THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2023

ADDITIONAL MATHEMATICS

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042 ADDITIONAL MATHEMATICS

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FOREWORD

This report presents Candidates' Items Response Analysis (CIRA) on the Certificate of Secondary Education Examination (CSEE) which was conducted in November 2023. The Certificate of Secondary Education Examination marks the end of four years of secondary education. It is a summative evaluation which, among other aspects, assesses the knowledge and skills acquired by the candidates in ordinary secondary education level. The report aims to provide feedback to all education stakeholders on the candidates' performance in the various question items in Additional Mathematics.

The Additional Mathematics paper consisted of fourteen (14) questions set from sixteen (16) topics. Generally, the overall performance of candidates in this paper was good. The good performance was enhanced by candidates' competence in the tested concepts; that is, they were equipped with the relevant knowledge and skills. However, few candidates had low performance which was attributed to inability of the candidates to perform basic operations, failure to follow instructions, and poor interpretation of the questions.

The intention of the National Examinations Council of Tanzania to prepare this report is to help ongoing students, teachers, and other education stakeholders set appropriate strategies for future improvement of candidates' performance in Additional Mathematics.

The National Examinations Council of Tanzania appreciates the contributions of examination officers and all those who participated in the preparation of this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report presents an analysis of candidates' responses who sat for 042 Additional Mathematics examination in the Certificate of Secondary Education Examination (CSEE) conducted in November 2023. The 042 Additional Mathematics paper was set according to the 2022 examination format derived from the 2010 Additional Mathematics Syllabus for Secondary Schools, Form I – IV. The paper was comprised of ten (10) questions in Section A, carrying six (6) marks each and four questions in Section B, carrying ten (10) marks each. The candidates were required to answer all the questions in both sections.

A total of 417 candidates sat for the CSEE Additional Mathematics examination in 2023. The analysis shows a decrease in the candidates' performance by 1.91 per cent in 2023 as compared to 2022. The summary of the candidates' performance for the two years is provided in Table 1.

Table 1: Candidates' Performance on Additional Mathematics CSEE 2022 and 2023

Year	Candidates Sat	Passed		Grades				
		No.	%	A	B	C	D	F
2022	394	393	99.75	73	84	195	41	01
2023	417	408	97.84	66	80	183	79	09

Table 1 indicates that, majority of the candidates scored good grades (A, B and C) in both years; 2022 and 2023.

Section 2.0 of this report provides a detailed analysis of candidates' performance for each item. The analysis briefly explains the requirements of each item as well as how the candidates' responded to. Extracts from the scripts of the candidates for both good and poor responses are included to justify the candidates' strengths and weaknesses. The factors for the good and poor performance of the candidates for each item are also given out. Furthermore, the analysis of the candidates' performance on each of the examined topic is presented in Section 3.0. Section 4.0 provides the recommendations that education stakeholders would take to improve the candidates' performance in the future examination.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH QUESTION

This section analyses candidates' performance in each part of the questions. In this analysis, the scores are categorized in the interval 0 – 29 per cent for poor performance, 30 – 64 per cent for average performance and 65 – 100 per cent for good performance. Similarly, the green, yellow and red colours have been used to represent good, average and poor performance respectively.

2.1 Question 1: Variations

The question consisted of parts (a) and (b). In part (a), the candidates were required to briefly explain the term joint variation and give one example of it. In part (b), the candidates were informed that "F varies directly as m and the square of v, and inversely as r." Then, they were required to:

(i) express the statement in the equation form.

(ii) use the equation in part (i) to find the values of a and b in the following table:

F	m	v	r
60	6	4	8
60	9	a	3
-25	b	-2	-4

The analysis of the data revealed that 22 candidates scored 0 to 1.5 marks, 33 candidates scored 2.0 to 3.5 marks and 362 candidates scored 4.0 to 6.0 marks. Therefore, 395 (94.72%) candidates scored 2.0 marks or more, indicating that the candidates' performance in this question was good. The summary of candidates' performances is given in Figure 1.

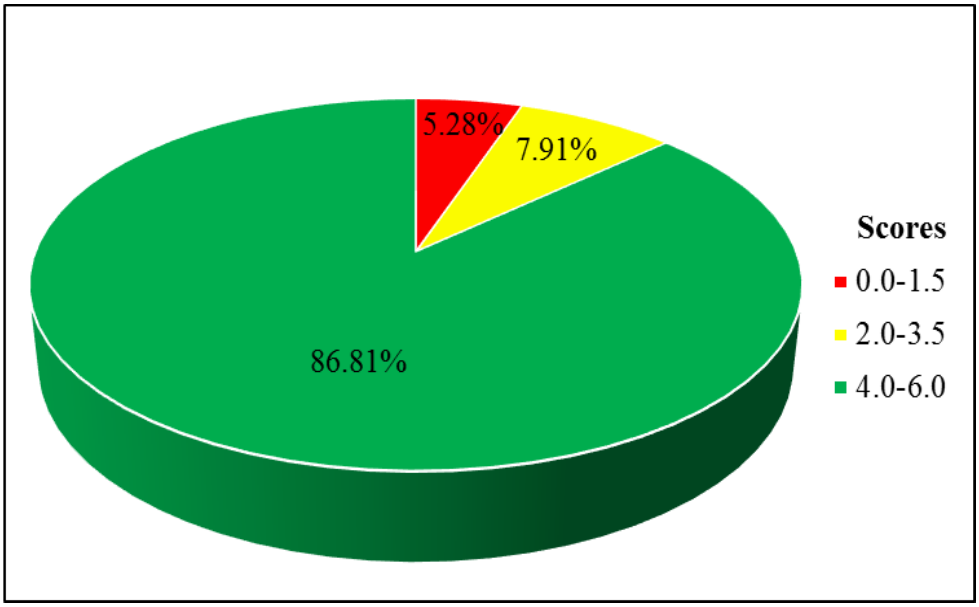


Figure 1: *Candidates' Performance on Question 1*

The data shows that 17.99 per cent of the candidates who managed to score 6.0 marks were able to provide the correct answers to all items. For instance, in part (a), the candidates briefly explained that; joint variation occurs when one variable depends on more than one variable. Then, they provided some examples, such as the facts that the mass (m) of a body depends on volume (v) and density (d), that is $m \propto vd$. In part (b) (i), the candidates correctly interpreted and wrote the word problem into the form $F \propto \frac{mv^2}{r}$. Thereafter, they introduced the proportionality constant (k) and obtained the equation $F = k\frac{mv^2}{r}$.

In part (b) (ii), the candidates correctly applied the mathematical equation; $F = k\frac{mv^2}{r}$ and consequently interpreted $\frac{F_1 r_1}{m_1 {v_1}^2} = \frac{F_2 r_2}{m_2 {v_2}^2}$. Then, they replaced $\frac{F_1 r_1}{m_1 {v_1}^2} = \frac{F_2 r_2}{m_2 a^2}$ with 60, 8, 6, 4, 60, 3, 9 and a in the equation and got $a = \pm 2$. Similarly, they replaced $\frac{F_3 r_3}{b {v_3}^2} = 5$ with -25, -4, and -2

in the equation to get the value of $b = 5$. Extract 1.1 is a sample response from one of the candidates who correctly responded to this question.

01. a) Joint variation refers to the variation ~~in~~ which shows relationship of more than one variables into comparison.

forexample;

$P \propto y, \quad P \propto \frac{1}{x2}$.

$= P \propto \frac{y}{v2}$.

b) i. Solution.

$F \propto m, \quad F \propto v^2, \quad F \propto 1/r$.

By combining,

$F \propto \frac{mv^2}{r}$.

Removing proportionality sign.

$F = \frac{K(mv^2)}{r}$.

$\therefore F = \frac{K(mv^2)}{r}$.

01. ii. from:

$$F = \frac{K(mv^2)}{r}$$

$$K = \frac{Fr}{mv^2}$$

$$K = \frac{60 \times 8}{6 \times 4^2}$$

$$= \frac{60 \times 8}{6 \times 16}$$

$$= 5$$

For a,

$$F = \frac{K(mv^2)}{r}$$

$$60 = \frac{5(9a^2)}{3}$$

$$\frac{5(9a^2)}{5} = \frac{180}{5}$$

$$\frac{9a^2}{9} = \frac{36}{9}$$

$$a^2 = 4$$

$$a = \sqrt{4}$$

$$a = \pm 2$$

$\therefore a = \pm 2$.

For b,

$$F = \frac{K(mv^2)}{r}$$

$$-25 = \frac{5(bx-2^2)}{-4}$$

$$100 = 5(4b).$$

$$\frac{100}{20} = \frac{20b}{20}, \quad b = 5.$$

01. $\therefore a = \pm 2$ and $b = 5$.

Extract 1.1: A sample of a correct response to question 1

In Extract 1.1, part (a), the candidate briefly explained the meaning of joint variation and gave an example of it. In part (b) (i), the candidate correctly formulated the equation combining F, m, v and r. In part (b) (ii) the candidate calculated the value of the proportionality constant (k) and used the equation $F = k\frac{mv^2}{r}$ to obtain the values of a and b.

In spite of the good performance, 22 (5.28%) candidates scored 1.5 marks or less. These candidates faced the following challenges: In part (a), most candidates failed to explain the meaning of joint variation. For example, the candidates wrote, "joint variation is a type of variation which is combined together to show variation or value." Then, they gave some examples, such as M varies and q^2. Other candidates wrote that, "joint variations are the events or occasions which happen together" and provided an example of farmers and union day. Also, other candidates defined joint variation as "a type of variation that comprises two directly or three directly to inversely proportional; example, x varies as y."

In part (b) (i), most candidates failed to write direct and inverse variation, resulting in an incorrect mathematical model. Some candidates wrote $F \propto \frac{m+v^2}{r}$ and consequently $F = \frac{k(m+v^2)}{r}$ instead of $F = k\frac{mv^2}{r}$, and others wrote $F \propto m + \frac{v^2}{r}$, resulting in $F = k\left(m+\frac{v^2}{r}\right)$ instead of $F = k\frac{mv^2}{r}$. Moreover, some candidates ignored the word "square" as they wrote $F = \frac{kmv}{r}$ instead of $F = k\frac{mv^2}{r}$.

The incorrect responses in part (b) (i) led to the incorrect responses in part (b) (ii). For instance, the candidates who substituted $F = 60$, $r = 8$, $m = 6$ and $v = 4$ into $F = \frac{k(m+v^2)}{r}$ obtained $k = \frac{240}{11}$ instead of $k = 5$. These candidates substituted $k = \frac{240}{11}$, $F = 60$, $m = 9$, $v = a$ and $r = 3$ into the incorrect equation and got $a = \frac{\sqrt{3}}{4}$. Furthermore, the analysis revealed that, some candidates substituted the incorrect values of the given variables in $F = \frac{kmv^2}{r}$. For instance, they substituted the values of $m = 5$ and $r = 6$ instead of $m = 6$ and $r = 8$ respectively, in the equation $F = \frac{kmv^2}{r}$ and got $k = \frac{45}{6}$ instead of $k = 5$ and consequently obtained the values of $a = \frac{\sqrt{40}}{3}$ and $b = \frac{5}{16}$. Extract 1.2 is a sample response from one of the candidates who faced difficulties when responding to the question.

1 a> Joint variation
This refer to the varration
Which contain many value of the letter
and jointly many value of letter
Example

$x \propto y - v \propto r$

$x \propto \frac{y-v}{r}$

$x = k\left(\frac{y-v}{r}\right)$

$\frac{\cancel{x}}{1} = k\left(\frac{y-v}{r}\right)$

$xr = k(y-v)$

$k = \frac{xr}{y-v}$

b> $f \propto mv \propto 1/r$

$f = \frac{mv}{r}$

$f = k\left(\frac{mv}{r}\right)$

$fr = k(mv)$

$k = \frac{fr}{mv}$

1 b) $k = \frac{fr}{mv}$

$$k = \frac{60 \times 8}{6 \times 4}$$

$$k = \frac{480}{24}$$

$$k = 20.$$

Then

$$k = \frac{fr}{mv}$$

$$20 = \frac{60 \times 3}{9 \times a}$$

$$20 = \frac{180}{9a}$$

$$\frac{180a}{180} = \frac{180}{180}$$

$$a = 1$$

and b

$$k = \frac{fr}{mv}$$

$$20 = \frac{-25 \times -4}{b \times -2}$$

$$20 = \frac{100}{-2b}$$

$$\frac{-40b}{-40} = \frac{100}{-40}$$

$$b = -2.5$$

Extract 1.2: A sample of an incorrect response to question 1

In Extract 1.2, part (a), the candidate failed to explain the term variation. In part (b) (i), the candidate misinterpreted the given statement as he or she failed to consider a square of v, resulting in an incorrect mathematical model. Then, in part (b) (ii), the candidates obtained the incorrect value of the proportionality constant (k), which lead to the wrong values of a and b.

2.2 Question 2: Statistics

The candidates were given the following frequency distribution table showing the age of 160 people who visited a certain public library.

Age (in years)	8 - 15	16 - 23	24 - 31	32 - 39	40 - 47	48 - 55	56 - 63
Frequency	12	29	40	44	20	12	3

The candidates were required to:

(a) use the class mark of the median class as an assumed mean, to calculate the mean correct to one decimal place.

(b) find the standard deviation.

The analysis of the data depicts that, out of 417 candidates who attempted this question, 122 (29.96%) scored 0 to 1.5 marks, 99 (23.74%) candidates scored 2.0 to 3.5 marks and 199 (47.00%) candidates scored 4.0 to 6.0 marks. Generally, the candidates' performance on this question was good. The summary of the candidates' performance is provided in Figure 2.

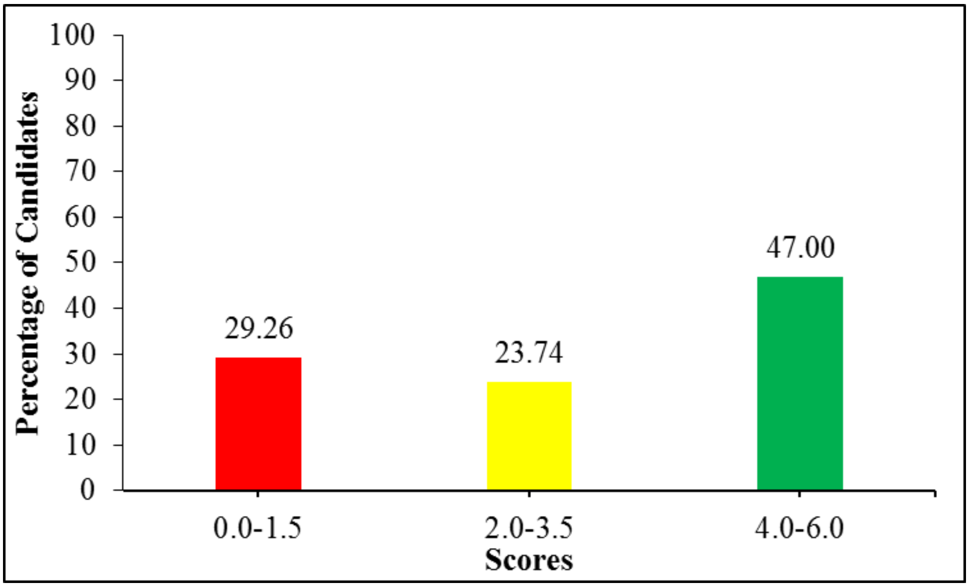


Figure 2: *Candidates' Performance on Question 2*

The data depicts that 70.74 per cent of the candidates scored 2.0 to 6.0 marks of which 19.18 per cent scored full marks. In part (a), the candidates constructed the frequency distribution table correctly and realized that $\sum f = 160$ and $\sum fd = 632$. Then, they correctly identified that the median class is 24.31 and the class mark is 27.5. Therefore, these candidates used an assumed mean, $A = 27.5$, and the formula $\bar{x} = A + \frac{\sum fd}{\sum f}$ to calculate the mean ($\bar{x}$) and got $\bar{x} = 31.5$, correct to one decimal place.

Similarly, in part (b), the candidates constructed a frequency distribution table and realized that $\sum f = 160$ and $\sum f\left(x - \bar{x}\right)^2 = 20352$. Thereafter, they substituted the values in the formula for calculating the standard deviation, $\delta = \sqrt{\frac{\sum f\left(x-\bar{x}\right)^2}{\sum f}}$, and obtained $\delta = 11.28$. Extract 2.1 provides a sample of the correct solutions from one of the candidates.

2..	c/i	f	x	c/f	$x - \bar{x}$ (d)	fd
	8-15	12	11.5	12	-16	-192
	16-23	29	19.5	41	-8	-232
*	24-31	40	27.5	81	0	0
	32-39	44	35.5	125	8	352
	40-47	20	43.5	145	16	320
	48-55	12	51.5	157	24	288
	56-63	3	59.5	160	32	96

$\Sigma fd = 632$

(a) $\bar{x} = A + \frac{\Sigma fd}{N}$

$\bar{x} = 27.5 + \frac{632}{160}$

$\bar{x} = 27.5 + 3.95$

$\bar{x} = 31.45$

Mean in one decimal place;

$\bar{x} = 31.4$.

(b)

$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
-19.95	398	4776
-11.95	142.8	4141.2
-3.95	15.6	624
4.05	16.4	721.6
12.05	145.2	2904
20.05	402	4824
28.05	786.8	2360.4

$\Sigma f(x-\bar{x})^2 = 20351.2$

2	Variance $= \dfrac{\sum f(x-\bar{x})^2}{N}$
	$= \dfrac{20351.2}{160}$
	$= 127.195$
	$S.D = \sqrt{\text{Variance}}$
	$S.D = \sqrt{127.195}$
	Standard Deviation $= 11.278$.

Extract 2.1: A sample of a correct response to question 2

In Extract 2.1, part (a), the candidate constructed a frequency distribution table and used it to get the assumed mean and consequently the mean. In part (b), the candidate correctly calculated the variance and used it to get the standard deviation.

Nevertheless, 13 (3.12%) candidates scored zero. In part (a), most candidates used an incorrect assumed mean. For instance, they used the assumed mean $A = 35.5$, which is the class mark of the middle class. Therefore, they applied the formula for calculating the mean, $\left(\bar{x}\right) = A + \dfrac{\sum fd}{N}$, with $\sum f = 160$ and $\sum fd = 648$, resulting in $\bar{x} = 39.5$ instead of 31.5. Moreover, some candidates applied an incorrect formula for calculating mean. For instance, $\left(\bar{x}\right) = A + \dfrac{\sum fx}{\sum f}$ and realized that $\sum fx = 5032$, $\sum f = 160$ and assumed mean $A = 35.5$, which resulted in $\bar{x} = 67.0$. Likewise, others used the formula $\bar{x} = A + \dfrac{\sum d}{\sum f}$ and got $\bar{x} = 20.2$ after substituting the assumed mean, A = 19.5, $\sum d = 112$ and $\sum f = 160$.

In part (b), some candidates applied the inappropriate formulae. For instance, the candidates used the formula for calculating variance, $\dfrac{\sum\left(x-\bar{x}\right)^2}{\sum f}$, in which they substituted $\sum f = 160$ and $\sum\left(x-\bar{x}\right)^2 = 1904$, resulted to 11.9. Also, a few candidates applied the incorrect formula $\sigma = \sqrt{\frac{\sum fd^2}{N}}$, and substituted $\sum f = 160$ and $\sum fd^2 = 83952$, resulting in $\sigma = 22.90$. Similarly, due to the incorrect mean obtained in part (a), the candidates got the incorrect standard deviation. For instance, some candidates wrote $\sum\left(x - \bar{x}\right)^2 = 40601.6$ instead of 1904, which was substituted into the formula $\sigma = \sqrt{\frac{\sum\left(x - \bar{x}\right)^2}{\sum f}}$, and obtained $\sigma = 253.76$ instead of 11.28. Extract 2.2 is a sample response from one of the candidates who failed to respond correctly to this question.

