



THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**CANDIDATES' ITEM RESPONSE ANALYSIS
REPORT ON THE DIPLOMA IN SECONDARY
EDUCATION EXAMINATION (DSEE) 2023**

MATHEMATICS



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740 MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is delighted to issue this report on Candidates' Item Response Analysis (CIRA) for Diploma in Secondary Education Examination (DSEE) 2023. This report has been prepared to deliver feedback to tutors, students, policymakers, and other educational stakeholders about the candidates' performance in Mathematics subject.

The report highlights the factors that made candidates to perform well in this examination. The factors include the ability to interpret the demand of the questions and follow the instructions as well as sufficient knowledge about the concepts and principles related to the subject. The report indicates that some of the candidates scored low marks because they failed to interpret the requirement of the questions and lacked sufficient knowledge and skills about the mathematical concepts examined, made errors while performing mathematical operations, failed to use relevant formulae, and the use of incorrect formulae.

The feedback provided in this report will serve as a basis for educational stakeholders to act effectively to improve teaching and learning in this subject. This will ultimately improve the candidates' performance in future examinations.

Finally, the National Examinations Council of Tanzania would like to extend sincere gratitude to everyone who participated in the preparation of this report.



Dr. Said A. Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report delivers the candidates' responses in Mathematics subject for the candidates who sat for the Diploma in Secondary Examination 2023. It gives a relevant feedback to educational stakeholder on the strengths and weaknesses of candidates' performance. A total of 528 candidates were registered in the 2023 DSEE in Mathematics subject out of which 523 (99.1%) candidates sat for the Examination.

The paper had a total of fourteen (14) questions separated into two sections, A and B. Section A consisted of 10 short answer questions, where candidates were required to answer all questions. Each correct answer had 4 marks, making a total of 40 marks. Section B consisted of four (4) essay questions and candidates were required to answer all questions from this section; each correct answer had 15 marks, making a total of 60 marks.

The analysis on the performance for each question in section A is categorised in three groups of scores, namely; 3-4 marks; good marks, 2-2.5 marks; average marks; and 0-1.5 marks; weak marks. In section B, the performance analysis for each question is also categorised in three groups of scores as follows: 10.5-15 marks; good marks, 6-10 marks; average marks; and 0-5.5 marks; weak marks. In addition, the analysis of performance was divided into three groups of intervals, which are 70%-100%, 40%-69%, and 0%-39% to represent good, average, and weak performance, respectively.

The analysis on candidates' responses to each question was prepared using data, figures, and extract of the sample of answers from the candidates. The figures of analysis on performance presented in this report used three colours to depict the performance as follows: green represents good performance, yellow represents average performance, and red represents weak performance.

2.0 ANALYSIS OF CANDIDATES' RESPONSES IN EACH QUESTION

2.1 Section A: Short Answer Questions

2.1.1 Question 1: Calculating Devices

The candidates were required to: (a) find the value of a and (b) use a non-programmable calculator to find the mean and standard deviation of the scores from the following data of 80 students:

| | | | | | | | | |
|--------------------|------|------|------|------|------|------|------|------|
| Class Marks | 90.5 | 80.5 | 70.5 | 60.5 | 50.5 | 40.5 | 30.5 | 20.5 |
| Frequency | 4 | 17 | 16 | 8 | a | 7 | 12 | 3 |

This question assessed candidates' ability to use non-programmable calculator as one of calculating devices.

A total of 523 (100%) candidates attempted this question, where 404 (77.2%) candidates had scores ranging from 2 to 4 marks. Therefore, the general performance of candidates in this question was good. Figure 1 shows the performance of candidates in this question.

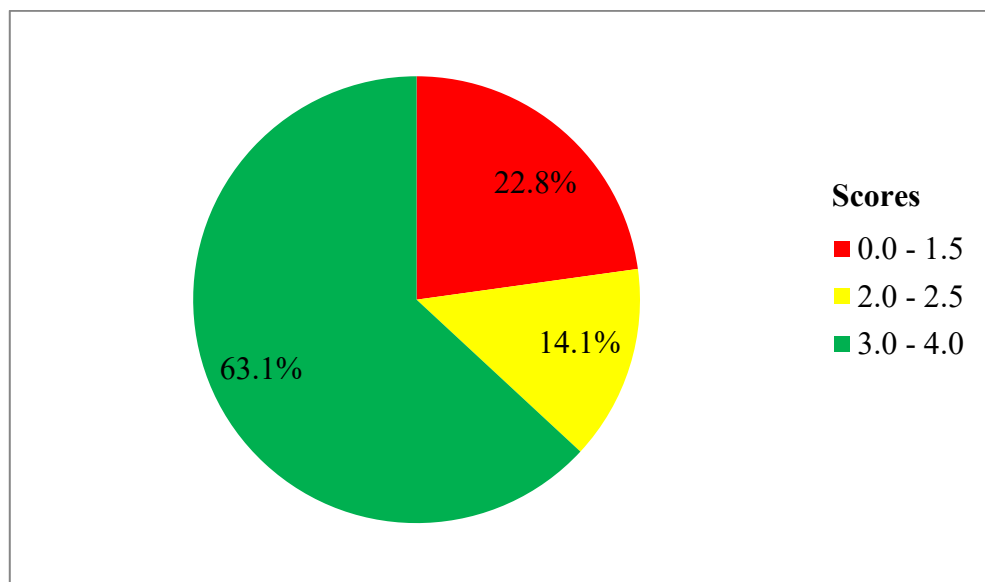


Figure 1: Performance of candidates in question 1

The data reveal further that, the candidates who correctly responded to this question realized that the summation of frequencies was equal to the number of students who sat for the test. That is, $4 + 17 + 16 + 8 + a +$

$7 + 12 + 3 = 80$, hence they obtain the value of a that was used to find the mean and standard deviation as shown in Extract 1.1.

| | |
|---|--|
| 1 | soln. |
| | a) |
| | $4+17+16+8+a+7+12+3=80$ |
| | $67+a=80$ |
| | $a+67-67=80-67$ |
| | $a=13$ |
| | Therefore the value of $a=13$. |
| | b) i) Mean = $58.875 \approx 58.9$. |
| | \therefore Mean of the score is 58.9 . |
| | ii) Standard deviation = 19.8 . |
| | \therefore The standard deviation of the score is 19.8 . |

Extract 1.1: A sample of correct responses to question 1.

In Extract 1.1, the candidate used the value of a , from part (a) and a non-programmable calculator to find the mean and standard deviation in part (b).

Despite the good performance of candidates in this question, it was observed that 119 (22.8%) candidates had scores between 0 to 1.5 marks. These candidates failed to realize that the sum of frequencies was equal to the number of students who sat for the test, that is $\sum f = 80$, hence they failed to find the value of a in part (a) of the question and consequently failed to compute the mean and standard deviation. These candidates lacked knowledge on the basic tenets of statistics. Some of them calculated wrongly the value of a in part (a) which led to wrong values of mean and standard deviation in part (b).

Other candidates stated the procedures for calculating using a non-programmable scientific calculator instead of calculating the value of a . Extract 1.2 shows the sample of incorrect responses to question 1 from one of the candidates.

| | |
|---|---|
| 1 | solution |
| | (i) shift mode then mode hence press 1 SD. to enter the data $X_i f$ when X_i class mark and f frequency. |
| | (ii) enters the data example go to press shift then the button \square hence $M+$ |

Extract 1.2: A sample of incorrect responses to question 1.

In Extract 1.2, the candidate stated the procedures for calculating using a non-programmable scientific calculator instead of calculating the value of a .

2.1.2 Question 2: Similarity and Congruence

This question assessed candidates' competence to apply congruence theorems to identify the related lines and angles. They were required to prove that the perpendicular line from the vertex B to the base \overline{AC} of an isosceles triangle ABC bisects the base and the angle ABC .

The analysis of statistical data shows that, 523 (100%) candidates attempted this question, whereby 457 (87.4%) candidates had score ranging from 0 to 1.5 marks. Hence, the candidates' performance in the question was weak. Figure 2 is a summary of the candidates' performance.

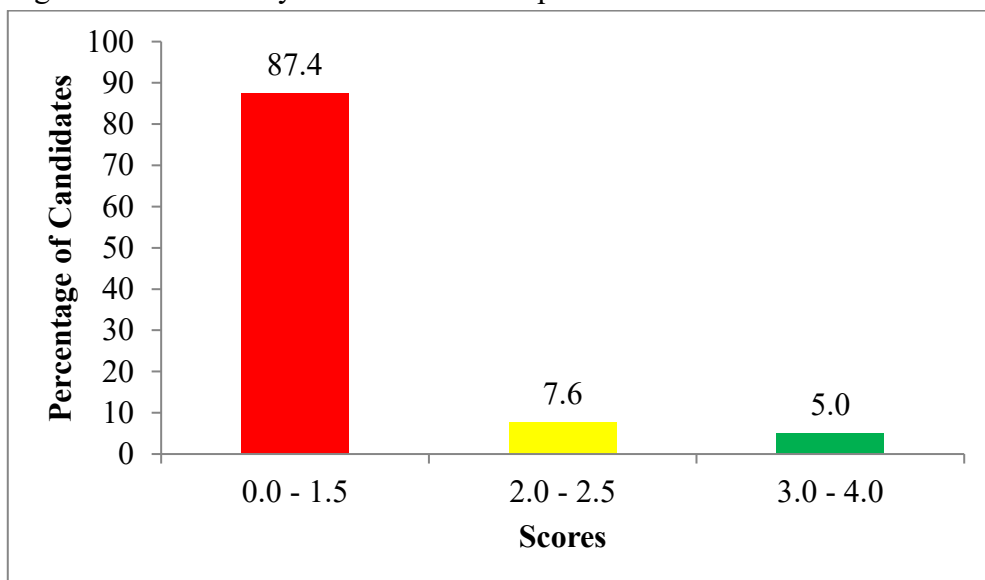


Figure 2: Performance of candidates in question 2

Performance analysis shows that, candidates who scored low marks had inadequate knowledge on congruence theorems and their applications in solving mathematical problems. Some candidates drew an equilateral triangle, labelled its sides and angles which was not the requirement of the question. Others were unable to translate the problem geometrically; this led them to write incorrect responses that could not lead to the required answer. Extract 2.1 shows a sample of candidate's responses who failed to translate the problem into a proper geometrical figures.

| | |
|---|--|
| 2 | |
| | <p>Consider the Bisector below:</p> |
| | |
| | <p>Intersecting lines are PQ and RS.</p> <p>They the bisector PQ bisects the opposite angle AOC and BOA at equal angle a.</p> <p>The bisector RS bisects the opposite angle COB and AOD at equal angle b.</p> <p>Then $a + b + b + a = 180^\circ$.</p> <p>$2a + 2b = 180^\circ$</p> <p>$2(a+b) = 180^\circ$, $a+b = 90^\circ$.</p> <p>* The bisectors are at perpendicular to each other, $a+b=90^\circ$.</p> |

Extract 2.1: A sample of incorrect responses to question 2.

In Extract 2.1, the candidate failed to understand the requirement of the question, consequently he/she could not translate the problem into a required geometrical figure.

Further analysis shows that, 26 (5.0%) candidates had scores ranging from 3 to 4 marks. The candidates who scored all 4 marks allotted to this question had gained the knowledge on congruence theorems and their applications. Extract 2.1 shows a sample of a correct response from one one of the candidates.

| | |
|---|---|
| 2 | <p>Required to prove.</p> <p>Considering the triangle ABC</p> <p>Required to prove Line from vertex B bisects AC</p> <p>Construction: Join BM</p> <p>Then from the Triangle.</p> <p>$\angle BMC = \angle BMA = 90^\circ$ - Right angle - given.</p> <p>$AB = BC$</p> <p>$BM = BM$ - Given ($\triangle ABC$ is an isosceles).</p> <p>BM - is common to both triangles</p> <p>Then, By</p> <p>RH Congruence theorem</p> <p>$\triangle ABM \cong \triangle CBM$</p> <p>and from the property of congruent triangles</p> <p>All sides (corresponding sides are equal)</p> <p>Then.</p> <p>$AM = CM$</p> <p>Since $AM = CM = \frac{1}{2} AC$</p> <p>Then</p> <p>Line from vertex B (ie BM) bisects the Base of $\triangle ABC$</p> |
|---|---|

Extract 2.2: A sample of correct responses to question 2.

In Extract 2.2, the candidate applied correctly the congruence theorem to produce the required answer.

