THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2024

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

Published by:
The National Examinations Council of Tanzania,
P. O. Box 2624,
Dar es Salaam, Tanzania.

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FOREWORD

The National Examinations Council of Tanzania is honoured to present this report on the Students' Item Response Analysis (SIRA) on the 2024 Form Two Basic Mathematics Assessment. The purpose of this report is to provide feedback to students, teachers, and other education stakeholders, highlighting key areas of achievement and challenges in Basic Mathematics after two years of secondary education. The insights aim to foster improvement in teaching, learning, and performance outcomes.

This analysis revealed that only 18.85 percent of students passed the assessment, with performance in 18 out of 20 topics classified as weak. *Numbers* and *Approximations* emerged as the strongest area, achieving an average performance of 45.54 percent. However, the topics of *Exponents*, *Radicals*, and *Logarithms* recorded the lowest performance of 5.56 percent. The main factors that contributed to low performance include; difficulties in computations, interpreting figures, applying formulas, and lack of skills for solving algebraic and geometric problems.

The report outlines practical recommendations, encouraging teachers to adopt engaging and resource-based methods, such as using charts, models, and participatory learning techniques. Students are urged to practice consistently and use available resources to strengthen their understanding of challenging concepts.

The Council believes that this report will serve as a roadmap for improvement, guiding teachers, students, and education stakeholders in addressing areas of concern. We extend our sincere gratitude to all contributors who worked diligently on this report, ensuring its value in improving the performance in future assessments.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report provides an analysis based on the statistics and description of students' performance in the Form Two National Assessment (FTNA) 2024 in Basic Mathematics. The statistical analysis focuses on students' performance, while the descriptive analysis examines the factors contributing to students' success or failure in providing correct responses.

The analysis of students' performance in Basic Mathematics reveals that 797,049 students sat for the FTNA in 2024. Of these, 150,117 students, equivalent to 18.85 percent, passed. In the FTNA 2023, a total of 695,145 students sat the Basic Mathematics assessment, and among them 146,362 students, equivalent to 21.08 percent, passed. Thus, the performance in FTNA 2024 has decreased by 2.23 percent compared to 2023.

The Basic Mathematics assessment paper consisted of 10 compulsory questions, each worth 10 marks. Performance on each question was categorized as Good, Average, or Weak based on the percentage of students who passed: 100–65 percent (Good), 64–30 percent (Average), and 29–0 percent (Weak). Based on the weaknesses identified in students' performance, this report provides useful recommendations for students and teachers to improve performance in future assessments.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section presents an analysis of students' performance on each question, categorized into three score intervals: 10.0–6.5 (good performance), 6.0–3.0 (average performance), and 2.5–0.0 (weak performance). These categories are visually represented using three colours: green for good performance, yellow for average performance, and red for weak performance. The section also provides a descriptive analysis to identify the factors contributing to students' success or failure in answering each question. The analysis is supported by samples of correct and incorrect responses, which illustrate the reasons behind the students' performance outcomes.

2.1 Question 1: Numbers and Approximations

This question had two parts, (a) and (b). Part (a) required students to find the Greatest Common Factor (GCF) and the Lowest Common Multiple (LCM) of 90 and 240. In part (b), students were required to estimate the value of $8108 \div 37$.

The data analysis shows that 362,981 (45.54%) students passed, out of whom 25,066 (3.14%) students scored full marks. This indicates that the students' performance on this question was average. Figure 1 shows the summary of the students' performance on question 1.

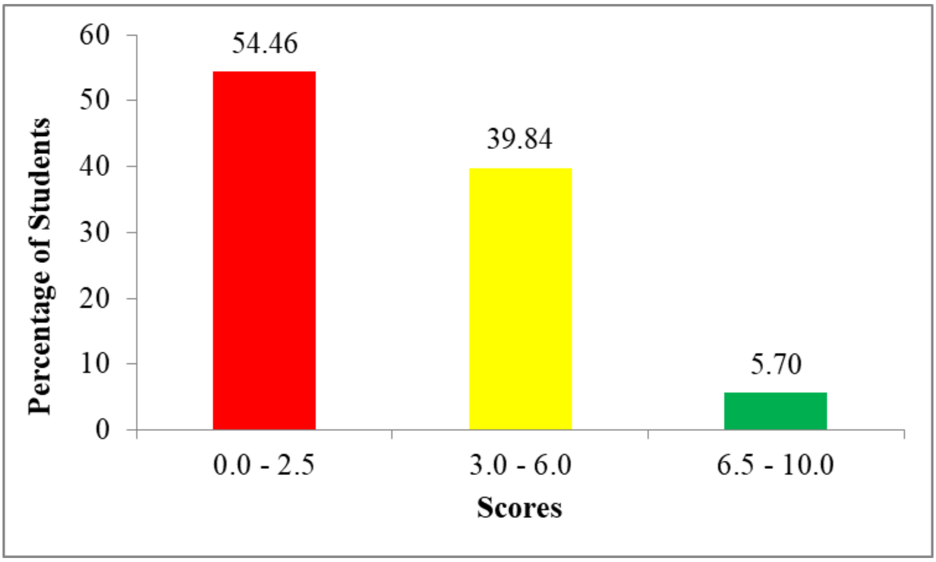

Figure 1: *Students' Performance on Question 1*

The students who answered this question correctly demonstrated knowledge and skills on the basic concepts of numbers and approximations. In part (a), the students were able to determine the factors, common factors, multiples and common multiples of 90 and 240. In part (b), they were able to round off the numbers 8108 and 37 to get 8000 and 40, respectively. They divided the resulting numbers correctly to get an estimated value, 200. Those students had adequate knowledge and skills on approximations, especially rounding off numbers. Extract 1.1 shows a sample of responses from a student who answered this question correctly.

1. (a) **Find the GCF and LCM of the numbers 90 and 240.**

2	90	240
2	45	120
2	45	60
2	45	30
3	45	15
3	15	5
5	5	5
	1	1

$$\begin{array}{r} 45 \\ \times 16 \\ \hline 270 \\ +45 \\ \hline 720 \end{array}$$

LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

LCM $= 16 \times 9 \times 5$

LCM $= 45 \times 16$

LCM $= 720$

GCF $= 2 \times 3 \times 5 = 30$

$\therefore$ LCM of 90 and 240 is 720 and GCF 30

(b) **Estimate the value of 8108 ÷ 37.**

Estimate: $8108 \div 37$

$8108 \approx 8000$ (Into the first significant figure)

$37 \approx 40$

$\Rightarrow 8108 \div 37$

$\approx 8000 \div 40$

≈ 200

$\therefore 8108 \div 37 \approx \underline{200}$

Extract 1.1: A sample of correct responses to question 1

In Extract 1.1, part (a), the student applied correctly the method of prime factorization to get the GCF and LCM of the given numbers. In part (b), the student was able to round off the numbers to one significant figure, so as to get a suitable estimated value.

Despite the average performance, one third of the students (33.24%) failed to get the correct responses, as a result they scored zero on this question. In part (a), some students wrongly computed the LCM of 90 and 240 by taking the product $3\times3\times3\times16$, $2\times2\times2\times3\times3\times5$, or $2\times2\times2\times3\times3$ instead of $2\times2\times2\times2\times3\times3\times5$. In finding the GCF, they failed to recognize the common factors of 90 and 240, hence, they wrote $\text{GCF}=24$, $\text{GCF}=432$ or $\text{GCF}=12$ instead of $\text{GCF}=30$ which is a correct answer. This shows that students lacked knowledge on the concepts of prime factorization of whole numbers.

In part (b), most of students evaluated $8108\div37$ by using the long division method to obtain 219.13, contrary to the requirement of the question. They failed to apply a suitable method of approximation by rounding off the numbers involved. Furthermore, there were students who estimated incorrectly 37 to 30 and 8108 to 8010 instead of 37 to 40 and 8108 to 8000 which was a necessary step to get the required answer. Extract 1.2 shows a sample of responses from a student who failed to answer this question correctly.

1. (a) Find the GCF and LCM of the numbers 90 and 240.

Solution

2	90	240
3	45	117
3	15	39
5	5	13
5	1	13
	1	1

The GCF is $= 2 \times 3 \times 3 \times 5$
$= 6 \times 15$
$= 90$ GCF

The LCM is $= 2 \times 3 \times 3 \times 5 \times 5$
$= 6 \times 15 \times 5$
$= 90 \times 5$
$= 450$ LCM

∴ The GCF is 90 and LCM is 450 of the numbers 90 and 240

(b) Estimate the value of $8108 \div 37$.

Soln.

12·34
37)8108
-14
114
-108
806
-403
403
-403

12•34

≃12·34

∴ The estimate the value is 12·34.

Extract 1.2: A sample of incorrect responses to question 1

In Extract 1.2, part (a) shows that, the student failed to perform the prime factorization of the given numbers. In part (b), the student failed to round off the given numbers before dividing. He/she divided the numbers contrary to the requirement of the question.

2.2 Question 2: Fractions, Decimals, and Percentages

The question consisted of two parts, (a) and (b). In part (a), students were required to simplify $3\frac{9}{10} \div \left(3\frac{3}{5} - 1\frac{1}{2}\right)$. In part (b), they were required to change $1.\dot{2}\dot{3}$ into mixed numbers.

The data analysis reveals that 580,621 (72.85%) students scored below 3.0 marks, out of whom 473,748 (59.44%) students scored zero on this question. Based on attained level, the students' performance on this question was weak. Figure 2 summarizes the students' performance on question 2.

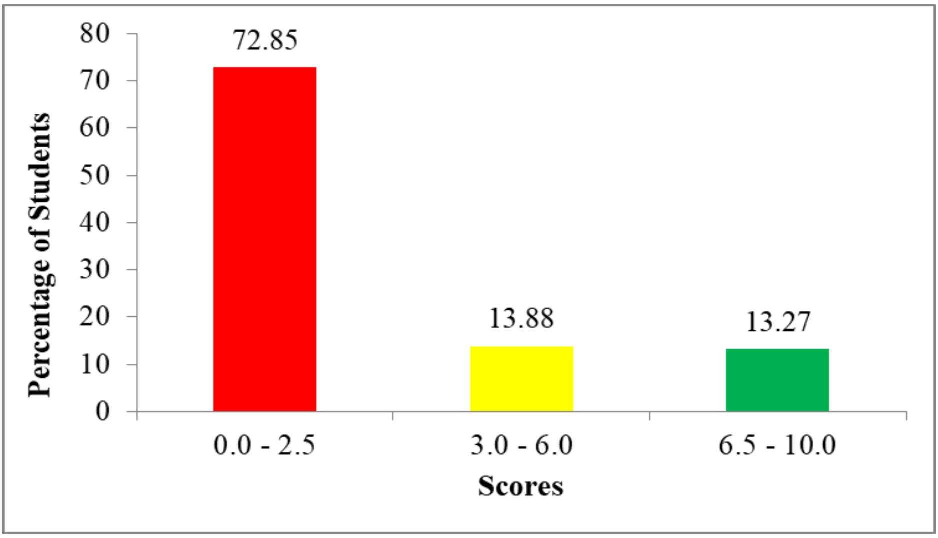

Figure 2: *Students' Performance on Question 2*

The analysis shows that more than half of the number of students (59.44%) who sat for the assessment failed to get the correct answers. In part (a), majority of the students lacked knowledge and skills of

$\frac{18}{5}-\frac{3}{2}=\frac{90-6}{10}$ instead of $\frac{36-15}{10}$. Those students failed to express the fractions under a common denominator due to lack of knowledge and skills on operations of fractions. Most of them were unable to express the mixed numbers involved in the given expression as improper fractions before performing the required operations.