2. a) FREQUENCY DISTRIBUTION TABLE

class Interval	Frequency	class Mark	Deviation, d=x-A	fd
8 - 15	12	11.5	-24	-288
16 - 23	29	19.5	-16	-464
24-31	40	27.5	-8	-320
32-39	44	35.5	0	0
40-47	20	40.5	5	100
48-55	12	48.5	13	156
56-63	3	56.5	21	63
	$\Sigma f = 160$			$\Sigma fd = -753$

2.(a) Mean $(\bar{X}) = A + \frac{\sum fd}{\sum f}$

$\bar{X} = \frac{35.5}{1} + \frac{-753}{160}$

$\bar{X} = \frac{5680 - 753}{160}$

$\bar{X} = \frac{4877}{160}$

$\bar{X} = 30.5$

2.(b) (S.D) Standard deviation: $\sqrt{\frac{\sum f(X-\bar{X})^2}{N}}$

$\bar{X} = 30.5$ $\quad \sum f = 160$

X	$X-\bar{X}$	$(X-\bar{X})^2$
11.5	-19	361
19.5	-11	121
27.5	-3	9
35.5	5	25
40.5	10	100
48.5	18	324
56.5	26	676
		= 1616

$S.D = \sqrt{\frac{160(1616)^2}{7}}$

$S.D = \sqrt{\frac{160 \times 2611456}{7}}$

$S.D = \sqrt{\frac{417832960}{7}}$

$S.D = \sqrt{59,690,422.86}$

$S.D = 7,725.957731$

Extract 2.2: A sample of an incorrect response to question 2

In Extract 2.2, part (a), the candidate failed to construct the correct frequency distribution table leading to the incorrect mean. In part (b), the candidate wrote $N = 7$ (the number of classes) instead of $N = 160$, leading to an incorrect standard deviation.

2.3 Question 3: Coordinate Geometry

The question consisted of parts (a) and (b). In part (a), the candidates were required to find the points of intersection of the three lines $y+x-8=0$, $y=2x-1$ and $2y-x-1=0$. In part (b), the candidates were asked to determine the equation of a circle passing through the points obtained in part (a).

The data shows that 336 (80.58%) candidates scored marks from 0 to 1.5 and 81 (19.42%) candidates scored marks from 2.0 to 6.0. Generally, the candidates' performance in this question was weak. Figure 3 presents the candidates' performance on this question.

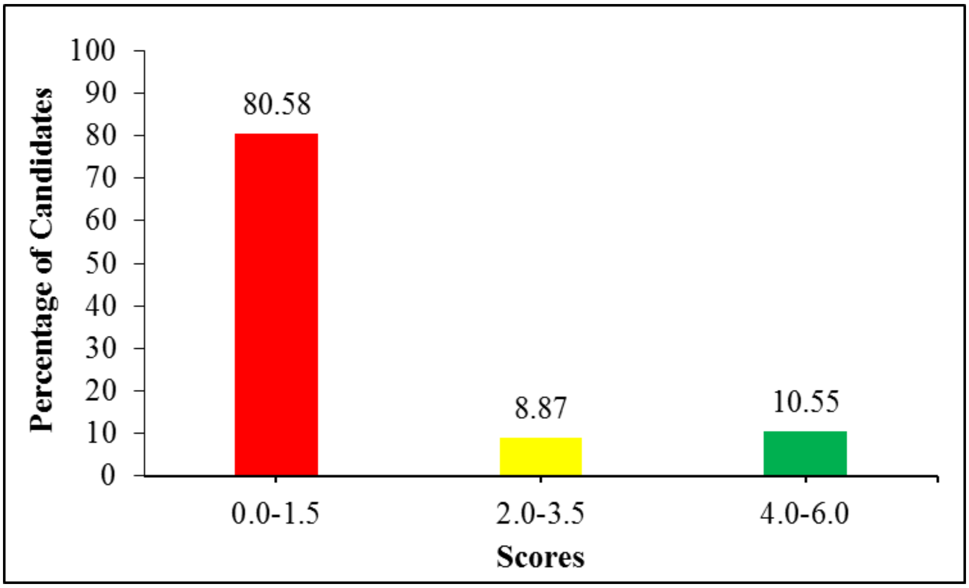


Figure 3: *Candidates' Performance on Question 3*

Figure 3 shows that 80.58 per cent of the candidates scored 1.5 marks or less, indicating that most of the candidates faced challenges in attempting it. In part (a), most candidates responded to the question by finding one point of intersection using the three equations. For instance, they solved for y by using the two equations $y+x-8=0$ and $y=2x-1$ to get $y=5$. Thereafter, they substituted the value of $y=5$ into $2y-x-1=0$ and got $x=9$. Then, they concluded that the point of intersection is $(9,5)$. Moreover, other candidates responded to the question by finding the values of one variable instead of the points of intersection. For example, some candidates solved for x using the two equations $y+x-8=0$ and $y=2x-1$ to obtain x = 3. Then, by using $2y-x-1=0$ and $y=2x-1$, they solved for x and got $x=1$. Similarly, they solved equations $y+x-8=0$ and $2y-x-1=0$ to obtain $x=5$. Therefore, the candidates obtained the values of $x=3$, $x=1$ and $x=5$.

In part (b), most candidates formulated the equation of a straight line instead of a circle. For example, some candidates applied the formula $m=\frac{y_2-y_1}{x_2-x_1}$ to find a slope by substituting incorrect points, particularly $(3, 0)$ and $(0, 2)$ to obtain a slope, $m=\frac{-2}{3}$ and consequently the incorrect equation $y=\frac{-2x}{3}+2$. Also, most of the candidates applied the distance formula, and one point was obtained instead of three points to find the equation of a circle. For instance, they used $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and the point $(5, 3)$ to obtain the incorrect equation of a circle: $r^2=x^2-y^2-10x-6y+34$ instead of $x^2+3y^2-16x-16y+26=0$. Extract 3.2, is a sample of the incorrect response from one of the candidates.

3a	$y+x-8=0$
	x-intercept $y=0$
	$0+x-8=0$
	$x=8$
	$(8,0)$ Y-intercept $x=0$
	$y+0-8=0$
	$y=8$
	$(8,8)$
	$y=2x-1$
	x-Intercept $y=0$
	$0=2x-1$
	$\frac{1}{2}=\frac{2x}{2}$
	$x=0.5$
	Y-Intercept $x=0$
	$y=2(0)-1$
	$y=-1$
	$(0.5,-1)$

3a	$m(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$
	$= \left(\frac{-0.25 + 4.3}{2}, \frac{-0.25 + 3.5}{2}\right)$
	$= (4.05, 3.25)$
	$\therefore$ The points where they intersect are $(4.05, 3.25)$
3b.	The equation of a circle
	$r = (x - a)^2 + (y - b)^2$
	$a = 4$ and $b = 3$
	$r = (x - 4)^2 + (y - 3)^2$
	$r = x^2 - 8x + 16 + y^2 - 6y + 9$
	$r = x^2 - 8x + y^2 - 16y + 25$
	$\therefore$ The equation of a circle is $r = x^2 - 8x + y^2 - 16y + 25$

Extract 3.1: A sample of an incorrect response to question 3

In Extract 3.1, part (a), the candidate computed the x and y intercepts for each line and the mid-point instead of the points of intersection. In part (b), the candidate formulated the equation of a circle by using the incorrect formula, then substituted one point (4, 3).

Despite the weak performance, 37 (8.87%) candidates scored all 6.0 marks. These candidates correctly responded to part (a) by solving the three pairs of equations $y + x - 8 = 0$, $y = 2x - 1$ and $2y - x - 1 = 0$ simultaneously, producing the three points of intersection $(3, 5)$, $(5, 3)$ and $(1, 1)$.

In part (b), the candidates substituted each point obtained in part (a) into $x^2 + y^2 - 2ax - 2by + c = 0$, the general equation of a circle with centre

(a, b). Then, they solved the equations simultaneously and got $a = \frac{8}{3}$, $b = \frac{8}{3}$ and $c = \frac{26}{3}$. Thereafter, they correctly substituted the values into the general equation of a circle and got $3x^2 + 3y^2 - 16x - 16y + 26 = 0$. Extract 3.1 is a sample response from one of the candidates who correctly responded to the question.

3. (a) Solution

Case 1:- Point of intersection of lines $y+x-8=0$ and $y=2x-1$

$$-\begin{cases} y + x = 8 \quad \text{--- (i)} \\ y - 2x = -1 \quad \text{--- (ii)} \end{cases}$$

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3} \qquad x = 3$$

$$\begin{matrix} -2 \\ 1 \end{matrix}\begin{cases} y + x = 8 \\ y - 2x = -1 \end{cases}$$

$$-\begin{cases} -2y - 2x = -16 \\ y - 2x = -1 \end{cases}$$

$$-3y - 0 = -15$$

$$\frac{-3y}{-3} = \frac{-15}{-3} \qquad y = 5$$

1st point $= (3, 5)$

3.(a). Case 2:- Point of intersection between lines

$y + x - 8 = 0$ and $2y - x - 1 = 0$

$$+\begin{cases} y + x = 8 \\ 2y - x = 1 \end{cases}$$

$$3y + 0 = 9$$

$$\frac{3y}{3} = \frac{9}{3} \quad y = 3$$

$$\begin{matrix} 2 \\ 1 \end{matrix}\begin{cases} y + x = 8 \\ 2y - x = 1 \end{cases}$$

$$-\begin{cases} 2y + 2x = 16 \\ 2y - x = 1 \end{cases}$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

$\therefore$ 2^{nd} point $= (5, 3)$

Case 3:- Point of intersection between the lines

$y = 2x - 1$ and $2y - x - 1 = 0$

$$\begin{matrix} 2 \\ 1 \end{matrix}\begin{cases} y - 2x = -1 \\ 2y - x = 1 \end{cases}$$

$$-\begin{cases} 2y - 4x = -2 \\ 2y - x = 1 \end{cases}$$

$$-3x = -3$$

$$\frac{-3x}{-3} = \frac{-3}{-3}$$

$$x = 1$$

3.(b) $-1\left\{ y - 2x = -1 \right.$
$-2\left\{ 2y - x = 1 \right.$

$$-\begin{cases} -y + 2x = 1 \\ -4y + 2x = -2 \end{cases}$$
$$3y - 0 = 3$$

$$\frac{3y}{3} = \frac{3}{3}$$

$$y = 1$$

$\therefore$ 3rd point $= (1, 1)$

(b) **Solution**

Equation of ~~locu~~ circle passing through 3 points

$$x^2 + y^2 + 2fx + 2gy + c = 0$$

At point $(3, 5)$,

$$3^2 + 5^2 + 2(3)f + 2(5)g + c = 0$$
$$9 + 25 + 6f + 10g + c = 0$$

$$6f + 10g + c = -34 \quad \text{---(i)}$$

At point $(5, 3)$,

$$5^2 + 3^2 + 2(5)f + 2(3)g + c = 0$$
$$25 + 9 + 10f + 6g + c = 0$$

$$10f + 6g + c = -34 \quad \text{---(ii)}$$

3(b) At point $(1,1)$,

$$1^2 + 1^2 + 2(1)f + 2(1)g + c = 0$$

$$2 + 2f + 2g + c = 0$$

$$2f + 2g + c = -2 \text{ --- (iii)}$$

Solve the equations simultaneously.

$$6f + 10g + c = -34,$$
$$10f + 6g + c = -34$$
$$2f + 2g + c = -2.$$

$$f = -8/3, \quad g = -8/3, \quad c = 26/3$$

Substitute f, g and c into the equation

$$3\left[x^2 + y^2 + 2(-8/3)x + 2(-8/3)y + 26/3\right] = 0 \times 3$$

$$3x^2 + 3y^2 + -16x + -16y + 26 = 0.$$

Extract 3.2: A sample of a correct response to question 3

In Extract 3.2, part (a), the candidate solved the three equations simultaneously and managed to get the three intersecting points. In part (b), the candidate substituted the intersection points into the general formula of the equation of a circle and finally got the correct equation of a circle.

2.4 Question 4: Locus

The candidates were required to find the equation of the locus of a point which is equidistant from the two points $A(1, 2)$ and $B(5, 4)$.

The data shows that 417 candidates attempted this question. Among them, 59 scored marks from 0 to 1.5, 37 candidates scored marks from 2.0 to 3.5, and 321 candidates scored marks from 4.0 to 6.0. Thus, the performance on this question was generally good. Figure 4 illustrates the summary of the candidates' performance on the question.

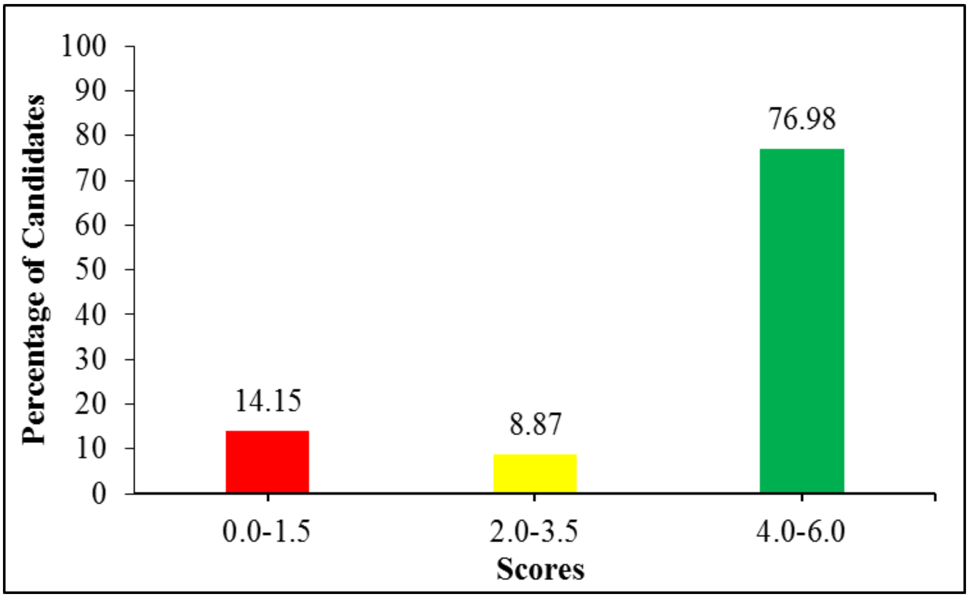
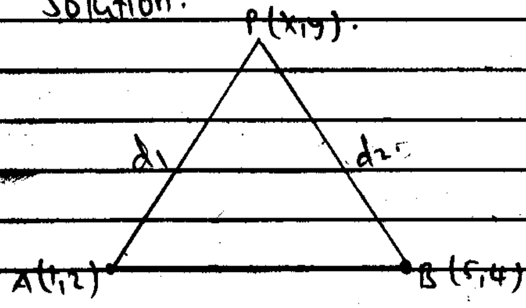


Figure 4: *Candidates' Performance on Question 4*

The candidates who scored full marks correctly applied the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. Then, they substituted the moving point $P(x,y)$ and the points $A(1,2)$ and $B(5,4)$. Thereafter, they substituted the points into the distance formula using the relation $\overline{PA}=\overline{PB}$ and got the equation of the locus of a point as $2x+y-9=0$. Extract 4.1 is a sample of the responses of the candidates who correctly determined the locus under discussion.

04 Solution.



$\overline{AP} = \overline{BP}$.

from.

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(x_2-x_1)^2+(y_2-y_1)^2 = (x_2-x_1)^2+(y_2-y_1)^2$.

$(x-1)^2+(y-2)^2 = (x-5)^2+(y-4)^2$.

$x^2-2x+1+y^2-4y+4 = x^2-10x+25+y^2-8y+16$.

$x^2-2x+y^2-4y+5 = x^2-10x+y^2-8y+41$

$x^2-x^2-2x+10x+y^2-y^2-4y+8y+5-41=0$.

$8x+4y-36=0$.

$\therefore$ $8x+4y-36=0$.

$2x+y-9=0$.

$\therefore$ The equation of the locus is

$2x+y-9=0$.

Extract 4.1: A sample of a correct response to question 4

In Extract 4.1, the candidate correctly wrote the relationship of points that are equidistant as $\overline{PA} = \overline{PB}$, then recalling the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and using it to get the equation of the locus $2x+y-9=0$.

However, the analysis also revealed that 33 candidates got zero. Most of these candidates formulated an equation for a straight line passing through points A and B. These candidates calculated the gradient of a line using the formula $m=\frac{y_2-y_1}{x_2-x_1}$ and the points $A(1,2)$ and $B(5,4)$ to get $m=\frac{1}{2}$ and formulated the equation of a line $x-2y+3=0$. Also, there were candidates who were not able to recall the correct formula. For example, the candidates used the formula $d=\sqrt{x^2+y^2}$ to find distance between the two points. They substituted the points $A(1,2)$ and $B(5,4)$ into the formula and got $d_1=\sqrt{5}$ and $d_2=\sqrt{41}$. Furthermore, other candidates failed to expand the terms correctly. For instance, some candidates correctly wrote $(x-1)^2+(y-2)^2=(5-x)^2+(4-y)^2$, but they incorrectly simplified the equation into $x^2+y^2+20x-10y-36=0$ instead of $2x+y-9=0$. Extract 4.2 is a sample response from one of the candidates who faced challenges when responding to the question.

Qn 4 $A(1,2)$ and $B(5,4)$

Gradient $= \frac{Y_2 - Y_1}{x_2 - x_1}$

$= \frac{4-2}{5-1}$

$= \frac{2}{4}$

Gradien $= \frac{1}{2}$

equation

$A(1,2)$ and (x,y)

$\frac{1}{2} = \frac{Y_2 - Y_1}{x_2 - x_1}$

$\frac{1}{2} = \frac{y-2}{x-1}$

$2(y-2) = x-1$

$2y-4 = x-1$

Extract 4.2: A sample of an incorrect response to question 4

In Extract 4.2, the candidate formulated the equation using the concept of gradient instead of the distance between two points.

2.5 Question 5: Algebra

The question had parts (a) and (b), which stated as follows:

(a) Factorize the expression $4x^2 - 4xy - 3y^2$.

(b) (i) Solve the following simultaneous equations:

$$\begin{cases} y = 3x - 7 \\ y = x^2 - 3x + 2 \end{cases}$$

(ii) Solve the equations $\log_x y = 2$ and $xy = 8$.

The data shows that 361 (86.57%) candidates scored from 2.0 to 6.0 marks and 56 (13.43%) candidates scored from 0.0 to 1.5 marks. Therefore, the candidates' performance in this question was good. Figure 5 presents the candidates' performance summary on this question by percentage.

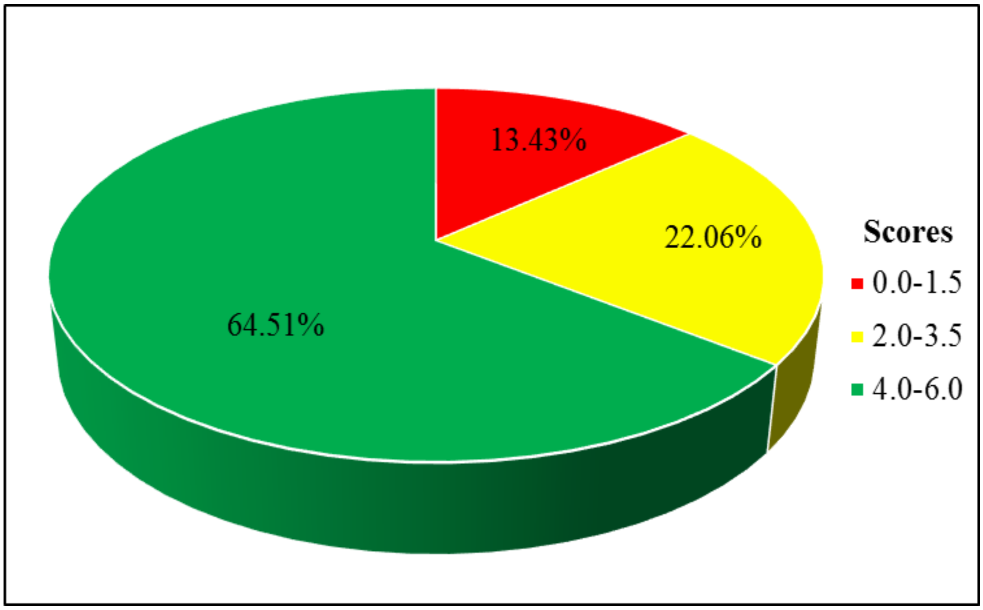


Figure 5: *Candidates' Performance on Question 5*

In part (a), the candidates who managed to score high marks correctly factorized the given expression by splitting the middle term. They rewrote the expression $4x^2 - 4xy - 3y^2$ in the form $4x^2 + 2xy - 6xy - 3y^2$. Then, they factored out the common terms to get $2x(2x+y)-3y(2x+y)$, which resulted in $(2x+y)(2x-3y)$.