2.1.3 Question 3: Coordinate Geometry II

This question examined candidates' ability to recognise and apply the condition that, the line $y = x - c$ touches the ellipse if the discriminant is equal to zero. They were required to find the possible value(s) of c and the coordinates of the point(s) of contact, if the line $y = x - c$ touches the ellipse $9x^2 + 16y^2 = 144$.

The question was attempted by 523 (100%) whereby 466 (89.1%) candidates scored from 0 to 1.5 marks. Therefore, the general performance of the candidates in this question was weak. Figure 3 shows the percentage of candidates who scored low, average and high marks.

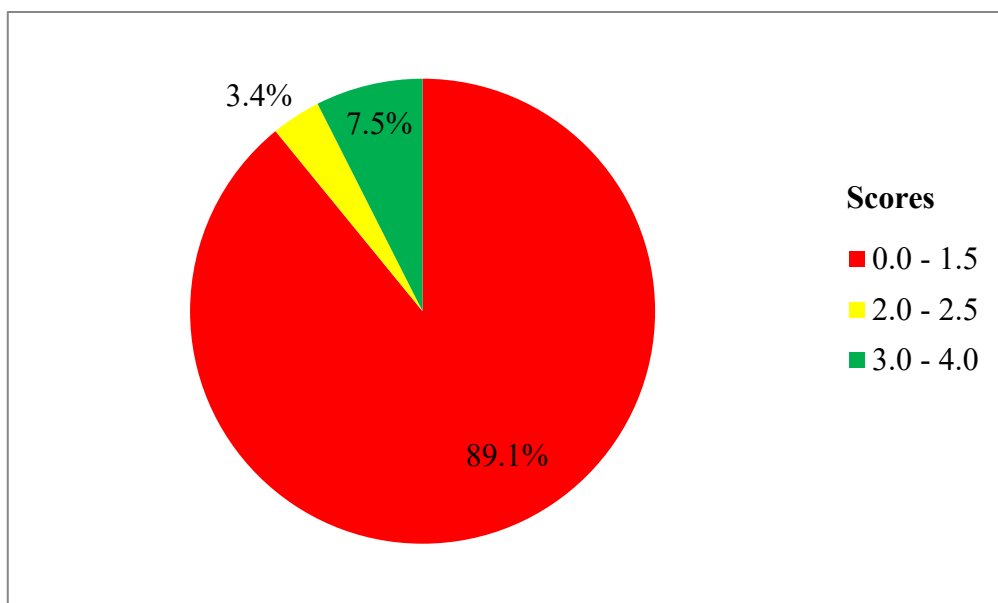


Figure 3: Performance of candidates in question 3

The analysis of candidates' responses shows that most of the candidates failed to understand the condition for the line to be a tangent to the ellipse. Some of them wrongly solved for the values of c by equating the equation of the ellipse and that of a straight line. Others were able to substitute $y = x - c$ in the ellipse $9x^2 + 16y^2 = 144$, but they failed to solve for c from the resulting equation. For example, a candidate calculated the value of c as follows; $(4y)^2 = (12)^2 - (3x)^2$ and wrongly obtained $4y = 12 - 3x$. The equations $4y = 12 - 3x$ and $y = x - c$ were compared to obtain $c = -12$. Another candidate compared $9y^2 + 16x^2 - 144 = 0$ with

$-x + y + c = 0$ to get $c = \pm 12$ which are wrong answers. Extract 3.1 is a sample of incorrect responses from one of the candidates.

| | |
|--|-------|
| 3 | Given |
| Equation: $b = x - c$. | |
| Ellipse equation: $9x^2 + 16y^2 = 144$. | |
| solution | |
| $9x^2 + 16y^2 = 144$ | |
| $9x^2 + 16b^2 - 144 = 0$. . . (1) | |
| $b = x - c$ | |
| $b - x + c = 0$. . . (ii) | |
| Compare equation (i) and (ii) | |
| $9x^2 + 16b^2 - 144 = 0$ | |
| $-x + b + c = 0$ | |
| $c = 144$ | |
| $c^2 = 12$ | |
| The value of c is $+12$ or -12 . | |
| The coord Coordinates of Point c . | |
| $9x^2 = x$ | |
| $9x = 1$ | |
| $x = \frac{1}{9}$ | |
| $x = \frac{-1}{9}$ | |
| $16b^2 = b$ | |
| $16b = 1$ | |
| $b = \frac{1}{16}$ | |
| The coordinate point points, $(\frac{-1}{9}, \frac{1}{16})$. | |

Extract 3.1: A sample of incorrect responses to question 3

In Extract 3.1, the candidate failed to apply the condition for a tangent line to the ellipse. He/she applied a wrong procedure in solving for the possible values of c . This propagated to wrong values of c and the corresponding coordinates of the points of contact.

On the other hand, analysis shows that 39 (7.5%) candidates scored high marks. These candidates managed to substitute $y = x - c$ in the equation of the ellipse $9x^2 + 16y^2 = 144$. They clearly understood and applied the condition for a line $y = x - c$ to touch the given ellipse, that is the discriminant must be equal to zero. Thus, they solved correctly to get the values of c as required. Extract 3.2 shows a correct response from one of the candidates.

| | |
|---|---|
| 3 | given, $y = x - c$ - - - (i) |
| | $9x^2 + 16y^2 = 144$ - - - (ii) |
| | Substitute eqn (i) into eqn (ii) |
| | $9x^2 + 16(x - c)^2 = 144$ |
| | $9x^2 + 16(x^2 - 2xc + c^2) = 144$ |
| | $9x^2 + 16x^2 - 32xc + 16c^2 = 144$ |
| | $25x^2 - 32cx + (16c^2 - 144) = 0$ |
| | Therefore |
| | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| | $x = \frac{-(-32c) \pm \sqrt{(-32c)^2 - 4(25)(16c^2 - 144)}}{2(25)}$ |
| | Condition |
| | for tangency: discriminant = 0 |
| | $(-32c)^2 - 4(25)(16c^2 - 144) = 0$ |
| | $1024c^2 - 4(400c^2 - 3600) = 0$ |
| | $1024c^2 - 1600c^2 + 14400 = 0$ |
| | $\frac{-576c^2}{-576} = \frac{-14400}{-576}$ |
| | $\sqrt{c^2} = \sqrt{25}$ |
| | $c = \pm 5$ |
| | $\therefore \underline{c = 5}$ or $\underline{c = -5}$ |
| | $x = \frac{32c}{50}$, for $c = 5$, $x = \frac{32 \times 5}{50} = \frac{16}{5}$ |
| | $x = \frac{32c}{50}$, for $c = -5$, $x = \frac{32 \times -5}{50} = -\frac{16}{5}$ |
| | Then, $y = \frac{16}{5} - 5 = -\frac{9}{5}$, $y = -\frac{16}{5} - 5 = -\frac{41}{5}$ |
| | Therefore: $(x, y) = \left(\frac{16}{5}, -\frac{9}{5}\right)$ or $(x, y) = \left(-\frac{16}{5}, -\frac{41}{5}\right)$ |

Extract 3.2: A sample of correct responses to question 3

In Extract 3.2, the candidate made correct substitution of y from the equation of a line into the equation of the ellipse and solved for the values of c .

2.1.4 Question 4: Teaching of Selected Topics

In this question, candidates were required to briefly explain the steps to follow in order to guide Form Three students on how to find the domain of the rational function $f(x) = \frac{1}{x-1}$. The question aimed at assessing candidates' ability to present the concept of domain of rational functions.

The data analysis shows that, 523 (100%) candidates attempted the question whereby 410 (78.4%) candidates had scores ranging from 0 to 1.5 marks. This shows that the general performance of candidates in this question was weak. Figure 4 gives a summary of candidates' performance in this question.

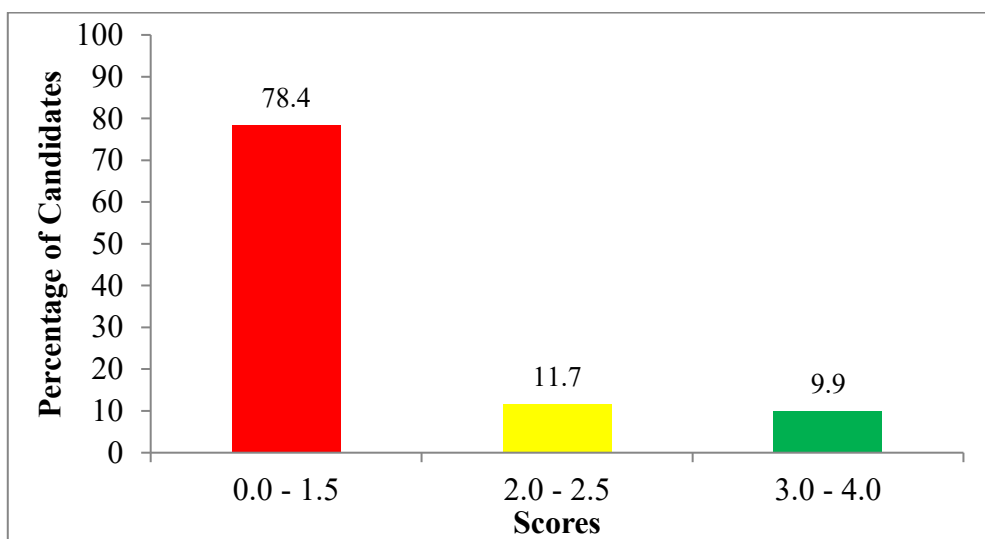


Figure 4: Performance of candidates in question 4

The responses analysis in this question indicates that most of the candidates failed to know the basic steps used in determining the domain of rational functions. They could not recognise that a rational function is defined when its denominator is not zero. That is, $f(x) = \frac{1}{x-1}$ is defined when $x-1 \neq 0$.

This important fact was necessary in explaining the steps on how to find the domain of a rational function.

Majority of the candidates were not familiar with the concept rational function as they failed to deduce that the domain of $f(x)$ is the set of x values for which the function is defined. For example, some candidates explained that the function $f(x)$ is defined at the point where the denominator is zero and wrote $x = 1$, $f(1) = \frac{1}{1-1} = 1$. Other candidates explained the steps of drawing the graph of a rational function as shown in Extract. 4.1.

| | |
|---|--|
| 4 | i/ construct the table of values by using the quadratic equations to find value of y axis eg or $x^2 - 2x + 1$ from $1 \leq x \leq 4$. |
| | ii/ Draw the graph of function in order to see the types of function either 1: one to one, or one to many function |
| | iii/. Join the point in order to know the graph spread downward, upward, leftward or rightward. |
| | iv/ Find the domain of the function by using $f(x) = \frac{1}{x-1}$ |

Extract 4.1: A sample of incorrect responses to question 4

In Extract 4.1, the candidate failed to know the requirement of the question, so he/she explained the redundant steps.

Despite the weak performance, 52 (9.9%) candidates answered this question correctly. These candidates demonstrated great understanding of the concept of rational functions, thus they were able to give important steps needed to guide Form Three student on how to find the domain of the given function. They stated that if the denominator of a rational function become zero then the function is undefined. Others used table of values to give the steps for determining the domain of $f(x)$ while others stated the domain by letting $x-1 \neq 0$ as shown in Extract 4.2.

