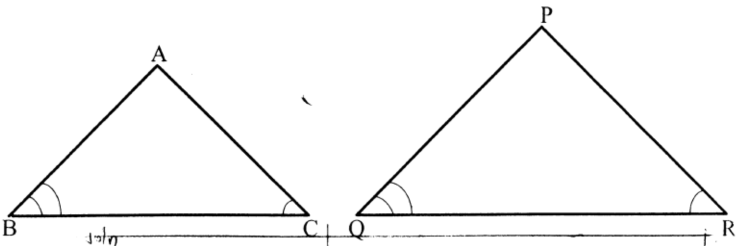
Required to prove that $\Delta PQS \equiv \Delta PRS$

Argument	Reason
$\overline{PQ} = \overline{PR}$	Given
$\overline{QS} = \overline{SR}$	Given
$\overline{PS} = \overline{PS}$	Common

$\Delta PQS \equiv \Delta PRS$ by (SSS - Theorem)

$\therefore$ Hence Proved

(b) In the following figure, $\Delta ABC \sim \Delta PQR$, $\overline{AC} = 4.8$ cm, $\overline{AB} = 4$ cm and $\overline{PQ} = 9\ cm$. Find $\overline{PR}$.



From similarity

If $\Delta ABC \sim \Delta PQR$

Then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\frac{AB}{PQ} = \frac{AC}{PR}$

Given: $AC = 4.8cm$ Required: PR

$AB = 4cm$

$PQ = 9cm$

$\frac{AB}{PQ} = \frac{AC}{PR}$

$PR = \frac{AC \times PQ}{AB}$

$= \frac{4.8 \times 9}{4} = 1.2 \times 9$

$= 10.8cm$

$\therefore \overline{PR} = 10.8cm$

In Extract 8.2, the student was able to interpret the information given in part (a) into a drawing and used correctly the SSS postulate in showing that triangle PQS is congruent to triangle PRS. Also he/she was able to apply the concept of similarity to obtain the length PR in triangle PQR, hence gaining full marks.

2.9 Question 9: Pythagoras Theorem and Trigonometry

The question had parts (a) and (b). In part (a), the students were asked to find the length $\overline{AD}$ in surd form given that the sides of an equilateral triangle ABC are 10 cm each and D is a midpoint on side BC. In part (b), they were asked to find the exact value of $\frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$ without using mathematical tables.

In this question, the majority of students (92.42%) scored below 3 out of 10 marks with 81.71 percent of them scoring a zero mark. The fact that there were a large number of students who scored very low marks in this question indicates that the topics on Trigonometry and Pythagoras theorem were not clear to most students. The performance of students is summarized in Figure 4.

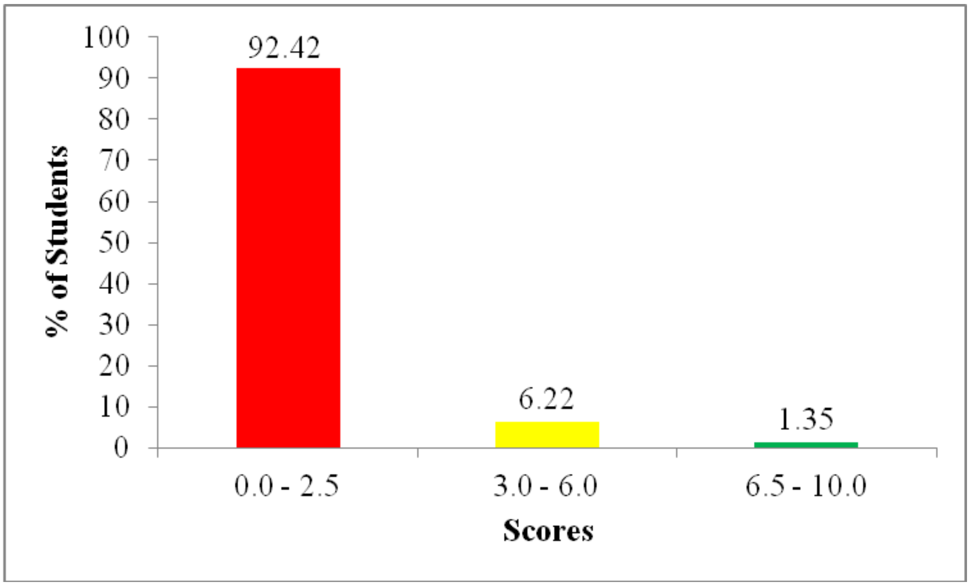
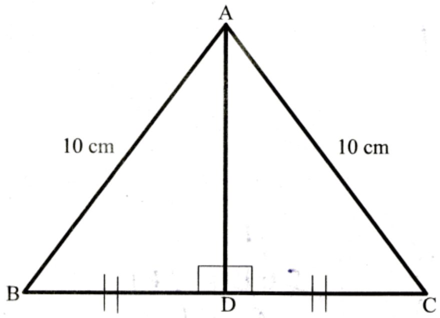