In part (b) (i), the candidates correctly applied the substitution method to solve the given simultaneous equations. They substituted $y=3x-7$ into $y=x^2-3x+2$ and simplified into $x^2-6x+9=0$. Then, they performed factorization on $x^2-6x+9=0$ which resulted in $(x-3)^2=0$ and consequently $x=3$. Thereafter, the candidates substituted $x=3$ into $y=3x-7$ to get $y=2$. In part (b) (ii), the candidates correctly converted $\log_x y=2$ into an exponential form $y=x^2$ and therefore replaced y in $xy=8$ with x^2 and obtained $x^3=8$, which led to $x=2$. They then substituted $x=2$ in $y=x^2$ and obtained $y=4$. A sample of a correct response from one of the candidates is given in Extract 5.1.

5(a) $4x^2-4xy-3y^2$

$4x^2-6xy+2xy-3y^2$

$4x^2+2xy-6xy-3y^2$

$2x(2x+y)-3y(2x+y)$

$(2x-3y)(2x+y)$

5(b)(i) $y=3x-7$ — eqn 1

$y=x^2-3x+2$ — eqn 2

substituting the value of y in eqn 2

$3x-7=x^2-3x+2$

$x^2-3x-3x+2+7=0$

$x^2-6x+9=0$

$ac=9\ (-3x-3)$

$x^2-3x-3x+9=0$

$x(x-3)-3(x-3)=0$

$(x-3)(x-3)=0$

$x-3=0$

$x=3$

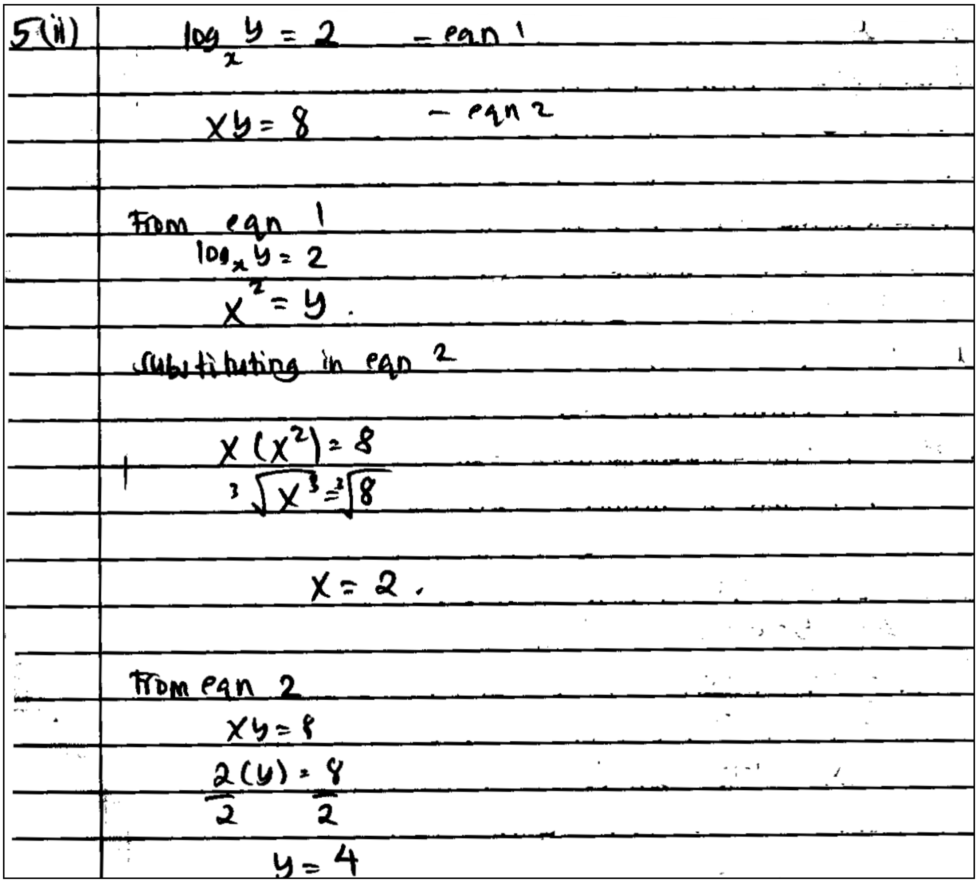
From eqn 1

$y=3x-7$

$y=3(3)-7$

$y=9-7$

$y=2$.



5(ii) $\log_x y = 2$ — eqn 1

$xy = 8$ — eqn 2

From eqn 1

$\log_x y = 2$

$x^2 = y$.

Substituting in eqn 2

$x(x^2) = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$.

From eqn 2

$xy = 8$

$\frac{2(y)}{2} = \frac{8}{2}$

$y = 4$

Extract 5.1: A sample of a correct response to question 5

In Extract 5.1, part (a), the candidate split the middle term, factored the expression and got the product of linear expressions. In part (b) (i), the candidate managed to formulate the equation and correctly solve it to get the values of x and y. In part (b) (ii), the candidate correctly converted the logarithm into an exponential form and solved the equations to get the values x and y.

Contrarily, 21 (5.04%) candidates scored zero due to various challenges. For example, in part (a), the majority of candidates failed to split the middle term when factorizing the given expression. For instance, some candidates wrote $4x(x-y)-y(3y)$ instead of $(2x+2y)(2x-3y)$. Other candidates only factored 4 from the first two terms of $4x^2-4xy-3y^2$ and obtained $4(x^2-xy)-3y^2$. Also, some of the candidates factored $4x$ in $4x^2-4xy-3y^2$, and hence, they got $4x(x-y)-3y^2$ and thereafter, they wrote $(x-y)-3y^2$. Futhermore, some candidates equated the expression to zero and solved the resulting equation, $4x^2-4xy-3y^2=0$. That is, they factorized $4x^2-4xy-3y^2=0$ to $4x(x-y)-3y^2=0$, then solved it and obtained $x=0$ and $y=0$.

In part (b) (i), the candidates made some computational errors. For instance, most candidates, correctly managed to make x the subject of $y=3x-7$ and obtained $x=\frac{y+7}{3}$. Then, they substituted it into $y=x^2-3x+2$ and got $y=\left(\frac{y+7}{3}\right)^2-3\left(\frac{y+7}{3}\right)+2$. However, they wrongly solved the equation, resulting in $y=0$ instead of $y=2$. Then, they substituted $y=0$ in the equation $y=3x-7$ to get $x=\frac{7}{3}$ instead of $x=3$. Moreover, other candidates replaced y in the equation $y=x^2-3x+2$ with 0 to obtain $x^2-3x+2=0$. Then, they simplified it into $(x-1)(x-2)=0$, which resulted in $x=1$ or $x=2$. Thereafter, they substituted $x=1$ and $x=2$ into $y=3x-7$ and got the values of $y=-4$ and $y=-1$.

In part (b) (ii), the analysis showed that most candidates failed to convert logarithmic equations into exponential form. For example, the candidates obtained $y=2^x$ instead of $y=x^2$ from $\log_x y=2$. Therefore, they substituted $y=2^x$ to the equation $xy=8$, resulting in $x(2^x)=2^3$, then wrongly got $x=3$. Likewise, the other candidates expressed $\log_x y=2$ as $y^x=2$, and obtained $y=\frac{8}{x}$ from $xy=8$. Then, they substituted $y=\frac{8}{x}$ into $y^x=2$ to obtain $\left(\frac{8}{x}\right)^x=2$, which was wrongly solved to get $x=4$ and $y=2$ instead of $x=2$ and $y=4$, respectively. In addition, other candidates applied the laws of logarithm inappropriately. For instance, they wrote $\log_x y=2$ as $\frac{y\log x}{\log x}=\frac{2}{\log x}$ instead of $\frac{\log y}{\log x}=2$. Thereafter, simplified it wrongly to get $xy = \frac{2}{\log x}$ and substituted into $xy = 8$ to obtain $8 = \frac{2}{\log x}$, and finally wrote $\log 8 = 2$ instead of $x = 2$ and $y = 4$. Extract 5.2 is a sample of an incorrect response from one of the candidates who incorrectly responded to this question.

6b ii) $\log_x y = 2$ and $xy = 8$

$y^x = 2$ --- (i)

$xy = 8$ -- (ii)

$\frac{xy}{x} = \frac{8}{x}$

$y = \frac{8}{x}$

$\left(\frac{8}{x}\right)^x = 2$

$\frac{8}{x} = \frac{2}{1}$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$xy = 8$

$\frac{4y}{4} = \frac{4}{4}$ $y = 2$

$\therefore$ The value of $x = 4$ and $y = 2$

Extract 5.2: A sample of an incorrect response to question 5

In Extract 5.2, the candidates failed to convert the logarithmic function into an exponential function, resulting in incorrect answers.

2.6 Question 6: Geometrical Constructions

The question consisted of parts (a) and (b). In part (a), the candidates were required to determine the sum of the interior angles of a six sided polygon. In part (b), the question stated that; the interior and exterior angles of a regular polygon are x and $\frac{x-36^0}{3}$, respectively. Then, the candidates were required to determine: (i), the value of x (ii), the size of each exterior angle.

The data shows that, 14 (3.36%) candidates scored marks from 0 to 1.5, 76 (18.23%) candidates scored marks from 2.0 to 3.5 and 327 (78.42%) candidates scored marks from 4.0 to 6.0. The summary of the candidates' performance is presented in Figure 6.

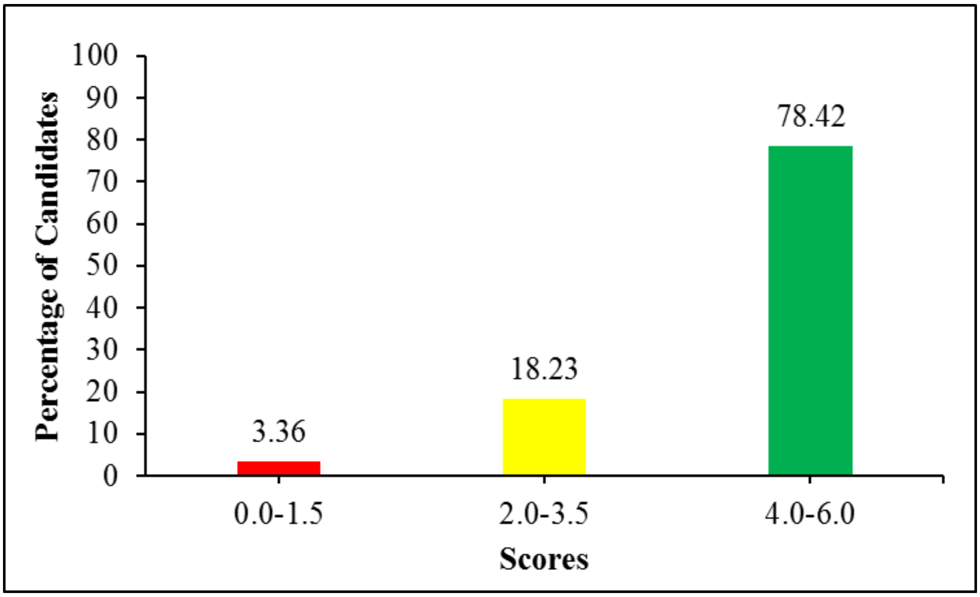


Figure 6: *Candidates' Performance on Question 6*

The general candidates' performance in this question was good because 403 (96.65%) candidates scored 2.0 to 6.0 marks. Those candidates in part (a) recognized that the number of sides of a six sided polygon (hexagon) is 6. Then, they used the formula $(n-2)180^0$ to calculate the sum of interior angles of a regular polygon and got 720^0.

In part (b) (i), the candidates recalled that the sum of the interior angle and exterior angle equals 180^0. Then, they wrote $x+\frac{x-36^0}{3}=180^0$ and solved for x, which resulting in $x=144^0$. In part (b) (ii), the candidates substituted the value of the interior angle $x=144^0$ obtained in part (b) (i) into $\frac{x-36^0}{3}$ to get the size of the exterior angle, 36^0. Extract 2.6 provides a sample of a correct response from one of the candidates.

6 a)

number of sides $n = 6$

Sum of Interior angles $= 180^\circ(n-2)$

$= 180(6-2)$

$= 180 \times 4$

$= 720^0$

$\therefore$ The sum of interior angles is 720°.

6 b)

i)

Interior angle $= x$

exterior angle $= \frac{x-36^0}{3}$

Interior angle + exterior angle $= 180^\circ$

$x + \frac{x - 36^\circ}{3} = 180^\circ$

$\frac{3x + x - 36^\circ}{3} = 180^\circ$

$\cancel{3} \times \frac{4x - 36^\circ}{\cancel{3}} = 180^\circ \times 3$

$4x - 36^\circ = 540^\circ$

$4x = 540^\circ + 36^\circ$

$\frac{4x}{4} = \frac{576^\circ}{4}$

$x = 144^\circ$

$\therefore$ The value of x is 144°

ii.

exterior angle $= \frac{x - 36^\circ}{3}$

$= \frac{144 - 36}{3}$

$= \frac{108}{3}$

$= 36^\circ$

$\therefore$ The size of each exterior angle is 36°

Extract 6.1: A sample of a correct response to question 6

In Extract 6.1, part (a), the candidate correctly identified the number of sides, $n = 6$. Then, he or she substituted $n = 6$ in the correct formula used to determine the sum of the interior angles of a polygon. In part (b), the candidate equated the sum of the interior and exterior angles to 180^0 and correctly solved for x and consequently the exterior angle.

Despite the strengths demonstrated by most candidates, there were 8 (1.92%) candidates who scored zero. In part (a), most of the candidates applied an incorrect formula to determine the sum of the interior angles. For instance, they used $\left(180^0-2\right)\times n$ instead of $\left(n-2\right)\times 180^0$. These candidates substituted $n=6$ in $\left(180^0-2\right)\times n$ and got the sum of the interior angles is 1068^0. Similarly, other candidates used the formula, interior angle $=\frac{n+2}{360^0}$ instead of $\frac{(n-2)\times 180^0}{n}$. Then, they substituted $n=6$ and obtained an interior angle of $\frac{8}{360^0}$. Finally, these candidates did not find the sum of interior angles.

In part (b) (i), some candidates did not recall the fact that the sum of interior and exterior angles at a particular vertex of a polygon is 180^0. Some candidates equated an expression of exterior angle to 180^0, that is $\frac{x-36^0}{3}=180^0$. Then, they solved for x and got $x=576^0$. Furthermore, some candidates wrote $x+\frac{x-36^0}{3}=360^0$ and obtained $x=279^0$. Also, other candidates correctly recalled the concept but committed errors in solving for x. For example, some of them wrote $x+\frac{x-360^0}{3}=180^0$ but ended up with $x=225^0$ instead of $x=144^0$. The difficulties the candidates faced in part (b) (i) led to an incorrect response in part (b) (ii). For example, the candidates substituted the value of $x=279^0$ in $\frac{x-36^0}{3}$ and got the exterior angle $=81^0$ instead of 36^0. Likewise, other candidates substituted $x=45^0$ into $\frac{x-36^0}{3}$ and concluded that the size of the exterior angle is 3^0 and for $x=576^0$, they got the size of the exterior angle is 180^0. Extract 6.2 provides a sample of incorrect responses from one of the candidates.

6a	$(180^\circ - 2)n$
	$n = 6$
	$(180 - 2)6$
	$(178)6$
	$= 1068^\circ$
	$\therefore$ The sum of interior angles are 1068°
6bi)	Exterior angle + Interior angle $= 360^\circ$
	$\frac{x - 36^\circ}{3} + \frac{x}{1} = 360^\circ$
	$\frac{x - 36^\circ}{3} + \frac{x}{1} = \frac{x - 36 + 3x}{3}$
	$\frac{4x - 36^\circ}{3} = \frac{360^\circ}{1}$
	$4x - 36 = 3(360^\circ)$
	$4x - 36 = 1080$
	$\frac{4x}{4} = \frac{1116^\circ}{4}$
	$x = 279^\circ$
	$\therefore$ The value of $x = 279^\circ$
6bii)	$\frac{x - 36^\circ}{3}$ $\quad x = 279$
	$\frac{279 - 36}{3} = \frac{243}{3} = 81^\circ$
	$\therefore$ The size of each exterior angle is 81°

Extract 6.2: A sample of an incorrect response to question 6

In Extract 6.2, part (a), the candidate used the incorrect formula for the sum of the interior angles of a polygon. In part (b), the candidate equated the sum of interior and exterior angles to 360^0 instead of 180^0, resulting in the wrong size of the exterior angle.

2.7 Question 7: Trigonometry

The question had parts (a) and (b) in which the candidates were required to:

(a) show that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$.

(b) solve the equation $3\cot^2 x = 2\cos x$ for x between -90^0 and 360^0 inclusive.

The analysis of the data revealed that 101 (24.23%) scored 2.0 to 6.0 marks and 316 (75.78%) candidates scored 0 to 1.5 marks. The summary of candidates' performance on this question is shown in Figure 7.

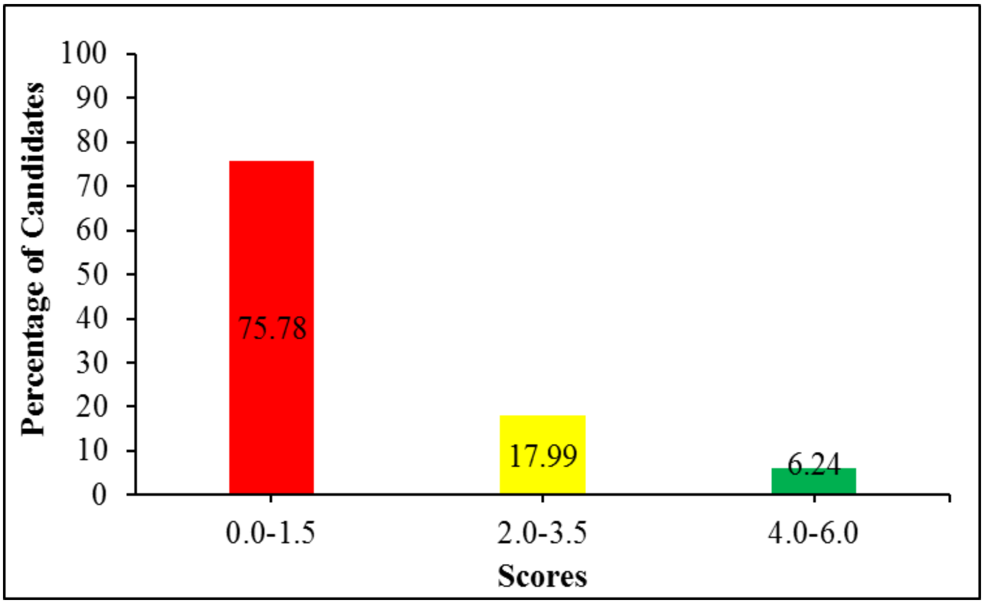


Figure 7: *Candidates' Performance on Question 7*

As Figure 7 shows, the performance of candidates in this question was weak. The candidates' weak performance was attributed to the following factors: In part (a), the candidates used the inappropriate approach of using a calculator instead of the inverse of trigonometrical expressions. Moreover, most candidates misinterpreted the inverse of the trigonometric expression as reciprocal. For instance, some candidates wrote $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$ as $\frac{1}{\tan\left(\frac{1}{2}\right)}+\frac{1}{\tan\left(\frac{1}{3}\right)}=\frac{\pi}{4}$. Thereafter, they substituted the reciprocal wrongly in $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ as $\frac{\tan\left(\frac{1}{2}\right)+\tan\left(\frac{1}{3}\right)}{\tan\left(\frac{1}{2}\right)\tan\left(\frac{1}{3}\right)}$, then wrote $\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{4}$, and finally, ended up with $\frac{\pi}{8}$.