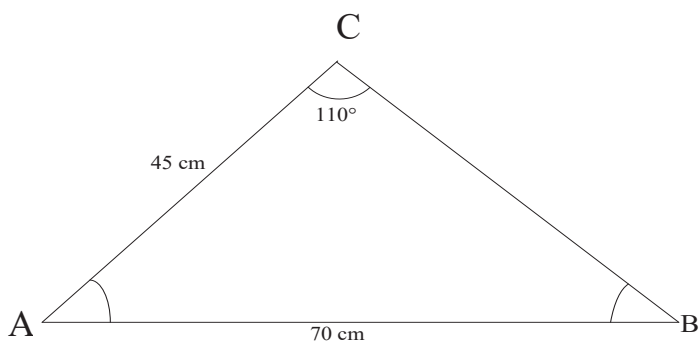
| | | | | | | | | | | | | | | | | | | | |
|----------|--|----------|------------|----|----------|----|-----|----|---|-------|-------|--------------|------|----|----------|---|-----|--|--|
| 4 | | | | | | | | | | | | | | | | | | | |
| | 4) The following are the steps to guide form III students to find the domain of the function $f(x) = \frac{1}{x-1}$ | | | | | | | | | | | | | | | | | | |
| | 1 st step; To teach the students on the meaning of domain; Domain is the value of x , real numbers | | | | | | | | | | | | | | | | | | |
| | 2 nd step; To develop the concept of table value which contains two rows but different columns. i.e. | | | | | | | | | | | | | | | | | | |
| | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>$f(x)$.</td> <td>column x</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3.</td> </tr> <tr> <td></td> <td>column y.</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> | $f(x)$. | column x | -3 | -2 | -1 | 0 | 1 | 2 | 3. | | column y . | | | | | | | |
| $f(x)$. | column x | -3 | -2 | -1 | 0 | 1 | 2 | 3. | | | | | | | | | | | |
| | column y . | | | | | | | | | | | | | | | | | | |
| | 3 rd step; To have concept on testing the $f(x) = \frac{1}{x-1}$ in the table of value | | | | | | | | | | | | | | | | | | |
| | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>$f(x)$</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y.</td> <td>-0.25</td> <td>-0.33</td> <td>-0.5</td> <td>-1</td> <td>∞</td> <td>1</td> <td>0.5</td> </tr> </table> | $f(x)$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | y . | -0.25 | -0.33 | -0.5 | -1 | ∞ | 1 | 0.5 | | |
| $f(x)$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | | | | | | | | | | | | |
| y . | -0.25 | -0.33 | -0.5 | -1 | ∞ | 1 | 0.5 | | | | | | | | | | | | |
| | 4 th step; To lead a form III student to determine the domain from the table value | | | | | | | | | | | | | | | | | | |
| | The result; The Domain are all real numbers (values) of x . $f(x) = \frac{1}{x-1}$ provide values except 1. | | | | | | | | | | | | | | | | | | |

Extract 4.2: A sample of correct responses to question 4

In Extract 4.2, the candidate recognised that students should be guided to understand the domain of a rational function is obtained when its denominator is not zero. Therefore, $f(x)$ is defined when $x-1 \neq 0$.

2.1.5 Question 5: Trigonometry

This question assessed candidates' ability to apply the knowledge of trigonometry ratios to calculate the area of a triangle. They were required to find the area of the following triangular field ABC , in square meters, correct to the nearest whole numbers.



A total of 523 (100%) candidates attempted this question, where 501 (95.8%) candidates had scores ranging from 0 to 1.5 marks. Therefore, the general performance of candidates in this question was weak. Figure 5 shows the performance of candidates in this question.

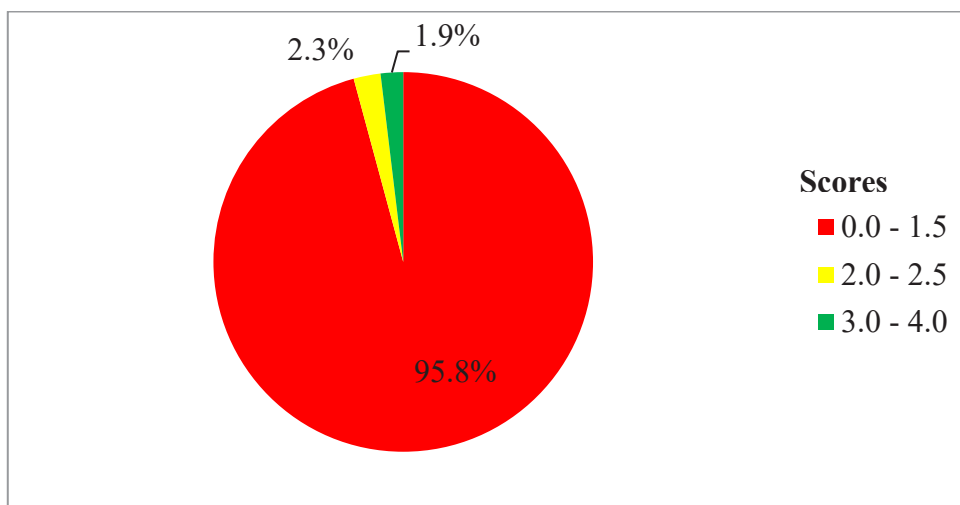


Figure 5: Performance of candidates in question 5

The analysis of data in this question shows that 12 (2.3%) candidates had scores ranging from 2 to 2.5 marks, while 10 (1.9%) candidates had scores ranging from 3 to 4 marks.

The candidates who failed to answer this question correctly did not know the formula for calculating the area of the rectangular field. That is, the area of a triangle whose angles are A, B and C with corresponding sides a , b and c , respectively is given by $\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$. They were not able to apply sine rule to find angle ABC which was necessary before applying the result and the formula to determine the required area. Some of the candidates assumed that angle ABC is equal to angle BAC, so

they wrongly wrote $\hat{B}\hat{A}C + \hat{A}\hat{B}C + 110^\circ = 180^\circ$. Others substituted the lengths of sides \overline{AC} , \overline{AB} , and $\angle ACB$ in the formula for area of a triangle. That is, $\text{Area} = \frac{1}{2} \overline{AC} \times \overline{AB} \sin A$, so they got wrong answers. Others applied directly the formula for calculating the area of a triangle, that is, $\text{Area} = \frac{1}{2} (\text{Base} \times \text{Height})$ as shown in Extract 5.1.

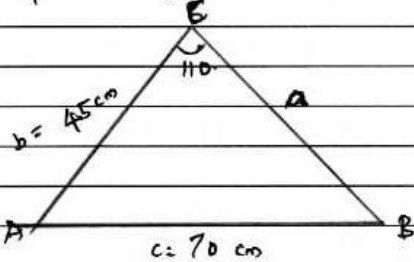
| | |
|---|---|
| 5 | $\text{Area} = \frac{1}{2} bh$ |
| | $= \frac{1}{2} AB \times AC $ |
| | $AC = 75 \text{ cm} = 0.75 \text{ m}$ |
| | $1 \text{ m} = 100 \text{ cm}$ |
| | $x = 75 \text{ cm}$ |
| | $AB = 70 \text{ cm} = 0.7 \text{ m}$ |
| | $\text{Area} = \frac{1}{2} \cdot 0.7 \text{ m} \times 0.75 \text{ m}$ |
| | $= \frac{0.525 \text{ m}^2}{2}$ |
| | $\text{Area} = 0.2625 \text{ m}^2$ |
| | $\therefore \text{The area is } 26.25 \times 10^{-2} \text{ m}^2$ |

Extract 5.1: A sample of incorrect responses to question 5

In Extract 5.1, the candidate applied directly the formula for finding the area of a triangle without computing the height of the triangle.

On the other hand, 22 (4.2%) candidates had scores ranging from 2 to 4 marks. Some candidates were able to apply sine rule to compute the value of angle ABC. They also applied the correct formula for calculating the area of the rectangular fields $\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$.

Extract 5.2 shows how a correct answer from one of the candidates.

| | |
|---|---|
| 5 | <p>Given the Triangular field. AC.</p> <p>with side AC = 45cm, AB = 70cm.</p> <p>and angle ACB = 110°.</p> <p>Required to find its Area.</p>  <p>Area of triangle ABC = $\frac{1}{2} \cdot AC \cdot AB \cdot \sin 110^\circ$.</p> <p>But since a can be found by sine Rule.</p> $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$ $\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b}$ $\frac{\sin 110^\circ}{70\text{cm}} = \frac{\sin \hat{B}}{45\text{cm}}$ $\sin \hat{B} = \frac{45\text{cm} \times \sin 110^\circ}{70\text{cm}}$ $\sin \hat{B} = 0.604$ <p>Then $\hat{B} = \frac{1}{2}$ angle $\hat{B} = \sin^{-1}(0.604)$.</p> $\hat{B} = 37.156^\circ$ <p>Then $\hat{A} + \hat{B} + \hat{C} = 180^\circ$.</p> $\hat{A} = 180 - 110 - 37.156$ $\hat{A} = 32.84^\circ$ <p>Then</p> $\sin \hat{A} = \sin(32.84^\circ)$ $\sin A = 0.5423$ |
| | $A = \frac{1}{2} AC \cdot AB \cdot \sin \hat{A}$ $= \frac{1}{2} \cdot 45\text{cm} \times 70\text{cm} \times 0.5423$ $\text{Area} = 854.12 \text{ cm}^2$ |

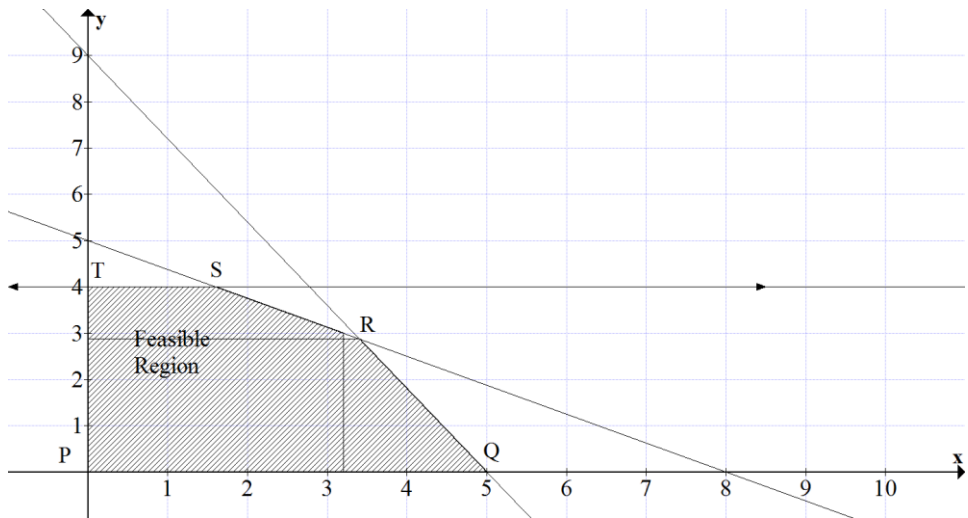
Extract 5.2: A sample of correct responses to question 5

In Extract 5.2, the candidate used the correct formula and made the correct substitution to get the answer.

2.1.6 Question 6: Linear Programming

This question assessed candidates' ability to formulate the constraints of a linear programming problem from the given graph. They were required to

formulate the constraints representing the feasible region shown in the following graph.



A total of 523 (100%) candidates attempted the question. Whereby 395 (75.5%) candidates scored from 0 to 1.5 marks. Hence, the general performance of candidates in this question was weak. Figure 6 shows the summary of candidates' performance in this question.

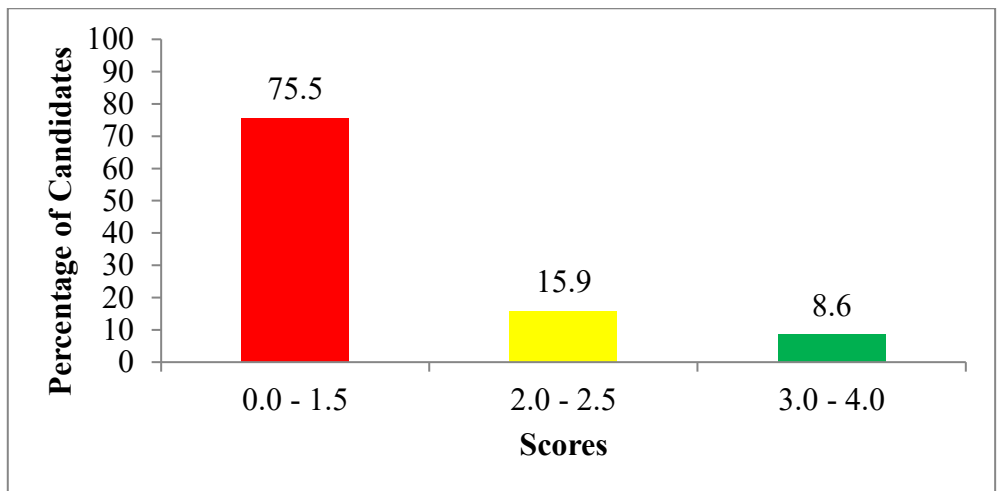


Figure 6: Performance of candidates in question 6

The data analysis shows that 83 (15.9%) candidates had scores ranging from 2 to 2.5 marks and 45 (8.6%) scored from 3 to 4 marks. The candidates who failed to get the correct answer lacked knowledge to determine the constraints representing the given feasible region. Some of the candidates were able to identify the points of intercepts from the given

graph but they failed to formulate the required inequalities. Extract 6.1 shows the response of one of the candidates who wrote wrong answers.

| | |
|---|--|
| 6 | <p>from the corner points</p> <p>Corner points (x, y)</p> <p>T (4, 0) .</p> <p>S (1.7, 4)</p> <p>R (3.5, 2.8)</p> <p>Q (5, 0) .</p> <p>let $x + y \geq 0$</p> <p>for point T (4, 0)</p> <p>$4x \geq 0$ ----- (i) .</p> <p>for point S (1.7, 4) .</p> <p>$1.7x + 4y \geq 0$ ----- (ii) .</p> <p>for point R (3.5, 2.8)</p> <p>$3.5x + 2.8y \geq 0$ ----- (iii) .</p> <p>for point Q .</p> <p>$5x \geq 0$ ----- (iv) .</p> <p>The linear inequalities are .</p> <p>$4x \geq 0$; $5x \geq 0$.</p> <p>$1.7x + 4y \geq 0$.</p> <p>$3.5x + 2.8y \geq 0$.</p> |
|---|--|

Extract 6.1: Sample of incorrect responses to question 6

In Extract 6.1, the candidate could not determine and use the intercepts to formulate the constraints representing the feasible region.