Figure 4: Students' Performance in question 10

In part (a), the majority of students did not understand the term equilateral triangle and hence failed to obtain the required length in surd form by using either the Pythagoras theorem or trigonometric ratios. It was observed that some students were unable to differentiate the hypotenuse from the adjacent and opposite sides thus making wrong substitutions into the Pythagoras formula such as $AD^2 = 5^2 + 10^2$. Other common errors involved using mathematical tables to find the value of $5\sqrt{3}$ to get 8.6603 contrary to the demands of the question, failure to identify the opposite and adjacent sides in the given figure and hence expressed $\sin 30° = \frac{AD}{10}$ instead of $\cos 30° = \frac{AD}{10}$, wrong substitution of $\sin 60°$ with $\frac{1}{2}$ instead of $\frac{\sqrt{3}}{2}$ in $\sin 60° = \frac{AD}{10}$ and using inappropriate trigonometric ratios such as $\cos 60° = \frac{AD}{10}$. Extract 9.1 illustrate one of these cases.

Extract 9.1

(a) The sides of an equilateral triangle ABC are 10 cm each. Find the length marked $\overline{AD}$ in surd form.



Solution:

$c^2 = a^2 + b^2$

$AD^2 = DC^2 + AC^2$

$AD^2 = 5^2 + 10^2$

$AD^2 = 25 + 100$

$\sqrt{AD^2} = \sqrt{125}$

$AD = 15$

$\therefore \overline{AD} = 15\text{cm}.$

In Extract 9.1, the student incorrectly applied the Pythagoras formula in finding length AD.

The analysis of students' responses revealed that in part (b), some students could neither recall trigonometric ratios for the tangent of the special angles $30°$ and $45°$ nor use both equilateral and isosceles triangles in order to find $\tan 30°$ and $\tan 45°$. However, it was pleasing to see that other students substituted the correct values of $\tan 45°$ and $\tan 30°$ into $\dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$ to get $\dfrac{1 + \frac{\sqrt{3}}{3}}{1 - \left(1 - \frac{\sqrt{3}}{3}\right)}$ but did not possess the algebraic skills needed to proceed to the correct answer. Some of them were unable to handle the rationalizing of the denominator involved to get simplified answer $2 + \sqrt{3}$. Others were unable to simplify the denominator $1-\left(1\times\frac{\sqrt{3}}{3}\right)$ as they got $-\frac{\sqrt{3}}{3}$ or $0\times\frac{\sqrt{3}}{3}$ instead of $1-\frac{\sqrt{3}}{3}$. It was observed that some students used Four Figure Mathematical tables to find the value of $\tan 30°$; that is $\tan 30° = 0.5774$ instead of expressing it as $\tan 30° = \frac{\sqrt{3}}{3}$. Such students ended up getting 3.7326 and were given no marks, see Extract 9.2.