Also, other candidates applied the incorrect formula. They wrote $\tan(A+B)=\frac{\tan A+\tan B}{\tan A\times\tan B}$ instead of $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$. Then, they replaced $\tan A$ and $\tan B$ with $\frac{1}{2}$ and $\frac{1}{3}$, respectively and obtained $\tan(A+B)=5$ and consequently $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=0.0875$

In part (b), most candidates committed computational errors. For instance, some candidates correctly obtained $3\left(\frac{\cos^2 x}{\sin^2 x}\right)-2\cos x=0$, then multiplied it by $\sin^2 x$ to only one term and obtained $3\cos^2 x-2\cos x=0$. Thereafter, they obtained $x=\cos^{-1}\left(\frac{2}{3}\right)$, resulting in $x=48^0$ instead of -90^0, -60^0, 60^0, 90^0, 270^0 and 300^0. Also, other candidates correctly formulated a cubic equation in terms of $\cos x$; $2\cos^3 x+3\cos^2 x-2\cos x=0$. However, they divided the equation by $\cos x$ to obtain $2\cos^2 x+3\cos x-2=0$. Instead of factoring out $\cos x$, they solved the quadratic equation and got $\cos x=\frac{1}{2}$ or $\cos x=-2$, resulting in $x=-60^0$, 60^0 and 300^0. Further, the analysis revealed that most candidates failed to recognize the given intervals of limits for x. For example, some candidates found the difference between the limits and then equated it with one of the given expressions. For instance, the candidates wrote $360^0--90^0=450^0$, then equated the result to $3\cot^2 x$ and obtained $\cot^2 x=150$. Thereafter, they substituted $\frac{1}{\tan^2 x}$ into $\cot^2 x$ to get the value of $x=0.00457$. Extract 7.1 is a sample response from one of the candidates who responded to the question wrongly.

7 ⓐ $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$:

$$\frac{1 \times \frac{1}{2}}{\tan} + \frac{1}{\tan}\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

$$\frac{1}{2} \div \tan + \frac{1}{3} \div \tan = \frac{\pi}{4}$$

$$\frac{1}{2} \times \frac{1}{\tan} + \frac{1}{3} \times \frac{1}{\tan} = \frac{\pi}{4}$$

$$\frac{1}{2\tan} + \frac{1}{3\tan} = \frac{\pi}{4}$$

$$\frac{1}{2\tan} + \frac{1}{3\tan} = \frac{180}{4}$$

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = 45$$

$$26^\circ 33' + 18^\circ 26' = 45^\circ$$

$$45^\circ = 45^\circ$$

Hence shown

$$\therefore \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}:$$

7 (b)

$$3\cot^2 x = 2\cos x$$

$$3(\cot x)(\cot x) = 2\cos x$$

$$3\left(\frac{1}{\tan x}\right)\left(\frac{1}{\tan x}\right) = 2\cos x$$

$$3\left(\frac{1}{\frac{\sin x}{\cos x}}\right)\left(\frac{1}{\frac{\sin x}{\cos x}}\right) = 2\cos x$$

$$3\left(1 \div \frac{\sin x}{\cos x}\right)\left(1 \div \frac{\sin x}{\cos x}\right) = 2\cos x$$

$$3\left(1 \times \frac{\cos x}{\sin x}\right)\left(1 \times \frac{\cos x}{\sin x}\right) = 2\cos x$$

$$3\left(\frac{\cos x}{\sin x} \times \frac{\cos x}{\sin x}\right) = 2\cos x$$

$$\frac{3}{3}\left(\frac{\cos^2 x}{\sin^2 x}\right) = \frac{2\cos x}{3}$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{2\cos x}{3}$$

$$\frac{\cos^{\cancel{2}} x \times 3}{\cancel{\cos x}} = \frac{\sin^2 x \times 2\cancel{\cos x}}{\cancel{\cos x}}$$

$$3\cos x = \sin^2 x$$

$$3\cos(-90) = \sin^2(360^\circ)$$

$$0 = 0$$

$$\therefore 0 = 0$$

Inclusive

Extract 7.1: A sample of an incorrect response to question 7

In Extract 7.1, part (a), the candidate considered the inverse of the trigonometric ratio as reciprocals. In part (b), the candidate failed to simplify the given equation using trigonometric identities.

Nevertheless, the data shows that 1.2 per cent of candidates answered all parts of this question correctly. In part (a), most of these candidates assumed that $\tan^{-1}\left(\frac{1}{2}\right)$ gives an angle A and $\tan^{-1}\left(\frac{1}{3}\right)$ gives an angle B. Thus, they realized that $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$, and then they rewrote $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$ in the form of $\tan(A+B)$. Thereafter, they introduced compound angle formula for tangent to write $\tan(A+B)$ as $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ in which they replaced $\tan A$ and $\tan B$ with $\frac{1}{2}$ and $\frac{1}{3}$ respectively and got $\tan(A+B)=1$. Then, they found the inverse of a tangent in the equation and obtained $A+B=\tan^{-1}(1)$, which equals 45^0. Hence, concluded that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$.

In part (b), the candidates were conversant with trigonometric identities as they correctly recalled that $\cot x=\frac{\cos x}{\sin x}$ and substituted it in the equation $3\cot^2 x=2\cos x$ to obtain $3\frac{\cos^2 x}{\sin^2 x}=2\cos x$ and consequently $3\cos^2 x=2\cos x\sin^2 x$. Further, they applied the identity $\sin^2 x=1-\cos^2 x$ into the equation, resulting in $2\cos^3 x+3\cos^2 x-2\cos x=0$. Which is equivalent to $\cos x(2\cos x-1)(\cos x+2)=0$. Therefore, they got $\cos x=0$, $\cos x=\frac{1}{2}$ or $\cos x=-2$, which yielded $x=-90^0$, -60^0, 60^0, 90^0, 270^0 and 300^0. Extract 7.2 shows a sample of a correct response provided by one of the candidates.

7 a) Soln.

$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$.

Consider LHS

$= \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$

~~Recall~~ let

$\tan^{-1}\left(\frac{1}{2}\right) = x$

$\tan x = \frac{1}{2}$

$\tan^{-1}\left(\frac{1}{3}\right) = \cancel{x}\ y$.

$\tan \cancel{y} = \frac{1}{3}$

Apply $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$

$= \frac{5}{6} \div \frac{5}{6}$

$\tan(x+y) = 1$, $\tan^{-1}(1) = 45$

Since $\frac{\pi}{4} = \frac{180^\circ}{4} = 45^\circ$

LHS = RHS Hence shown.

b) Soln.

$3\cot^2 x = 2\cos x$

$3\left(\cancel{\sin^2 x}\ \dfrac{\cos^2 x}{\sin^2 x}\right) = 2\cos x$

$\dfrac{3\cos^2 x}{\sin^2 x} = 2\cos x$

$3\cos^2 x = 2\cos x \sin^2 x$

$3\cos^2 x = 2\cos x\,(1 - \cos^2 x)$

$3\cos^2 x = 2\cos x - 2\cos^3 x$

$2\cos^3 x + 3\cos^2 x - 2\cos x = 0$.

$\cos x = \frac{1}{2}$ or $\cos x = -2$ or $\cos x = 0$.

From, General eqn.

$x = 360^\circ n \pm \cos^{-1}\alpha$

$x = 360^\circ n \pm \cos^{-1}\left(\frac{1}{2}\right)$

$x = 360^\circ n \pm \cos^{-1}(-2)$

$x = 360^\circ n \pm \cos^{-1}(0)$

$x = 60^\circ, -60^\circ, 300^\circ, 90^\circ, -90^\circ, 270^\circ$

$\therefore$ The value of x

$-90, -60^\circ, 60^\circ, 90^\circ, 270^\circ$ and 300°.

Extract 7.2: A sample of a correct response to question 7

In Extract 7.2, part (a), the candidate was able to recall the compound angle formula for tangent and correctly verify that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$. In part (b), the candidate was conversant with trigonometric identities and managed to get values of $x=-90^0$, -60^0, 60^0, 90^0, 270^0 and 300^0.

2.8 Question 8: Numbers

The question comprised parts (a) and (b). In part (a), the candidates were required to determine the rule governing the pattern of the numbers $0, 1, 1, 2, 3, ...,$ hence write the seventh number for this sequence. In part (b), they were required to use the divisibility rules to show that 420672 is divisible by 6.

The analysis of the data depicts that 123 (29.50%) scored marks from 0 to 1.5, 118 (28.30%) scored marks from 2.0 to 3.5 and 176 (42.21%) candidates scored marks from 4.0 to 6.0. Therefore, the candidates' performance in this question was good. Figure 8 summarises the candidates' performance on this question.

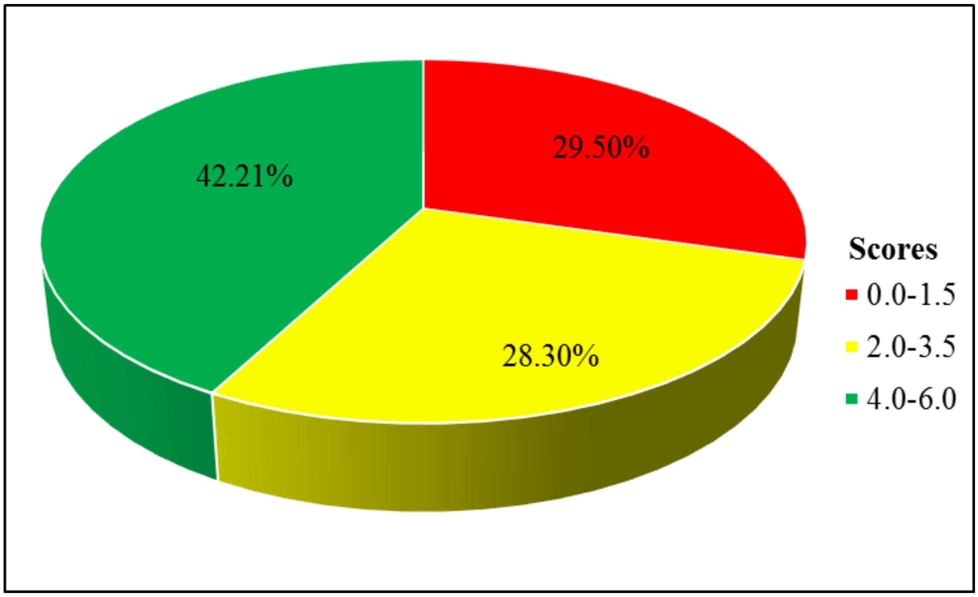


Figure 8: *Candidates' Performance on Question 8*

In part (a), the candidates who correctly responded to this question recognized that the given sequence is Fibonacci, in which each term is obtained by summing up the two preceeding terms. For instance, the third number (1) is the sum of the first and second numbers $(0+1)$, the fourth number (2) is the sum of the second and third numbers $(1+1)$ and the fifth number (3) is the sum of the third and fourth numbers $(1+2)$. Therefore, they determined the sixth term by summing up the fourth and fifth terms $(2+3)$ and got 5. Then, they determined the seventh term by summing up the fifth and sixth terms $(3+5)$ and got 8. In part (b), the candidates correctly stated that, the given number 420672 is divisible by 2 since the last digit is an even number. Also, they recognized that the number is divisible by three since the sum of its digits is 21, which is divisible by 3. Therefore, they concluded that 420672 is divisible by 6 since it is divisible by both 2 and 3. Extract 8.1 is a sample response from one of the candidates who correctly responded to this question.

8 (a)

0, 1, 1, 2, 3

The rule governing the sequence above is that to get the next term ~~the~~ the two consecutive terms are added together

0 1 1 2 3

F_1 F_2 F_3 F_4 F_5 F_6 F_7

$F_7 = F_6 + F_5$

for $F_6 = F_5 + F_4$

$= 3 + 2 = 5$

$F_7 = F_6 + F_5$

$= 5 + 3$

$= 8$

$\therefore$ The seventh number is 8

8 (b)

for a number to be divisible by six then the last digit should be even or zero and the sum of its digits must be divisible by three.

420672

The last digit is even

for sum of digits

$= 4+2+0+6+7+2$

$= 21$

$= \frac{21}{3} = 7$

The sum of digits are divisible by three.

$\therefore$ 420672 is divisible by six since it obeys all conditions for a number to be divisible by six

$420672 \div 6 = 70112$

Extract 8.1: A sample of a correct response to question 8

In extract 8.1, part (a), the candidate recognized that the given sequence is a Fibonacci sequence, hence finding the seventh term correctly. In part (b), the candidate correctly stated and applied the divisibility rule for a number to be divisible by 6 to verify that 420672 is divisible by 6.

Despite the strengths demonstrated by most candidates, 23 (5.52%) candidates scored zero. In part (a), most of these candidates provided the incorrect rules governing the pattern of the numbers, such as $(-1+n)$. They identified $n=7$ and substituted in $(-1+n)$ to obtain the seventh term which is 6 instead of 8. Also, other candidates stated that the rule governing the given pattern is arithmetic progression. Thus, they substituted $A_1 = 0$, $n = 7$ and $d = 1$ into $A_n = A_1 + (n-1)d$ and got the seventh term is 6.

In part (b), most of these candidates used the incorrect divisibility rule. For example, some candidates stated that, when a number is completely divisible by another number, the quotient is a whole number, and the remainder must be zero. Then, they applied an inappropriate approach, the long division of the numbers as $6\overset{70112}{\overline{\smash{)}420672}}$, hence they concluded that; since the remainder is zero, the given number is divisible by 6. Also, other candidates stated that, a number is divisible by 6 if the sum of its digits is divisible by 6. Thereafter, found the sum of the digits and obtained 21; then, divided by 6 and got 3 with a remainder of 3. Hence, they recommended that the number is not divisible by 6. Extract 8.2 is a sample of a response from one of the candidates who faced difficulties when responding to the question.

8 a) The rule is Arthmetic progression

mom

7th $A_n = A_1 + (n-1)d)$

$A_7 = 0 + (7-1)1)$

$A_7 = 0 + (6)1$

$A_7 = 0 + 6$

$A_7 = 6$

∴ The seventh number will be 6.

b) soln

$$\begin{array}{r|l} & 70112 \\ \hline 6 & 420672 \\ & \underline{42} \\ & 0 \\ & \underline{0} \\ & 6 \\ & \underline{6} \\ & 7 \\ & \underline{6} \\ & 12 \end{array}$$

∴ The number 420672 is divisible by 6.

Extract 8.2: A sample of an incorrect response to question 8

In Extract 8.2, part (a), the candidate stated the incorrect rule governing the given pattern, $A_n = A_1 + (n-1)d$. In part (b), the candidate performed division instead of applying the divisibility rule.

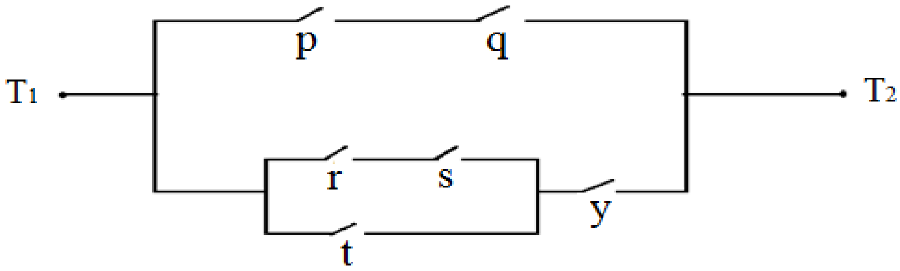
2.9 Question 9: Logic

The question comprised the following parts:

(a) (i) Construct a truth table for the compound statement $\sim(p \wedge q) \vee (\sim p \leftrightarrow q)$.

(ii) Test the validity of the argument "If I study hard, I will not fail Mathematics. If I am not a truant, then I will study Mathematics. I failed Mathematics. Therefore, I was truant."

(b) Find the compound statement which is represented by the following electrical network.



According to the analysis of the data, it was observed that 80 candidates scored from 0 to 1.5 marks, 94 candidates scored from 2.0 to 3.5 marks and 243 candidates scored from 4.0 to 6.0 marks. The summary of the candidates' performance in percentage on this question is presented in Figure 9.

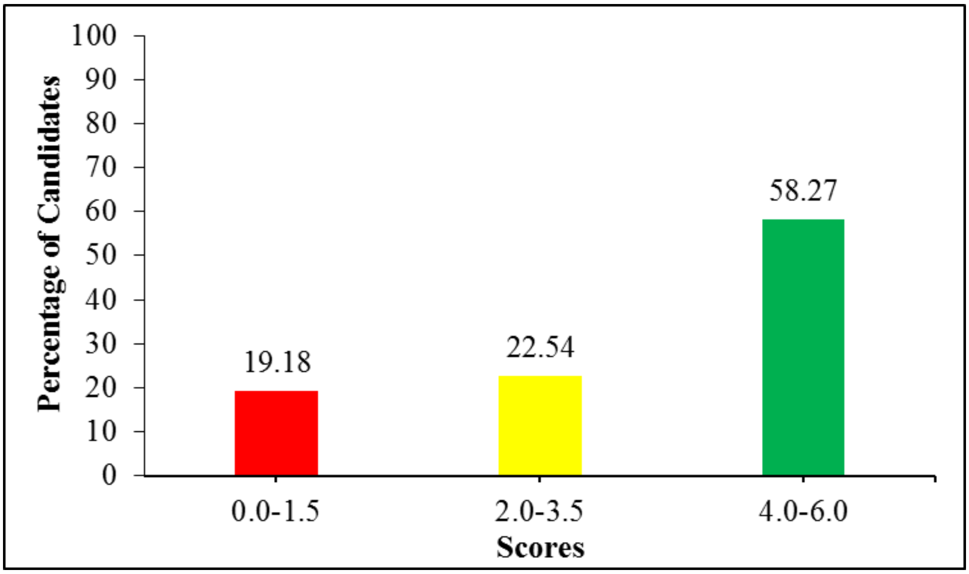


Figure 9: *Candidates' Performance on Question 9*

Figure 9 shows that 80.81 per cent of the candidates scored from 2.0 to 6.0 marks; hence, the candidates' performance on this question was good. Further analysis shows that 116 (27.82%) candidates scored all 6.0 marks. These candidates showed adequate knowledge and skills in logic, whereby in part (a), they recognized the number of columns and rows required for the given compound statement $\sim(p \wedge q) \vee (\sim p \leftrightarrow q)$. Thereafter, they correctly performed logical operations by assigning the correct truth values, as Extract shows. In part (b), the candidates correctly formulated the compound statement $(p \wedge q) \vee [((r \wedge s) \vee t) \wedge y]$ from the given electrical network. Extract 9.1 is a sample of a correct response from one of the candidates who correctly attempted the question.

9 (a)

(i)	p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim p \leftrightarrow q$	$\sim(p \wedge q) \vee (\sim p \leftrightarrow q)$
	T	T	T	F	F	F	F
	T	F	F	T	F	T	T
	F	T	F	T	T	T	T
	F	F	F	T	T	F	T

9 (b)

$(p \wedge q) \vee (((r \wedge s) \vee t) \wedge y)$

Extract 9.1: A sample of a correct response to question 9

In Extract 9.1, part (a), the candidate constructed the correct truth table for the given compound statement. In part (b), the candidate correctly formulated a compound statement.