On the other hand, the candidates who answered this question correctly determined and used the intercepts of the lines to formulate the required

constraints of the given linear programming problem. Extract 6.2 shows a correct response to this question.

| | |
|---|--|
| 6 | <p style="text-align: center;">soln</p> <p>line of point A and contain (5,0) and (0,9)</p> <p>sgn $\frac{y_2 - y_1}{x_2 - x_1} = \text{slope}$</p> $\frac{9 - 0}{0 - 5} = -\frac{9}{5}$ <p>from $y = mx + c$</p> $y = -\frac{9}{5}(x - x_1) + y_1$ $y = -\frac{9}{5}(x - 5) + 0$ $y = -\frac{9}{5}x + 9$ $5y + 9x \leq 45 \quad \text{--- (i)}$ <p>point B (8,0) and (0,5)</p> $\text{slope} = \frac{5 - 0}{0 - 8} = -\frac{5}{8}$ $y = -\frac{5}{8}(x - 8) + 0$ $y = -\frac{5}{8}x + 5$ $8y + 5x \leq 40 \quad \text{--- (ii)}$ <p>point (0,4)</p> $y \leq 4 \quad \text{--- (iii)}$ $x \geq 0, y \geq 0 \quad \text{--- (iv)}$ <p>\therefore linear inequality represent the feasible region is</p> $5y + 9x \leq 45$ $8y + 5x \leq 40$ $y \leq 4$ $x \geq 0, y \geq 0$ |
|---|--|

Extract 6.2: A sample of correct responses to question 6

In Extract 6.2, the candidate formulated the correct constraints representing the feasible region. He/she was able to use the intercepts to determine the equation of the lines defining the feasible region.

2.1.7 Question 7: Assessment in Mathematics

This question examined candidates' ability to recognize the importance of keeping records of students' mathematics learning. The question required the candidates to support briefly by giving four reasons the statement that; "Students' progressive report keeping is an important aspect for motivation in learning Mathematics".

A total of 523 (100%) candidates attempted this question, whereby 505 (96.6%) candidates had scores ranging from 2 to 4 marks. Hence, the general performance of candidates in this question was good. Figure 7 represents the performance of the candidates in this question.

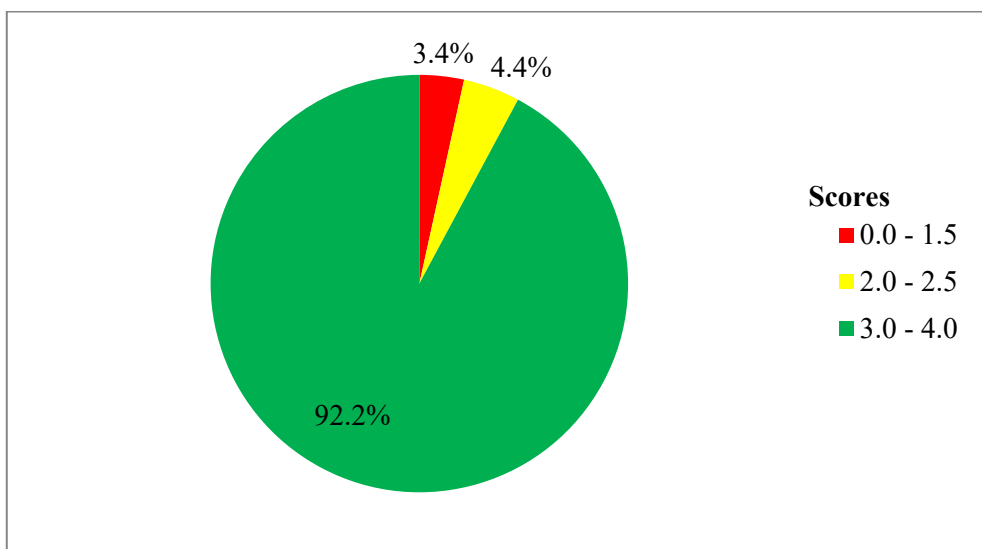


Figure 7: Performance of candidates in question 7

The data analysis shows that 18 (3.4%) candidates had scores ranging from 0 to 1.5 marks, 23 (4.4%) candidates had scores ranging from 2 to 2.5 marks, and 482 (92.2%) candidates had scores ranging from 3 to 4 marks.

The candidates who had good performance were able to demonstrate the importance of keeping records of students' mathematics learning. Extract 7.1 shows one of the correct responses to this question.

| | |
|---|--|
| 7 | <p>i) It creates a spirit for the student to keep on struggling. This is in order to ensure that he is maintaining his performance.</p> <p>ii) It equip a student with references. A student can review from the past documents in order to remember</p> <p>iii) Promotes Competition among students. Due to the report, the student can compete in studying hence performing better</p> <p>iv) It Create history of learners in a particular school / institution. Some informations can be preserved for future use.</p> |
|---|--|

Extract 7.1: A sample of correct responses to question 7

In Extract 7.1, the candidate demonstrated good knowledge of the importance of keeping students progressive report in the process of learning mathematics.

The candidates who failed to answer this question correctly failed to understand its requirement. For example, some candidates responded by listing the incorrect reasons such as; *Use of teaching aid, conducive environment during learning mathematics, teaching from simple to complex, bring students attention in learning mathematic, providing good environment, by using participatory methods, more solving and good teaching approach.* Extract 7.2 shows one of the candidate's incorrect responses in question 7.

| | |
|---|---|
| 7 | <p>Because:</p> <p>i) Making connection.</p> <p>ii) Different students skills</p> <p>iii) Structure lesson.</p> <p>iv) Articulating goal.</p> <p>v) forestimating engagement.</p> |
|---|---|

Extract 7.2: A sample of incorrect responses to question 7

In Extract 7.2, the candidate lacked knowledge about the requirement of the question, thus he/she gave wrong responses to the question.

2.1.8 Question 8: Algebra

This question assessed candidates' ability to use the concept of roots of a quadratic equation to establish the relationships among the coefficients of the quadratic equation. The question had two parts (a) and (b). In part (a) candidates were given that, the roots of the quadratic equation $ax^2 + bx + c = 0$ differ by 2, then they were required to show that $4ac = b^2 - 4a^2$. In part (b), they were given that; $x + 2$ and $2x - 1$ are factors of the quadratic equation $ax^2 + x - c$, then they were required to find the values of a and c .

The question was attempted by 523 (100%) candidates, whereby 308 (58.9%) candidates had scores ranging from 0 to 1.5 marks. Hence, the general performance in this question was average. Figure 8 shows the candidates' performance in this question.

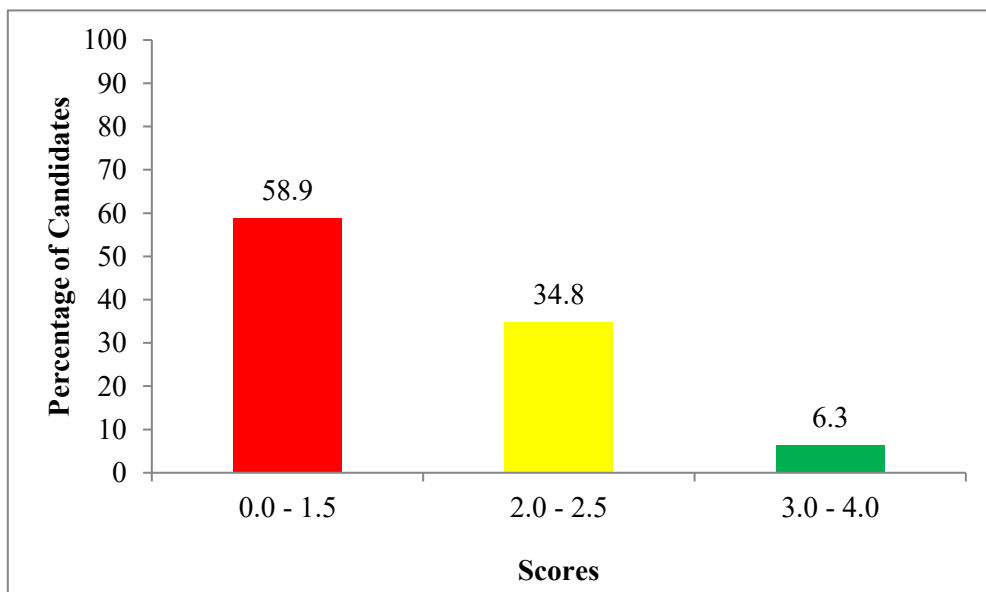


Figure 8: Performance of candidates in question 8

The data analysis shows that 308 (58.9%) candidates had scores ranging from 0 to 1.5 marks, 182 (34.8%) candidates had scores ranging from 2 to 2.5 marks, and 33 (6.3%) candidates had scores ranging from 3 to 4 marks.

Out of 308 (58.9%) candidates who scored between 0 and 1.5 marks in this question, 87 (16.6%) candidates scored zero.

This failure indicates that candidates lacked the knowledge on basic concepts of algebra. Some candidates applied the concept of sum and products of the roots of a quadratic equation inappropriately leading them to wrong answer. Other candidates applied the concept of completing the square on $ax^2 + x - c$ to obtain $(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$. Next, they let $\frac{b^2 - 4ac}{4a^2} = 1$ to get $4ac = b^2 - 4a^2$ which is not correct. In part (b), candidates failed to apply the concept of remainder theorem. For example, one candidate replaced the coefficient of x by $x + 2$ and c by $2x - 1$ in the equation $ax^2 + x - c$. The resulting quadratic equation was compared to $x^2 + x - c$ to obtain incorrect values of a and c . Extract 8.1 shows one of the incorrect answer to question 8.

| 8 | |
|--------------------------------|--|
| $ax^2 + bx + c = 0$ | b) $x+2$ and $2x-1$ |
| But let $x = 2$. | When $x+2$ |
| $a(2)^2 + b(2) + c = 0$. | $x = -2$ |
| $4a + 2b + c = 0$ | $ax^2 + x - c = 0$ |
| From: | $a(-2) = 2 - c = 0$ |
| $ax^2 + bx + c = 0$ | $=$ |
| $b^2 = 4ac$ | $-2c - c = 2$ — (i) |
| $b^2 = 4(2)c$ | When: |
| $b^2 = 4ac$ | $x = \frac{1}{2}$ |
| $b^2 = 4ac$ | $a(\frac{1}{2}) + \frac{1}{2} - c = 0$ |
| From: | $a + 1 - 2c = 0$ |
| $4ac = b^2 - 4a^2$ | $a - 2c = -1$ — (ii) |
| But $b^2 = 4ac$. | To solve simultaneous |
| $4ac = 4ac - 4a^2$ | equation: |
| $4a^2 = 4ac - 4ac$ | $-2c - c = 2$ |
| $\frac{4a^2}{4} = \frac{0}{4}$ | $a - 2c = -1$ |
| $a^2 = 0$ | The value of |
| $a = 0$ | a and c are |
| The value of $a = 0$ | $(a, c) = -1, 0$ |
| | The value of |
| | $a = -1$, and |
| | $c = 0$ |

Extract 8.1: A sample of incorrect responses to question 8

In Extract 8.1, the candidate used the general quadratic equation to substitute imaginary values to solve for the unknown variables a , b and c .

