THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2022

ADDITIONAL MATHEMATICS

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042 ADDITIONAL MATHEMATICS

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FOREWORD

This report presents Candidates' Items Response Analysis (CIRA) on Form Four Additional Mathematics National Examination which was conducted in November 2022. The report aims to provide feedback to all educational stakeholders on the factors that contributed to the candidates' performance in Additional Mathematics.

The Certificate of Secondary Education Examination (CSEE) is a summative evaluation which intends to provide feedback that teachers, students and other educational stakeholders can use to improve teaching and learning. This analysis shows justification for the candidates' performance in the Additional Mathematics subject. The candidates who attained high scores had adequate knowledge and skills of the tested concepts, correct interpretation of word problems into mathematical model, computation skills, understanding of the questions asked, and the ability to recall the appropriate formulae and laws. However, candidates who scored on average faced difficulties in responding to the questions due to their insufficient knowledge of the tested concepts.

This report will help the on going students to identify strengths and weaknesses encountered by the candidates for them to improve learning before sitting for their Certificate of Secondary Education Examination (CSEE). It will help teachers to identify the challenging areas and take appropriate measures during teaching and learning.

The National Examinations Council of Tanzania (NECTA) expects that the feedback provided in this report will highlight the challenges which education stakeholders should take proper measures to improve teaching and learning the Additional Mathematics subject. Consequently, students will acquire knowledge, skills and competence indicated in the syllabus for better performance in future examinations.

The Council appreciates the contribution of all those who prepared this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is an outcome of the analysis of the candidates' items responses in Additional Mathematics on the Certificate of Secondary Education Examination (CSEE), 2022. It is prepared to provide feedback to stakeholders about the candidates' performance especially in the areas where the candidates faced challenges and the areas in which the candidates performed well.

The examination paper was set according to the 2019 Examination format which is based on the 2010 Ordinary Level Additional Mathematics syllabus. It comprised sections A and B. Section A had ten (10) compulsory questions, each carrying six (6) marks. Section B consisted of four (4) compulsory questions, each carrying ten (10) marks.

For the year 2022, a total of 394 candidates sat for the Additional Mathematics examination. In 2021, 320 candidates sat for this examination, comperatively there is an increase of 18.78 per cent of the candidates who sat for this examination in 2022. Table 1 presents the summary of the candidates' performance in Additional Mathematics for the years 2021 and 2022.

Table 1: Candidates' Performance in Additional Mathematics CSEE 2021 and 2022

Year	Students Sat	Passed		Grades				
		No.	%	A	B	C	D	F
2021	320	315	98.75	72	63	126	54	04
2022	394	393	99.75	73	84	195	41	01

Table 1 shows that the candidates' performance in Additional Mathematics was good as 99.75 per cent of those who sat for examination passed, of whom 73 (18.53%) candidates scored A, 84 (21.32 %) scored B, 195 (49.49%) scored C, 41 (10.41%) scored D while 01 (0.25 %) scored F. Generally, the candidates' performance was good since 393 (99.75%) candidates passed the examination. Comparatively, in 2021 a total of 315 (98.75%) candidates passed the examination. This shows that the candidates' performance in 2022 has increased by 1.00 per cent compared to that of 2021.

The percentages of candidates who passed the examination in Additional Mathematics in different grades are shown in Figure 1.