In spite of the good performance, 80 (19.18%) candidates scored below 2.0 marks of which 30 (7.19%) scored zero. In parts (a), some of these candidates constructed truth tables lacking some basic columns, including the columns for p, q and $\sim p$. Instead, some candidates drew the truth table with $(\sim p \vee \sim q) \vee (\sim p \leftrightarrow q)$ being combined. Also, some candidates drew a truth table with three columns $\sim(p \wedge q)$, $\sim p \leftrightarrow q$ and $(\sim p \vee \sim q) \vee (\sim p \leftrightarrow q)$ instead of seven columns, while disregarding columns for p, q, $\sim p$ and $(p \wedge q)$. Furthermore, some candidates performed the logical operations of negation (~), conjunction ($\wedge$), disjunction ($\vee$) and double implication $(\leftrightarrow)$ incorrectly. They constructed the truth table with all columns, but some truth values were incorrect (Extract 9.2).

In part (b), most candidates could not recognize the appropriate logic connectives that define series or parallel connections of switches. For instance, some candidates wrote $(p \wedge q) \vee [(r \wedge s \wedge t) \wedge y]$ and others wrote

$(p \wedge q) \vee (r \wedge s) \wedge (t \wedge y)$ instead of $(p \wedge q) \vee [((r \wedge s) \vee t) \wedge y]$. Extract 9.2 is a sample of an incorrect response from one of the candidates who was not able to respond correctly to the question.

9: a) i) $\sim(p \wedge q) \vee (\sim p \leftrightarrow q)$

p	q	$\sim p$	$p \wedge q$	$\overset{a}{\sim(p \wedge q)}$	$p \leftrightarrow q$	$\overset{b}{\sim p \leftrightarrow q}$	$a \vee b$
T	T	F	T	F	F	$\cancel{F}$ T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	F	F	F
F	F	T	T	F	$\cancel{F}$ T	T	T

ii) $p \wedge \sim q$

iii) Compound statement that represented by electrical networke.

$p \wedge q$, $r \wedge s$

$(p \wedge q) \vee (r \wedge s) \vee t \wedge \cancel{x}\, y$.

$\therefore$ the compound statement is $(p \wedge q) \vee (r \wedge s) \vee t \wedge y$.

Extract 9.2: A sample of an incorrect response to question 9

In Extract 9.2, part (a), the candidate failed to perform logical operations correctly which led to an incorrect truth table. In part (b), the candidates failed to write series and parallel connections of switch in symbolic form, resulting in the wrong compound statement.

2.10 Question 10: Sets

The question consisted of parts (a) and (b). In part (a), the candidates were required to simplify $(A \cup B)' \cap (A \cap B)'$ using the properties of sets operations. In part (b), the candidates were informed that; A survey of 500 students pursuing at least one of the courses in Business, Mathematics and Economics in one academic year revealed that 83 study Business and

Mathematics, 63 study Mathematics and Economics, 217 study business and Economics. If 295, 186 and 329 students study Economics, Mathematics and Business respectively. Then, they were required to represent the given information on a Venn diagram and hence calculate the number of students pursuing Business or Economics but not Mathematics.
The analysis of the data shows that 219 candidates scored from 0 to 1.5 marks and 198 scored 2.0 to 6.0 marks. Therefore, the performance of candidates on this question was average. Figure 10 provides a summary of the candidates' performance on this question.

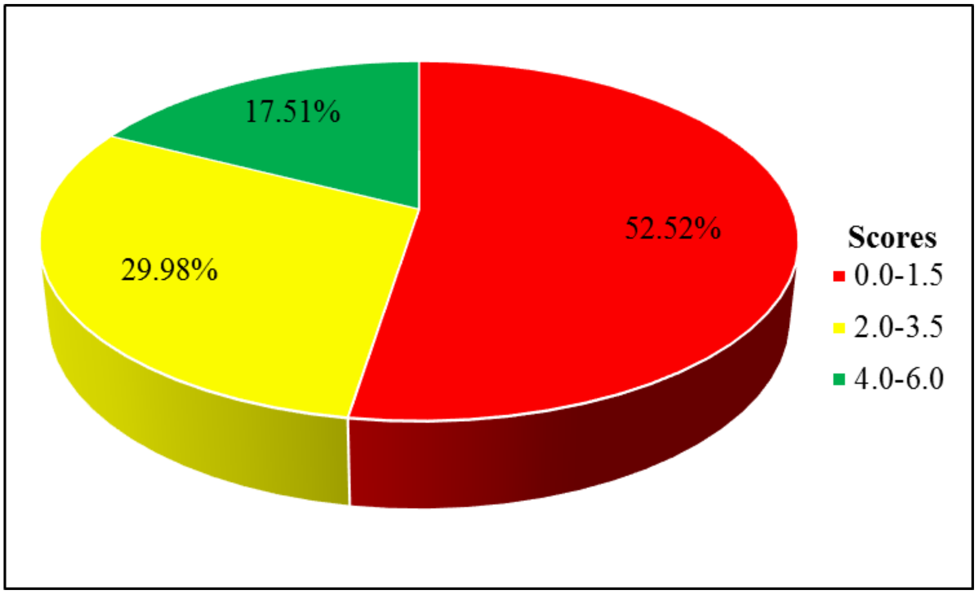


Figure 10: *Candidates' Performance on Question 10*

The candidates who correctly responded to part (a) correctly recalled and applied the properties of sets operations; De'Morgan, distributive, associative, commutative and idempotent laws to simplify $(A\cup B)' \cap (A\cap B)'$ and ended up with $A'\cap B'$ or $(A\cup B)'$. In part (b), the candidates represented the given information in the Venn diagram (see Extract 10.1). Finally, they obtained the number of students pursuing Business or Economics but not Mathematics by summing up the number of students pursuing only Business (82), only Economics (68) and only Business and Economics (164) and got 314. Extract 10.1 illustrates a sample response from one of the candidates who correctly attempted this question.

10. (a) $(A \cup B)' \cap (A \cap B)'$ ---- Given.

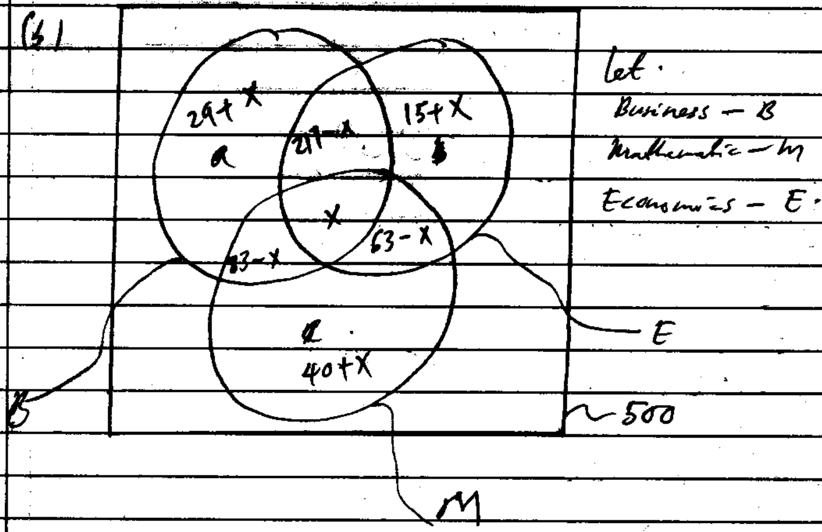
$(A' \cap B') \cap (A' \cup B')$ ---- Demorgan's law.

$A' \cap (B' \cap (A' \cup B')$ ----- Associative property.

$A' \cap B'$ ------- Absorption property.

$(A \cup B)'$ ------- De morgan's law.

(b)



~~29 + x +~~

$29 + x + 217 - x + 15 + x + 83 - x + x + 63 - x + 40 + x = 500.$

$x = 53.$

$= 29 + x + 217 - x + 15 + x$

$= 29 + 53 + 217 - 53 + 15 + 53 = 314.$

∴ Business or economics but not mathematics is 314.

Extract 10.1: A sample of a correct response to question 10

In Extract 10.1, part (a), the candidate correctly applied the properties of sets operations to simplify $(A \cup B)' \cap (A \cap B)'$ into $(A \cup B)'$. In part (b), the candidate drew a Venn diagram, inserted all the entries in their respective regions, and hence got the correct number of students pursuing Business or Economics but not Mathematics.

Despite the good performance, 104 (24.94%) candidates scored less than 1 mark. In part (a), most candidates did not state the laws involved in each step of simplifying the expression $(A \cup B)' \cap (A \cap B)'$. Also, other candidates used the basic properties of sets inappropriately. For example, some candidates responded as follows:

$(A \cup B)' \cap (A \cap B)'$ . . . Morgan law

$(A' \cap B') \cap (A' \cup B')$ . . . Complement law

$(A' \cap A')' \cap (B' \cap B')'$ . . . . Identity law (unknown)

$A \cap B$ . . . Associative law (unknown)

In part (b), some candidates failed to find the region represented by students taking only one subject. For instance, the candidates wrote $29 + 217 - x + x + 63 - x + 15 + 83 - x + 40 = 500$, resulted in $-2x = 53$ instead of $x = 53$. Therefore, the Venn diagram was drawn with the wrong entries in the respective regions. Furthermore, other candidates applied the formula contrary to the instructions of the question instead of using the Venn diagram. In addition, some of these candidates applied incorrect formula $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) - n(A \cap B \cap C)$ instead of $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$. Therefore, they substituted the data into the wrong formula, resulting in $n(A \cap B \cap C) = -53$ rather than 53. Extract 10.2, illustrates a response from one of the candidates who faced difficulties in responding this question.

10. a) Soln.

Given; $(A\cup B)' \cap (A\cap B)'$

$(A\cup B)' \cap (A\cap B)'$ . . . . Given

$(A'\cap B') \cap (A'\cup B')$ . . . . Compliment law

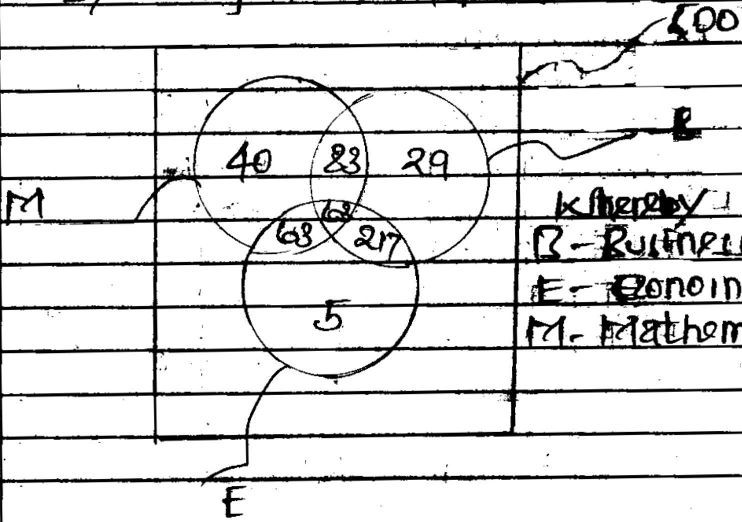
$(A'\cap A') \cap (B'\cup B')$ . . . . Identity law

$A \cap B$ . . . . Associative law

$\therefore (A\cup B)' \cap (A\cap B')) = A\cap B$ by Associative law/property.

b) Soln.

By using Venn diagram:



Where by

B - Business

E - Economics

M - Mathematics.

$\therefore$ 217 students persuing Business or Economics but not Mathematics.

Extract 10.2: A sample of an incorrect response to question 10

In Extract 10.2, part (a), the candidate applied inappropriate properties of sets. In part (b), the candidate failed to represent the given information on the Venn diagram.

2.11 Question 11: Functions

The question had parts (a), (b) and (c). In part (a), the candidates were required to find the value of $(\alpha-\beta)^2$ without solving the equation, given that α and β are the roots of $4x^2+8x-1=0$. In part (b), the candidates were required to use the remainder theorem to find the remainder when the polynomial $p(x)=4x^3-5x+4$ is divided by $2x-1$. In part (c), they were required to sketch the graph of $y=\dfrac{x+2}{x^2-9}$, and use the graph to determine the domain and range.

The data shows that 43 (10.31%) candidates scored from 0 to 2.5 marks, 172 (41.25%) scored from 3.0 to 6.0 marks and 202 (48.44%) candidates scored from 6.5 to 10 marks. Figure 11 illustrates the candidates' performance summary on this question.

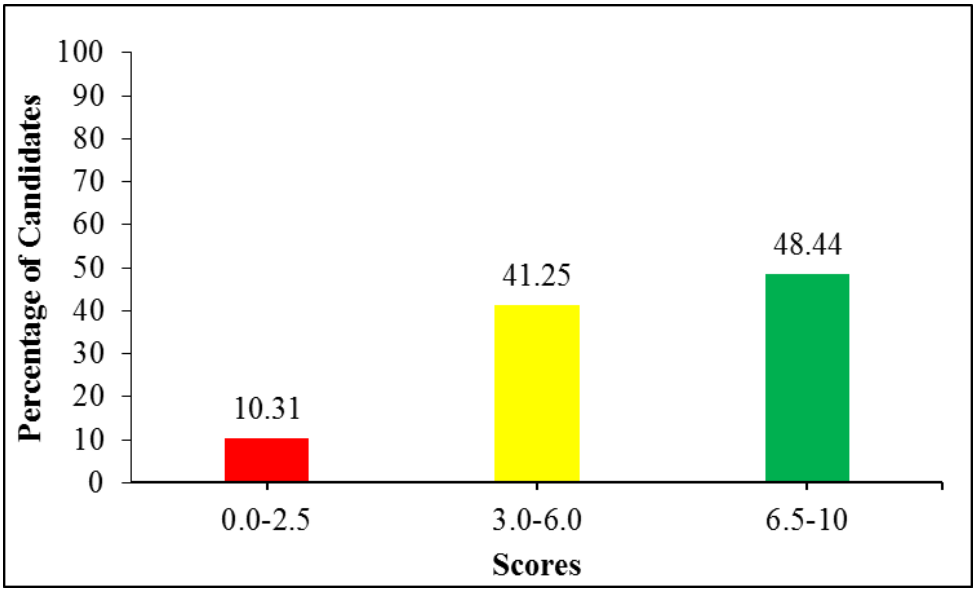


Figure 11: *Candidates' Performance on Question 11*

As Figure 11 shows, the candidates' performance in this question was generally good because 89.69 per cent of the candidates scored from 3.0 to 10 marks. In part (a), the candidates recalled the conditions for the sum and product of the roots of quadratic equation $ax^2+bx+c=0$ as $(\alpha+\beta)=\dfrac{-b}{a}$

and $\alpha\beta = \frac{c}{a}$, respectively. Thus, they realized that for the given equation $4x^2 + 8x - 1 = 0$, $a = 4$, $b = 8$ and $c = -1$, therefore, $(\alpha + \beta) = -2$ and $\alpha\beta = -\frac{1}{4}$. Thereafter, the candidates expressed $(\alpha - \beta)^2$ in the form $(\alpha + \beta)^2 - 4\alpha\beta$, then, replaced $\alpha + \beta$ and $\alpha\beta$ with –2 and $-\frac{1}{4}$, respectively, and correctly simplified the expression to get $(\alpha - \beta)^2 = 5$. In part (b), the candidates were conversant with the remainder theorem that, when a polynomial $p(x)$ is divided by $x - a$, the remainder is p(a). Thus, they expressed the divisor $2x - 1$ in the form $2\left(x - \frac{1}{2}\right)$, and hence, they evaluated $p\left(\frac{1}{2}\right)$ from $p(x) = 4x^3 - 5x + 4$ and got the remainder of 2.

In part (c), the candidates correctly determined $x -$ intercept and $y -$ intercept. Thus, from the given function, $y = \frac{x+2}{x^2 - 9}$, they obtained $y -$intercept $-\frac{2}{9}$, and $x -$ intercept -2. Also, they determined that, the vertical asymptote, $x = \pm 3$ and the horizontal asymptote $y = 0$, since the highest degree of x is in the denominator. Therefore, the candidates correctly sketched the graph of the function (Extract 11.1). By studying the graph, the candidates determined that the domain includes all real numbers except –3 and 3 $(\{x \in \Re : x \neq \pm 3\})$ and range includes all numbers $(\{y : y \in \Re\})$. Extract 11.1 is a sample of a response from one of the candidates who was able to answer this question correctly.

11 a) Equation:

$4x^2+8x-1=0$

$ax^2+bx+c=0$

On comparing;

$a=4$

$b=8$

$c=-1$

Roots;

α and β

Recall;

Sum of roots $= -b/a$

$\alpha+\beta = -8/4$

$\alpha+\beta=-2$

Product of roots $= c/a$

$\alpha\beta = -1/4$

Then for $(\alpha-\beta)^2$

$(\alpha-\beta)^2 = ?$

$(a-b)^2 = a^2-2ab+b^2.$

$(\alpha-\beta)^2 = \alpha^2-2\alpha\beta+\beta^2$

$= \alpha^2+\beta^2-2\alpha\beta$

but

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

Then;

$= \cancel{\alpha}(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$

$=(\alpha+\beta)^2-4\alpha\beta$

$=(-2)^2-4\left(-\frac{1}{4}\right)$

$=4+1$

$= 5$

$\therefore$ The answer is 5

11 b) $p(x) = 4x^3 - 5x + 4$

Equate the divisor to zero

$2x - 1 = 0$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{1}{2}$

$x = \frac{1}{2}$

$P\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right) + 4$

$= 4\left(\frac{1}{8}\right) - \frac{5}{2} + 4$

$= \frac{4}{8} - \frac{5}{2} + 4$

$= 2$

$\therefore$ The remainder is 2

11 c) $y = \frac{x+2}{x^2 - 9}$

$y = \frac{x+2}{x^2 - 3^2}$

$y = \frac{x+2}{(x-3)(x+3)}$

V.A:

Denominator $= 0$

$x - 3 = 0$

$x = 3$

$x + 3 = 0$

$x = -3$

H.A:

$n < m$

$y = 0$

x-intercept; $(y = 0)$

$x = -2$

Point $(-2, 0)$

y-intercept; $(x = 0)$

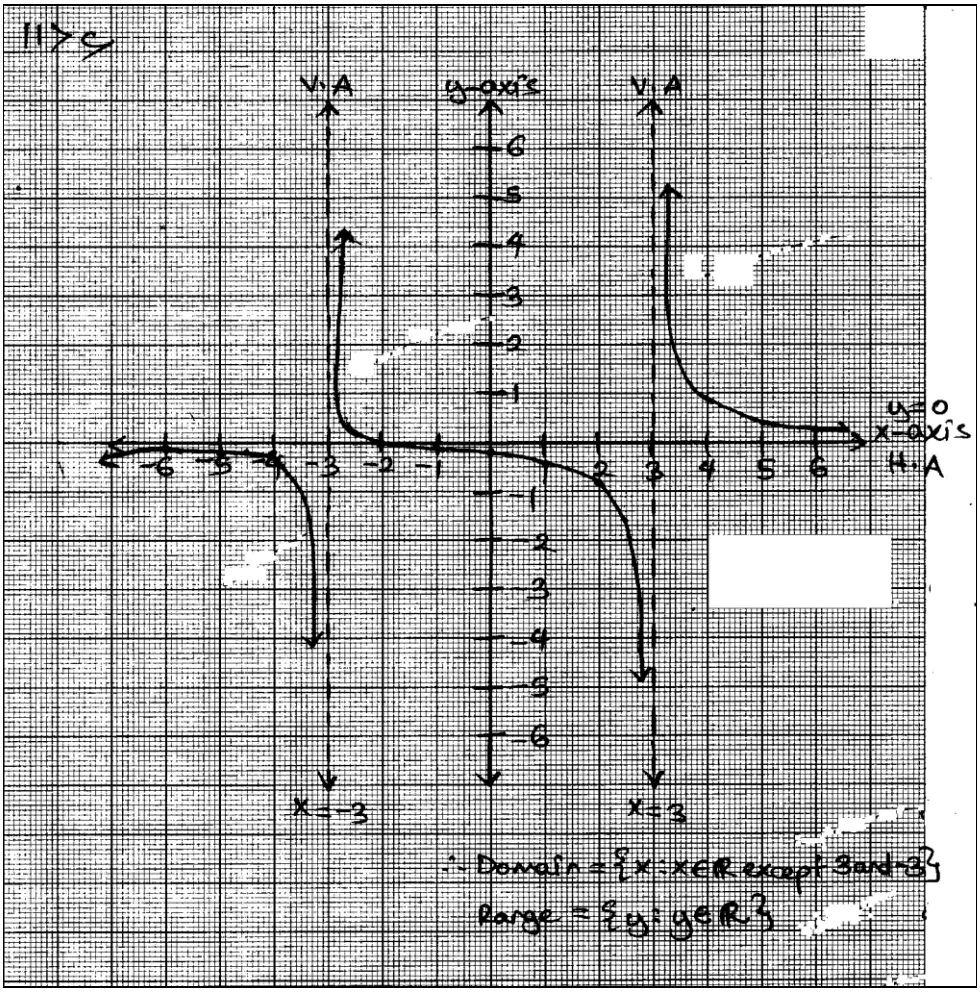
$y = -\frac{2}{9}$

Point $\left(0, -\frac{2}{9}\right)$

No hole

Intervals; $x = \{-2, -3, 3\}$

	$x < -3$ (-4)	$-3 < x < -2$ (-2.5)	$-2 < x < 3$ (0)
$x+2$	$-ve$	$-ve$	$+ve$
$(x-3)(x+3)$	$+ve$	$-ve$	$-ve$
	$-ve$	$+ve$	$-ve$



Extract 11.1: A sample of a correct response to question 11

In Extract 11.1, part (a), the candidate correctly applied the condition for the sum and product of roots of a quadratic equation. In part (b), the candidate correctly applied the remainder theorem, which led to getting the remainder, $r = 2$. In part (c), the candidate sketched the graph of the given function and used it to determine the domain and range.