However, the candidates who answered this question correctly were able to establish the relationships among the coefficients of quadratic equation. They applied properly the concept of sum and product of the roots to obtain the correct answer as shown in Extract 8.2.

| | |
|---|---|
| 8 | (a) $ax^2+bx+c=0$. |
| | $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ |
| | Let the roots of the equation be α and β . |
| | Given, $\alpha - \beta = 2$. |
| | Sum of roots = $-\frac{b}{a}$. |
| | $\alpha + \beta = -\frac{b}{a}$ — (1) |
| | Product of roots = $\frac{c}{a}$. |
| | $\alpha\beta = \frac{c}{a}$ — (2) |
| | but $\alpha = \beta + 2$ |
| | From $\alpha - \beta = 2$ squaring both sides. |
| | $(\alpha - \beta)^2 = 2^2$. |
| | $\alpha^2 - 2\alpha\beta + \beta^2 = 4$. |
| | but $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$. |
| | $(\alpha + \beta)^2 - 4\alpha\beta = 4$. |
| | $(-\frac{b}{a})^2 - 4\frac{c}{a} = 4$ |
| | $\frac{b^2}{a^2} - \frac{4c}{a} = 4$. |
| | $b^2 - 4ac = 4a^2$. |
| | $\therefore 4ac = b^2 - 4a^2$. |
| | (b) If $x+2$ and $2x-1$ satisfy the eqn, then $(x+2)(2x-1)=0$. |
| | $2x^2+3x-2=0$ |
| | solving quadratic eqn, $x=0.5$ or -2 . |
| | when $x=0.5$, $a(0.5)^2+0.5-c=0$ |
| | $\frac{a}{4}+\frac{1}{2}=c \Rightarrow a+2=4c$ — (1) |
| | when $x=-2$, $a(-2)^2+-2-c=0$ |
| | $4a-2=c$ — (2) |
| | Solving eqn (1) and (2) simultaneously, |
| | $a = \frac{2}{3}$ and $c = \frac{2}{3}$. |

Extract 8.2: A sample of correct responses to question 8

In Extract 8.2, the candidate used the correct procedures and made a good substitution of values to get the correct answers.

2.1.9 Question 9: Hyperbolic Functions

This question was set to examine candidates' ability to plot the graphs of hyperbolic functions. The candidates were given the function $f(x) = \cosh^{-1} x$, and they were required to; (a) sketch the locus of the function using the table of values such that $1 \leq x \leq 4$ and (b) determine for the values of x and y where the function is defined.

A total of 523 (100%) candidates attempted this question whereby 438 (83.0%) candidates scored from 0 to 1.5 marks. Therefore, the general performance in the question was weak. Figure 9 presents a summary of candidates' performance in this question 9.

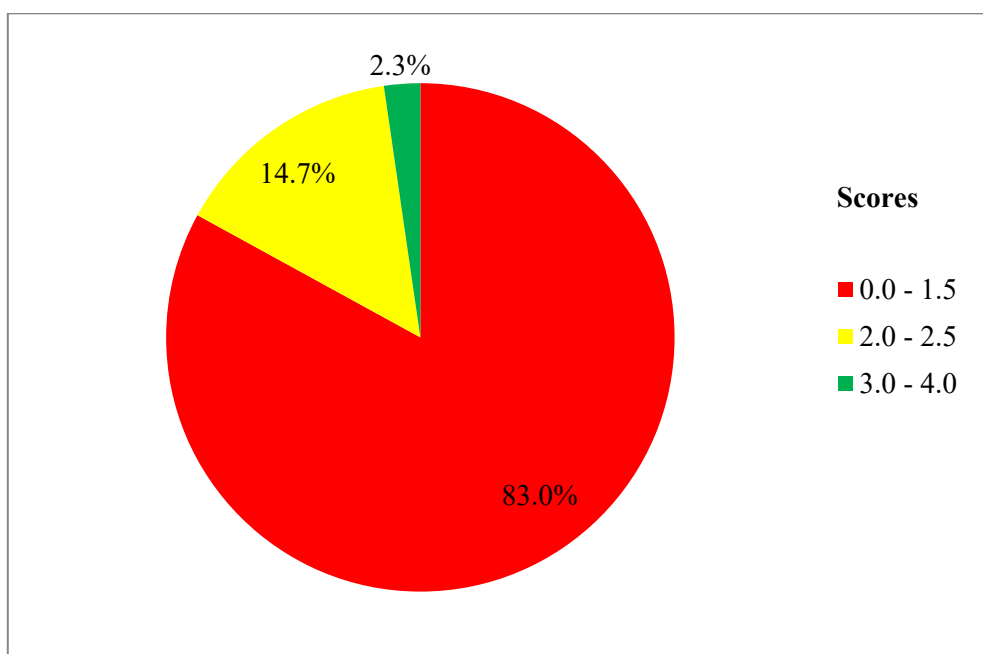


Figure 9: Performance of candidates in question 9

The data analysis shows that 77 (14.7%) candidates had scores ranging from 2 to 2.5 marks and 12 (2.3%) candidates who scored from 3 to 4 marks. Many candidates failed to answer this question correctly because they were unable to construct a table of value leading to failure in plotting the graph. For example, some candidates constructed wrong table of values for $1 \leq x \leq 4$ and $f(x) = \cosh^{-1} x$. Extract 9.1 shows an incorrect response to this question.

9

$y = \cosh^{-1} x$
 $y = \cosh^{-1} x$

$x = \cosh y, \cosh^{-1} x = y = \frac{1}{2} \ln(2x-1)$

Table of Value.

| | | | | |
|--------------------|------|------|------|------|
| X | 1 | 2 | 3 | 4 |
| $y = \cosh^{-1} x$ | 0.00 | 0.55 | 0.80 | 0.97 |

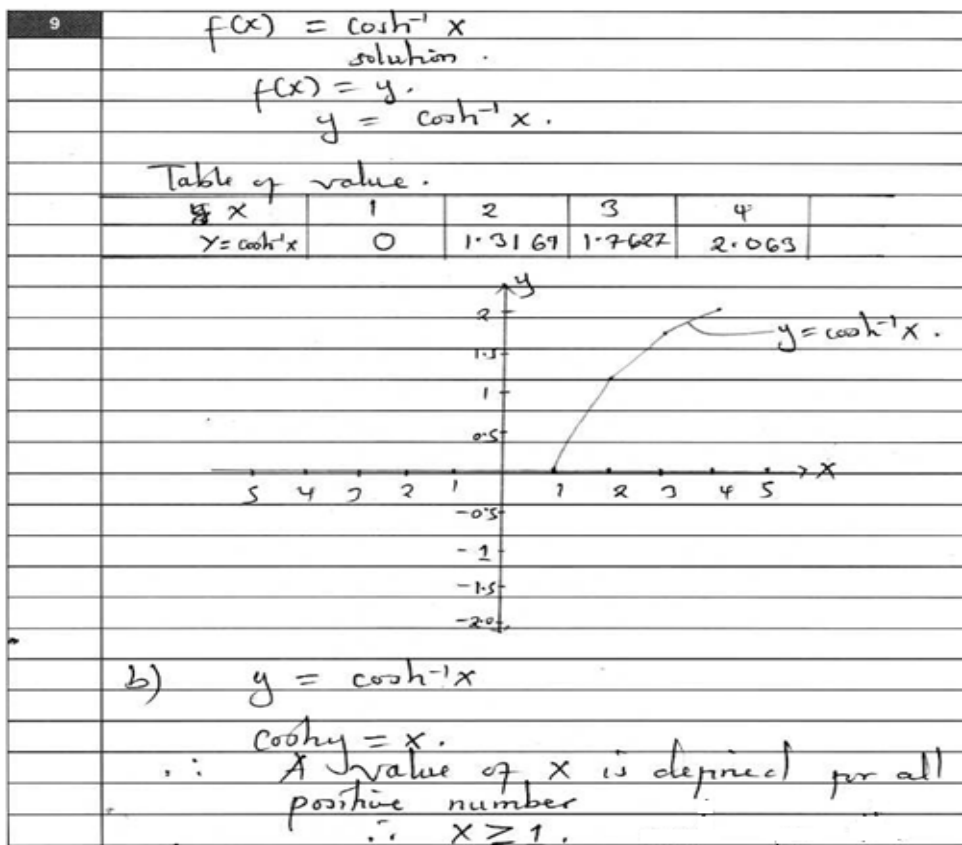
a)

b) The function is defined when the value of x is 1 and y is 0.

Extract 9.1: A sample of incorrect responses to question 9

In Extract 9.1, the candidate constructed a wrong table of values, which led to a wrong graph.

On the other hand, 89 (17.0%) candidates managed to sketch the correct graph because of their ability to construct a correct table of values. Some of them were also able to identify the region in the graph where the function is undefined as required. Extract 9.2 shows a correct response to this question.



Extract 9.2: A sample of correct responses to question 9

In Extract 9.2, the candidate constructed a correct table of values and was able to plot the graph of the function.

2.1.10 Question 10: Integration

This question examined the candidates' ability to apply integration techniques to find the value of unknown limit. The candidates were required to find the value of a in surd form if $\int_1^a (x + \frac{1}{2}) dx = \int_0^{\frac{\pi}{4}} \sin^2 x dx.$

The question was attempted by 523 (100%) candidates, where by 500 (95.6%) candidates had scores ranging from 0 to 1.5 marks. Hence, the general performance of candidates in this question was weak. Figure 10 shows the performance of candidates in this question.

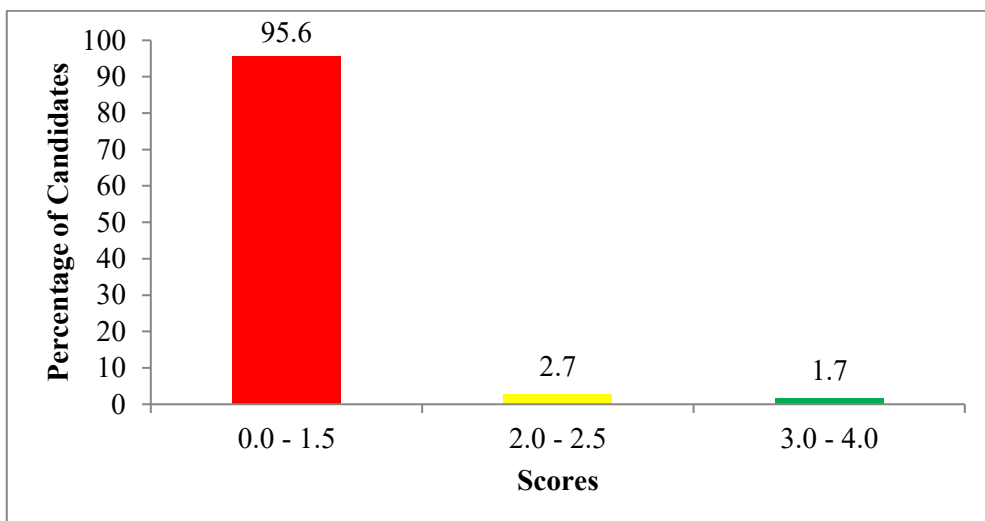


Figure 10: Performance of candidates in question 10

The data analysis shows that 14 (2.7%) candidates had scores ranging from 2 to 2.5 marks and 9 (1.7%) candidates had scores ranging between 3 to 4 marks.

Most of the candidates failed to answer this question because they lacked integration techniques, especially when the integrand is a trigonometric function. They could not find $\int_0^{\frac{\pi}{4}} \sin^2 x dx$. Some candidates substituted the

limits without integrating, that is, from $\int_1^a (x + \frac{1}{2}) dx = \int_0^{\frac{\pi}{4}} \sin^2 x dx$; they got

$\left(a + \frac{1}{2}\right) - \left(1 - \frac{1}{2}\right) = \left(\sin^2 \frac{\pi}{4} - \sin^0 0\right) \Rightarrow a = 0.010766$ which is a wrong

answer. Other candidates made inappropriate computations such as,

$\int_0^{\frac{\pi}{4}} \sin^2 x dx = \sin \int_0^{\frac{\pi}{4}} x dx = \sin \int_0^{\frac{\pi}{4}} 1 + c = \sin \frac{\pi}{4} - \sin 0 \Rightarrow a = 1$. Extract 10.1

shows one of the incorrect responses to this question.

$$\int_1^a (x + 1/2) dx = \int_0^{\pi/4} \sin^2 x dx.$$

$$\int_1^a (\pi/4 + 1/2) dx = \int_1^a (0 + 1/2).$$

$$\text{but } \int_0^{\pi/4} \sin^2 x dx = \sin \int_0^{\pi/4} x dx + C.$$

$$= \sin \int_0^{\pi/4} 1 dx + C.$$

$$= \sin \pi/4 - \sin 0 + C$$

$$= 1 + C.$$

\therefore The value of $a = 1$.