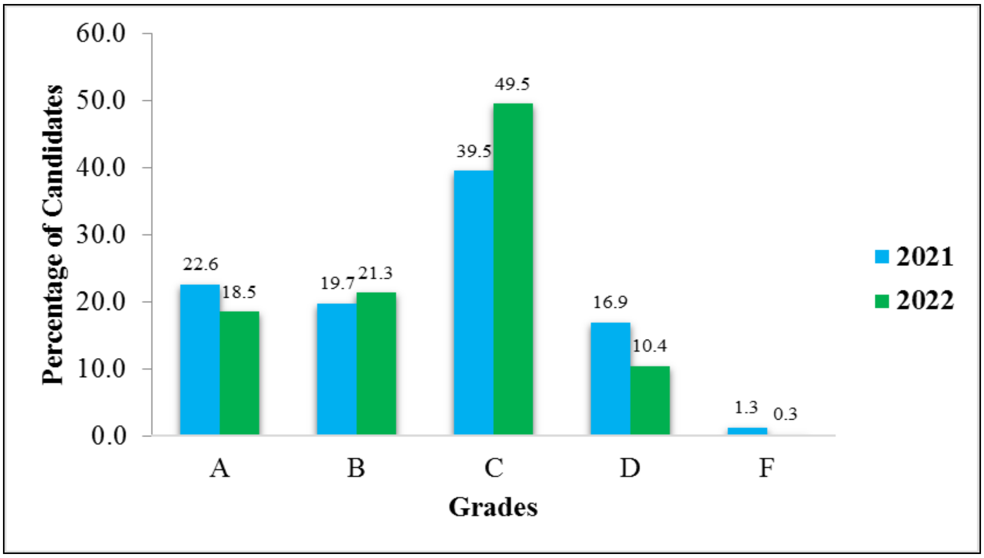


Figure 1: *Performance of Candidates in Different Grades*

As Figure 1 shows, the quality of performance for the year 2022 increased as compared to that of 2021. For example, the number of candidates who scored grade F decreased by 1.00%.

Section 2.0 presents the analysis of candidates' performance on each question. The section describes the requirements of the questions and the candidates' responses. It also presents extracts which show the strengths and the weaknesses demonstrated by the candidates while responding to each question.

The candidates' performance on each question is categorized by using percentage of candidates who scored 30 per cent or more of the total marks allocated to a particular question. The performance is categorized in three groups: 65 to 100 per cent for good performance; 30 to 64 per cent for average performance and 0 to 29 per cent for weak performance. Furthermore, green, yellow and red colours are used to indicate good, average and weak performance respectively.

The analysis of candidates' responses per topic and the factors which led to average performance on some topics are provided in Section 3.0, while Section 4.0 presents the recommendations for improving performance in future examinations.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH QUESTION

This section presents the analysis of the candidates' performance on each question. The national examination results were based on the score intervals 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 which are equivalent to excellent, very good, good, satisfactory and fail respectively. For the purposes of this report, the percentage of candidates who scored 30 per cent or more of the total marks for each question determines whether the performance on that question was good, average, or weak in the percentage intervals of 65 – 100, 30 - 64 and 0 – 29 respectively.

2.1 Question 1: Variations

This question had two parts (a) and (b). In part (a), the candidates were required to find the value of p when $q=4$ and $r=6$, if p is directly proportional to the product of q and r, and $p=5$ when $q=3$ and $r=4$. In part (b), the question was; The number of hours (t) needed by workers to assemble machines varies directly as the number of machine (N) and inversely as the number of workers (w). The candidates were required to find the number of workers needed to assemble 16 machines in 4 hours given that 5 workers can assemble 10 machines in 2 hours.

The analysis of data reveals that out of 394 (100%) candidates who attempted this question, 1 (0.3%) candidate scored 0 to 1.5 marks, 6 (1.5%) candidates scored 2.0 to 3.5 marks while 387 (98.2%) candidates scored 4.0 to 6.0 marks. Therefore, the candidates' performance on this question was generally good. The summary of candidates' performance on this question is presented in Figure 2.