Nevertheless, 43 (10.31%) candidates scored below 3.0 marks, of which 1.20 per cent scored zero. In part (a), most candidates failed to express $(\alpha - \beta)^2$ as the sum and product of α and β. For example, some candidates wrote $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta$, instead of $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$. These candidates substituted $\alpha + \beta = -2$ and

$\alpha\beta = \frac{-1}{4}$ into $(\alpha+\beta)-2\alpha\beta$ and got $(\alpha-\beta)^2 = 4.5$ instead of 5. Moreover, other candidates expressed $(\alpha-\beta)^2$ as $2\alpha\beta-\alpha+\beta$ and hence got $(\alpha-\beta)^2 = 1.5$. Also, some candidates wrongly recalled the concepts of the sum and product of the roots. Most of these candidates wrote $\alpha+\beta = \frac{a}{b}$ and $\alpha\beta = \frac{a}{c}$ instead of $\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$. Therefore, these candidates obtained $\alpha+\beta = \frac{1}{2}$, and $\alpha\beta = -4$ instead of $\alpha+\beta = \frac{-b}{a} = -2$ and $\alpha\beta = \frac{c}{a} = \frac{-1}{4}$, which resulted in $(\alpha-\beta)^2 = \frac{81}{4}$ instead of $(\alpha-\beta)^2 = 5$.

In part (b), most candidates applied long division method instead of the remainder theorem. However, most of these candidates incorrectly worked out and ended up with incorrect remainders, particularly $-3x-1$. Furthermore, other candidates failed to express the divisor $2x-1$ in the standard form $b(x-a)$. Therefore, these candidates substituted the incorrect value of a in computing $p(a)$. For instance, some of these candidates computed $p\left(\frac{1}{3}\right)$ and got the remainder $\frac{67}{4}$ instead of 5 from $p\left(\frac{1}{2}\right)$.

In part (c), most candidates committed computational errors. For instance, some candidates got y intercept $\frac{2}{9}$ instead of $-\frac{2}{9}$, indicating errors in performing basic operations. Similarly, some candidates incorrectly computed the $x-$intercept and got $x = 2$ instead of $x = -2$. Further, some candidates calculated the vertical asymptote by considering the numerator instead of the denominator and hence obtained $x = -2$. Furthermore, some candidates calculated the horizontal asymptote by dividing each term by x instead of x^2. Most of these candidates also incorrectly evaluated the value of $\frac{x-2}{x^2-9}$ as x approached to infinite (∞) and got the wrong horizontal asymptote $y=1$. As a result, they drew an incorrect graph. Similarly, other candidates used a table of values to sketch the graph of the given rational functions. For instance, the following is a table of value constructed by one of the candidates:

x	–2	–1	0	1	2
$y=x^2-9$	–10.5	–8	0	–6	–0.5

Unfortunately, all the values entered were incorrect, and the candidate used for sketching the graphs. Furthermore, these candidates got incorrect domains and ranges, whereby the common responses noted from the candidates included $\text{Domain}=\{x : x=-9\le x\le 9\}$ and $\text{Range}=\{y : y\ge 1\}$; others wrote $\text{Domain}=\{x : x=\pm 3\}$ and $\text{Range}=\{y : y=0\}$. Extract 11.2 is a sample of an incorrect response from the candidate who faced difficulties in attempting the question.

11 a) Solution

Given that

$-4x^2+8x-1=0$

$(\alpha+\beta)^2$ = required

from

$(\alpha+\beta)^2=(\alpha-\beta)^2$

$(\alpha+\beta)^2=\left(\frac{-b}{2a}\right)^2$

where

$a=4$

$b=8$

Then

$(\alpha-\beta)^2=\left(\frac{-8}{2\times 4}\right)^2$

$=\left(\frac{-8}{8}\right)^2$

$=\frac{64}{64}$

$(\alpha-\beta)^2=1$

$\therefore (\alpha-\beta)^2=1$

b/ $\underline{\text{soln}}$.

$p(x) = 4x^3 - 5x + 4$ by $2x-1$

by using

$x = \frac{1}{2}$ | 4   0   -5   4

           2   -1   -3

      4   -2   -6   1.

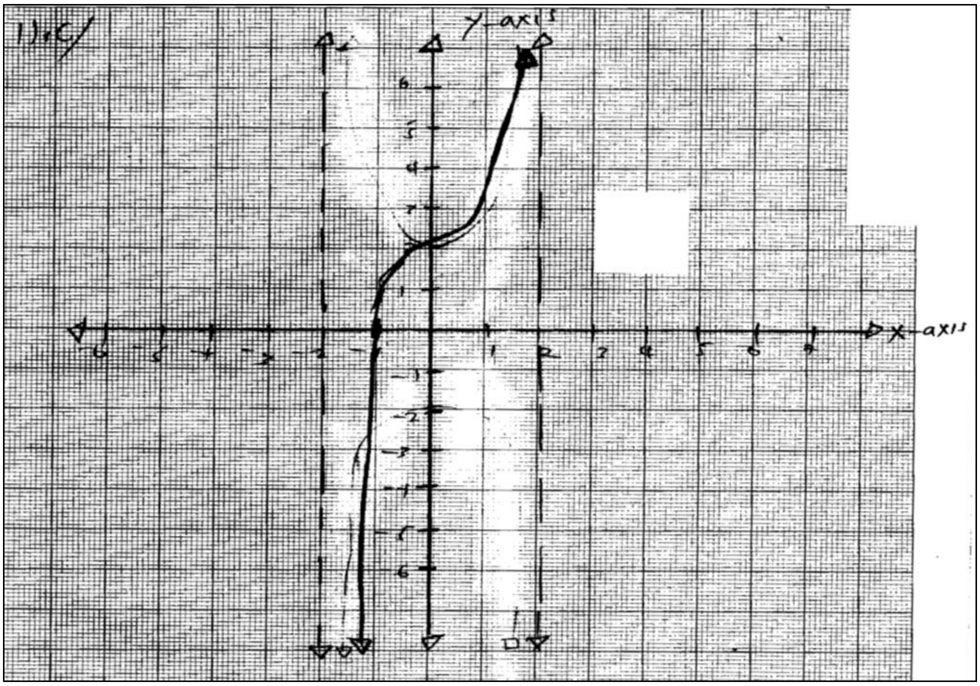
Quostrent $= 4x^2 + -2x - 6$

remaind $= 1$.

$\therefore$ The remainder $= 1$.

c/ $\underline{\text{soln}}$.

$y = \dfrac{x+2}{x^2-9}$



Extract 11.2: A sample of an incorrect response to question 11

In Extract 11.2, part (a), the candidate wrote $\alpha + \beta = -\dfrac{b}{2a}$ instead of $\alpha + \beta = -\dfrac{b}{a}$. In part (b), the candidate applied synthetic division instead of remainder theorem, and in part (c), the candidate failed to determine the correct asymptotes and intercepts, hence drawing an incorrect graph.

2.12 Question 12: Integration and Differentiation

The question consisted of three parts (a), (b) and (c). In part (a), the candidates were required to prove that the tangents at P and R are parallel given that, the curve $y=(x-2)(x-3)(x-4)$ crosses the $x-$ axis at the points $P(2,0)$, $Q(3,0)$ and $R(4,0)$. In part (b), the candidates were required to find the equation of a normal to the curve $y=x^3-6x^2+12x+2$ at which the tangent to the curve is parallel to the line $y=3x$. In part (c) required the candidates to find the value of t such that $\int_0^2 tx(2-x^2)^2 dx=1$.

The data depicts that 128 (30.70%) candidates scored from 3.0 to 10 marks and 289 (69.30%) scored from 0 to 2.5 marks. Generally, the candidates performed averagely on this question. The performance summary on this question is presented in Figure 12.

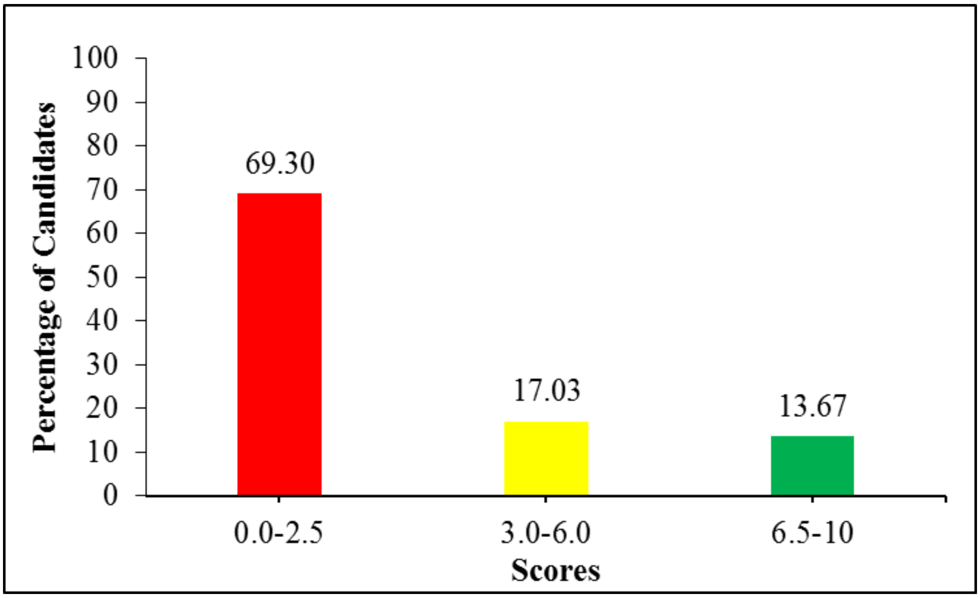


Figure 12: *Candidates' Performance on Question 12*

The data shows that 57 (13.67%) candidates scored high marks (6.5 – 10). In part (a), many candidates expanded the equation $y=(x-2)(x-3)(x-4)$ into $y=x^3-9x^2+26x-24$ and differentiated it to obtain $\frac{dy}{dx}=3x^2-18x+26$. Then, they computed the slope by substituting

$(x, y) = (2, 0)$ into $\frac{dy}{dx}$ and obtaining $m_1 = 2$. Likewise, they calculated the slope of the curve (m_2) by substituting $(x, \mathrm{y}) = (4, 0)$ into $\frac{dy}{dx}$ and obtained $m_2 = 2$. Therefore they concluded that the tangents at $P(2, 0)$ and $R(4, 0)$ are parallel as they have the same slope, $m_1 = m_2 = 2$.

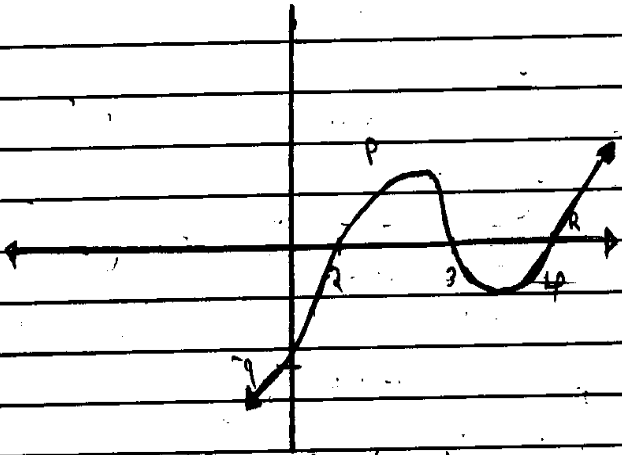
In part (b), the candidates recall the fact that the tangents and normals to the curve are perpendicular, and thus the product of their slope is -1 $(m_t \times m_n = -1)$. These candidates also realized that since the tangent is parallel to the line $y = 3x$, its slope is the same as the slope of the line, that is, $m_t = 3$ and consequently $m_n = -\frac{1}{3}$. Then, these candidates performed differentiation on $y = x^3 - 6x^2 + 12x + 2$ and got $\frac{dy}{dx} = 3x^2 - 12x + 12$. They also identified that $\frac{dy}{dx} = 3$, as the slope of the tangent is equivalent to the first derivative of the curve at the particular point. Therefore, they wrote $3x^2 - 12x + 12 = 3$ and solved for x to obtain $x = 1$ or $x = 3$. Thus, they substituted $x = 1$ and $x = 3$ into $y = x^3 - 6x^2 + 12x + 2$ and obtained $y = 9$ and $y = 11$, respectively. Then, they realized that the normal passes through the points $(1, 9)$ and $(3, 11)$; hence, they determined the equation of a normal to the curve using the slope $m_n = -\frac{1}{3}$ and the point $(1, 9)$ resulting in $x + 3y = 28$. Likewise, they used the slope and the point $(3, 11)$ resulting in $x + 3y = 36$.

In part (c), the candidates performed integration on $\int_0^2 tx(2 - x)^2\, dx = 1$ by applying the substitution technique and obtained $\left[-\frac{t}{2} \times \frac{(2 - x^2)^3}{3}\right]_0^2 = 1$. Then, they solved for t, resulting in $t = \frac{3}{8}$. Extract 12.1 is a sample of a correct response from one of the candidates who were able to attempt the question.

11. (a) $y = (x-2)(x-3)(x-4)$

$P(2,0) \quad Q(3,0) \quad R(4,0)$

Soln.



$y = (x^2 - 5x + 6)(x - 4)$

$y = x^3 + -4x^2 - 5x^2 + 20x + 6x - 24$

$y = x^3 - 9x^2 + 26x - 24$

$dy/dx = 3x^2 - 18x + 26$

dy/dx at $P = 3(2)^2 - 18 \times 2 + 26$

$dy/dx = 2$

dy/dx at $R = 3(4)^2 - 18 \times 4 + 26$

$dy/dx = -24 + 26 = 2.$

Since gradient (dy/dx) at point P and R are same $= 2$ then they are parallel.

c) $\int_0^2 tx(2-x^2)^2\,dx = 1$

Soln

$tx(2-x^2) = 1$

let $u = 2-x^2$

$du/dx = -2x$

12c $\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$\int_0^2 tx(2-x^2)\,du/_{-2x} = 1$

$-\frac{t}{2}\int_0^2 (u)^2\,du = 1$

$-\frac{t}{2}\left[\frac{u^3}{3}\right]_0^2 = 1$

$\left(-\frac{t}{2}\right)\left[\frac{(2-x^2)^3}{3}\right]_0^2 = 1$

$\left(-\frac{t}{2}\right)\left[\left(\frac{2-4}{3}\right)^3 - \left(\frac{(2-0)^3}{3}\right)\right] = 1$

$\left(-\frac{t}{2}\right)\left\{-\frac{8}{3} - \frac{8}{3}\right\} = 1$

$\left(-\frac{t}{2}\right)\left(-\frac{16}{3}\right) = 1$

$\frac{16t}{6} = 1$

$\frac{16t}{16} = \frac{6}{16}$

$t = 3/8.$

$t = 3/8.$

Extract 12.1: A sample of a correct response to question 12

In Extract 12.1, part (a), the candidate multiplied the factors of the curve to obtain its equation, then differentiated it to get the slope of the curve, $\left(\frac{dy}{dx}\right)$. Thereafter, she or he proved that the tangents are parallel. In part (c), the candidate solved for t when integrating the given integral.

On the other hand, 158 (37.89%) candidates got zero. In part (a), some candidates calculated the slope of the tangent by substituting the points $P(2,0)$ and $R(4,0)$ into the formula, slope $(m) = \frac{y_2 - y_1}{x_2 - x_1}$ and obtained $m = 0$. Other candidates substituted the coordinates of the points $P(2,0)$, $Q(3,0)$ and $R(4,0)$ in $y = (x-2)(x-3)(x-4)$, obtained $y = 0$ and concluded that the tangents at P and R are parallel. Moreover, the analysis also shows that some candidates committed errors in performing basic operations. For instance, a few candidates wrongly expanded $y = (x-2)(x-3)(x-4)$, and obtained $y = x^3 - 9x^2 + 29x - 24$ instead of $y = x^3 - 9x^2 + 26x - 24$.

In part (b), some candidates deduced the slope of the tangent by equating the derivative to zero, that is, $\frac{dy}{dx} = 0$. These candidates correctly obtained $\frac{dy}{dx} = 3x^2 - 12x + 12$ from $y = x^3 - 6x^2 + 12x + 2$ and therefore they formulated the equation $x^2 - 4x + 4 = 0$ and correctly solved it to obtain $x = 4$ or $x = 1$. These candidates finally obtained the incorrect equation of normal, commonly $y = 9(x-1) + 9$ instead of $y = -\frac{1}{3}(x-1) + 9$. In addition, other candidates substituted $x = 0$ in $\frac{dy}{dx} = 3x^2 - 12x + 12$ and obtained the slope of the tangent, $m_1 = 12$ and consequently the slope of the normal, $m_2 = -\frac{1}{12}$. They also worked on the equation by substituting $x = 0$ into $y = x^3 - 6x^2 + 12x + 2$ and getting $y = 2$. Thereafter, they substituted the point (0, 2) and $m_2 = -\frac{1}{12}$ into $m = \frac{y - y_1}{x - x_1}$ and got $x + 12y - 24 = 0$.

In part (c), many candidates failed to evaluate $\int_0^2 tx\left(2-x^2\right)^2 dx$ as they got $\left[2tx\left(2-x^2\right)\right]_0^2$ and hence $\left[2tx\left(2-x^2\right)\right]_0^2 = 1$, resulting in $t = \frac{-1}{8}$ instead of $t = \frac{3}{8}$. Furthermore, other candidates correctly applied the substitution technique; however, they failed to apply the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$. For example, some candidates obtained $\left[\frac{-1}{2}t\left(\frac{u^2}{3}\right)\right]_0^2 = 1$ from $\int_0^2 tx\left(2-x^2\right)^2 dx = 1$ instead of $-\frac{t}{2}\times\frac{u^3}{3}\Big|_0^2 = 1$, whereby the exponent of u was not added by 1. Then, after substituting $u = 2-x^2$, they wrote $\left[\frac{-t}{2}\left(\frac{2-x^2}{3}\right)\right]_0^2 = 1$, which resulted in $t = \frac{3}{2}$ instead of $t = \frac{3}{8}$. Extract 12.2 is a sample response from one of the candidates who was not able to respond to the question correctly.