Extract 10.1: A sample of incorrect responses to question 10

In Extract 10.1, the candidate lacked the knowledge on integration techniques. He/she made irrelevant computations to obtain a wrong answer.

However, there were 23 (4.4%) candidates who had scores ranging from 2 to 4 marks. The candidates who scored full marks demonstrated good knowledge on integration techniques. They were able to find the value of a using the concept of definite integrals. Extract 10.2 shows a sample of correct responses from one of the candidates.

| 10 | Solution. |
|----|---|
| | $\int_1^a (x + \frac{1}{2}) dx = \int_0^{\frac{\pi}{4}} \sin^2 x dx$, Required Value of a . |
| | $\int_1^a x dx + \int_1^a \frac{1}{2} dx = \int_0^{\frac{\pi}{4}} \left(\frac{1 - \cos 2x}{2} \right) dx$. |
| | $\left[\frac{x^2}{2} \right]_1^a + \frac{1}{2} \left[x \right]_1^a = \int_0^{\frac{\pi}{4}} \frac{1}{2} dx - \int_0^{\frac{\pi}{4}} \frac{\cos 2x}{2} dx$ |
| | $\left[\frac{x^2}{2} \right]_1^a + \frac{1}{2} [x]_1^a = \frac{1}{2} \int_0^{\frac{\pi}{4}} dx - \frac{1}{2} \int_0^{\frac{\pi}{4}} \cos 2x dx$. |
| | $\left[\frac{x^2}{2} \right]_1^a + \frac{1}{2} [x]_1^a = \frac{1}{2} [x]_0^{\frac{\pi}{4}} - \frac{1}{2} \left[\frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}}$. |
| | $\left(\frac{a^2 - 1^2}{2} \right) + \frac{1}{2} [a - 1] = \frac{1}{2} \left[\frac{\pi}{4} - 0 \right] - \frac{1}{4} \left[\sin \left(2 \times \frac{\pi}{4} \right) - \sin 2(0) \right]$. |
| | $\left(\frac{a^2 - 1}{2} \right) + \frac{1}{2} (a - 1) = \frac{\pi}{8} - \frac{1}{4} \left(\frac{\sin \pi}{2} - \sin 0 \right)$ |
| | $\left(\frac{a^2 - 1}{2} \right) + \frac{1}{2} (a - 1) = \frac{\pi}{8} - \frac{1}{4} (1 - 0)$ |
| | $\frac{1}{2} [(a^2 - 1) + a - 1] = \frac{1}{2} \left[\frac{\pi}{4} - \frac{1}{2} \right]$. |
| | $a^2 - 1 + a - 1 = \frac{\pi}{4} - \frac{1}{2}$ |
| | $a^2 + a - 2 = \frac{\pi}{4} - \frac{1}{2}$ |
| | $a^2 + a - \frac{3}{2} - \frac{\pi}{4} = 0$. |
| | $a^2 + a - \frac{(6 + \pi)}{4} = 0$; by using general formula |
| | $a = \frac{-1 \pm \sqrt{1 - 4 \cdot \left(\frac{6 + \pi}{4} \right)}}{2}$ |
| | $a = \frac{-1 \pm \sqrt{1 + 6 + \pi}}{2}$ |
| | $a = \frac{-1 \pm \sqrt{7 + \pi}}{2}$; \therefore Value of a is $\frac{-1 \pm \sqrt{7 + \pi}}{2}$ |

Extract 10.2: A sample of correct responses to question 10

In Extract 10.2, the candidate applied properly the techniques of integration of definite integrals to obtain the correct answer.

2.2 Section B: Essay Questions

2.2.1 Question 11: Vectors

The question examined candidates' knowledge on the concepts of cross and dot products of vectors. The question had three parts (a), (b) and (c). In part (a), they were required to determine the values of λ and μ such that the points $(-1, 3, 2)$, $(-4, 2, -2)$ and $(5, \lambda, \mu)$ lie on a straight line. In part (b), they were given that; If $|\underline{A} + \underline{B}| = 60$, $|\underline{A} - \underline{B}| = 60$, and $|\underline{B}| = 46$, then they were asked to find $|\underline{A}|$, and in part (c), they were required to find the angle

between the vectors $2\mathbf{i}+6\mathbf{j}+3\mathbf{k}$ and $12\mathbf{i}-4\mathbf{j}+3\mathbf{k}$, giving the answer correct to two decimal places.

The question was attempted by 523 (100%) candidates, where 265 (50.7%) candidates had scores ranging from 0 to 5.5 marks. Hence, the general performance of candidates in this question was average. Figure 11 shows the performance of candidates in this question.

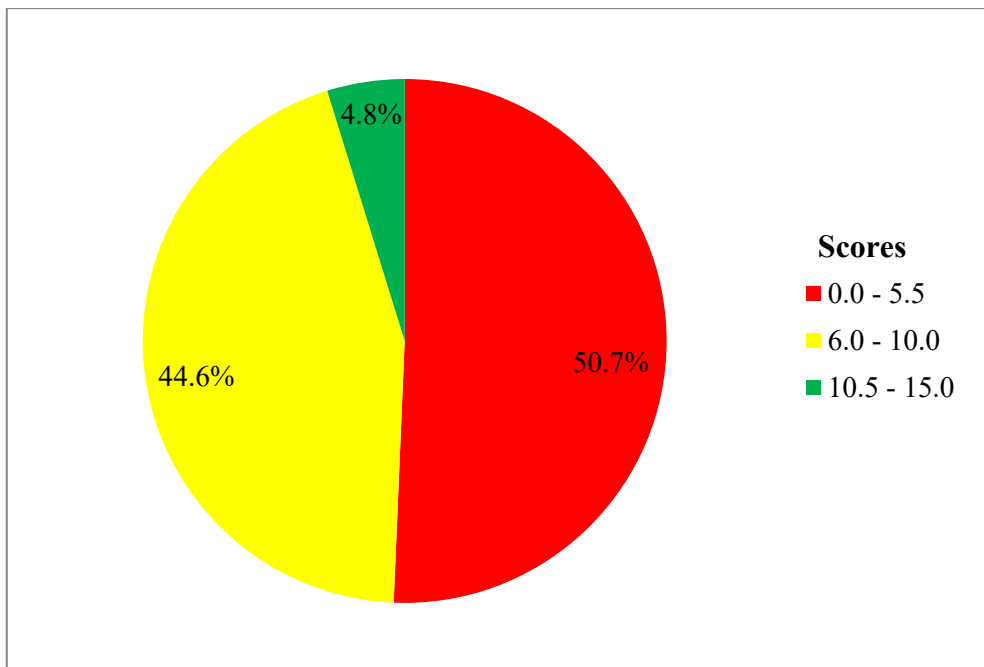


Figure 11: Performance of candidates in question 11

The data analysis shows that 233 (44.6%) candidates had scores ranging from 6 to 10 marks and 25 (4.8%) candidates had scores ranging from 10.5 to 15 marks. The candidates who scored low marks lacked knowledge on cross product, dot product and the concept of collinear vectors. For example, some candidates in part (a), calculated $(-1,3,2) + (-4,2,-2) + (5,\lambda,\mu)$ to get;

$$\begin{aligned} -1 + -4 + 5 &= 3 + 2 + \lambda \\ 0 &= 5 + \lambda \\ \lambda &= -5 \\ -1 + -4 + 5 &= 2 + -2 + \mu \end{aligned}$$

$\mu = 0$, which was a wrong procedure. Others used points the A, B, C instead of the vectors $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ and $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$. That is, $A \times B = 0$ instead of $\overrightarrow{AB} \times \overrightarrow{BC} = 0$ for collinear vectors.

In addition, the candidates failed to calculate the modulus of vectors in part (b). For example, one candidate calculated $|A|$ as follows;

$$\begin{aligned} |A + B| &= |A| + |B| - |A - B| \\ 60 &= |A| + 46 - 60 \\ |A| &= 120 - 46 = 74. \end{aligned}$$

Others assumed $|A + B|^2 = |A - B|^2 = 4|A||B| = 0$ implying $|A| = 0$. Also, some candidates solved simultaneously $a + b = 60$ and $a - b = 60$ where $|B| = 60$ to obtain the value of $|A| = 14$.

In part (c), some of them used a wrong formula $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\sin\theta$ for dot product instead of $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$. Extract 11.1 shows an incorrect response from one of the candidates.

| | |
|----|--|
| 11 | (b) If $ \underline{A} + \underline{B} = 60$, $ \underline{A} - \underline{B} = 60$ and $ \underline{B} = 46$ Find $ \underline{A} $ |
| | Solution: |
| | $a + b = 60$ --- eqn (i) |
| | $a - b = 60$ --- eqn (ii) |
| | where by $ \underline{B} = 46$. |
| | Take eqn (i) |
| | $a + b = 60$ |
| | $a = 60 - b$ |
| | where by $ \underline{B} = 46$ |
| | $a = 60 - 46$ |
| | $ a = 14$ |
| | <u>$\underline{A} = 14$</u> |
| | (c) Solution: |
| | $\frac{12\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}}{2\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}}$ divide by $\frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{2\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}}$ |
| | $\frac{6\mathbf{i} - 2\mathbf{j}}{1\mathbf{i} + 3\mathbf{j}}$ |
| | $6 - 0.6$ |

$$6 - 0.6$$

$$= \frac{5.4 \times 60}{360 \div 2}$$

The angle is 30°

(a) From the point $(-1, 3, 2)$,
 $(-4, 2, -2)$ and
 $(5, 1, \mu)$

Solution $A = (3 + 2)$
 $= 5$

$$\mu = \frac{-2}{2}$$

$$= -1$$

Answer $A = 5$ and $\mu = -1$

Extract 11.1: A sample of incorrect responses to question 11

In Extract 11.1, the candidate used wrong formula and incorrect procedures to calculate the modulus and the unknown parameters.

On the other hand, the candidates had scores ranging from 6.5 to 15 marks demonstrated good knowledge on the concept of dot product and cross product of vectors. They used the formula $\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$ which lead them to obtain an angle between the two vectors. They also calculated well the modulus and cross product of the vectors to obtain the correct answer. Extract 11.2 shows part of the correct answer to this question.

11

(a) Let vector $A = -1, 3, 2$
 $B = -4, 2, -2$
 $C = 5, \lambda, \mu$

$$AB = B - A$$

$$AB = (-4, 2, -2) - (-1, 3, 2)$$

$$AB = -3, -1, -4$$

$$BC = C - B$$

$$BC = (5, \lambda, \mu) - (-4, 2, -2)$$

$$BC = (9, \lambda - 2, \mu + 2)$$

From the collinear vector

$$|AB \times BC| = 0$$

| | | | |
|----|---------------|-----------|-----|
| i | j | k | |
| -3 | -1 | -4 | = 0 |
| 9 | $\lambda - 2$ | $\mu + 2$ | |

$$(\mu + 2 - (-4\lambda + 8))i - (-3(\mu + 2) - 36)j +$$

$$(-3(\lambda - 2) - 9)k = 0$$

$$(\mu + 2 + 4\lambda - 8)i - (-3\mu - 6 + 36)j + (-3\lambda + 6 + 9)k$$

$$= 0$$

$$4\lambda + (\mu + 4\lambda - 6)i - (-3\mu + 30)j + (-3\lambda + 15)k = 0$$

$$(\mu + 4\lambda - 6)i = 0$$

$$(\mu + 4\lambda - 10)j = 0$$

$$-4 + 4\lambda = 10 \quad \text{--- (1)}$$

$$3\lambda - 30 = 0$$

$$3\lambda = 30$$

| 11 Cont. | |
|----------|--|
| | $u = 10$ |
| | $(-3\lambda + 15)k = 0$ |
| | $-3\lambda + 15 = 0$ |
| | $-3\lambda = -15$ |
| | $\lambda = 5$ |
| | $\therefore \lambda = 5 \text{ and } u = 10$ |
| | (b) From |
| | $ A+B ^2 = (A+B)^2$ |
| | But $ A+B = 60$ |
| | $(60)^2 = A ^2 + 2 A B + B ^2$ |
| | $3600 = A ^2 + 2 A B + B ^2$ |
| | $3600 = A ^2 + B ^2 + 2 A B \quad \dots \text{--- (i)}$ |
| | Again |
| | From $ A-B ^2 = (A-B)^2$ |
| | But $ A-B = 60$ |
| | $(60)^2 = A ^2 - 2 A B + B ^2$ |
| | $3600 = A ^2 + B ^2 - 2 A B \quad \dots \text{--- (ii)}$ |
| | Add eqn(i) and (ii) |
| | $ A ^2 + B ^2 + 2 A B = 3600$ |
| | $ A ^2 + B ^2 - 2 A B = 3600$ |
| | $2 A ^2 + 2 B ^2 = 7200$ |
| | divid by 2 both side |
| | $ A ^2 + B ^2 = 3600$ |
| | But $ B = 46$ |
| | $ A ^2 + (46)^2 = 3600$ |
| | $ A ^2 + 2116 = 3600$ |
| | $ A ^2 = 3600 - 2116$ |
| | $ A ^2 = 1484$ |
| | Apply square root both side |

Extract 11.2: A sample of correct responses to question 11

In Extract 11.2, the candidate performed the correct calculations to obtain the required values.