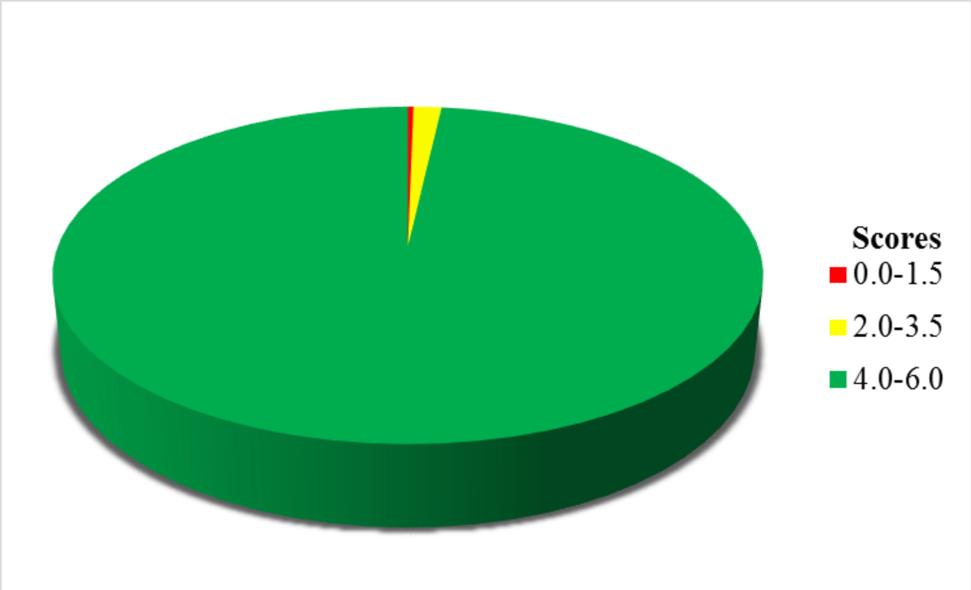


Figure 2: *Candidates' Performance on Question 1*

In part (a), the analysis shows that the candidates correctly transformed the variation problem into mathematical statement, applied the proportional sign α and correctly wrote $p \,\alpha\, qr$. After that, k was introduced as a proportionality constant to get $p = kqr$, and the values of $p=5$, $q=3$ and $r=4$ were substituted in the equation to get $5 = k\times 3\times 4 \Rightarrow k = \frac{5}{12}$. Thereafter, they worked on $p = kqr$ to get the value of p when $k=\frac{5}{12}$, $q=4$ and $r=6$. Thus, they got $p=\frac{5}{12}\times 4\times 6$ and then, simplified it to obtained $p=10$ as the required value.

In part (b), the candidates correctly translated the word problem and formulated the mathematical model as $t\alpha \frac{N}{w}$. Then, they introduced a constant k and got $t=\frac{kN}{w}$. Following that, they correctly identified the values of $w=5$, $N=10$ and $t=2$ then substituted them in the equation as $2=\frac{k\times 10}{5}$ which resulted to $k=1$. Furthermore, they worked on w when $N=16$ and $t=4$ that is, $\frac{w\times 4}{16}=1$ which resulted to $w=4$. Extract 1.1

shows a sample of a response from one of the candidates who responded correctly to this question.

1\. (a) $p \propto qr$

$p = kqr$

$(5) = k(3)(4)$

$5 = 12k$

$k = 5/12$

Then

$p = 5/12\ qr$

$p = 5/12 \times 4 \times 6$

$= \frac{5 \times 24}{12}$

$= 5 \times 2$

$p = 10$

$\therefore p = 10.$

1\. (b) $t \propto \frac{N}{w}$

$t = \frac{kN}{w}$

$k = \frac{tw}{N}$

1\. (b) $K = \frac{2 \times 5}{10}$

$K = 1$

Then

$t = \frac{1N}{w}$

$t = N/w$

$w = \frac{N}{t}$

$= \frac{16}{4}$

$= 4$ workers

$\therefore$ 4 workers are needed.

Extract 1.1: A sample of the candidate's correct responses to question 1

In Extract 1.1, the candidate transformed the variation statement in mathematical symbols and then used it to obtain the correct value of p in part (a). In part (b), the candidate was able to interpret correctly the given word problem which was solved to obtain 4 numbers of workers.

On the other hand, some candidates performed poorly on this question as they scored low marks. Those candidates had limited knowledge on variation. In part (a), some of the candidates failed to correctly transform the given variation statement into a mathematical model, which was $p \, \alpha \, q \, r$. For example, a one of the candidates mistook the term "product" for an addition, and hence formulated the wrong expression $p \, \alpha \left(q + r\right)$. Following that, introduced the proportionality constant k to obtain $p = k\left(q + r\right)$, then substituted $p = 5$, $q = 3$ and $r = 4$ to get $k = \frac{5}{7}$ instead of $k=\frac{5}{12}$. Some candidates substituted $k=\frac{5}{7}$, $q=4$ and $r=6$ into $p=k\left(q+r\right)$ to obtain the value of $p=7.14$ instead of $p=10$.

Some other candidates managed to correctly interpret the given statement as $p\,\alpha\,q\,r$, then introduced the proportionality constant k to obtain $p=kqr$, but failed to use the proper basic operations of multiplication and division. As a result, they got incorrect values of p such as $p=57.6$ instead of $p=10$.

Furthermore, the analysis shows that, few candidates incorrectly interpreted the statement by separating the variables. For example, one candidate separated as $p\,\alpha\,q$ and $p\,\alpha\,r$ then, introduced k_1 and k_2 as proportionality constants to obtain $p=k_1q$ and $p=k_2r$ respectively. Later, he/she substituted $p=5$ and $q=3$ into $p=k_1q$ as well as $p=5$ and $r=4$ into $p=k_2r$ to obtain $k_1=\frac{5}{3}$ and $k_2=\frac{5}{4}$ respectively. Finally, the candidate substituted the given values of $q=4$ and $r=6$ in both equations and obtained incorrect values, $p=6.7$ and $p=7.5$.