12	a/ soln.
	$y = (x-2)(x-3)(x-4)$
	$P(2,0), Q(3,0), R(4,0)$
	$y = (2-2)(2-3)(2-4) = 0 \times 1 \times 2)\ y_1 = 0$
	$y = (3-2)(3-3)(3-4) = 1 \times 0 \times 1 = y_2 = 0$
	$y = (4-2)(4-3)(4-4) = 0 \times 0 \times 0 = y_3 = 0$
	So Tangent P and R are parallel
12.	b/ soln.
	$y = x^3 - 6x^2 - 12x + 2$ Line $y = 3x$.
	$y = 3x$
	$\int x^3 - 6x^2 - 12x + 2$
	slope = 3.
	$M_1 = M_2$
	$M_1 = 3$.
	$\frac{3}{1} = \left(\frac{y-3}{x-2}\right)$
	$y - 3 = 3x - 6$
	$y - 3x + 6 = 0$
	The eqn $y - 3x + 6 = 0$.

12 c/ soln

$$\int_0^2 tx(2-x^2)^2\,dx = 1.$$

$$\int_0^2 \left(2x - \frac{x^3}{3}\right)^2 tx = 1.$$

$$\left(2(2) - \frac{2^3}{3}\right)^2 t2 -$$

$$\left(4 - \frac{8}{3}\right)^2 2t - \left(0 - \frac{0^3}{3}\right)^2 t0 = 1.$$

$$\left(4 - \frac{8}{3}\right)^2 2t = \cancel{0} = 1.$$

$$\frac{4}{3} \times \frac{16}{9} \times 2t = 1 \times 9$$

$$16 \times 2t = 9$$

$$\frac{32t}{32} = \frac{9}{32}$$

$$t = \frac{9}{\cancel{32}} \times \cancel{32} \quad 9 - \cancel{7} = 2$$

$$t = 2.$$

Extract 12.2: A sample of an incorrect response to question 12

In Extract 12.2, part (a), the candidate substituted the given points into the equation of a curve before differentiation. In part (b), the candidate considered the tangent and normal to be parallel, thus using the slope of the tangent and the point $(2, 3)$ to find the incorrect equation. In part (c), the candidate failed to apply the substitution techniques; she or he multiplied the expression in the brackets by x, then substituted the limit without integrating the polynomial.

2.13 Question 13: Probability

The question consisted of parts (a), (b) and (c) which stated as follows:

(a) Using one example explain the meaning of “independent events” as applied in probability.

(b) Two dice whose sides are labeled 1, 2, 3, 4, 5 and 6 each are thrown simultaneously at once. What is the probability that the sum of the sides of two dice is less than 10?

(c) Find the number of permutations in all letters of the word TERRITORY.

The analysis of the data revealed that 127 candidates scored marks ranging from 0 to 2.5, 67 scored marks from 3.0 to 6.0 and 223 candidates scored marks from 6.5 to 10. Therefore, the candidates' performance on this question was generally good. The summary of the candidates' performance on this question is presented in Figure 13.

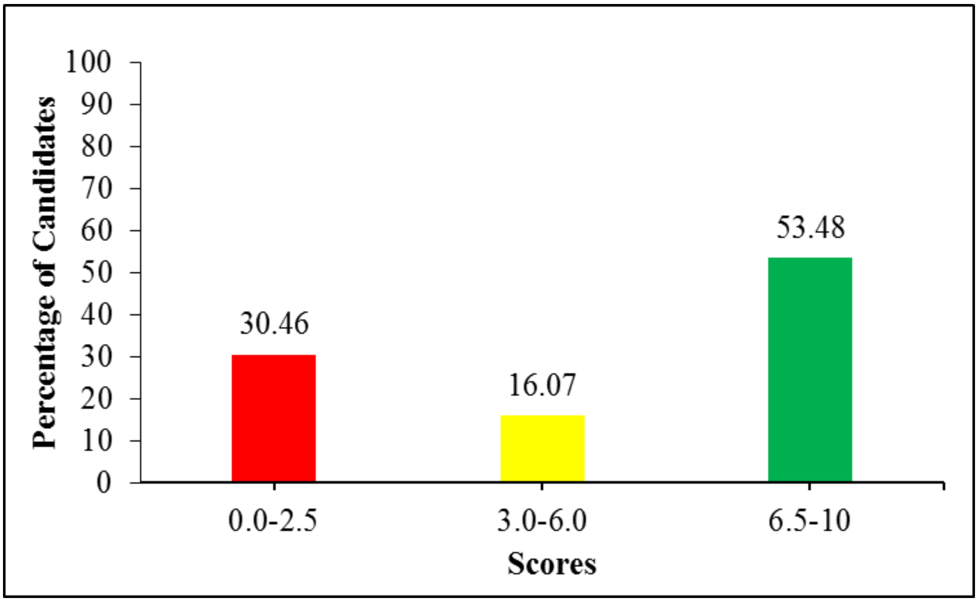


Figure 13: *Candidates' Performance on Question 13*

The data analysis shows that 68 (16.31%) candidates managed to score full marks. In part (a), the candidates explained that independent events are "the events when the actual happening of the one does not influence the happening of the other." They also managed to provide relevant example; for instance, the results obtained by throwing a die are independent of the results obtained by drawing a king from a deck of cards. In part (b), most candidates tabulated the outcomes of the two dice, as Extract 13.1 shows. From the table, they identified the number of sample space, $n(S)=36$ and the number of outcomes whose sum is 10, 11 or 12 and got $n(E)=6$, which led to get $P(E)=\frac{1}{6}$. After that, they found the probability of an event where the sum of the sides of two dice is less than 10 using the formula $P(B)=1-P(E)$ and got $P(B)=\frac{5}{6}$.

In part (c), the analysis showed that the candidates applied the formula for arranging n objects, of which r are of type 1 and q are of type 2. Then, the candidates correctly obtained the total number of all letters using $n! = 9!$. Then, for letters of the same kind, they wrote $T's, r! = 2!$ and $R's, q! = 3!$. Thereafter, they calculated the number of ways by using $\frac{p!}{r!q!} = \frac{9!}{2!3!}$, which resulted in 30240 ways. Extract 13.1 illustrates the correct solution to this question from one of the candidates' responses.

13 (a). Independent events are events whose outcome doesn't affect the outcome of another event. For example ~~throwing a dice~~ rolling a dice and tossing a coin are independent events.

(b). Possible combinations when throwing two dice.

Dice 02 / Dice 01	1	2	3	4	5	6
1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

Sum of the two dices is 10

let E be the event that the sum of the sides of two dices is less than 10

$n(E) = 30$

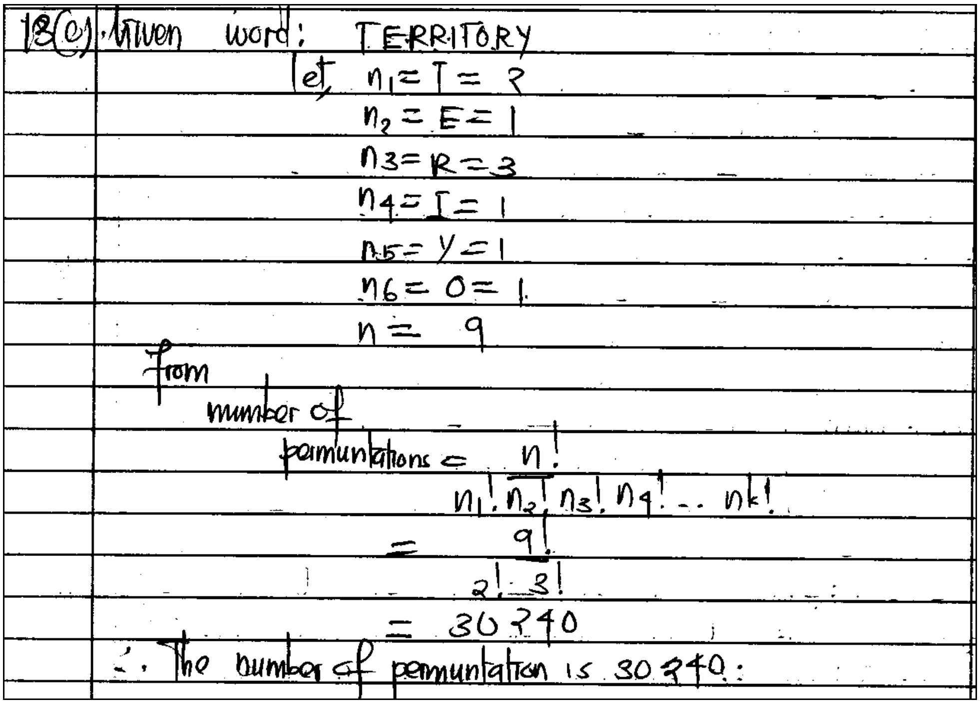
S represent total number of outcomes

$n(S) = 36$

from $P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{30}{36} = \frac{5}{6}$

$\therefore$ The probability that the sum of the sides of the dices is less than 10 is $\frac{5}{6}$.


13(c). Given word: TERRITORY

let, $n_1 = T = 2$

$n_2 = E = 1$

$n_3 = R = 3$

$n_4 = I = 1$

$n_5 = Y = 1$

$n_6 = O = 1.$

$n = 9$

from number of permuntations $= \dfrac{n!}{n_1!\, n_2!\, n_3!\, n_4! \ldots n_k!}$

$= \dfrac{9!}{2!\; 3!}$

$= 30240$

$\therefore$ The number of permuntation is 30240.

Extract 13.1: A sample of a correct response to question 13

In Extract 13.1, part (a), the candidate correctly explained the meaning of independent events with relevant example. In part (b), the candidate employed a table on which she or he drew a table with a first die and a second die. Then, correctly found the probability that the sum of the sides of two dice was less than 10. In part (c), the candidate recognized and applied correctly the technique of arrangement and got the number of ways of arrangement.

On the contrary, the analysis of the data shows that, among 127 candidates who scored low marks, 63 (15.11%) scored zero. This was due to some difficulties they faced while attempting the question. In part (a), most candidates were not able to define independent event as used in probability. For instance, they wrote, “independent events are the events which do not depend to any sources of the data.” Likewise, others wrote that “independent events are the events which are multiple by the repeated events” and gave an example of finding the number of ways of arranging letters in the word BARAZA as $\frac{6!}{3!} = \frac{6\times5\times4\times3\times2\times1}{3\times2\times1}$.

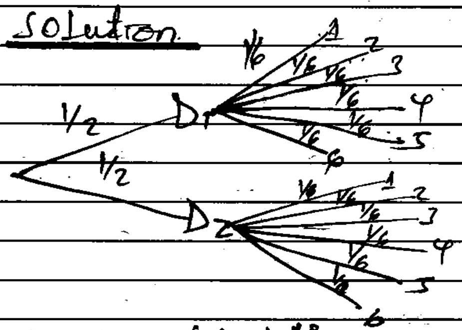
In part (b), the analysis shows that some candidates were not able to determine the number of events and sample space and failed to recognize the formula for the probability of an event. Furthermore, other candidates responded by using a tree diagram but failed to consider the two dice as combined events. They separated the branches for each die, then in the first branch, they added all six numbers to 1, as $1+1=2$, $1+2=3$, $1+3=4$, $1+4=5$, $1+5=6$ and $1+6=7$. While in the second branch they added all six numbers to 2, as $2+1=3$, $2+2=4$, $2+3=5$, $2+4=6$, $2+5=7$ and $2+6=8$. Thereafter, without showing the number of events or sample space it was concluded that the probability that the sum of the sides is less than 10 is $\frac{27}{31}$.

In part (c), the analysis depicts that, the candidates applied inappropriate formula. For example, they applied the formula ${}^{n}P_{r}=\frac{n!}{(n-r)!}$ used to arrange r objects from n unlike objects. For instance, from the word TERRITORY, most candidates identified $n=9$ and $r=3$, subsequently they substituted into the formula ${}^{n}P_{r}=\frac{n!}{(n-r)!}$ and got 504 instead of 30240 ways. Additionally, the analysis shows that some candidates had knowledge of permutation but failed to use the formula for arranging n objects, of which r are of type 1 and q are of type 2. For instance, they correctly identified the number of each letter in the word TERRITORY by writing $T=2$, $E=1$, $R=3$, $I=1$, $O=1$ and $Y=1$. Then, they wrote $9p_2+9p_1+9p_3+9p_1+9p_1+9p_1$ and obtained 612 ways instead of 30240 ways. Extract 13.2 is a sample response from one of the candidates who faced difficulties when responding to the question.

13 (a) Independent events, These are event that the normally occur without depend on any matter

example: Every living organism will die

13. (b) Solution



$\frac{1}{2} \times \frac{1}{6} = \left(\frac{1}{12}\right) \times 12 = \times 12$

$= 1$

13 (c) Solution

Given that

TERRITORY

let

n = number of all letters

n = 9

but

nP = n!

Then

9!

= 9X8X7X6X5X4X3X2X1

= 362,880

∴ The number of permutation is 362,880

Extract 13.2: A sample of an incorrect response to question 13

In Extract 13.2, in part (a), the candidate failed to give the meaning of independent events. In part (b), she or he failed to combine the two dice properly resulting in a failure to get the correct answer. In part (c), the candidate calculated the number of permutations of n unlike objects in the word TERRITORY and did not consider the repeated letters (T,R).

2.14 Question 14: Vectors, Matrices and Transformations

The question was composed of parts (a), (b) and (c) which stated as follows:

(a) Given that $\underline{a} = -2\underline{i} + 5\underline{j} - 3\underline{k}$ and $\underline{b} = 3\underline{i} - \underline{j} + 2\underline{k}$, find:

(i) $\underline{a} \times \underline{b}$

(ii) $(\underline{a} \times \underline{b}) \cdot \underline{a}$

(b) (i) Find the value of t which satisfies the equation

$$\begin{vmatrix} t+3 & 5 & 6 \\ -1 & t-3 & -1 \\ 1 & 1 & t+4 \end{vmatrix} = 0$$

(ii) Given that $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$, show that $\det(AB) = \det(A)\det(B)$.

(c) Find the matrix corresponding to the linear reflection of a point $P(x, y)$ in the line $y - x = 0$ and use it to find:

(i) the image of the line $x + 2y = 6$

(ii) the point whose image under the reflection is $(3, -2)$

The analysis of the data revealed that 278 (66.67%) candidates scored from 3.0 to 10 marks and 139 (33.33%) scored from 0 to 2.5 marks. Therefore, the performance of the candidates on this question was good. Figure 14 provides the candidates' performance summary on this question.

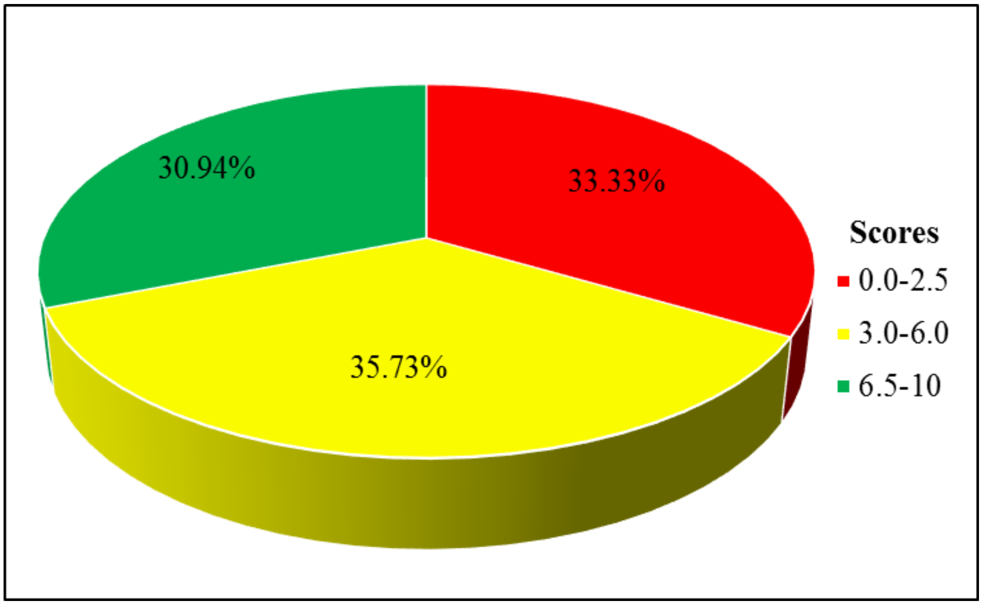


Figure 14: *Candidates' Performance on Question 14*

The analysis show that the candidates who correctly responded to part (a) (i), correctly computed $\underline{a}\times\underline{b}$ by arranging the vectors in matrix form, that is, $\underline{a}\times\underline{b}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ -2 & 5 & -3\\ 3 & -1 & 2\end{vmatrix}$ then, computed it correctly and managed to get $\underline{a}\times\underline{b}=7\underline{i}-5\underline{j}-13\underline{k}$. In part (a) (ii), the analysis revealed that, the candidates correctly used the result of $\underline{a}\times\underline{b}$ from part (a) (i) to find the dot product with $\underline{a}$. For instance, they correctly wrote $(7i-5j-13k)\cdot(-2i+5j-3k)$, to find $(\underline{a}\times\underline{b})\cdot\underline{a}$. Thereafter, they computed and managed to get $(\underline{a}\times\underline{b})\cdot\underline{a}=0$.

In part (b) (i), most candidates computed the determinant of a 3×3 matrix by expanding about any row or any column. Therefore, they got $(t+3)\begin{vmatrix}t-3 & -1\\ 1 & t+4\end{vmatrix}-5\begin{vmatrix}-1 & -1\\ 1 & t+4\end{vmatrix}+6\begin{vmatrix}-1 & t-3\\ 1 & 1\end{vmatrix}=0$, which resulted in $t^3+4t^2-9t-6=0$ and consequently $t=2$. In part (b) (ii), the candidates recognized that, matrix multiplication is possible for the given orders of the given matrices. Therefore, they correctly multiplied the matrices A and B

to get matrix $AB = \begin{pmatrix} 10 & -4 & 12 \\ 3 & 3 & 0 \\ 11 & -5 & 9 \end{pmatrix}$. Then, they worked on the determinant of matrix AB and got $\det(AB) = -198$. They also computed the determinants of matrices A and B, resulting in $\det(A) = 33$ and $\det(B) = -6$ respectively, which gives $\det(A) \times \det(B) = -198$. Therefore, they concluded that $\det(AB) = \det(A) \times \det(B) = -198$.

In part (c), the candidates correctly recalled the formula for finding the image under reflection, which is $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$. They also correctly realized that, for the line $y - x = 0$, the size of an angle is 45^0. Therefore, they substituted $\alpha = 45^0$ in the formula, and got $M = \begin{pmatrix} \cos 90^0 & \sin 90^0 \\ \sin 90^0 & -\cos 90^0 \end{pmatrix}$, and consequently $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. In part (c) (i), the candidates correctly determined the x and $y-$intercepts of the line $x + 2y = 6$ and got $(6, 0)$ and $(0, 3)$ respectively. Then, they found the respective images of the points under the reflection and got $\begin{pmatrix} x_1' \\ y_1' \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} x_2'' \\ y_2'' \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$. Thus, they worked on these images to $y = -2x + 6$. In part (c) (ii), these candidates substituted $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ into the formula $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, resulting in $\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ and consequently $(x, y) = (-2, 3)$. Extract 14.1 provides a sample of a response from one of the candidates who correctly attempted the question.