In part (b), majority of students multiplied the assigned variable for the recurring decimal, $1.\dot{2}\dot{3}$ by 10 or 1000 instead of 100. This led them to have difficulties in computations, hence they were not able to get the correct answer. The students' incorrect responses indicate lack of the intended competences that were expected to be acquired after learning the basic concepts of decimals and fractions. Extract 2.1 shows a sample of responses from a student who failed to answer this question correctly.

2. (a) Simplify $3\frac{9}{10} \div \left(3\frac{3}{5} - 1\frac{1}{2}\right)$.

Solution

$3\,{}^{9}/_{10} \div (3\,{}^{3}/_{5} - 1\,{}^{1}/_{2})$

$3\,{}^{3}/_{5} - 1\,{}^{1}/_{2} = \frac{9-6}{10} = {}^{3}/_{10}$

$3\,{}^{9}/_{10} \div ({}^{3}/_{10})$

$3\,{}^{9}/_{10} \times {}^{10}/_{3} = \frac{18 \times 100}{30} = \frac{1800}{30}$

$60.$

$\therefore$ Answer is 60.

(b) Change $1.\dot{2}\dot{3}$ into mixed numbers.

Solution

Let $x = 1.\dot{2}\dot{3}$

$100x = 1.23$

$100x = 1.232\dot{3}$

$100x - x = 1.232\dot{3} - 1.23$

$\frac{99x}{99} = \frac{23}{99}$

$x = {}^{23}/_{99}$

$\therefore 1.\dot{2}\dot{3}$ in mixed number is ${}^{23}/_{99}$

Extract 2.1: A sample of incorrect responses to question 2

In Extract 2.1, part (a), the student failed to subtract mixed numbers correctly. In part (b), he/she failed to multiply by 100 throughout the equation.

On the other hand, the response analysis shows that, 66,390 (8.33%) students managed to answer this question correctly. In part (a), the students applied the BODMAS rule accordingly to simplify

$3\frac{9}{10} \div \left(3\frac{3}{5} - 1\frac{1}{2}\right)$ $1\frac{6}{7}$, which is the correct answer.

In part (b), the students were able to convert the recurring decimal, $1.\dot{2}\dot{3}$ into a fraction. They assigned a suitable variable to the recurring decimal and performed appropriate computations to get the correct answer. Extract 2.2 shows a sample of responses from a student who answered this question correctly.

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2. (a) Simplify $3\frac{9}{10} \div \left(3\frac{3}{5} - 1\frac{1}{2}\right)$.

Solution

$$3\tfrac{9}{10} \div \left(3\tfrac{3}{5} - 1\tfrac{1}{2}\right) = \frac{39}{10} \div \left(\tfrac{18}{5} - \tfrac{3}{2}\right)$$
$$= \frac{39}{10} \div \left(\frac{(2\times18) - (5\times3)}{10}\right)$$
$$= \tfrac{39}{10} \div \left(\frac{36 - 15}{10}\right)$$
$$= \tfrac{39}{10} \div \left(\tfrac{21}{10}\right)$$
$$= \frac{39}{10} \div \frac{21}{10}$$
$$= \frac{39}{10} \times \frac{10}{21}$$
$$= \frac{39}{21}$$
$$= 1\tfrac{6}{7}$$
$$\therefore 3\tfrac{9}{10} \div \left(3\tfrac{3}{5} - 1\tfrac{1}{2}\right) = 1\tfrac{6}{7}$$

(b) Change $1.\dot{2}\dot{3}$ into mixed numbers.

Solution

let $y = 1.\dot{2}\dot{3}$ ...(i)

Multiply by 100 since the recurring decimal is in Tenths and hundredths.

$100 \times y = 1.\dot{2}\dot{3} \times 100$

$100y = 123.\dot{2}\dot{3}$ ...(ii)

Subtract (i) from (ii)

$100y - y = 123.\dot{2}\dot{3} - 1.\dot{2}\dot{3}$

$99y = 122$

$y = \frac{122}{99}$

$\therefore$ $1.\dot{2}\dot{3}$ into mixed number is $1\frac{23}{99}$

Extract 2.2 A sample of correct responses to question 2

In Extract 2.2, part (a), the student applied properly the BODMAS rule to get the correct answer. In part (b), he/she defined a variable to the given decimal and made suitable and relevant computations to obtain the correct mixed number.

2.3 Question 3: Units, Ratios, Profit and Loss

This question consisted of two parts, (a) and (b). In part (a), students were required to find the amount of money which would have been lent in order to get the interest of Tsh 36,000 at a rate of 5% per annum for 6 months. In part (b), students were required to find the amount of milk in litres that Nyanjara bought, if she bought 50 bottles of milk for 70 children, given that the capacity of each bottle was 300 ml.

The analysis shows that a total of 623,927 students, which is equivalent to more than three quarters (78.28%) scored 2.5 marks or less. It was further noted that, 465454 (58.40%) students scored zero on this question. In contrast, 51,106 (6.41%) students scored 6.5 to 10 marks. The general performance on this question was weak. Figure 3 shows the students' performance on question 3.

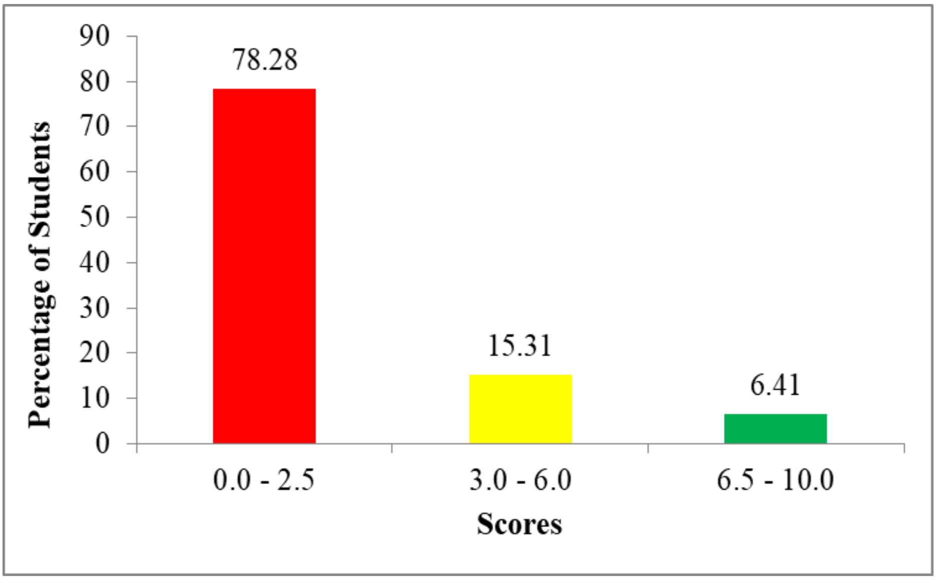


Figure 3: *Students' Performance on Question 3*

The analysis of responses shows that, the students' failure to answer this question correctly was attributed to inadequate knowledge and skills of solving real life problems on ratios, profit and loss as well as the basic concept of metric units. In part (a), most of the students applied incorrect formulae for finding the principal amount from the simple interest such as $I = \frac{PT}{100}$ $I = \frac{PRT}{100}$, where I

P is the principle amount, R is the interest rate and T is the time. The incorrect formulae led to irrelevant computations, resulting in incorrect answers. Moreover, there were students who used $T = 6$ instead of $T = \frac{6}{12} = \frac{1}{2}$ year which led to incorrect answers. This was due to lack of knowledge of converting units of time, particularly months into years. Other students failed to compute the principal amount after making the substitutions in the equation $\text{sh } 36000 = \frac{P \times 5 \times \frac{1}{2}}{100}$. They failed to transpose the formulae in order to solve for the value of P. This indicates a lack of knowledge on the basic concepts of ratios, profit and loss.

In part (b), majority of the students failed to convert 300 ml into litres. For instance, some students wrote 300 ml as 0.03 litres instead of 0.3 litres which was an essential step to get the required answer. This misconception resulted from the failure to clearly interpret the given word problem. Extract 3.1 shows a sample of responses from a student who failed to answer this question correctly.

3. (a) How much money will you have to lend in order to get the interest of sh. 36,000 at 5% per annum if you lend it for 6 months?

Interest = 36,000
Rate = 5%
Time = 6m

$\frac{5}{100} \times 36{,}000 \times \frac{6}{12}$

$1800 \times \frac{6}{12}$

∴ 900shs

(b) Nyanjara bought 50 bottles of milk for 70 children. If the capacity of each is 300 *ml*, find the amount of milk in litres that Nyanjara bought.

Solution

50 bottles of milk for 70 children
Capacity of bottle = 300ml
Find amount of milk in litres

1 ~~litres~~ mL = 100000 litres
300mL = L?x

1mLx = 100000 L × 300mL

$\frac{1mLx}{300mL} = \frac{100000L \times 300mL}{300mL}$

$\frac{300}{300} = \frac{10000000L}{300}$

~~1 = 1000~~ L = 333

Extract 3.1: A sample of incorrect responses to question 3

In Extract 3.1, part (a), the student applied a wrong procedure for finding the simple interest. In part (b), the student used an incorrect relation of the units of volume, and ended up with incorrect answer.

On the other hand, 4.50 percent of the students performed well on this question and scored full marks. In part (a), the students applied the correct formula and made relevant computations to get the correct answer. In part (b), the students converted the metric units correctly. Extract 3.2 shows a sample of responses from the student who answered this question correctly.

3. (a) How much money will you have to lend in order to get the interest of sh. 36,000 at 5% per annum if you lend it for 6 months?

Data given.

$P = ?$

$I = Sh. 36{,}000$.