In part (b), other candidates correctly responded to the given statement as $t\alpha\frac{N}{w}$ and introduced the constant k to obtain $t=\frac{kN}{w}$. However, they made the incorrect substitutions, such as $t=2$, $w=15$ and $N=10$ instead of $t=2$, $w=5$ and $N=10$. This led to the incorrect value, $k=3$ and got 12 which was the incorrect number of workers.

Moreover, other candidates failed to formulate the variation model $t\alpha\frac{N}{w}$. Instead, they wrote $t\alpha\frac{w}{N}$, then incorrectly changed to equation without introducing the proportionality constant k that resulted to $w=tN$. Finally, they substituted $N=16$ and $t=4$ to obtain 64 as number of workers. On the other hand, some candidates managed to formulate the correct mathematical model $t\alpha\frac{N}{w}$. Then, they introduced the proportional constant k but they incorrectly substituted the values. For example, one of the candidates substituted $N = 16$ and $t = 4$ into $t = \frac{w}{N}$ instead of $t = \frac{kN}{w}$ which resulted into $w = 64$ workers. Extract 1.2 is a sample response selected from one of the candidates who faced challenges in attempting this question.

1. a) To find the value of P

solution

$P \propto q \ast r$	$P = 5$	$P = ?$
	$q = 3$	$q = 4$
	$r = 4$	$r = 6$

$P \propto q + r$

$\frac{P}{q+r} = \frac{k(q+r)}{q+r}$

$k = \frac{P}{q+r}$

$k = \frac{5}{3+4}$

$k = 5/7$

from $P = ?$

$k = \frac{P}{q+r}$

$5/7 = \frac{P}{4+6}$

$5/7 \times \frac{P}{10}$

$10 \times 5 = 7P$

$\frac{7P}{7} = \frac{50}{7}$

$P = 50/7$

$\therefore$ The value of $P = 7.14$

Extract 1.2: A sample of the candidate's incorrect responses to question 1

In Extract 1.2, the candidate misinterpreted the given variation statement as an addition, hence formulated the incorrect expression $p\alpha(p+r)$ which resulted to an incorrect response.

2.2 Question 2: Statistics

The candidates were given the following table which summarizes the scores obtained by 50 students in Additional Mathematics test:

Class mark (x)	32	37	42	47	52	57	62	67	72	77
Frequency	2	4	t	6	$2t$	8	$t+1$	5	3	1

Then they were asked to find:

(a) the value of t

(b) the mean score using the class marks and frequency of the table.

This question was attempted by 394 (100%) candidates, out of whom 6 (1.5%) candidates scored 0 to 1.5 marks. Meanwhile 42 (10.7%) candidates scored 2.0 to 3.5 marks and 346 (87.8%) candidates scored 4.0 to 6.0 marks. The candidates' performance summary is presented in Figure 3.