Extract 9.2

(b) Without using mathematical tables, find the exact value of $\frac{\tan 45° + \tan 30°}{1-\tan 45° \tan 30°}$.

Soln

$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

$= \frac{1.0000 + 0.5774}{1 - 1.0000 \times 0.5774}$

$= \frac{1.5774}{1 - 0.5774}$

$= \frac{1.5774}{0.4226}$

$= 3.7328$

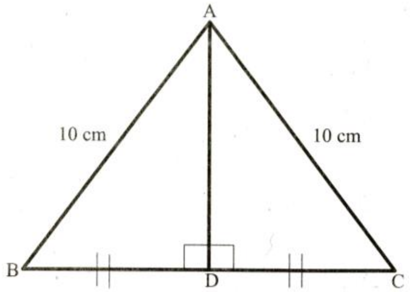
$\therefore$ exact value of tables $= 3.7328$

In Extract 9.2, the student attempted part (b) of this question using mathematical table despite the instruction that he/she should not use it.

Only 1,675 out of the 486,108 students who attempted this question managed to score full marks. A sample answer from one of those students to show how they were able to apply correctly the knowledge and skills on the topics of Trigonometry and Pythagoras' theorem is shown in Extract 9.3.

Extract 9.3

9. (a) The sides of an equilateral triangle ABC are 10 cm each. Find the length marked $\overline{AD}$ in surd form.



soln.

Let a be $\overline{AD}$
b be $\overline{DC}$
c be $\overline{AC}$

$a^2 + b^2 = c^2$
$x^2 + 5^2 = 10^2$
$x^2 = 100 - 25$
$x^2 = 75$
$x = \sqrt{75} = 5\sqrt{3}$

$\therefore$ The length of $\overline{AD}$ is $5\sqrt{3}$ cm.

(b) Without using mathematical tables, find the exact value of $\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$.

soln.

$\tan 45^\circ = 1$, $\tan 30^\circ = \sqrt{3}/3$

$\frac{1 + \sqrt{3}/3}{1 - (1 \times \sqrt{3}/3)} = \frac{\frac{3+\sqrt{3}}{3}}{1 - \sqrt{3}/3}$

$\frac{3+\sqrt{3}}{3} \div 1 - \sqrt{3}/3 = \frac{3+\sqrt{3}}{3} \times \frac{3}{3-\sqrt{3}}$

$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9 + 3\sqrt{3} + 3\sqrt{3} + 3}{9-3} = \frac{12 + 6\sqrt{3}}{6}$

$= 2 + 1\sqrt{3}$

$\therefore 2 + \sqrt{3}$

Extract 9.3 shows how a student was able to correctly apply the Pythagoras theorem to find the length $\overline{AD}$ in part (a). In part (b), the student showed all the necessary steps and had adequate knowledge on how to apply the trigonometric ratio for special angles in finding the value of the given expression.

2.10 Question 10: Statistics and Sets

This question was attempted by all students whereby a significant number of students (50.96%) scored below 3 out of 10 marks with 20.7 percent of them scoring zero. However, the percentage of students who scored from 3 to 10 marks was 49 and only 1.45 percent scored all 10 marks. The data analysis shows that the students' performance was average. Figure 5 shows these data graphically.