$R = 5\%$

$T = \frac{1}{2}$ year

$$I = \frac{PRT}{100}$$

$$36{,}000 = \frac{P \times 5 \times \frac{1}{2}}{100}$$

$$\frac{5}{2}P = 36000 \times 100$$

$$\frac{5p}{2} = 3600000$$

$$\frac{5p}{5} = \frac{2 \times 3{,}600{,}000}{5}$$

$$P = 1440{,}000$$

$\therefore$ You will have to lend Sh. 1,440,000.

(b) Nyanjara bought 50 bottles of milk for 70 children. If the capacity of each bottle is 300 *ml*, find the amount of milk in litres that Nyanjara bought.

1 bottle = 300 mL

50 bottles ?

$50 \times 300 = 15{,}000$ mL

1 L = 1000 mL

? 15,000 mL

$$\frac{15{,}000}{1000} = 15\text{ L}$$

$\therefore$ Nyanjara bought 15 Litres of milk.

Extract 3.2: A sample of correct responses to question 3

In Extract 3.2, part (a) demonstrates that the student correctly applied the formula for simple interest. In part (b), the student accurately converted the given metric units of volume to get the required answer.

2.4 Question 4: Geometry, Perimeters and Areas

This question had two parts, (a) and (b). In part (a), the students were required to find the size of the two angles which are complementary such that one angle is twice the other. In part (b), students were given a task to sketch an isosceles triangle with a perimeter of 15 cm and a base of length 7 cm, and then find the length for each of the remaining equal sides.

The data analysis reveals that a total of 693,419 (87.00%) students scored below 3 marks, indicating that the students' performance on this question was weak. However, 40,442 (5.07%) students scored 6.5 to 10 marks. Figure 4 summarizes the students' performance on question 4.

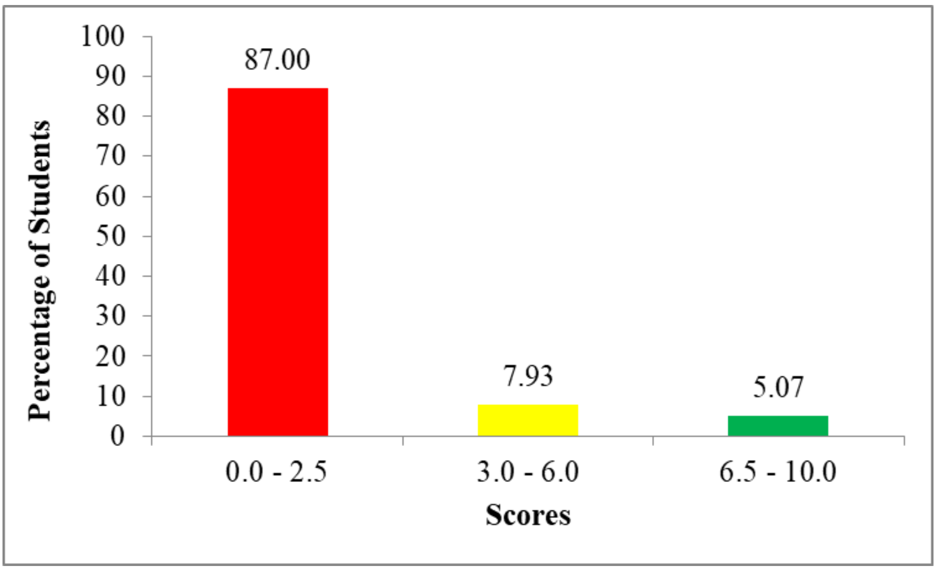


Figure 4: *Students' Performance on Question 4*

Further analysis shows that more than three quarters of the students (79.57%) scored zero on this question due to failure to provide the correct responses. In part (a), most of the students failed to recognize the properties of complementary angles. They formulated a wrong equation $2x + x = 180°$ instead of $2x + x = 90°$. This shows that the students had inadequate knowledge on the properties of angles. In part (b), the students failed to apply the properties of an isosceles triangle in formulating a relevant equation using the formulae for finding the perimeter of a triangle. Some students wrote incorrect formula for calculating the perimeter of triangle such as $a^2 + b^2 = c^2$, $P = 2(L + W)$, or $A = \frac{1}{2}bh$ instead of $P = L_1 + L_2 + L_3$, where L_1, L_2 and L_3 are the lengths of the sides of a triangle. Thus, they got incorrect answers. These responses highlight a lack of knowledge and skills on the basic concepts of perimeters and geometry. Extract 4.1 shows a sample of responses from a student who failed to answer this question correctly.

4. (a) Two complementary angles are such that, one angle is twice the other. Find the size of those angles.

Solution

let those angles be x and angle one be $2x$, angle 2nd be x

$2x + x = 180°$

$\frac{3x}{3} = \frac{180°}{3}$

$x = 60°$

$2x$ but $x = 60$

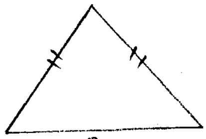
$2 \times 60° = 120°$

∴ The first angle 120° and the size of the second angle is 60°

(b) The perimeter of an isosceles triangle is 15 cm. If the base is 7 cm long, represent this information in a diagram and then find the length for each of the remaining equal sides.

Solution

Perimeter = 15cm
base = 7cm



7cm

Perimeter = $\frac{1}{2} \times h \times$ base

Perimeter = $\frac{1}{2} \times 7cm \times L$

$= \frac{7}{2} cm \times L$

$= \frac{14cm}{2} = \frac{2L}{2}$

$= 7cm$

∴ The other remaining equal side length 7cm

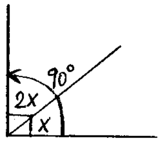
Extract 4.1: A sample of incorrect responses to question 4

In Extract 4.1, part (a), the student wrongly used the property of supplementary angles instead of complementary angles. In part (b), the student applied the formula for finding the area of the triangle instead of the formula for finding the perimeter. Hence, he/she performed irrelevant computations.

Further analysis shows that, 3.43 percent of the students answered this question correctly. In responding to part (a), the students applied the property of complementary angles to formulate the equation $x + y = 90°$ $y = 2x$ x and y to get $x = 30°$ and $y = 60°$

perimeter. That is, they formulated the equation $x + x + 7\,\text{cm} = 15\,\text{cm}$. Then, they solved for the value of x to get $x = 4\,\text{cm}$, which is the correct answer. Extract 4.2 shows a sample of responses from a student who answered this question correctly.

4. (a) Two complementary angles are such that, one angle is twice the other. Find the size of those angles.



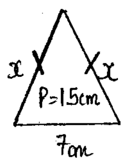
from:
Obeying the laws of angles.
Complementary angles form a total of 90° from the Point

$$2x + x = 90^\circ$$
$$\frac{3x}{3} = \frac{90^\circ}{3}$$
$$x = 30^\circ$$

first Angle = $x = 30^\circ$
Second Angle: $2x = 2 \times 30^\circ = 60^\circ$

$\therefore$ The size of Angles that form 90° is 60° and 30° respectively

(b) The perimeter of an isosceles triangle is 15 cm. If the base is 7 cm long, represent this information in a diagram and then find the length for each of the remaining equal sides.



from. Similarity.
The ratio of two corresponding sides in an Isosceles are equal to 1.
$x/x = 1$

Required to find x
from. Sum of Sides = Perimeter of triangle.
But.
P. triangle = $x + x + 7\,\text{cm}$
$15\,\text{cm} = 2x + 7\,\text{cm}$
$\frac{8\,\text{cm}}{2} = \frac{2x}{2}$
$x = 4\,\text{cm}$

$\therefore$ The length of each of the remaining Sides is 4cm

Extract 4.2: A sample of correct responses to question 4

In Extract 4.2, part (a), the student correctly used the property of complementary angles to calculate the measures of the two angles. In part (b), the student applied the perimeter formula along with the properties of an isosceles triangle to determine the lengths of its two equal sides.

2.5 Question 5: Algebra and Quadratic Equations

This question had two parts, (a) and (b). In part (a), students were required to solve for x in the equation $1-\frac{x+2}{2}=x-3$. In part (b), they were required to find a constant term that must be added to $n^2+1\frac{1}{2}n=0$ in order to make the equation a perfect square.

The data analysis reveals that, a total of 750,904 (94.21%) students scored 2.5 marks or less on this question, with 646,875 (81.16%) students scoring zero. These results indicate that the students' performance on this question was weak. A summary of the students' performance on question 5 is presented in Figure 5.

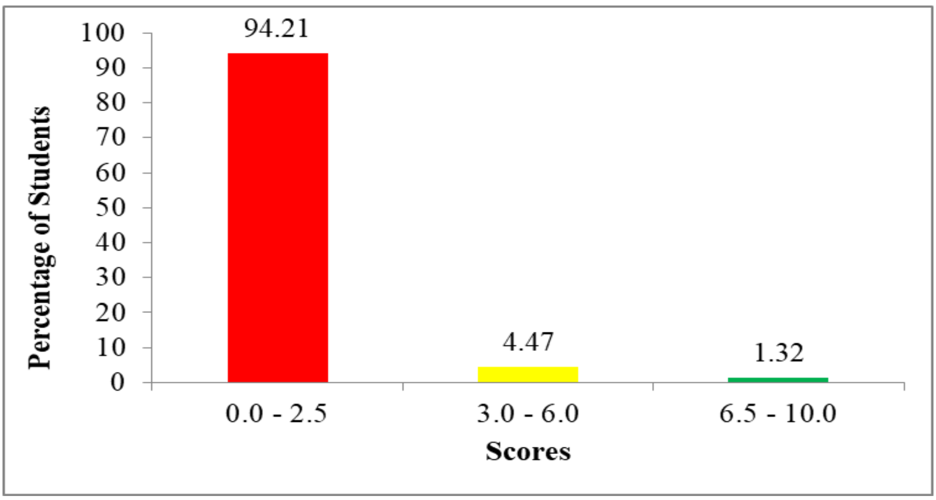


Figure 5: *Students' Performance on Question 5*

The analysis of responses suggests that several factors contributed to the students' inability to respond correctly to this question. In part (a), some students struggled to rearrange the equation to effectively collect like terms. Others were unable to express the left-hand side of the equation under a common denominator or perform cross-multiplication correctly. In part (b), majority of the students were unable to identify the condition for perfect squares. They wrote $b = 4ac$ instead of $b^2 = 4ac$ for an equation of the form $ax^2 + bx + c = 0$. Other students were unable to make proper substitution of the value of c. They substituted $\left(\frac{3}{2}\right)^2 = 4\times1\times0$ instead of $\left(\frac{3}{2}\right)^2 = 4\times1\times c$, which led to incorrect value of the constant c. These responses show a lack of mastery of the basic principles of algebraic and quadratic equations. Therefore, the learning outcomes on these concepts were not achieved. Extract 5.1 shows a sample of responses from one of the students who failed to answer this question correctly.

5. (a) Solve for x in the equation $1 - \frac{x+2}{2} = x - 3$.