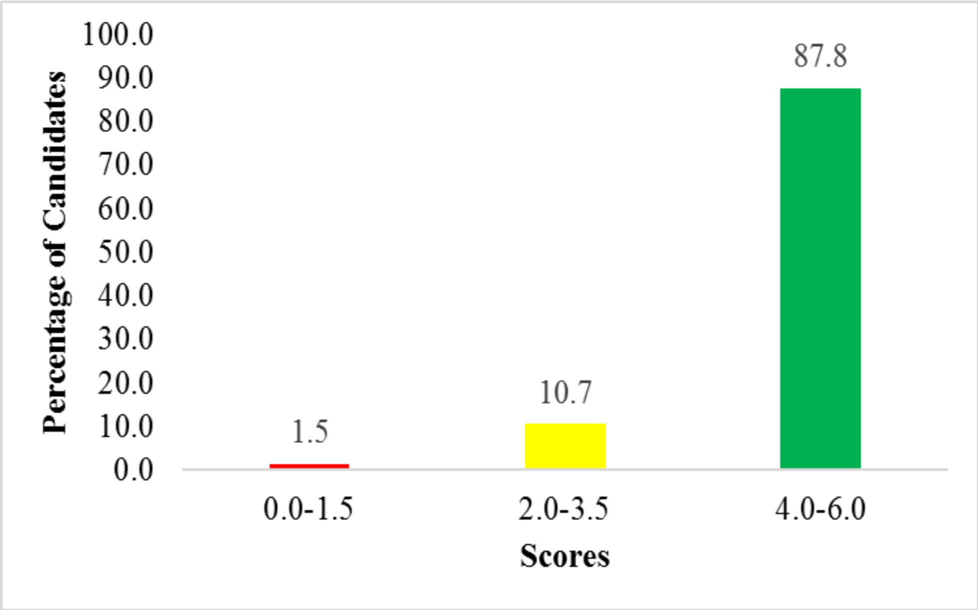


Figure 3: *Candidates' Performance on Question 2*

The analysis shows the candidates' performance on this question was generally good as it is observed from Figure 3 that, 98.5 per cent of the candidates scored 2.0 to 6.0 marks.

In part (a), the analysis of data depicts that the candidates who responded correctly to this question managed to score all the marks alloted to this question. The candidates recognized that the sum of all frequencies equals to 50 candidates who were scheduled for Additional Mathematics test. Thus, $\sum f = f_1 + f_2 + f_3 + \ldots\ldots + f_n = 50$, where f denotes frequency. Later, they substituted the given values as $50 = 2+4+t+6+2t+8+t+1+5+3+1$ after which they got $4t = 20$ and finally $t = 5$ which was the required value.

In part (b), the candidates recalled correctly the formula for the mean $(\bar{x}) = \dfrac{\sum fx}{\sum f}$ and constructed a frequency distribution table with columns of x, f and fx. Then, they determined $\sum f = 50$ and $\sum fx = 2680$ from the table and correctly substituted all the identified values in the formula as $(\bar{x}) = \dfrac{\sum fx}{\sum f} = \dfrac{2680}{50}$ and simplified to 53.6. Extract 2.1 is a sample response from one of the candidates who correctly responded to this question.

2.	Frequency distribution Table				
	Intervals	f	x	fx	
	30 − 34	2	32	64	
	35 − 39	4	37	148	
	40 − 44	5	42	210	
	45 − 49	6	47	282	
	50 − 54	10	52	520	
	55 − 59	8	57	456	
	60 − 64	6	62	372	
	65 − 69	5	67	335	
	70 − 74	3	72	216	
	75 − 79	1	77	77	
		N = 50		Σfx = 2680	

2.	(a) $2+4+t+6+2t+8+t+1+5+3+1=50$
	$29+4t+1=50$
	$30+4t=50$
	$4t=20$
	$t=20/4$
	$\therefore t=5$
	(b) Mean
	From
	$\bar{x}=\frac{\Sigma fx}{\Sigma f}$
	$=\frac{2680}{50}$
	$=53.6$
	$\therefore$ The mean score $=53.6$.

Extract 2.1: A sample of the candidate's correct responses to question 2

In Extract 2.1, the candidate managed to find the value of t by recognizing that the sum of all frequencies equals to 50 candidates in part (a). Also, in part (b), the candidate calculated the mean value by applying the appropriate formula of mean, $\bar{x}=\frac{\sum fx}{\sum f}$.

Despite the good performance shown by most of the candidates, there were some candidates who failed to answer the question as they lacked adequate knowledge on statistics, and computation skills.

In part (a), some of the candidates made some computational errors while responding to the question. For example, one of candidates equated the sum of individual values of frequencies as $2+4+t+6+2t+8+t+1+5+3+1=50$, but ended up with the incorrect value of $t=\frac{20}{3}$ instead of $t=5$. Also, some candidates wrongly attempted the question by solving for different values of t. For instance, one of the candidates calculated t_1, t_2 and t_3 as , $52+2t_1=29$, $42+t_2=29$ and $62+1+t_3=29$. Then such a candidate arrived at $t_{total}=1.15+13+34=48.15$ rather than the required value of 5.

In part (b), other candidates responded to the question by using the incorrect formula such as $\text{Mean}\left(\bar{x}\right)=\frac{\sum x}{\sum f}$ instead of $\text{Mean}\left(\bar{x}\right)=\frac{\sum fx}{\sum f}$. The incorrect formula produced a mean of $\frac{545}{50}$, which was simplified to 10.9 rather than 53.6. Additionally, some candidates perceived the question as individual data, thus applying the incorrect mean formula such as $\bar{x}=\frac{\sum f}{n}$. Then, they added the frequencies to obtain $\sum f=50$ which corresponded to the number of data $n=10$. Thereafter, the candidates substituted the values in the formula $\bar{x}=\frac{\sum f}{n}=\frac{50}{10}$, which then resulted to incorrect value, mean $(\bar{x})=2$. Extract 2.2 is a sample of response from one of the candidates who responded to the question incorrectly.