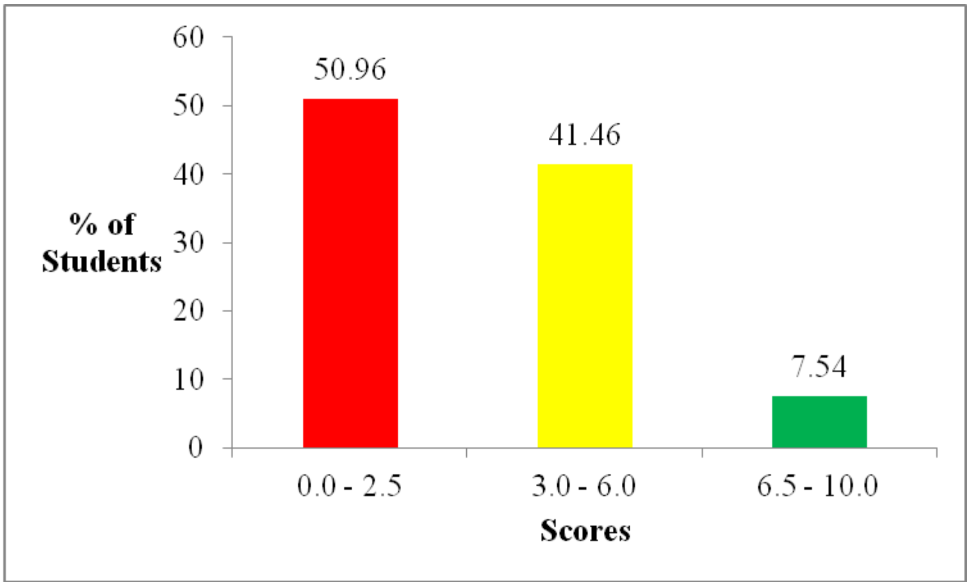


Figure 5: Students' Performance in question 10

In part (a), the students were given that "In a primary school of 150 pupils, 50 study Hisabati, 70 study Sayansi and 40 study both subjects", then they were asked to use an appropriate formula to calculate the number of pupils who study neither Hisabati nor Sayansi.

Most students performed poorly in this part as they failed to apply the formulae $n(H\cup S)=n(H)+n(S)-n(H\cap S)$ and $n(H\cup S)'=n(\mu)-n(H\cup S)$ to find the required number of pupils who study neither Hisabati nor Sayansi. The analysis of students' responses revealed that a number of students calculated only the number of pupils who study either Hisabati or Sayansi using the formula $n(H\cup S)=n(H)+n(S)-n(H\cap S)$ but did not proceed to obtain the required number of pupils who study neither Hisabati nor

Sayansi using the formula $n(H \cup S)' = n(\mu) - n(H \cup S)$. It was further noted that several students did not understand the requirements of the question and hence provided poor answers or did not respond to this part. For example many added 150, 50, 70 and 40 to get 310. Extract 10.1 is a sample answer from one of such students which illustrates this case.

Extract 10.1

10. (a) In a primary school of 150 pupils 50 study Hisabati, 70 study Sayansi and 40 study both subjects. By using the appropriate formula, calculate the number of pupils who study neither Hisabati nor Sayansi.

Data given

150 pupils (u)

n(H) 50

n(S) ~~1~~70

n(H∩S)' = 40

Formula

n(H ∪ B) = n(A) + n(B) − n(A∩B)

n(H ∪ B) = 50 + 70 − 40

120 − 40 = 80 ∴ The students who study neither Hisabati nor sayansi the are 80.

In Extract 10.1, the student did not follow the instruction given as he/she calculated the number of students who study either Hisabati or Sayansi.

In part (b), the students were given the following frequency distribution table showing the marks of 61 students:

Marks in %	30	35	45	50	60	75	80	85	90
Number of students	3	5	7	10	18	9	4	3	2

They were then required to find (i) the marks scored by few students, (ii) the highest mark, (iii) how many students passed the examination if the pass mark was 50% and (iv) which mark was scored by many students.

The analysis of the students' responses shows that most students answered this part correctly. These students managed to apply the concepts of Statistics in answering this question. A sample answer from one of those students is shown in Extract 10.2.

Extract 10.2

(b) The marks of 61 students are represented in the following table:

Marks in %	30	35	45	50	60	75	80	85	90
Number of students	3	5	7	10	18	9	4	3	2

From the table answer the following questions:

(i) Which mark was scored by few students?

90% mark was scored by few students.

(ii) What was the highest mark?

90%. was the highest mark.

(iii) If 50% was the pass mark in the examination, how many students passed the examination?

46 students passed the examination.