Solution

$1 - \frac{x+2}{2} = x - 3$

$\frac{x+2}{2} \quad \frac{2x}{2}$

$1 - x = x - 3$

$x = -3 + 1x$

$\frac{1x}{1x} = \frac{4x}{1x}$

$x = \frac{4x}{1x}$

$x = 4$

the value of $x = 4$

(b) What term must be added to $n^2 + 1\frac{1}{2}n = 0$ in order to make the equation a perfect square?

Solution

$n^2 + 1\frac{1}{2} n = 0$

$n = b \pm \sqrt{b^2 - 4ac}$

$n = \frac{1\frac{1}{2} \pm \sqrt{1\frac{1}{2} - 4ac}}{2 \times 1}$

$n = \frac{1\frac{1}{2} \pm \sqrt{2\frac{2}{4} - 4 \times 0 \times 0}}{2 \times 1}$

$n = \frac{1\frac{1}{2} \pm \sqrt{\frac{10}{4} - 4}}{2}$

$n = \frac{1}{2} \pm \sqrt{\frac{6}{4}}$

$n = \frac{1}{2} \pm \sqrt{6} + \sqrt{4}$

Extract 5.1: A sample of incorrect responses to question 5

Extract 5.1, part (a) shows that the student failed to express $1 - \frac{x+2}{2}$ under a common denominator. In part (b), the student applied the quadratic formula to solve for the value of n contrary to the requirement of the question.

On the other hand, the analysis shows that 0.91 percent of the students answered this question correctly. Those students had gained sufficient knowledge and skills after learning the basic concepts of algebra and quadratic equations. In part (a), they managed to collect like terms and perform mathematical operations correctly to obtain $x = 2$. In part (b), the students were able to recall the condition for a quadratic equation to be a perfect square, that is $b^2 = 4ac$. They used this property to get the correct answer. Extract 5.2 shows a sample of responses from a student who answered this question correctly.

5. (a) Solve for x in the equation $1-\frac{x+2}{2}=x-3$.

Solution

$1-\frac{x+2}{2}=x-3$ [Collect like terms]

$1+3=x+\frac{x+2}{2}$

$4=\frac{x}{1}+\frac{x+2}{2}$

$4=\frac{2x+x+2}{2}$

$4=\frac{3x+2}{2}$ [Multiply by 2 both sides]

$4\times 2=3x+2$

$8=3x+2$

$8-2=3x$

$\frac{6}{3}=\frac{3x}{3}$ [Divide by 3 both sides]

$x=2$

$\therefore x=2$

(b) What term must be added to $n^2+1\frac{1}{2}n=0$ in order to make the equation a perfect square?

Solution

from: $n^2+1\frac{1}{2}n=0$

$a=1, b=1\frac{1}{2}, c=?$

from: $b^2=4ac$

$\left(\frac{3}{2}\right)^2=4(1)(c)$

$\frac{9}{4}=4c$

$c=\frac{9}{4}\div 4$

$c=\frac{9}{16}$

$\therefore \frac{9}{16}$ should be added so as to make the equation a perfect square.

Extract 5.2: A sample of correct responses to question 5

Extract 5.2, part (a) shows that the student had sufficient knowledge of solving linear equations. In part (b), the student clearly understood the requirement of the question and used the condition for an expression to be a perfect square to compute the constant term that must be added.

2.6 Question 6: Geometrical Transformations and Coordinate Geometry

This question had two parts, (a) and (b). In part (a), students were required to find the image of the point $(5, -1)$ under the same translation which moves the point $\mathrm{P}(-2, 2)$ into $\mathrm{P}'(0, 0)$. In part (b), students were assessed if they can find the equation of a straight line which passes through the point $(1, 2)$ and cuts the y-axis at the point $(0, 2)$.

The analysis indicates that, out of 681,165 (85.46%) students who scored below 3.0 marks, 573,973 (72.01%) students scored zero. Based on these scores, the students' performance on this question was weak. On the contrary, 10,119 (1.27%) students scored all the marks allotted to the question. Figure 6 summarizes the students' performance on question 6.

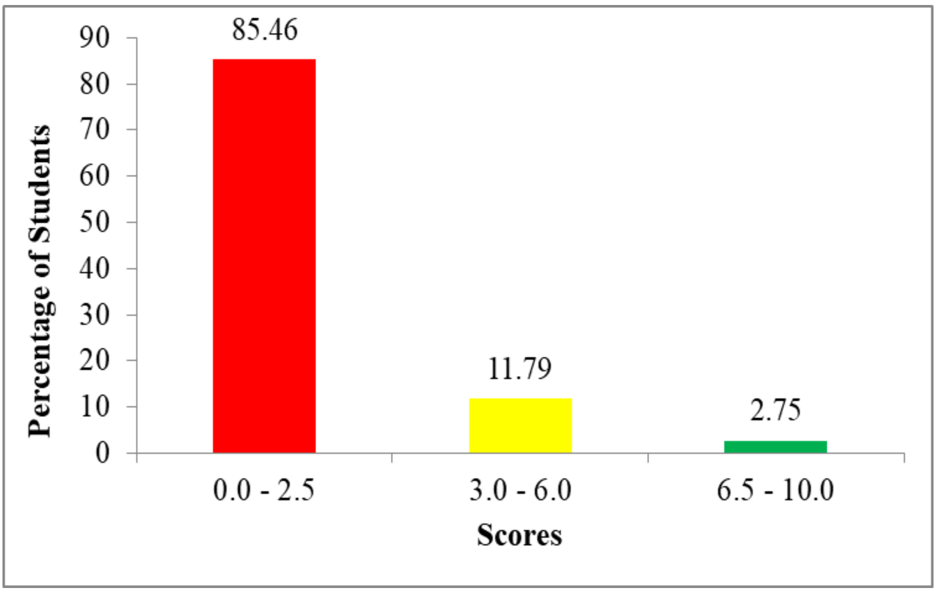
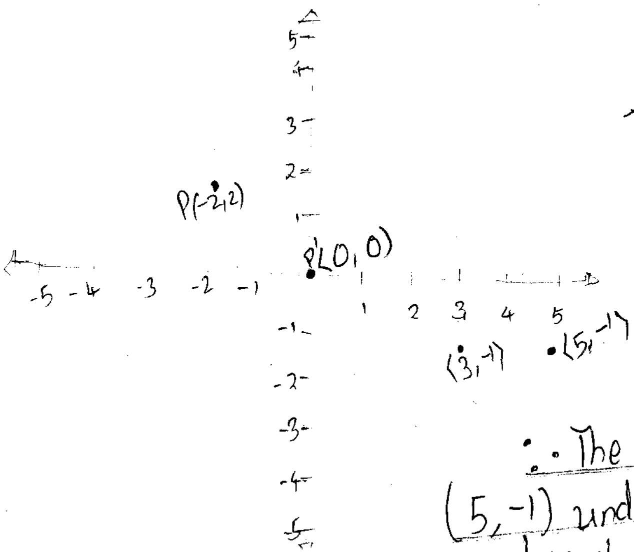


Figure 6: *Students' Performance on Question 6*

The analysis of responses shows that majority of students failed to answer this question correctly due to several reasons, including the following: in part (a), some students failed to write the correct formula for finding the image of a point under a translation. They wrote $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$ instead of $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$. Other students failed to get the correct translation factor $\begin{pmatrix} a \\ b \end{pmatrix}$, they got $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ instead of $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$. Moreover, there were students who failed to distinguish between the object and the image from the question. They interchanged the object and its image in the formula. For instance, they wrote $(2,-2)$ as an image instead of $(0,0)$. As a result, they got incorrect answers. This indicates that the students lacked knowledge of finding the image of a point under a translation.

In part (b), most of the students failed to write the correct formula for finding the gradient (slope) of a line. For instance, they wrote $m = \frac{x_2 - x_1}{y_2 - y_1}$, $m = \frac{y_2 + y_1}{x_2 + x_1}$ instead of $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$. This led to an incorrect gradient. The mistakes in the responses indicate that the students lacked knowledge and skills on the concept of geometrical transformations and coordinate geometry. Extract 6.1 shows a sample of responses from a student who failed to answer this question correctly.

6. (a) A point P' (0, 0) is the image of P (−2, 2) under a translation T. What is the image of the point $(5, -1)$ under the same translation?



$$\begin{array}{r} (5,-1) \\ +(-2,2) \\ \hline (3,-1) \end{array}$$

∴ The image of point (5, −1) under the same translation is (3, −1)

(b) If a straight line passes through the point (1, 2) and cuts the y-axis at the point (0, 2), find its equation.

Soln.

y-axis $x = 0$

$P_y(x, y) = P'(x, y)$

$P_y(x, y) = P'(-x, y)$

$P_y(1, 2) = P'(-1, 2)$

$\therefore P_y(1, 2) = P'(-1, 2)$

y-axis $x = 0$

$P_y(x, y) = P'(-x, y)$

$\therefore P_y(0, 2) = P'(0, 2)$.

Extract 6.1: A sample of incorrect responses to question 6

In Extrct 6.1, part (a), the student added the given points contarry to the requirement of the question. In part (b), the student did not understand the demand of the question since he/she used the concept of geometrical trannsformation instead of coordinate geometry.

Further analysis of the responses shows that 1.27 percent of the students performed well and scored full marks on this question. The students demonstrated good understanding of the basic concepts of geometrical transformations and coordinate geometry. In part (a), they managed to write the required formula for finding the image of the point under a translation and worked out all necessary calculations which led to the correct answer. This shows that the students had gained sufficient knowledge on geometrical transformation. In part (b), the students applied the correct formula to compute the gradient of the line to get $m = 0$. They used the obtained gradient to determine the equation of the line, that is $y = 2$. The reason attributed to their success was their competence on finding the gradient of the line to form the equation of the line using the formula $y = m(x - x_1) + y_1$. Extract 6.2 shows a sample of responses from a student who answered this question correctly.

6.	(a)	A point P' (0, 0) is the image of P (−2, 2) under a translation T. What is the image of the point $(5, -1)$ under the same translation?

Solution

Given,

Point $P'(0,0)$ image of $P(-2,2)$
(x', y') (x, y)

Required; image of point $(5,-1)$ under Translation $T(a,b)$

for translation,

from,

$$(x', y') = (x, y) + T(a, b)$$

$$T(a,b) = (x', y') - (x, y)$$

$$= (0,0) - (-2, 2)$$

$$= (2, -2)$$

for image of point $(5,-1)$,

$$(x', y') = (x, y) + T(a, b)$$

$$\text{image}(x', y') = (5, -1) + (2, -2)$$

$$= (7, -3)$$

$\therefore$ The image of point $(5,-1)$ is $(7,-3)$

	(b)	If a straight line passes through the point (1, 2) and cuts the y-axis at the point (0, 2), find its equation.