2. a) soln:

CLASS MARK (X)	FREQUENCY	FX
32	2	64
37	4	148
42	t	42t
47	6	282
52	2t	
57	8	
62	t+1	
67	5	
72	3	
77	1	
	N = 29	

$52+2t=29$

$2t=52-29$

$\frac{2t}{2}=\frac{23}{2}$

$t=1.15$

2 a) $42 + t = 29$

$t = 42 - 29$

$t = 13$

$62 + t + 1 = 29$

$62 + 1 + t = 29$

$63 + t = 29$

$t = 63 - 29$

$t = 34$

The value of $t = 1.15 + 13 + 34$

$t = 48.15$

$\therefore$ Value of t is 48.15.

b) Mean $(\bar{x}) = \frac{\Sigma x}{\Sigma f}$

$\bar{x} = \frac{32+37+42+47+52+57+62+67+72+77}{50}$

$\bar{x} = \frac{545}{50}$

$\bar{x} = 10.9$

$\therefore$ Mean score is 10.9

Extract 2.2: A sample of the candidate's incorrect responses to question 2

In Extract 2.2 , the candidate attempted the question incorrectly by solving for different values of t in part (a) while in part (b), the candidate applied the incorrect formula for mean and got an incorrect answer.

2.3 Question 3: Coordinate Geometry

This question consisted of parts (a) and (b). The candidates were given the vertices of the triangle as $A(0,0)$, $B(6,4)$ and $C(9,8)$. Then, they were asked to;

(a) calculate the perpendicular distance from point A to the line segment $\overline{BC}$.

(b) use the results obtained in part (a), to find the area of the triangle.

The analysis shows that a total of 394 candidates attempted this question, out of whom 53 per cent of candidates scored 0 to 1.5 marks while 11.7 per cent of candidates scored 2.0 to 3.5 marks. However, 35.3 per cent of candidates scored 4.0 to 6.0 marks. Therefore, the candidates' performance on this question was on average. Figure 4 presents the candidates' performance summary on this question.

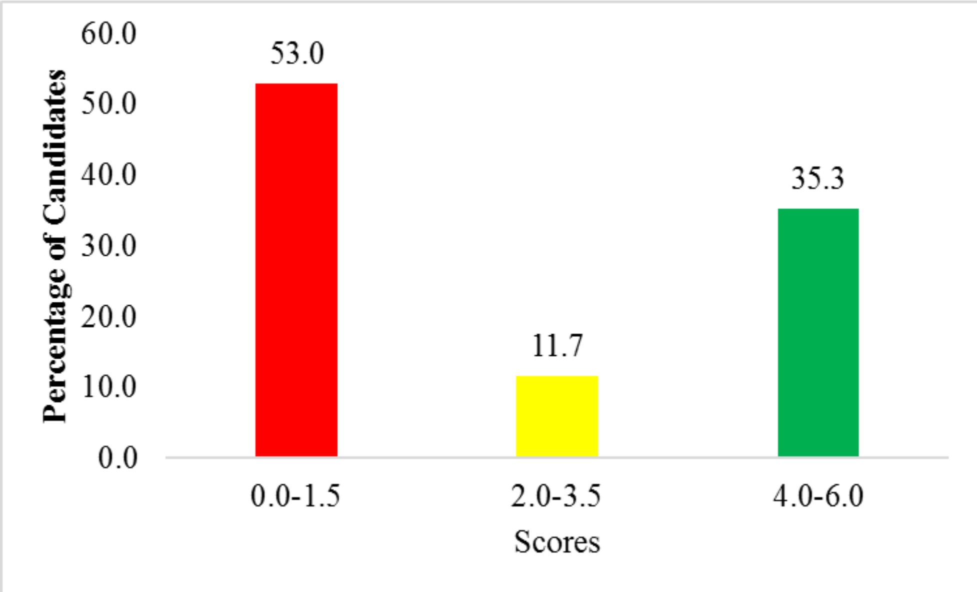
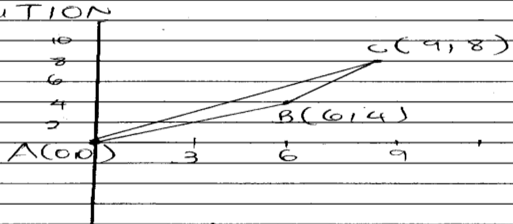