14 a $a = -2i + 5j - 3k.$

$b = 3i - j + 2k$

soln:

i) $a \times b$

$$a \times b = \begin{vmatrix} i & j & k \\ -2 & 5 & -3 \\ 3 & -1 & 2 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} 5 & -3 \\ -1 & 2 \end{vmatrix} i - \begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix} j + \begin{vmatrix} -2 & 5 \\ 3 & -1 \end{vmatrix}$$

$a \times b = (10 - 3) i - (-4 + 9) j + (2 - 15) k.$

$a \times b = 7i - 5j - 13 k.$

$\underline{a \times b = 7i - 5j - 13 k.}$

ii) $(a \times b) \cdot a$

$= (7i - 5j - 13k) \cdot (-2i + 5j - 3k)$

$= -14 - 25 + 39$

$= 0$

14 (b) i) soln

$(t+3)$

$\text{Det } (X) = 0$

$$0 = \begin{vmatrix} t-3 & -1 \\ 1 & t+4 \end{vmatrix} t+3 + 5\begin{vmatrix} -1 & -1 \\ 1 & t+4 \end{vmatrix} + 6\begin{vmatrix} -1 & t-3 \\ 1 & 1 \end{vmatrix}$$

$$((t^2+t-12)+1)\ t+3 \ -5(-t-4+1) + 6(-1-(t-3)) = 0$$

$$(t^2+t-11)(t+3) + 5t + 15 + 6-(-1+3-t) = 0$$

$$t^3 + 3t^2 + t^2 + 3t - 11t - 33 + 5t + 15 + 12 + 6t = 0$$

$$t^3 + 4t^2 + -9t + -6 = 0$$

$$t^3 + 4t^2 - 9t - 6 = 0$$

$$(t-2)(t+5.45)(t-0.55) = 0$$

$$t = 2 \text{ or } {}^{-}5.45 \text{ or } 0.55.$$

ii) $$AB = \begin{pmatrix} 1 & 3 & 5 \\ 2 & ^{-}1 & 0 \\ 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 1 & ^{-}3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2+3+5 & 0+{}^{-}9+5 & 1+6+5 \\ 4+{}^{-}1+0 & 0+3+0 & 2+{}^{-}2+0 \\ 8+2+1 & 0+{}^{-}6+1 & 4+4+1 \end{pmatrix} = \begin{pmatrix} 10 & ^{-}4 & 12 \\ 3 & 3 & 0 \\ 10 & ^{-}5 & 9 \end{pmatrix}$$

4b ii)

$$AB = \begin{pmatrix} 10 & -4 & 12 \\ 3 & 3 & 0 \\ 11 & -5 & 9 \end{pmatrix}$$

$$Det(AB) = 10\begin{vmatrix} 3 & 0 \\ -5 & 9 \end{vmatrix} + 4\begin{vmatrix} 3 & 0 \\ 11 & 9 \end{vmatrix} + 12\begin{vmatrix} 3 & 3 \\ 11 & -5 \end{vmatrix}$$

$$Det(AB) = (10 \times 27) + (4 \times 27) + 12 \times -48$$

$$Det(AB) = -198$$

$$Det(A) = 1\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}$$

$$Det(A) = (1 \times -1) + 3(2) + (5 \times 8)$$

$$Det\ A = 33.$$

$$Det\ B = 2\begin{vmatrix} -3 & 2 \\ 1 & 1 \end{vmatrix} - 0 + 1\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix}$$

$$Det\ B = -6$$

$$Det(AB) = Det\ A \times Det(B)$$

$$-198 = 33 \times -6$$

$$-198 = -198.$$

$\therefore$ $Det(AB) = Det\ A \times Det\ B$ Hence shown.

149 ii $y - x = 0$

$y = +x$

$y = x.$

$y/x = 1$

$\tan a = 1$

$a = \tan^{-1}(45)$

$a = 45°$

$$M_{y=x} = \begin{pmatrix} \cos 2a & \sin 2a \\ \sin 2a & -\cos 2a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$M_{y=x} = \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & -\cos 90 \end{pmatrix}$$

$$\text{matrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{matrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

ii $x + 2y = 6$

$x = 6$ $(6, 0)$ and $(0, 3)$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

14 b i)

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$m = \frac{0 - 6}{3}$$

$$m = {}^{-}2.$$

$$m = \frac{y - y_1}{x - x_1}$$

$$-2 = \frac{y - 6}{x - 0}$$

$$y - 6 = -2x$$

$$y + 2x = 6$$

$\therefore$ Equation is $y + 2x = 6.$

ii) $\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$\therefore y = 3$

and $x = {}^{-}2.$

$\therefore (x, y) = ({}^{-}2, 3)$

$\therefore (x, y) = ({}^{-}2, 3)$

Extract 14.1: A sample of a correct response to question 14

In Extract 14.1, part (a) (i), the candidate computed the cross product $\underline{a} \times \underline{b}$ and got the correct answer. In part (b) (i), the candidate solved for t by computing the determinant of a 3×3 matrix. In part (b) (ii), the candidate realized that the matrix multiplication is possible and correctly shown, $\det(AB) = \det(A) \times \det(B)$. In part (c), the candidate correctly used the matrix of the linear reflection to determine the image of the line in part (c) (i) and also in part (c) (ii) to obtain the required point.

Conversely, the data shows that 33.33 per cent of the candidates scored 2.5 marks or less. In part (a), most candidates were not knowledgeable about how to perform the cross product (vector product). For instance, some candidates wrote $\underline{a}\times\underline{b}=(-2+5-3)(3-1+2)$, resulting in $\underline{a}\times\underline{b}=0$. These candidates summed up the coefficients of $\underline{i}$, $\underline{j}$ and $\underline{k}$ of each vector $\underline{a}$ and $\underline{b}$, then multiplied the sums. Also, other candidates performed the dot product. These candidates multiplied the corresponding components of the two vectors, $\underline{a}\times\underline{b}=(-2\underline{i}+5\underline{j}-3\underline{k})\times(3\underline{i}-\underline{j}+2\underline{k})$, and obtained $\underline{a}\times\underline{b}=-6\underline{i}-5\underline{j}-6\underline{k}$. Furthermore, some candidates ignored the unit basis vectors of $\underline{i}$, $\underline{j}$ and $\underline{k}$. They wrote $\underline{a}\times\underline{b}=(10+3)-(-4+9)+(2-15)$, and hence they ended up with a scalar 5 instead of a vector $7\underline{i}-5\underline{j}-13\underline{k}$. The incorrect responses obtained in part (a) (i) affected the responses in part (a) (ii). For instance, the candidates who obtained $\underline{a}\times\underline{b}=0$ wrote $(\underline{a}\times\underline{b})\cdot\underline{a}=0\cdot(2\underline{i}+5\underline{j}-3\underline{k})=0$. Similarly, those who obtained $\underline{a}\times\underline{b}=-2\underline{i}+5\underline{j}-3\underline{k}$ got $(\underline{a}\times\underline{b})\cdot\underline{a}=12\underline{i}-25\underline{j}+18\underline{k}$ instead of a scalar value 0.

In part (b) (i), some candidates were not able to find the determinant of a 3×3 matrix. For instance, they only considered the determinant of the reduced 2×2 matrix to formulate the equation. For example, some candidates wrote $\begin{vmatrix} t-3 & -1 \\ 1 & t+4 \end{vmatrix}=0$, resulting in $t=-2.8$ instead of $t=2$. Further, other candidates committed errors in performing the basic operations. For example, the candidates obtained the incorrect equation $t^3+4t^2-13t-6=0$, and hence they got $t=2.41$ or -6 rather than $t=2$. In part (b) (ii), many candidates committed computational errors, either in multiplication of the matrices or in finding determinants of the matrices. For example, some candidates wrote $|A|=-1\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}+3\begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}-5\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}$ instead of $|A|=1\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}-3\begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}+5\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}$, and thus they got $|A|=-33$ instead of $|A|=33$. Other candidates failed to multiply 3×3 matrices. They obtained $AB=\begin{bmatrix}10 & -4 & 12\\ 3 & 3 & 0\\ 9 & -5 & 8\end{bmatrix}$ instead of $AB=\begin{bmatrix}10 & -4 & 12\\ 3 & 3 & 0\\ 11 & -5 & 9\end{bmatrix}$, resulting in an incorrect determinant, particularly $\det(AB)=-360$ instead of -198. Likewise, they wrongly found $\det(B)$ as -5 instead of -6; thus when comparing $\det(A)\det(B)=33\times-5=-155$ and $\det(AB)=-360$, they concluded that $|AB|\neq|A||B|$.

In part (c), the analysis revealed that most candidates failed to recognize the angle formed by the line $y-x=0$. For instance, some candidates identified angle $\alpha=90^o$ instead of $\alpha=45^o$. Therefore, they substituted $\alpha=90^o$ into the reflection matrix $M=\begin{pmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{pmatrix}$, resulting in $M=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ instead of $M=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. Other candidates recalled the wrong formula, $\mathrm{M}=\begin{pmatrix}\cos 2\alpha & -\sin 2\alpha\\ \sin 2\alpha & \cos 2\alpha\end{pmatrix}$ instead of $\mathrm{M}=\begin{pmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{pmatrix}$. Thus, they substituted $\alpha=45^o$ into the formula and obtained $M=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ instead of $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. In part (c) (i), some candidates only computed the intercepts. For instance, from line $x+2y=6$, they obtained $x=6$ and $y=3$, then concluded that the image point is $(6,3)$. Furthermore, some candidates failed to get the image due to challenges they faced in finding the reflection matrix. For example, some candidates used $M=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ with the correct x and y intercepts $(6,\ 0)$ and $(0,\ 3)$ respectively. Thereafter, they applied it to find the image of intercepts and obtained $(-6,3)$ instead of $(3,\ 6)$ without finding the image of the line. Similarly to part (c) (i), in part (c) (ii), some candidates wrongly considered $(3,-2)$ as the object instead of the image. These candidates wrote

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ instead of $\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, and therefore they got the incorrect point $(-3,-2)$ instead of $(-2, 3)$. Most of these candidates recalled the incorrect matrix of reflection, $M = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ instead of $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Extract 14.2 is the sample response from one of the candidates who faced difficulties when responding to the question.

14. (a) soln

$\underline{a} = -2i + 5j - 3k$

$\underline{b} = 3\underline{i} - j + 2k$

i) $\underline{a} \times \underline{b}. = (2i + 5j - 3k) \times (3\underline{i} - j + 2k)$

$= -6i + (-5j) - 6k$

$= -6\dot{i} - 5j - 6k$

$\therefore \underline{a} \times \underline{b} = -6\underline{i} + 5\underline{j} - 6\underline{k}$

ii) $(\underline{a} \times \underline{b}) \cdot a$

soln

$(\underline{a} \times \underline{b}) = \sqrt{-6^2 + (-5^2) + 6^2}$

$= \sqrt{36 + 25 + 36}$

$(\underline{a} \times \underline{b}) = \sqrt{97}$

$|\underline{a} \times \underline{b}| \cdot a = \sqrt{97}\,(-2\underline{i} + 5\underline{j} - 3\underline{k})$

$\therefore |\underline{a} \times \underline{b}| \cdot a = \sqrt{97}\,(-2\underline{i} + 5\underline{j} - 3\underline{k})$

(b) Soln.

$A = \begin{bmatrix} t+3 & 5 & 6 \\ -1 & t-3 & -1 \\ 1 & 1 & t+4 \end{bmatrix} = 0$

let

$|A| = + \begin{bmatrix} t-3 & -1 \\ 1 & t+4 \end{bmatrix} - \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & t-3 \\ 1 & 1 \end{bmatrix}$

$|A| = ((t-3)(t+4) + 1) - (-1 + 1) + (-1 - t + 3)$

14. (b)

$$|A| = (t^2-3t+4t-12+1)+(-1-t+3)$$

$$\cancel{|A| = t^2-3t}$$

$$|A| = t^2+t-11-t+2$$

$$|A| = t^2+t-t-11+2$$

$$|A| = t^2-9$$

$$A = 0$$

$$0 = t^2-9$$

$$\sqrt{t^2} = \sqrt{9}$$

$$t = \pm 3$$

$\therefore$ The value of $t = 3$ or $t = -3$

15) Soln.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$$

$$|A| = (-1-0)-(2-0)+(4+4)$$

$$|A| = -1-2+8$$

$$|A| = 5$$

Then

$$|B| = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}$$

$$|B| = (-3-2)-(1-2)+(1+3)$$

$$|B| = -5+1+4$$

14 (b) ii)

$|B| = B - 5 + 5$

$|B| = 0$

$\det(AB) = \det(A)\det(B)$

$\det(AB) = 5 \times 0$

$\det(AB) = 0$

$\therefore \det(AB) = 0$

(c) Soln

$P(x, y)$

$y = x$

i) $x + 2y = 6$

$(x, y) = (6, 3)$

$y = x = (x, y) = (y, x)$

$(x, y)' = (3, 6)$

$\therefore$ The image of the line $x + 2y = 6$ is $(3, 6)$

ii) $(3, -2)$, $M_{y=x}(x, y) = (x, -y)$

$M_y(3, -2) = (3, 2)$

$\therefore$ The image of this reflection $(3, -2)$ is under the image reflection of $(3, 2)$

Extract 14.2: A sample of an incorrect response to question 14

In Extract 14.2, part (a) (i), the candidate performed the dot product while maintaining the unit vectors. In part (a) (ii), the candidate computed the magnitude of the resulted vector in part (a) (i) and multiplied it by $\underline{a}$. In part (b), the candidate wrongly found the determinant of a 3×3 matrix without multiplying the elements of the first row by the remaining 2×2, as well as failed to get the determinant of AB and consequently failed to get the answer. In part (c) (i), the candidate only computed the image of the x and y intercepts, concluding that they are coordinates of the image of the line. Similarly, in part (c) (ii), the candidate incorrectly assumed that the image was reflected along the x axis.

3.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH TOPIC

The Additional Mathematics paper in CSEE 2023 had sections A and B with a total of fourteen (14) questions. A total of sixteen (16) topics had been examined, including *Numbers*, *Variations*, *Statistics*, *Locus*, *Coordinate geometry*, *Geometrical Constructions*, *Logic*, *Sets*, *Algebra*, *Trigonometry*, *Functions*, *Differentiation*, *Integration*, *Probability*, *Vectors* and *Matrices and Linear Transformation*. The analysis depicts that, among these topics, eleven (11) had good performance. These topics are *Geometrical Constructions* (96.65%), *Variations* (94.72%), *Functions* (89.69%), *Algebra* (86.57%), *Locus* (85.85%), *Logic* (80.81%), *Statistics* (70.74%), *Numbers* (70.51%), *Probability* (69.55%) and *Vectors* and *Matrices and Transformation* (66.67%). The candidates' good performance in these topics was attributed to competence in the tested concepts, their ability to identify the requirements of the questions, their ability to translate word problems into mathematical models, and their proper recall and use of appropriate formulae, facts, laws and theorems.

Further analysis shows that three (3) topics, *Sets* (47.49%) and *Differentiation* and *Integration* (30.70%) were performed averagely. The candidates' average performance in these topics was due to some candidates' inability to recall and apply properties of sets operations, failure to interpret word problems involving sets, inability to draw and use Venn diagrams, failure to understand the relationship between the derivative and tangent of a curve, as well as the gradient and derivative, and failure to integrate polynomials. Furthermore, the candidates had weak performance in the two topics, *Trigonometry* (24.23%) and *Coordinate Geometry* (19.42%). The candidates' weak performance was associated with a lack of competence in the tested concepts, the use of inappropriate approaches while answering the questions, the inability to recall the correct and appropriate formulae, failure to interpret the inverse of trigonometric expressions, the failure to translate the intervals of limits, the misinterpretation of word problems into Mathematical models, errors in doing basic operations (computational errors) and the failure to adhere to the requirements of the questions. The analysis of the candidates' performance on each topic is presented in Appendix I of the report.

Further analysis shows that there is an increase in the performance of the candidates in five (5) topics of *Algebra* (30.99%), *Logic* (16.85%), *Geometrical Constructions* (6.55%), *Functions* (5.17%) and *Locus* (3.36%) for 2023 as compared to 2022. Likewise, there are eleven (11) topics that show a decrease in candidates' performance as compared to 2022. These topics are *Variations* (5.03%), *Numbers* (15.53%), *Probability* (20.04%), *Vectors* and *Matrices and Linear Transformation* (24.19%), *Trigonometry* (26.53%), *Coordinate Geometry* (27.53%), *Statistics* (27.74%), *Differentiation* and *Integration* (28.18%), and *Sets* (39.82%). Appendix II provides a comparison of candidates' performance per topic in 2022 and 2023.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The performance of the candidates in the Additional Mathematics in CSEE 2023 was generally good, as 97.84 per cent of the candidates passed. The good performance was enhanced by candidates' competence in the tested concepts; that is, they were equipped with the relevant knowledge and skills. However, few candidates had low performance which was attributed to inability of the candidates to perform basic operations, failure to follow instructions, and poor interpretation of the questions. The Candidates' Item Response Analysis (CIRA) has been prepared in order to give feedback to education stakeholders about the strengths and weaknesses shown by candidates' responses so as to take appropriate measures to improve more candidates' performance on future Additional Mathematics examinations.

4.2 Recommendations

In order to improve candidates' performance in future Additional Mathematics examinations, the following measures are recommended:

(i) students be encouraged to solve many problems, from which challenges may be observed and dealt with before the examination season.

(ii) Due to a lack of candidates' competence in some areas of *Sets*, *Differentiation*, *Integration*, *Trigonometry* and *Coordinate Geometry*, student should study and solve questions on all the topics stipulated in the syllabus. As well as to make effective use of text books to enhance understanding of the concepts taught in Additional Mathematics.

(iii) Teachers should lead students to discuss the derivation of the equation of a circle and using it in solving mathematical problems in order to build more competence in *Coordinate Geometry*.

(iv) Teachers should lead students to derive trigonometric identities and solve simple trigonometric equations up to a second degree in order to build more competence in *Trigonometry*.

(v) Teachers should lead students to discuss the relationship between tangent to the curve and the normal as well as to guide students to find equation of a line normal to a given curve.

(vi) In order to build more competence in *Integration*, it is recommended that, teachers lead students to integrate indefinite polynomials and simple trigonometric functions.

Appendix I: Analysis of Candidates' Performance on Each Topic

S/N	Topics	Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks
1.	Geometrical Constructions	6	96.65	Good
2.	Variations	1	94.72	Good
3.	Functions	11	89.69	Good
4.	Algebra	5	86.57	Good
5.	Locus	4	85.85	Good
6.	Logic	9	80.81	Good
7.	Statistics	2	70.74	Good
8.	Numbers	8	70.51	Good
9.	Probability	13	69.55	Good
10.	Vectors, Matrices and Transformations	14	66.67	Good
11.	Sets	10	47.49	Average
12.	Integration and Differentiation	12	30.70	Average
13.	Trigonometry	7	24.23	Weak
14.	Coordinate Geometry	3	19.42	Weak

Appendix II: Comparison of Candidates' Performance on Each Topic in 2022 and 2023

S/N	Topics	2022			2023		
		Question Number	**Percentage of Candidates who Scored an Average of 30% or Above**	**Remarks**	**Question Number**	**Percentage of Candidates who Scored an Average of 30% or Above**	**Remarks**
1.	Geometrical Constructions	6	90.10	Good	6	96.65	Good
2.	Variations	1	99.75	Good	1	94.72	Good
3.	Functions	11	84.52	Good	11	89.69	Good
4.	Algebra	5	55.58	Average	5	86.57	Good
5.	Locus	4	82.49	Good	4	85.85	Good
6.	Logic	9	63.96	Average	9	80.81	Good
7.	Statistics	2	98.48	Good	2	70.74	Good
8.	Numbers	8	86.04	Good	8	70.51	Good
9.	Probability	13	89.59	Good	13	69.55	Good
10.	Vectors, Matrices and Transformations	14	90.86	Good	14	66.67	Good
11.	Sets	10	87.31	Good	10	47.49	Average
12.	Integration and Differentiation	12	58.88	Average	12	30.70	Average
13.	Trigonometry	7	50.76	Average	7	24.23	Weak
14.	Coordinate Geometry	3	46.95	Average	3	19.42	Weak