Solution

Given,

Points $(1, 2)$ and $(0, 2)$
x_1, y_1 $x_2\ y_2$

Required, equation of line.

from,

$$\text{Gradient, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-2}{0-1}$$

$$= \frac{0}{-1}$$

$$= 0$$

for equation of line,
Take $m = 0$ and point $(0, 2)$
x_1, y_1

from, $m = \frac{y_0 - y_1}{x_0 - x_1}$

$$\frac{0}{1} = \frac{y-2}{x-0}$$

$$0(x-0) = y-2$$

$$0 = y-2$$

$\therefore$ Its equation is $y - 2 = 0$

Extract 6.2: A sample of correct responses to question 6

In Extract 6.2, part (a), the student used the point P $(-2, 2)$ and its image, P' $(0, 0)$ to determine the translation factor and used it to find the image of the point $(5, -1)$. In part (b), the student applied the correct formula to determine the gradient of the line. He/she used the obtained gradient to find the equation of the line.

2.7 Question 7: Exponents, Radicals, and Logarithms

The question had two parts, (a) and (b). In part (a), students were given $x=\sqrt{3}$, $y=\sqrt{2}$ and $z(x-y)=2$. They were required to express z in the form of $z=a\left(\sqrt{b}+\sqrt{c}\right)$. In part (b), they were required to simplify the expression $\dfrac{\log 8-2\log 4}{\log 4-\log 2}$.

The data analysis indicates that 752,769 (94.44%) students scored below 3.0 marks, out of whom 658,233 (82.58%) students scored zero on this question. This confirms that the students' performance on this question was weak. Figure 7 summarizes the students' performance on question 7.

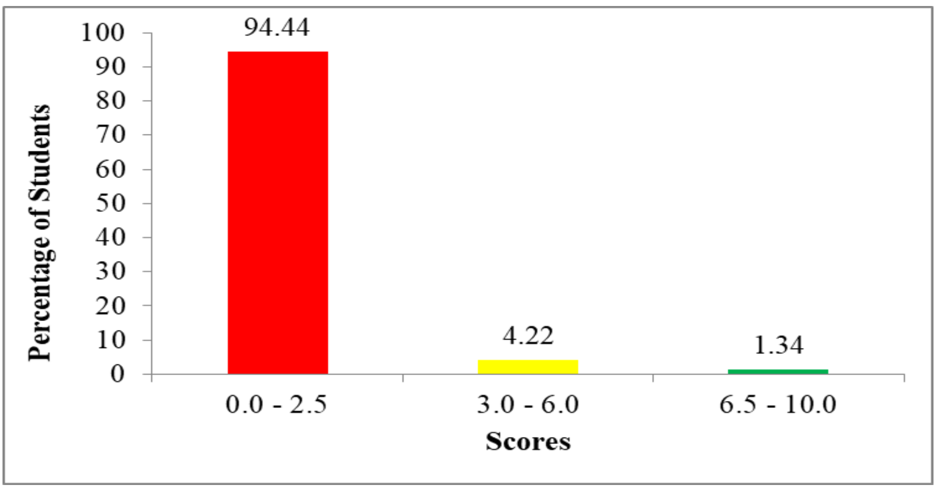


Figure 7: *Students' Performance on Question 7*

The analysis of the responses shows that the students who failed to answer this question had insufficient knowledge on the basic tenets of exponents, radicals and logarithms. In part (a), the students failed to write z in terms of the other variables. Furthermore, there were students who managed to express z in terms of other variables, but failed to choose a correct rationalizing factor. They used incorrect rationalizing factors such as $\sqrt{3}-\sqrt{2}$ instead of $\sqrt{3}+\sqrt{2}$. Thus, they wrote $z=\left(\frac{2}{\sqrt{3}-\sqrt{2}}\right)\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$ instead of $z=\left(\frac{2}{\sqrt{3}-\sqrt{2}}\right)\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$, which resulted to an incorrect answer.

In part (b), most of the students failed to apply the laws of logarithms to simplify the given expression. For example, some students applied the wrong formula $\log a-\log b=\frac{\log a}{\log b}$ instead of $\log a-\log b=\log\left(\frac{a}{b}\right)$. Others, wrote incorrect expressions like $\frac{\log 8}{\log 16}-\frac{\log 4}{\log 2}$ instead of $\log\left(\frac{8}{16}\right)\div\log\left(\frac{4}{2}\right)$. Moreover, there were students who applied an incorrect quotient such as $\left(\frac{3\log 2}{4\log 2}\right)\div\left(\frac{2\log 2}{\log 2}\right)$ which led to an incorrect answer. Extract 7.1 shows a sample of responses from a student who failed to answer this question correctly.

7. (a) If $x = \sqrt{3}$, $y = \sqrt{2}$ and $z(x-y) = 2$, express z in the form $z = a\left(\sqrt{b}+\sqrt{c}\right)$.

$x=\sqrt{3}$
$y=\sqrt{2}$
$z=?$
$z(x-y)=2$
$z(\sqrt{3}-\sqrt{2})=2$
$z=a(\sqrt{b}+\sqrt{c}$
$z=2\sqrt{2}-\sqrt{3}.$
$\therefore$ z inform of $z=a(\sqrt{b}+\sqrt{c}$ is $z=2\sqrt{2}-\sqrt{3}$

(b) Simplify $\frac{\log 8 - 2\log 4}{\log 4 - \log 2}$.

Solution
$\frac{\log 8 - 2\log 4}{\log 4 - \log 2}$
$\log_{10}(8-4)$
$\log_{10}4$
then
$2\log 4 - \log 2$
$\log_{10}(4^2-2)$
$\log_{10}(16-2)$
$\log_{10}14$
$\log_{10}14 - \log_{10}4$
$\log_{10}(14-4)$
$\log_{10}10$
$=1$ $\therefore$ $\frac{\log 8 - 2\log 4}{\log 4 - \log 2} = 1$

Extract 7.1: A sample of incorrect responses to question 7

Extract 7.1, part (a), the students made irrelevant substitutions and computations. In part (b), the student wrote $\log 8 - 2\log 4 = \log(4^2 - 2)$ instead of $\log 8 - 2\log 4 = \log\left(8 \div 4^2\right)$ due to failure to apply the quotient rule of logarithms.

Despite the weak performance on this question, the analysis of responses shows that 0.40 percent of the students answered this question correctly and scored full marks. The students demonstrated their competence on the basic concepts of exponents, radicals and logarithms. In part (a), the students transposed the formula correctly. They chose a suitable rationalizing factor and simplified the equation to get the required answer. In part (b), they applied the laws of exponents and logarithms to simplify the given expression correctly. Extract 7.2 shows a sample of responses from a student who managed to answer this question correctly.

7. (a) If $x=\sqrt{3}$, $y=\sqrt{2}$ and $z(x-y)=2$, express z in the form $z=a\left(\sqrt{b}+\sqrt{c}\right)$.

Solution

$$\frac{z(x-y)}{(x-y)}=\frac{2}{(x-y)}$$

$$z=\frac{2}{(x-y)}$$

where,

$x=\sqrt{3}$

$y=\sqrt{2}$

so,

$$z=\frac{2}{\sqrt{3}-\sqrt{2}}$$

The rationalizing factor is $\sqrt{3}+\sqrt{2}$

$$z=\frac{2}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$=\frac{2\sqrt{3}+2\sqrt{2}}{3+\sqrt{6}-\sqrt{6}-2}$$

$$=\frac{2\sqrt{3}+2\sqrt{2}}{1}$$

$$=2\sqrt{3}+2\sqrt{2}$$

$$=2(\sqrt{3}+\sqrt{2})$$

$\therefore z=2(\sqrt{3}+\sqrt{2})$

(b) Simplify $\frac{\log 8 - 2\log 4}{\log 4 - \log 2}$.

$$\frac{\log 8 - 2\log 4}{\log 4 - \log 2}$$

$$= \frac{\log 2^3 - 2\log 4}{\log 2^2 - \log 2}$$

$$= \frac{\log(8/16)}{\log(4/2)}$$

$$= \frac{\log \frac{1}{2}}{\log 2}$$

$$= \frac{\log(2)^{-1}}{\log 2}$$

$$= \frac{-1\cancel{\log 2}}{\cancel{\log 2}}$$

$$= -1$$

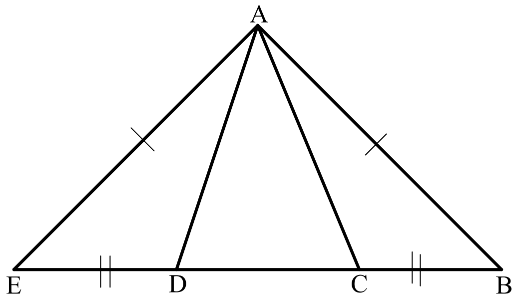
$$\therefore \frac{\log 8 - 2\log 4}{\log 4 - \log 2} = \underline{-1}$$

Extract 7.2: A sample of correct responses to question 7

In Extract 7.2, part (a), the student transposed the formula correctly and chose a relevant rationalizing factor to get the correct answer. In part (b), the student applied the laws of exponents and logarithms to simplify the given expression correctly.

2.8 Question 8: Congruence and Similarity

This question had two parts, (a) and (b). In part (a), the students were given two similar triangles ΔPQR and ΔLMN, where $M\hat{N}L = 40°$ and $Q\hat{P}R = 60°$. They were required to find the measure of angle $R\hat{Q}P$. In part (b), they were required to use the following figure to prove that $\Delta ABC \equiv \Delta AED$.



According to data analysis, 610,214 (76.56%) students scored below 3 marks, and among them 486,476 (61.03%) students scored zero. This indicates that the students' performance on this question was weak. In spite of weak performance, 20,617 (2.59%) students scored full marks on this question. Figure 8 summarizes the students' performance on question 8.