Figure 4: *Candidates' Performance on Question 3*

In part (a), the analysis shows that some candidates were able to respond to this question and scored all marks alloted to this question. They were competent in finding the perpendicular distance from point $A(0,0)$ to the line segment BC. They correctly remembered the gradient formula $M_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ and substituted points $B(6,4)$ and $C(9,8)$ in the formula as $M_{BC} = \frac{8-4}{9-6} = \frac{4}{3}$. Then, they correctly used the gradient to find the equation of $\overline{BC}$ in the form $ax + by + c = 0$. Thus, they determined the equation of $\overline{BC}$ by substituting the obtained gradient, $m = \frac{4}{3}$ and point $B(6,4)$ in the formula such that, $\frac{4}{3} = \frac{y-4}{x-6}$. Then, they rightly simplified it to obtain $4x-3y-12=0$. Then, they correctly recalled the formula for perpendicular distance as $d=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$ and identified all the required data as $x=0$, $y=0$, $a=4$ and $b=-3$ then substituted them in the formula $d=\frac{|4(0)-3(0)-12|}{\sqrt{4^2+(-3)^2}}$, which resulted to $d=\frac{12}{5}$ units as the required distance.

In part (b), the candidates showed their competence as they recognized that the height of the given triangle was the perpendicular distance from $A(0,0)$ to the line segment $\overline{BC}$, thus the height was to be $h=\frac{12}{5}$ units. Then, they applied the distance formula to find the length of the base $\overline{BC}$, which was $\overline{BC}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. Then, they substituted $B(5,4)$ and $C(9,8)$ to get $\overline{BC}=\sqrt{(9-6)^2+(8-4)^2}=5$ units, which was the length of the base. Then, they correctly applied the formula for the area of a triangle, which was $\frac{1}{2}\times b\times h$. Then, they substituted the value of $h=\frac{12}{5}$, $b=5$ in the formula and got $A=\frac{1}{2}\times 5\times\frac{12}{5}=6$ square units. Finally, they were able to conclude that the area of triangle was 6 square units. Extract 3.1 provides a sample of a response from one of the candidates who responded correctly to this question.

3 (a) SOLUTION



From the equation of the line

Slope BC (m) $= \frac{\Delta y}{\Delta x}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8-4}{9-6} = \frac{4}{3}.$

3 (a) equation of BC

$m = \frac{4}{3}$

$\frac{4}{3} \times \frac{y-4}{x-6}$

$3y - 12 = 4(x-6)$

$3y - 12 = 4x - 24.$

$0 = 4x - 3y - 24 + 12$

$4x - 3y - 12 = 0.$

$a = 4, b = -3, c = -12$

Perpendicular linear distance From A to BC.

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$(x_1 y) = A(0,0)$

$$d = \left| \frac{4(0) + -3(0) - (12)}{\sqrt{4^2 + -3^2}} \right|$$

$$= \left| \frac{0 + 0 - 12}{\sqrt{16 + 9}} \right|$$

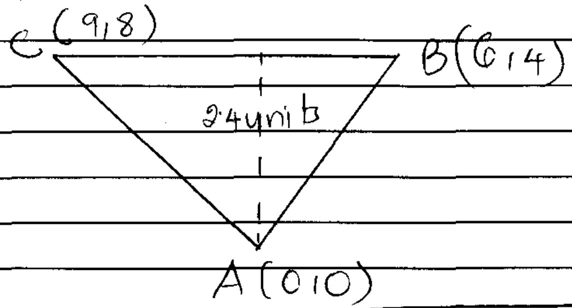
$$= \left| \frac{-12}{\sqrt{25}} \right|$$

$$= \frac{12}{5} \text{ units}$$

3 (a) ∴ The perpendicular distance from point A to line segment BC is 2·4 units

(b) SOLUTION.

From illustration



Length, $\overline{BC} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

From the distance formula

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\overline{BC} = \sqrt{(9-6)^2+(8-4)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= \sqrt{25} \text{ units}$$

$\overline{BC}$ = 5 units of length

but from

Area of Δ ABC = ½ base × height

but height, h = d = 2·4 units

base, BC = 5 units

3	(b) Now
	Area = $\frac{1}{2}$ x BC x perpendicular distance
	= $\frac{1}{2}$ x 5 x 2·4
	= 6 square units.
	∴ The area of a triangle ABC is 6 square units.