(iv) Which mark was scored by many students?

60% mark was scored by many students

Extract 10.2 shows that the student was able to interpret the given frequency distribution table correctly.

However, there were a few students who scored low marks in this part. These students were unable to interpret correctly the information from the given frequency distribution table. For instance, several students computed the number of students who failed the examination in (iii) contrary to the requirements of the question. Such students ended up getting 15 students instead of 46 students. This indicates that they lacked knowledge and skills on statistics. Further analysis of students' responses indicated that there were many answers that were totally un-related to the question asked. The students in this category provided meaningless calculations, an indication that they lacked knowledge and skills to answer this question. Extract 10.3 illustrate this case.

Extract 10.3

(b) The marks of 61 students are represented in the following table:

Marks in %	30	35	45	50	60	75	80	85	90
Number of students	3	5	7	10	18	9	4	3	2

From the table answer the following questions:

(i) Which mark was scored by few students?

∴ The mark was scored by few student is 62÷0=1

(ii) What was the highest mark?

∴ highest mark = ~~550~~. = 550÷61 = 27·5

(iii) If 50% was the pass mark in the examination, how many students passed the examination?

how many student passed the examination is 62.

(iv) Which mark was scored by many students?

Which mark was scored by many student is 550

Extract 10.3 shows how the student lacked knowledge and skills on the topic of statistics as a result performed calculations which were not related to the demands of the question.

3.0 CONCLUSION

The students' performance in the 2017 Basic Mathematics Form Two National Assessment questions was overall poor. Out of 10 questions that were assessed, the students performed well in only one (01) question on *Numbers* and averagely in four (04) questions that were set from the topics of *Fractions and Decimals; Statistics and Sets; Units, Profit and Loss* and *Geometry, Perimeters and Areas.*

On the other hand, the students had weak performance in five (05) questions on *Radicals and Logarithms; Coordinate Geometry and*

Transformation; Congruency and Similarity; Algebra and Exponents and *Pythagoras theorem and Trigonometry*.

The analysis of the students' performance for each topic is presented in the Appendix. In this Appendix, green, yellow and red colours represent good, average and weak performance respectively.

The factors which have contributed to weak performance in this assessment include: lack of knowledge and skills on the assessed topics, inability to use concepts/formulas/laws correctly, failure of the students to identify the demands of the questions and lack of skills to interpret word problems mathematically or diagrammatically.

4.0 RECOMMENDATIONS

In order to improve the standard of performance in this subject it is recommended that;

(a) Students must show all working to enable method marks be awarded.

(b) Teachers should encourage the students to build the habit of reading the questions carefully and identify the requirements before performing any task.

(c) Teachers should use simple methods of teaching and learning so as to allow the students to understand mathematical concepts and formulae.

(d) Teachers should make sure that all topics in the syllabus are covered before the assessment.

(e) During the learning process, teachers are advised to identify students with learning difficulties so that they can be given extra coaching.

(f) The School Quality Assurers should make close monitoring on the teaching and learning of the subject in order to identify areas that need special attention.

(g) Finally, the Ministry of Education, Science and Technology is advised to use the information in this report to make sure that there is close monitoring on how the teaching and learning are conducted in schools so as to raise the standard of performance in this subject.

Appendix

Analysis of Students' Performance per Topic in Basic Mathematics

S/n	Topic	Question Number	Percentage of Students who Scored 30 marks	Remarks
1	Numbers	1	68.71	good
2	Fractions and Decimals	2	50.07	average
3	Statistics and Sets	10	49	average
4	Units, Loss and Profit	3	38.59	average
5	Geometry, Perimeter and Areas	4	33.68	average
6	Radicals and Logarithms	7	21.89	weak
7	Coordinate geometry and Transformations	6	19.93	weak
8	Congruency and similarity	8	14.58	weak
9	Algebra and Exponents	5	12.55	weak
10	Trigonometry and Pythagoras theorem	9	7.57	weak