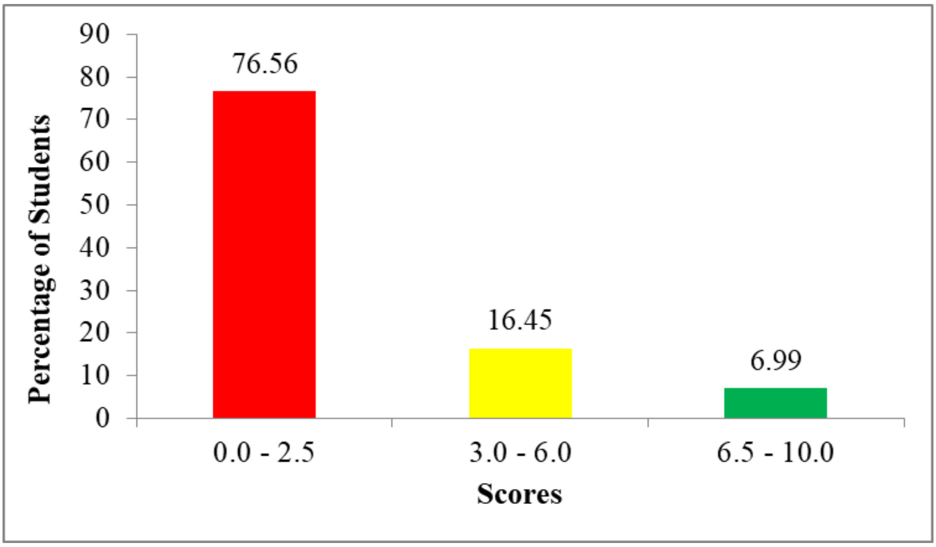


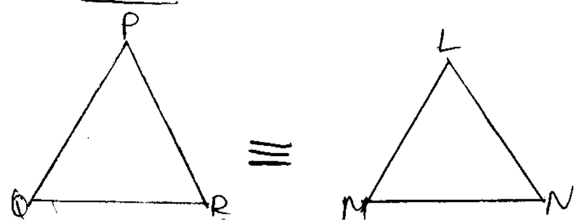
Figure 8: *Students' Performance on Question 8*

The analysis of responses shows that students who failed to answer this question correctly had insufficient knowledge about the concepts on congruence and similarity. In part (a), the students failed to use the given information to find the measure of angle $R\hat{Q}P$

$Q\hat{P}R = 60^\circ$ and $M\hat{N}L = 40^\circ$ to get $R\hat{Q}P = 100^\circ$, which is an incorrect answer. Other students subtracted the measure of the two angles, that is $60^\circ - 40^\circ = 20^\circ$ which was an incorrect answer. In part (b), the students failed to recall the properties of congruent figures. Thus, they failed to provide logical arguments to prove that $\Delta ABC \equiv \Delta AED$. There were students who wrote that $\frac{AB}{AE} = \frac{ED}{CB}$ instead of $\overline{AB} = \overline{AE}$ and $\overline{BC} = \overline{ED}$. Extract 8.1 shows a sample of responses from a student who failed to answer this question correctly.

8. (a) If ΔPQR and ΔLMN are similar, find $R\hat{Q}P$ given that $M\hat{N}L = 40^\circ$ and $Q\hat{P}R = 60^\circ$.

Soln



$40^\circ + 60^\circ$

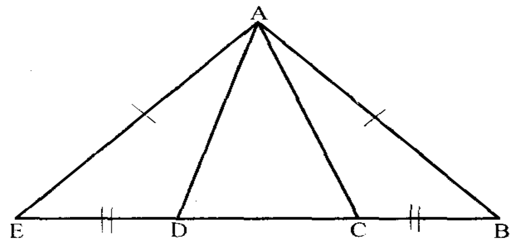
100°

$100^\circ \div 2$

$$\begin{array}{r} 50 \\ 2\overline{)100} \\ -10 \\ \hline 0 \\ -0 \\ \hline \end{array}$$

$\therefore R\hat{Q}P = 50^\circ$

(b) Use the following figure to prove that $\Delta ABC \equiv \Delta AED$:



$\Delta ABc \equiv \Delta AED$

Consider

• $\Delta AB = \Delta AE$ {given}

• $\Delta Bc = AED$ {given}

• $\Delta AD = \Delta Ac$ {Common}

Hence shown

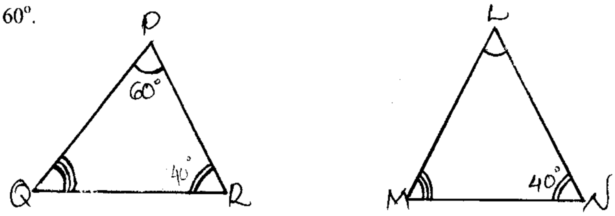
$\Delta ABc \equiv \Delta AED$

Extract 8.1: A sample of incorrect responses to question 8

In Extract 8.1, part (a), the student calculated the average of the two angles contrary to the requirement of the question. In part (b), the student provided illogical arguments and failed to use a suitable postulate to prove the congruence of the two triangles in the given figure.

On the other hand, the analysis of responses shows that the students who performed well on this question had sufficient knowledge on the concepts of congruence and similarity. In part (a), they used the properties of similar triangles to calculate the measure of angle $R\hat{Q}P$. They were able to recall that the sum of interior angles of a triangle is $180°$. In part (b), the students provided relevant and logical arguments to prove the congruence of the two triangles. They either used the Side-Side-Side (SSS) postulate or Side-Angle-Side (SAS) postulate to make their conclusion that the two triangles are congruent. Extract 8.2 shows a sample of responses from a student who answered this question correctly.

8. (a) If ΔPQR and ΔLMN are similar, find $R\hat{Q}P$ given that $M\hat{N}L = 40^{\circ}$ and $Q\hat{P}R = 60^{\circ}$.



Since the triangles are similar

$P\hat{R}Q = L\hat{N}M = 40^{\circ}$

Data given.

$Q\hat{P}R = 60^{\circ}$

$P\hat{R}Q = 40^{\circ}$

$R\hat{Q}P = ?$

$Q\hat{P}R + P\hat{R}Q + R\hat{Q}P = 180^{\circ}$

$60^{\circ} + 40^{\circ} + P\hat{Q}P = 180^{\circ}$

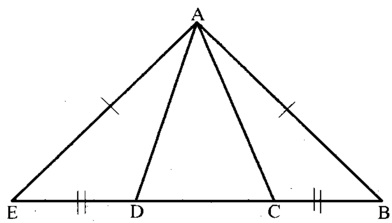
$100^{\circ} + R\hat{Q}P = 180^{\circ}$

$R\hat{Q}P = 180^{\circ} - 100^{\circ}$

$R\hat{Q}P = 80^{\circ}$

$\therefore R\hat{Q}P = 80^{\circ}$

(b) Use the following figure to prove that $\Delta ABC \equiv \Delta AED$:



Required to prove that $\Delta ABC \equiv \Delta AED$.

$\overline{AB} = \overline{AE}$ - - (given)

$\overline{CB} = \overline{DE}$ - - - (given)

$A\hat{B}C = A\hat{E}D$ (angles on the base line of the isosceles triangle)

$\therefore \Delta ABC \equiv \Delta AED$

Hence proved!

(by SAS postulate)

Extract 8.2: A sample of correct responses to question 8

Extract 8.2, part (a), the student recognized the corresponding angles of two similar triangles. He/she managed to determine the measure of angle $R\hat{Q}P$. In part (b), the student used the Side-Angle-Side (SAS) postulate to write a logical and correct proof.

2.9 Question 9: Pythagoras' Theorem and Trigonometry

This question consisted of two parts, (a) and (b). In part (a), the students were given a real life word problem which states that "A student walks from home to school, first eastwards to a road junction 14 km from home, then southwards to school. If the shortest distance from home to school is 20 km", how far is the school from the road junction? (Express your answer to three decimal places). In part (b), students were required to find the value of $2\sin 60° + \cos 30° - \tan 60°$, giving the answer in radical form.

The data analysis shows that 718,114 (90.10%) scored 2.5 marks or less, and out of them 642,962 (80.67%) students scored zero, indicating that the students' performance on this question was weak. However, 4,060 (0.51%) students scored full marks. The students' performance on question 9 is summarized in Figure 9.

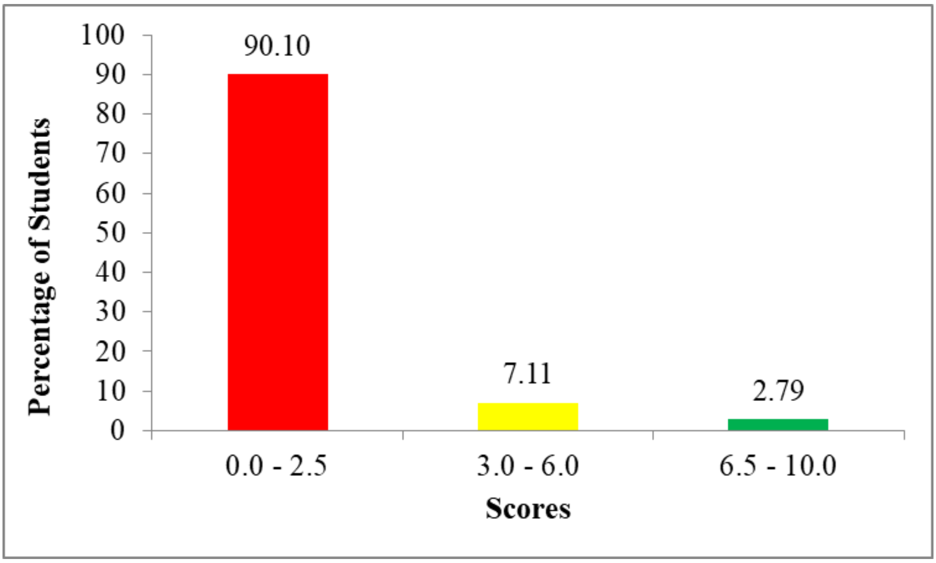
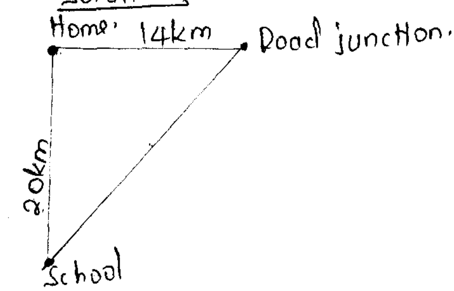


Figure 9: *Students' Performance on Question 9*

The analysis of responses shows that the students who failed to answer this question lacked sufficient competence on the basic concepts of Pythagoras' theorem and trigonometry. In part (a), majority of students labelled the measurement in the diagram wrongly. Thus, they were not able to represent correctly the given information in a diagram. Some students lacked adequate knowledge and skills of finding the square roots of a number which led to incorrect answers. In part (b), most of the students demonstrated inadequate skills on calculating the values of special angles. For example, they wrote $2\sin\frac{\sqrt{3}}{2}+\cos\frac{1}{2}-\tan\frac{\sqrt{3}}{2}$ instead of $2\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}-\sqrt{3}$. It was also noted that, most of them lacked skills of performing mathematical operations on radicals. Thus, they made mistakes in their computations. Extract 9.1 shows a sample of responses from a student who failed to answer this question correctly.

9. (a) A student walks from home to school, first eastwards to a road junction 14 km from home, then southwards to school. If the shortest distance from home to school is 20 km, how far is the school from the road junction? Express your answer correct to 3 decimal places.

Solution:



From,

$a^2 + b^2 = c^2$

$14^2 + 20^2 = c^2$

$196 + 400 = c^2$

$\sqrt{596} = \sqrt{c^2}$

$c = 24.4131$

$\therefore$ From school to road junction is 24.413 km.