2.2.2 Question 12: Differentiation

This question examined candidates' knowledge on the application of differentiation. The word problem stated that; "An open rectangular box with square ends is fitted with an overlapping lid, which covers the top and front face". The candidates were required to determine the maximum volume of the box if 6 m^2 of metal are used to make it.

The question was attempted by 523 (100%) candidates and all of them had scores ranging from 0 to 4 marks. Thus, the general performance in this question was weak. Figure 12 shows the performance of candidates in this question.

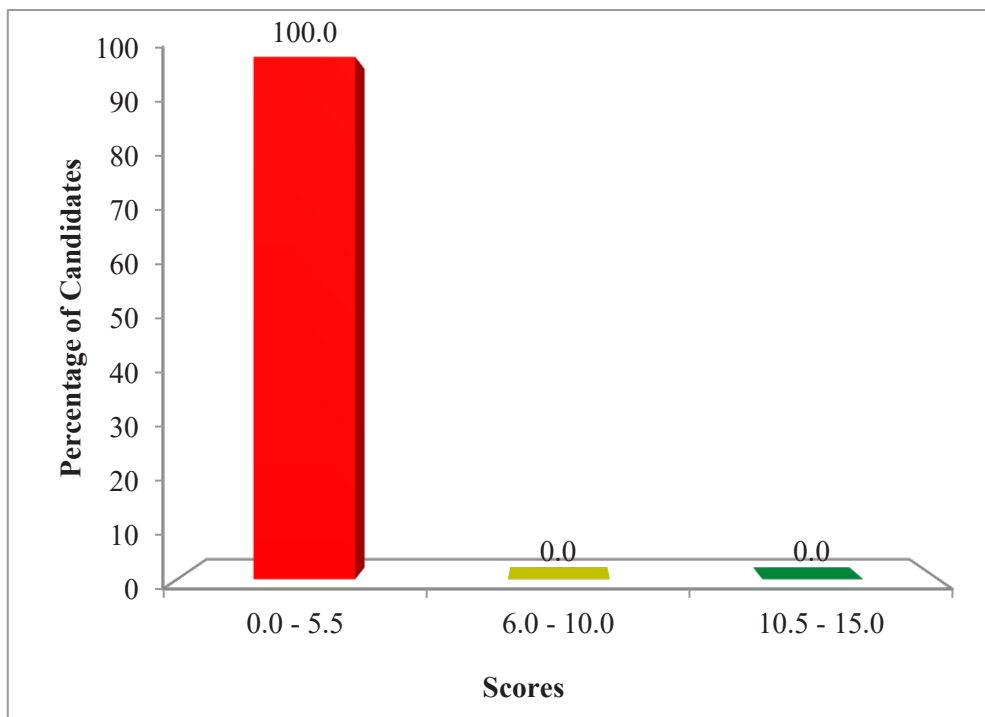


Figure 12: Performance of candidates in question 12

Further data analysis shows that 467 (89.3%) candidates scored zero and 56 (10.7%) candidates had scores ranging from 0.5 to 4 marks. Most of the candidates failed to translate the given word problem into mathematical equation. This shows lack of competence in solving word problems on applications of differentiation. For example, one candidate sketched a square figure whose sides are 6 m^2 and used it to find the volume. Another candidate drew a rectangular figure with sides x and $x - 1$ so that Area = Length \times Width. He/she made wrong calculations as follows;

$$6\text{ cm}^2 = x(x - 1)$$

$$\sqrt{6} = (x + 1)^2$$

$$x = \sqrt{6 + 1} = 3.4$$

$$\text{Volume} = \frac{1}{3}\pi r^2$$

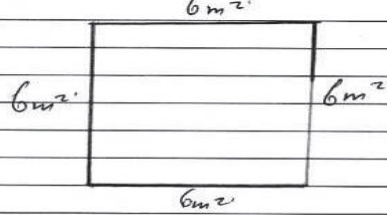
Some of the candidates calculated the volume of a rectangular box, that is;

$$\text{Volume} = w \times h \times l = 60\text{m}^2 \times h$$

$$\text{Volume} = (60H - 60h)\text{m}^3$$

Volume = $60(H - h)\text{m}^3$ without defining the variable H .

Although the sketch from the word problem represents a rectangular box, some candidates applied the formula $V = \frac{1}{3}\pi r^2 \times 2$ or $V = \frac{4}{3}\pi r^3$ to find the volume of the box, where r was regarded as the diameter of the box, that is, $r = 6$ cm. Extract 12 shows an incorrect response from one of the candidates.

| 12 | |
|-----|--|
| 12. | <p style="text-align: center;">soln.</p> <p>Rectangular box with square ends.</p> <p>Given $6m^2$ of metal are used to make it.</p> <p>Required to find or determine the maximum volume of the box.</p> <p>consider rectangular box below.</p> <div style="text-align: center;">  </div> <p>maximum volume of the box = $L \times width \times base$ $= 6 \times 6 \times 6$ $= 216$ $= length \times width \times base$ $= 6 \times 6 \times 6$ $= 216 m^3$</p> <p>\therefore This maximum volume of the box = $216 m^3$</p> |

Extract 12: A sample of incorrect responses to question 12

In Extract 12, the candidate calculated the volume of a square, which is not applicable in Geometry.

It is important to note that, the data analysis displays that no candidate answered this question correctly.

2.2.3 Question 13: Planning and Preparation for Teaching Mathematics

The question examined the candidates' ability to plan and prepare the lesson to teach mathematics. The word problem stated that; "Suppose you

are preparing to teach a topic on Sequence and Series to Form Two students, prepare a detailed 80 minutes lesson plan to teach the concept about the sum of first n terms of an arithmetic progression”.

The question was attempted by 523 (100%) candidates whereby, 449 (85.8%) candidates had scores ranging from 6 to 15 marks. Therefore, the general performance of candidates in this question was good. Figure 13 shows a summary of candidates' performance in the question.

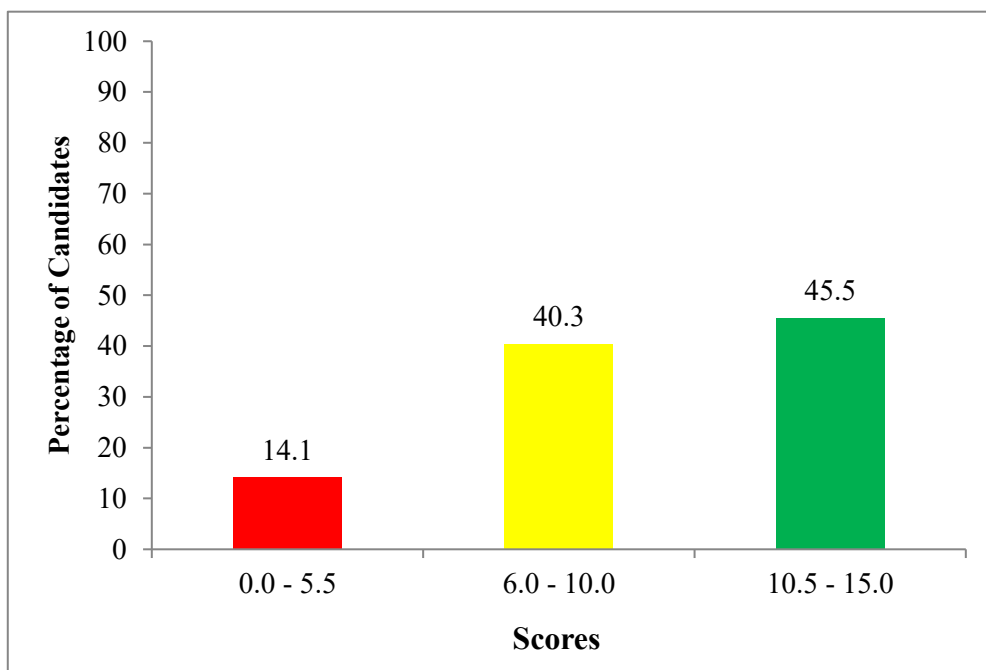


Figure 13: *Performance of candidates in question 13*

The data analysis shows that 74 (14.1%) candidates had scores ranging from 0 to 5.5 marks, 14 (2.7%) candidates scored zero, 211 (40.3%) candidates had scores ranging from 6 to 10 marks, and 238 (45.5%) candidates had scores ranging from 10.5 to 15 marks.

The candidates, who scored at least 10.5 marks, were able to plan and prepare the lesson. This shows that the candidates understood the format of the lesson plan. Also, they were able to state appropriately the teaching, learning, and assessment activities of the lesson. Extract 13.1 shows a correct response from one of the candidates.

NAME OF SCHOOL: IPALAMAGANG SECONDARY SCHOOL
 NAME OF THE TEACHER: BARAKA CYPRIAN
 SUBJECT NAME: BASIC MATHEMATICS

| Date | Class | Period | Time | Number of Students | | | | | |
|-----------|---------|--|-------------------|--------------------|-------|-------|---------|-------|-------|
| | | | | Registered | | | Present | | |
| 2021/2022 | Form II | 1 st and 2 nd | 8:00 - 9:20 am | Boys | Girls | Total | Boys | Girls | Total |
| | | | | 25 | 20 | 45 | | | |

Competence: Students to show the ability to find the arithmetic progression.

General objective: Student should understand to find the sum of the terms of arithmetic progression.

Topic: Sequence and series

Sub-topic: Arithmetic progression

Specific objective: By the end of 80 minutes, each student should be able to find the sum of first n terms of an arithmetic progression.

Teaching and Learning Aid: A chart showing the series of numbers

Teaching and learning method: Group discussion method.

References: Tanzania Institute of Education, (2021), ~~Maths~~ MATHEMATICS FOR SECONDARY SCHOOLS, Form Three, Student's Book, Tanzania Institute of Education Dar es Salaam, Tanzania.

LESSON DEVELOPMENT

| 13 Cont. | Stages | Time | Teaching activities | Learning activities | Assessments. |
|----------|---------------|----------|--|---|---|
| | Introduction. | 10 mins | Guiding the students to state the sequences of arithmetic progression | Students stating the sequences of arithmetic progression | Observing if the student is able to state correctly sequences of the arithmetic progression. |
| | New knowledge | 45 mins | Leading the students in groups to discuss on how to find the sum of first n terms of an A.P (arithmetic progression) - Making more clarifications from their discussion | Discussing on how to find the sum of first n terms of an arithmetic progression - Listening from teacher's clarification | Checking if the student is able to find the sum of the first n terms of an arithmetic progression (AP) correctly. |
| | Reinforcement | 15 mins. | Displaying the chart with the series of number and leading the student to find the sum of the terms. | Finding the sum of the terms displayed. | Checking and observing if the student is able to find the sum of the terms given correctly. |
| | Reflection | 5 mins. | Leading the student to state the application of what was learnt in the real situation | Stating the applications. | Checking if the student is able to state the applications of what learnt. |

| 3 Cont. | Stages | Teaching activities | Learning activities | Assessments |
|---------|-----------------------|--|--|-------------------------------------|
| | Consolidation | Guiding the student to comment on the lesson and providing exercise. | Commenting on the lesson the way it was and responding to the exercise given | Noting down comments from students. |
| | Student's evaluation: | | | |
| | Teacher's evaluation: | | | |
| | Remarks: | | | |

Extract 13.1: A sample of correct responses to question 13

In Extract 13.1, the candidate demonstrated good understanding of the format of a lesson plan and wrote appropriate teaching, learning, and assessment activities of the lesson.