(b) Find the value of $2\sin 60^0 + \cos 30^0 - \tan 60^0$. **Give your answer in radical form.**

soln.

$2\sin 60^\circ + \cos 30^\circ - \tan 60^\circ$

$2(0.8660) + 0.8660 - 1.7321$

$1.7320 + 0.8660 - 1.7321$

$1.7320 + 0.8660 - 1.7321$

$= 0.8659$

Extract 9.1: A sample of incorrect responses to question 9

Extract 9.1, part (a) shows that the student failed to interpret the word problem. He/she labelled the diagram incorrectly. This mistake consequently propagated in his/her response. In part (b), the student

$\sin 60°$, $\cos 30°$ and $\tan 60°$ instead of writing the exact values in radical form.

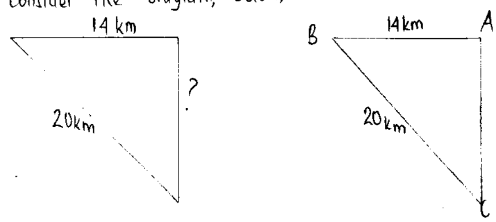
On the contrary, the analysis of responses shows that the students who answered the question correctly had gained sufficient competence on the basic principles of Pythagoras' theorem and trigonometry. In part (a), the students were able to present the information in a diagram and applied the Pythagoras' theorem. In part (b), the students recognised the values of the special angles given in the expression. They substituted the values in the expression $2\sin 60° + \cos 30° - \tan 60°$ and performed the basic operations on radicals, which led to the correct answer. Extract 9.2 shows a sample of responses from a student who answered this question correctly.

9. (a) **A student walks from home to school, first eastwards to a road junction 14 km from home, then southwards to school. If the shortest distance from home to school is 20 km, how far is the school from the road junction? Express your answer correct to 3 decimal places.**

Solution

From the data given,

consider the diagram, below,



from pythagoras theorem,

$\overline{AC}^2 + \overline{AB}^2 = \overline{BC}^2$

$\overline{AC}^2 = \overline{BC}^2 - \overline{AB}^2$

$\overline{AC}^2 = (20km)^2 - (14km)^2$

$\overline{AC}^2 = 400km^2 - 196km^2$

$\overline{AC}^2 = 204km^2$

$\sqrt{\overline{AC}^2} = \sqrt{204\ km^2}$

$\overline{AC} = 14.28 km$

$\approx 14.280 km$

∴ The school is 14.280km from the road junction.

(b) Find the value of $2\sin 60^0 + \cos 30^0 - \tan 60^0$. Give your answer in radical form.

Solution

Required to find the value of $2\sin 60^\circ + \cos 30^\circ - \tan 60^\circ$

where,

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 60^\circ = \sqrt{3}$

so,

$$2\sin 60^\circ + \cos 30^\circ - \tan 60^\circ = 2 \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \sqrt{3}$$

$$= \sqrt{3} + \frac{\sqrt{3}}{2} - \sqrt{3}$$

$$= \frac{\sqrt{3}}{2}$$

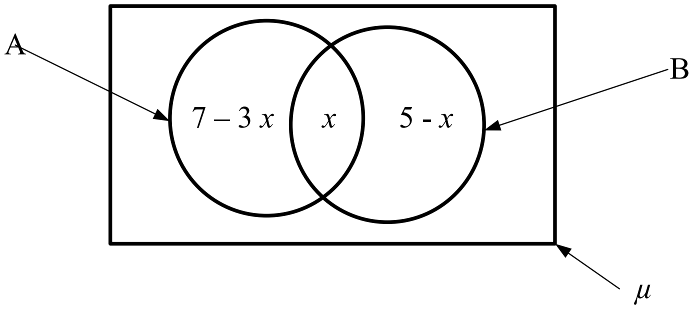
$$\therefore 2\sin 60^\circ + \cos 30^\circ - \tan 60^\circ = \frac{\sqrt{3}}{2}$$

Extract 9.2: A sample of correct responses to question 9

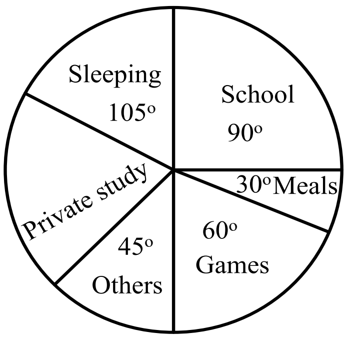
Extract 9.2, part (a), the student was able to present the word problem in a diagram. He/she applied correctly the Pythagoras' theorem to get the correct answer. In part (b), the student substituted the correct values of the special angles in the given expression. He/she had relevant skills on operations of radicals.

2.10 Question 10: Sets and Statistics

This question had two parts, (a) and (b). In part (a), students were given the Venn diagram below which shows the number of elements in each of the two joint sets, A and B, where the number of elements in A is equal to the number of elements in B. They were required to find $n(A \cup B)$ and $n(A)$.



In part (b), students were given the pie chart below which represents the time spent by John in doing different activities on every Monday. They were required to determine: (i) the number of hours he spends for private study and (ii) the number of hours he sleeps.



The data analysis reveals that, a total of 684,039 (85.82%) students scored less than 3.0 marks, and among them 487,257 (61.13%) students scored zero. This shows that the students' performance on this question was weak. Despite the weak performance, 5,882 (0.74%) students scored full marks on this question. The summary of the students' performance on question 10 is indicated in Figure 10.

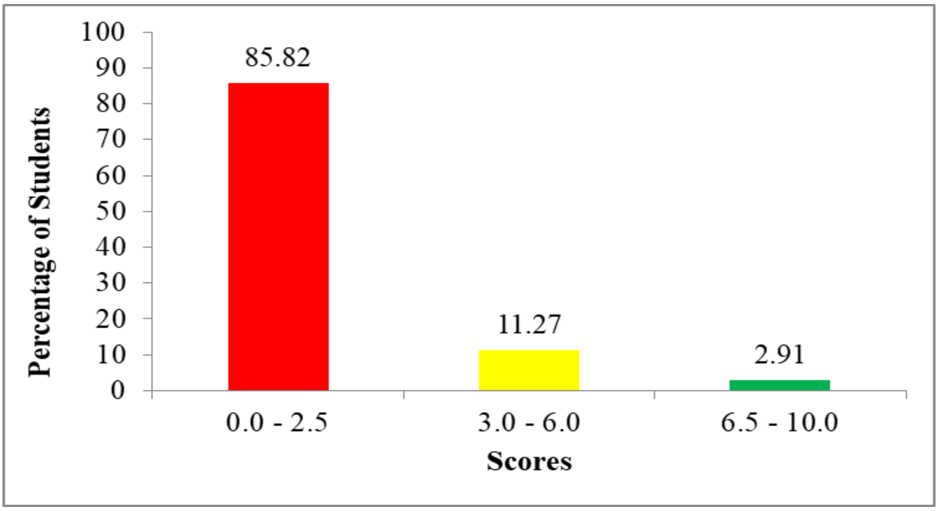


Figure 10: *Students' Performance on Question 10*

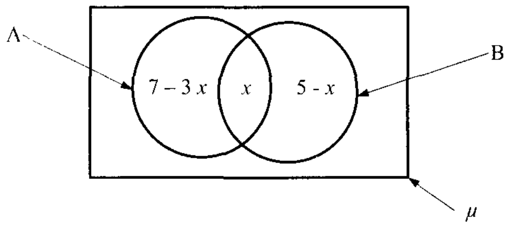
The analysis of responses shows that, the students who failed to answer the question correctly lacked the knowledge on basic concepts of sets theory and statistics. In part (a), they could not represent the given information in an algebraic equation. They used a wrong expression $n(A)+n(B)-n(A\cap B)+n(A\cup B)$ to get an incorrect answer. Furthermore, some students used a wrong formula for the number of elements in $A\cup B$ such as $n(A)+n(B)-n(A\cap B)+n(A\cup B)$ instead of $n(A\cup B)=n(A)+n(B)-n(A\cap B)$. Moreover, there were students who wrote the number of elements in set A as $7-3x+x+5-x$ instead of $7-3x+x$. Others wrote the number of elements in set $A\cup B$ as $7-3x+x+5-x+x$ instead of $7-3x+x+5-x$. This indicates that the students have not acquired the intended competences after learning the concepts of sets and statistics.

In part (b), some students failed to calculate the angle of the sector representing private study and made wrong interpretation of information

$1° = 15\,\text{min}$,

$\frac{30°}{360°} \times 60\,\text{min}$, or $\frac{105°}{360°}$ by 12 hours instead of $1 \text{ day} = 24 \text{ hours}$; $\frac{30°}{360°} \times 24 \text{ hours}$. Extract 10.1 shows a sample of responses from a student who failed to answer this question correctly.

10. (a) The following figure shows the number of elements in each subset. If the number of elements in set A is equal to the number of elements in set B, find n(A ∪ B) and n(A).



Soln.

$n(A \cup B) = n(A)$

$n(A \cup B) = 7-3x+5+x+x.$

$n(A \cup B) = 7-3x+5+x+x\ .$

$n(A \cup B) = 7-5+3x+x+x.$

$n(A \cup B = \frac{14}{5} = \frac{5x}{5}$

$x = 2.4$

Answer $n(A \cup B) = \cancel{2.4}\ 24$

Answer $n(A \cup B) = 24$

Soln.

$n(A)$

~~14 + 2 + 704 + 204~~

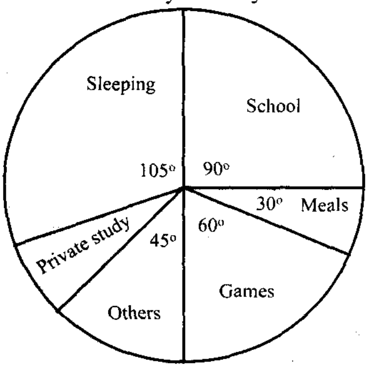
~~21 + 6 - 2.4 = 24~~

$n(A) = 7 - 3x$

$n(A) = 7 - 3 \times 24 = 79$

Answer $n(A) = 79$

(b) The given Pie chart represents the time spent by John in doing different activities on every Monday.



(i) How many hours does he spend for private study?

$105° + 90° + 30° + 60° + 45° + x = 360°$

$330 + x = 360$

$x = 360 - 330$

$x = 130°$

$\Rightarrow 130°$ ~~hours~~

$360° = 12\text{hr}$

$130° = x$

$\frac{360° \times x}{360°} = \frac{130° \times 12\text{hr}}{360°}$

$\therefore 4\frac{3}{4}$ hours does he spend for private study

(ii) How many hours does he sleep?