The candidates (14.1%) who scored low marks in this question failed to plan and prepare the lesson. Some of them wrote the preliminary part of the lesson plan, drew a matrix of the lesson development but could not write the statements of teaching, learning, and assessment activities. Others had misconception to the question requirements as they wrote an essay on the details of the lesson plan instead of preparing the lesson plan. Some of the candidates could not differentiate the statements of competence and general objective. For instance, one candidate stated the statements of competence and general objective as follows:

Competence: Student should be able to understand concept of sequence and series with real application in real life situation.

General objective: Student should be able to understand the concept about the sum of the first n terms of an arithmetic progression.

Further analysis shows that some candidates prepared a Scheme of work instead of a lesson plan. Extract 13.2 shows one of the incorrect answer in this question.

| | |
|----|---|
| 13 | Soln |
| | from first n term |
| | General sum of n is $S_n = \frac{n(n-1)d}{2}$ |
| | $A_n = A_1 + (n-1)d$ |
| | When you give |
| | $2 + 6 + 12 + 20 \dots$ |
| | $1 \times 2 + 2 \times 2 + 3 \times 4 + 4 \times 5$ |
| | So |
| | $1 + 2 + 3 + 4 \dots n$ |
| | and |
| | $2 + 3 + 4 + 5 \dots n$ |
| | from |
| | $A_{n1} = A_1 + (n-1)d$ |
| | $A_{n2} = A_2 + (n-1)d$ |
| | $A_1 = 1$ and A_2 |
| | but $d = ?$ |
| | $A_{n1} = 1 + (n-1)d$ |
| | and |
| | $A_{n2} = 2 + (n-1)d$ |
| | Then find summation |
| | $A_{n1} + A_{n2} = A_1 + A_2 + (n-1)d + (n-1)d$ |
| | $\frac{2S_n}{2} = \frac{2A_1 + 2(n-1)d}{2}$ |

Extract 13.2: A sample of incorrect responses to question 13

In Extract 13.2, the candidate failed to understand the requirements of the question. Therefore, he/she performed irrelevant calculations.

2.2.4 Question 14: Analysis of Mathematics Curriculum Materials

This question examined candidates' ability to analyse briefly curriculum materials. The question stated that, "If the head of a school intends to purchase Mathematics reference books, then analyse five factors he/she should consider before purchasing suitable books".

The question was attempted by 523 (100%) candidates, whereby 482 (92.1%) candidates had scores ranging from 6 to 15 marks. Hence, the general performance of candidates in this question was good. Figure 14 illustrates the performance of candidates in the question.

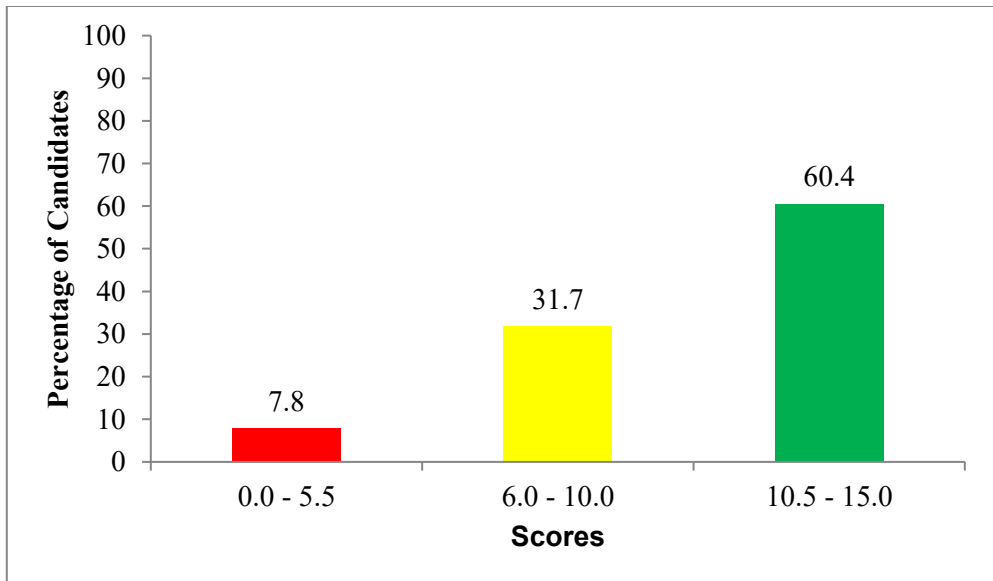


Figure 14: *Performance of candidates in question 14*

The data analysis shows that 41 (7.8%) candidates had scores ranging from 0 to 5.5 marks, 166 (31.7%) had scores ranging from 6 to 10 marks, and 316 (60.4%) had scores ranging from 10.5 to 15 marks. Most of the candidates answered this question correctly because of good pedagogical skills on mathematics curriculum materials. Extract 14.1 shows a correct response from one of the candidates.

Mathematic reference books; Refer to the extra books which are used during teaching and learning process. For example of mathematics reference books such as journal, pamphlet and newspaper which are used during conducting teaching and learning process. Also reference books can be able to expand our knowledge, skills, and understanding when we used it well.

The following are the factors he/she should consider so as to purchase suitable books

Level of the learner; when he or she considering the suitable books should look the level of the learner of he or she is teaching so as to understand strength and weakness of that book he or she purchase during teaching and learning process

language used; When the head of school is choosing a suitable books he or she should consider language used to that books when the language used is simple or complex so as to understand the understanding of your learners during conducting teaching and learning process.

Picture, diagram and illustration; During the consideration of mathematics suitable books you to consider also the picture, diagram and illustration used to that books in order to create skills and knowledge of learners during teaching and learning process

Durability pages; Also you have to look the page durability when considering the mathematics reference books in order to be suitable when they are using in the class so that to stay for long time when they are using in studying because others learners are not careful in protecting books during teaching

| | |
|----------|--|
| 14 Cont. | and learning process |
| | Syllabus; The suitable books should lie on syllabus during teaching and learning process when choosing a suitable books the number of book topics should be same with that of syllabus in order to go specific with syllabus and learners understand well during conducting teaching and learning process. |
| | Generally; Through considering a suitable books we can be able to create knowledge, skills and understanding during teaching and learning process through ideas obtained from the suitable books. |

Extract 14.1: A sample of correct responses to question 14

In Extract 14.1, the candidate demonstrated good knowledge on mathematics curriculum materials and he/she was able to analyse the factors to consider before purchasing suitable books.

On the other hand, the candidates who had poor performance lacked the knowledge on mathematics curriculum materials. Some of them mentioned the author of the book, publisher's name, area of publication, and level of the author as the factors to consider. Others explained that lack of reference books, inadequate of different materials, interest of the learners to study different materials, increased performance of the students and motivation of learners to study are the factors to consider before purchasing mathematics book. Extract 14.2 is a sample response of a candidate who failed to understand the demand of the question.

| | |
|----|---|
| 14 | The following are the factors consider to purchase suitable book; |
| | i/ Author of the book. |
| | ii/ Year of but publication. |
| | iii/ A publisher name. |
| | iv/ Area of publication. |
| | v/ level of the author. |

Extract 14.2: A sample of incorrect responses to question 14

In Extract 14.2, the candidate mentioned the details of a book instead of analysing the factors to be conserved before purchasing suitable books.

3.0 THE ANALYSIS OF CANDIDATES PERFORMANCE PER TOPIC

The analysis of candidates' performance per topic showed that four out of 14 topics examined had a good performance. These topics are; *Assessment in Mathematics* (96.6%), *Analysis of Mathematics Curriculum Materials* (92.1%), *Planning and preparation for teaching Mathematics* (85.8%), and *Calculating Devices* (77.2%).

Two topics had an average performance, namely; *Vectors* (49.4%) and *Algebra* (41.1%). Further analysis shows that the candidates had weak performance in eight topics, which are *Linear Programming* (24.5%), *Teaching of Selected Topics* (21.6%), *Hyperbolic Functions* (17.0%), *Similarity and Congruence* (12.6%), *Coordinate Geometry II* (10.9%), *Integration* (4.4%), *Trigonometry* (4.2%), and *Differentiation* (0%). This weak performance was due to candidates' lack of knowledge about the formulae, failure to understand the requirements of the questions and lack of awareness on the basic concepts in these topics.

Further analysis shows that one topic had good performance for three consecutive years, which is *Analysis of Mathematics Curriculum Materials* having a performance of 98.1 percent in 2021, 97.1 percent in 2022 and 92.1 percent in 2023. This performance was due to the reason that the questions from this topic have been a part of candidates' daily activities in college classroom. For the topics with poor performance, the candidates scored low marks because of lack of knowledge on the basic formula associated.

4.0 CONCLUSION

The general performance for this subject in 2023 examination has increased by 3.9% compared to that of 2022 with an overall average of 44.9%, while that of 2022 had an overall average score of 41.0%. The performance of candidates on *Assessment in Mathematics* topic has improved from weak in 2022 to good in 2023. In 2022, the performance was 2.7 percent while in 2023 the performance was 96.6 percent.

5.0 RECOMMENDATIONS

In order to improve the performance of candidates in future examinations especially in the topics, which has weak performance, the National Examinations Council of Tanzania suggest that:

- (a) Tutors should use individual exercises on solving problems involving similarity theorems of triangles and life experiences.
- (b) Tutors and students should demonstrate on the proof of congruence of triangles during teaching and learning in the classroom.
- (c) Students should use group discussion, gallery walk, demonstration, and practical in their learning.
- (d) Tutors should enable the students to use internet or library search on the concept of ellipse with real life.
- (e) Tutors should use the brainstorming, pair experimentation and pair reflection teaching strategies.
- (f) Tutors should provide the project work on designing the mathematics activities using principles during teaching and learning process.
- (g) Students should be encouraged to use individual demonstration, microteaching and self-oral presentation.

APPENDIX

**SUMMARY OF THE CANDIDATES' PERFORMANCE IN
MATHEMATICS SUBJECT**

| 2022 | | | | | | 2023 | | | |
|------|---|-----------------|----------------------------------|-----------------------------------|---------|-----------------|----------------------------------|-----------------------------------|---------|
| S/N | Topic | Question Number | Performance in Each Question (%) | Average Performance Per Topic (%) | Remarks | Question Number | Performance in Each Question (%) | Average Performance Per Topic (%) | Remarks |
| 1. | Assessment in Mathematics | 6 | 2.7 | 2.7 | Weak | 7 | 96.6 | 96.6 | Good |
| 2. | Analysis of Mathematics Curriculum Materials | 13 | 98 | 97.1 | Good | 14 | 92.1 | 92.1 | Good |
| | | 14 | 96.2 | | | | | | |
| 3. | Planning and preparation for teaching Mathematics | 3 | 34.5 | 51.6 | Average | 13 | 85.8 | 85.8 | Good |
| | | 10 | 68.6 | | | | | | |
| 4. | Calculating Devices | 2 | 46.7 | 46.7 | Average | 1 | 77.2 | 77.2 | Good |
| 5. | Vector | 7 | 18.3 | 18.3 | Weak | 11 | 49.4 | 49.4 | Average |
| 6. | Algebra | 5 | 24.5 | 24.5 | Weak | 8 | 41.1 | 41.1 | Average |
| 7. | Linear Programming | 9 | 83.5 | 83.5 | Good | 6 | 24.5 | 24.5 | Weak |
| 8. | Teaching of Selected Topics | - | - | - | - | 4 | 21.6 | 21.6 | Weak |

| 2022 | | | | | | 2023 | | | |
|------|---------------------------|-----------------|----------------------------------|-----------------------------------|---------|-----------------|----------------------------------|-----------------------------------|---------|
| S/N | Topic | Question Number | Performance in Each Question (%) | Average Performance Per Topic (%) | Remarks | Question Number | Performance in Each Question (%) | Average Performance Per Topic (%) | Remarks |
| 9. | Hyperbolic Functions | 11 | 45.8 | 45.8 | Average | 9 | 17.0 | 17.0 | Weak |
| 10. | Similarity and Congruency | - | - | - | - | 2 | 12.6 | 12.6 | Weak |
| 11. | Coordinate Geometry II | 4 | 0.8 | 0.8 | Weak | 3 | 10.9 | 10.9 | Weak |
| 12. | Integration | 12 | 3.8 | 3.8 | Weak | 10 | 4.4 | 4.4 | Weak |
| 13. | Trigonometry | - | - | - | - | 5 | 4.2 | 4.2 | Weak |
| 14. | Differentiation | 8 | 12 | 12 | Weak | 12 | 0 | 2.1 | Weak |