Soln

$\frac{105}{360} \times 12hr.$

$= 3.5hr$

$3\frac{1}{2}$ hrs.

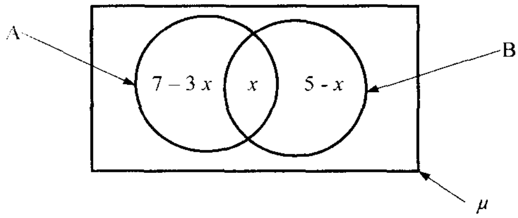
$\therefore$ 3.5hr does he sleep.

Extract 10.1: A sample of incorrect responses to question 10

In Extract 10.1, part (a), the student failed to recall and apply the properties of equal sets to form an algebraic equation. In part (b), the student failed to perform the conversion of units of time.

Although the performance was weak, analysis of responses shows that 0.73 percent of the students answered this question correctly because they were able to apply the competences gained after learning the basic concepts of sets and statistics. In part (a), they were able to formulate an algebraic equation using the information given in the question, that is $n(A)=n(B)$ such that $7-3x+x=x+5-x$. They solved for the value of x to get $x=1$. Finally, they substituted the value of x to get $n(A)=5$ and $n(A\cup B)=9$, which are the correct answers. In part (b), students were able to compute the degrees for private study and obtained $30°$. They converted the fractions of $30°$ and $105°$ into hours correctly. Extract 10.2 shows a sample of responses from a student who answered this question correctly.

10. (a) The following figure shows the number of elements in each subset. If the number of elements in set A is equal to the number of elements in set B, find n(A ∪ B) and n(A).



Solution

Given that,

number of elements in set A = number of elements in set B

Required: n(A∪B) and n(A)

from,

number of elements in set A = number of elements in set B

$$n(A) = n(B)$$

$$(7-3x)+x = (5-x)+x$$

$$7-3x+x = 5-x+x$$

$$7-2x = 5-0$$

$$7-2x = 5$$

$$7-5 = 2x$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$x = 1$$

So, number of elements in set A, n(A)

$$= 7-3x+x$$

$$= 7-3(1)+1$$

$$= 7-3+1$$

$$= 7-2$$

$$= 5$$

and, $n(A \cup B) = 7-3x+x+5-x$

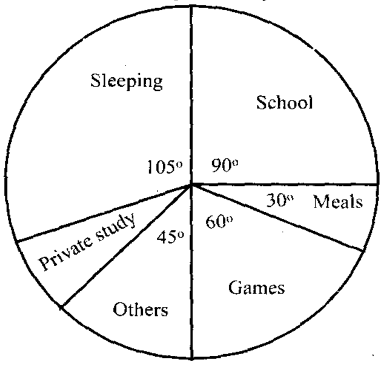
$$= 7-3(1)+1+5-1$$

$$= 7-3+1+5-1$$

$$= 9$$

$\therefore$ $n(A \cup B) = 9$ and $n(A) = 5$

(b) The given Pie chart represents the time spent by John in doing different activities on every Monday.



(i) How many hours does he spend for private study?

Solution

from the pie chart.

Private study $= 360° - (105° + 90° + 30° + 60° + 45°)$

$= 360° - 330°$

Private study $= 30°$

But,

1 day $= 24$hrs

So,

private study $= \frac{30°}{360°} \times 24$ hrs

$= \frac{1}{12} \times 24$hrs

$= \frac{24\text{hrs}}{12}$

$= 2$hrs

$\therefore$ He spends 2 hours for private study.

(ii) How many hours does he sleep?

Solution

Sleeping: 105°

But,

1 day = 24 hrs

So,

Sleeping $= \frac{105^\circ}{360^\circ} \times 24$ hrs

$= \frac{105}{360} \times 24$ hrs

$= \frac{2520}{360}$ hrs

$= 7$ hrs

$\therefore$ He sleeps for 7 hours

Extract 10.2: A sample of correct responses to question 10

Extract 10.2, part (a), shows that the student interpreted the information correctly and formulated the corresponding algebraic equation. He/she was able to solve the equation to get the correct answers. In part (b), the student analysed the pie chart properly. He/she performed the relevant conversions to get the required number of hours in all the items.

3.0 ANALYSIS OF STUDENTS' PERFORMANCE ON EACH TOPIC

The Basic Mathematics assessment paper consisted of 10 questions which were set from 20 topics. The analysis shows that only two (2) topics of *Numbers* and *Approximations* had an average performance of 45.54 percent. The remaining 18 topics had a weak performance ranging from 27.15 to 5.56 percent. The topics include: *Fractions*, *Decimals* and *Percentages* (27.15%), *Congruence* and *Similarity* (23.44%), *Units*, *Ratios*, *Profit* and *Loss* (21.72%), *Geometrical Transformations* and *Coordinate Geometry* (14.54%), *Sets* and *Statistics* (14.18%), *Geometry*, *Perimeters* and *Areas* (13.00%), *Pythagoras' Theorem* and *Trigonometry* (9.90%), *Algebra* and *Quadratic Equations* (5.79%) as well as *Exponents*, *Radicals* and *Logarithms* (5.56%). The Appendix on page 52 summarizes the students' performance on each topic.

4.0 CONCLUSION

The analysis reveals that 18.85 percent of the students passed the Basic Mathematics assessment in FTNA 2024. It was further noted that 45.54 percent of the students had the highest performance in the topics of *Numbers* and *Approximations* in 2024 Basic Mathematics assessment while 33.4 percent of the students had the highest performance on *Trigonometry* and *Pythagoras' theorem* in 2023 Basic Mathematics assessment.

However, a total of 16 topics had constantly been poorly performed in both 2024 and 2023. Those topics are; *Fractions*, *Decimals and Percentages*, *Congruence*, *Similarity*, *Units*, *Ratio, Profit and Loss*, *Geometrical Transformations*, *Coordinate Geometry*, *Sets*, *Statistics*, *Geometry*, *Perimeters and Areas*, *Algebra*, *Quadratic Equations*, *Exponents and Radicals*, and *Logarithms*. This report highlighted the main factors which contributed to weak performance in these topics and recommended suitable measures to achieve the intended learning outcomes highlighted in the syllabus based on the role of teachers and students in the teaching and learning process of Basic Mathematics subject.

5.0 RECOMMENDATIONS

The analysis shows that the students' performance was generally weak because 18 out of 20 topics had weak performance in Basic Mathematics assessment. Based on the findings, the following are recommended in order to improve the students' performance in future assessments:

(a) On the topic of Numbers, teachers should guide learners in finding factors, common factors, multiples and common multiples of numbers by using a factor tree diagram, factor chart and number chart. Teachers should design relevant activities that reflect the use of factors and multiples in real life. Also, they should give learners home works, exercises and tests. Learners are advised to do the exercises, especially in the textbooks in order to master the intended concepts.

(b) Teachers should demonstrate and guide students on how to do approximations on numbers by means of number charts, manila paper and other available materials. Teachers should prepare hands-on activities that show the application of approximations in everyday life.

(c) On the topic of fractions, teachers should choose appropriate teaching and learning methods using resources such as oranges and a bar of soap in guiding learners on how to do basic operations on fractions. They should also elaborate well the procedures of converting recurring decimals into fractions through well designed activities.

(d) For learners to gain the competences on the topics of units, ratios, profit and loss, teachers should elaborate well the concepts of units especially conversion of units of time and capacity using clocks, timetables, bottles of litres and other containers. In the case of ratios, profit and loss, they should use physical items like bank statements, money and work sheets to demonstrate the concept of simple interest through appropriate activities that reflect applications in real life situation.

(e) Teachers should use teaching and learning resources such as beam balance, coloured chalks and work sheets to design relevant activities to guide students to solve different algebraic equations involving fractions. On quadratic equations, they should use multiplication charts, factor trees and coloured chalks to demonstrate perfect squares. Also, teachers should enlighten learners the use of quadratic equations in different fields and in everyday life.

(f) In teaching geometrical transformations and coordinate geometry; teachers should use graph papers, manila papers, rubber bands, geo boards, mathematical sets and other related materials to design relevant learning activities that elaborate basic concepts on

xy-plane. They should also guide learners in groups to use graph papers, geo boards and rubber bands to find slope and equations of straight lines. Also, they should provide activities that reflect learners' experiences on the applications of these concepts.

(g) Teachers should apply participatory methods in guiding students on how to simplify radicals, perform basic operations on radicals and rationalizing the denominator as well as transposition of formulae using suitable hands-on activities and resources such as Manila paper, Multiplication tables and Mathematical formulae. Also, on logarithms, they should guide students to discuss laws of logarithms and applying the knowledge of exponents in solving different real life problems.

(h) On Congruence and Similarity, teachers should design learning activities using models and real objects to demonstrate conditions for congruence of triangles, illustrate logical proofs of congruence of triangles and applying theorems of congruence of triangles. Resources such as geometrical figures, manila papers, mathematical sets, photographs, paper cuttings as well as marker pens can be used to achieve the intended competence. They should also use mirror, similar objects and other related resources to demonstrate on how to identify similar polygons to prove theorems and solve problems related to similarity.

(i) On Pythagoras' Theorem and Trigonometry, teachers should guide students on how to illustrate, prove and solve problems using flat objects in shape of right angled triangle, manila paper, marker pens square cuttings, square root tables and square tables by applying knowledge of Pythagoras' Theorem. They should also demonstrate on how to find and use the special angles by employing the knowledge of Trigonometry. In this case, it is important for teachers to design activities that correspond to learners' day to day experience on the application of these topics.

(j) On the topic about Sets, teachers should demonstrate to students on how to list the members, determine number of elements and identify types of sets. Also, they should give relevant activities and examples on operation on sets and interpretation of information from Venn diagrams using real objects such as playing cards, pictures, dinner sets and other physical available resources that are commonly used in real life situations.

(k) On statistics, teachers should prepare hands-on activities and demonstrate how to use resources such as protractors, rulers, pair of compasses and coloured chalks to represent and interpret statistical data using graphs, charts and tables.

***APPENDIX*: Analysis of Students' Performance Per Topic – FTNA 2024**

S/N	Topics	Question Number	Percentage of Students who Scored 30 or Above	Remarks
1	Numbers and Approximations	1	45.54	Average
2	Fractions, Decimals, and Percentages	2	27.15	Weak
3	Congruence and Similarity	8	23.44	Weak
4	Units, Ratios, Profit and Loss	3	21.72	Weak
5	Geometrical Transformations and Coordinate Geometry	6	14.54	Weak
6	Sets and Statistics	10	14.18	Weak
7	Geometry, Perimeters and Areas	4	13.00	Weak
8	Pythagoras' Theorem and Trigonometry	9	9.90	Weak
9	Algebra and Quadratic Equations	5	5.79	Weak
10	Exponents, Radicals, and Logarithms	7	5.56	Weak