Extract 3.1: A sample of the candidate's correct responses to question 3

In Extract 3.1, the candidate calculated correctly the perpendicular distance of a point from a line segment in part (a). In part (b), the candidate demonstrated high level of competence in finding the area of the triangle.

In spite of the high competence demonstrated by some of the candidates, 209 (53%) were not able to respond to the question correctly as they scored below 2 out of 6 marks. These candidates faced the following difficulties.

In part (a), some of the candidates responded to the question using irrelevant formula such as the mid point formula, which was, $E = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Then, they substituted the coordinates of the point $B(6, 4)$ and $C(9, 8)$ in the formula that resulted to $(7.5, 6)$. After that, they substituted the obtained points in the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and got $d = \sqrt{(7.5)^2 + 6^2} = 9.6$, which was the incorrect perpendicular distance.

Some other candidates wrongly found the perpendicular distance from point $A(0,0)$ to line segment $\overline{BC}$ by subtracting points from $\overline{BC}$. For example, one candidate calculated $\overline{BC} = (9, 8) - (6, 4) = (3, 4)$, then substituted the coordinates of $(3, 4)$ in the distance formula and got $d = \sqrt{(3-0)^2 + (4-0)^2}$. Then, such a candidate simplified incorrectly and got $\sqrt{13}$ units as a perpendicular distance instead of $\frac{12}{5}$ units.

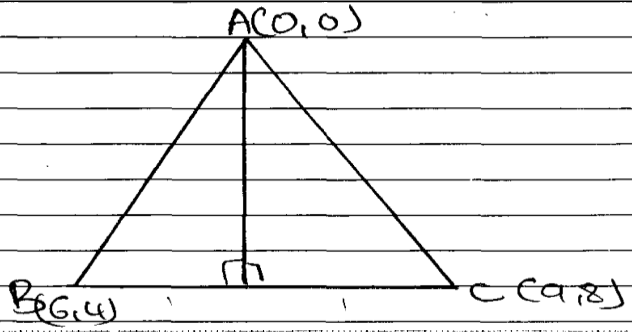
Further analysis shows that, some candidates correctly recalled the formula for perpendicular distance as $d = \left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right|$ but substituted the incorrect values, such as used $a = 3$, $b = -4$ and $c = 0$ instead of $a = 4$, $b = -3$ and $c = 12$. Then, they worked on $d = \frac{3x + -4y + 0}{\sqrt{3^2 + (-4)^2}}$ and ended up with $\frac{3x - 4y + 0}{\sqrt{25}} = \frac{3x - 4y}{5}$.

In part (b), some of the candidates applied the formula for area of the triangle $A = \frac{1}{2}ab\sin\theta$ and substituted the obtained values in part (a) such as $h = 9.6$ and $a = 2.5$ to obtain $A = \frac{1}{2}\times 2.5\times 9.6$. Thereafter, they computed and obtained incorrect value of 12 units, instead of 6 square units.

Furthermore, some of the candidate found the base $\overline{BC}$ correctly using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, then substituted the coordinates of vertices $B(6, 4)$ and $C(9, 8)$ and got the correct distance. However, they substituted the obtained value of $d = 5$ and wrong value of $h = 2$ in the formula of finding the area of the triangle as $A = \frac{1}{2}\times 5\times 2$. Then, they simplified and obtained the area of 5 square units instead of 6 square units.

Moreover, there were some candidates who incorrectly applied the vector competence to find the area of a triangle. For instance, they arranged the vertices in rows and columns as $A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 4 & 1 \\ 9 & 8 & 1 \end{vmatrix}$. Then, they computed $A = \frac{1}{2}\times 9.6\times 1.5$ and finally ended up with incorrect response of 7.2 square units instead of 6 square units. Extract 3.2 is a sample response from one of the candidates who incorrectly responded to this question.

3 a/



3a/ Perpendicul = Mid point
let the mid point be E
BE = CE

Mid point = $\left(\frac{x_1+x_2}{2}\right), \left(\frac{y_1+y_2}{2}\right)$

$= \left(\frac{6+9}{2}\right), \left(\frac{8+4}{2}\right)$

$= (7.5, 6)$

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(x_2)^2+(y_2)^2}$

$= \sqrt{(7.5)^2+(6)^2}$

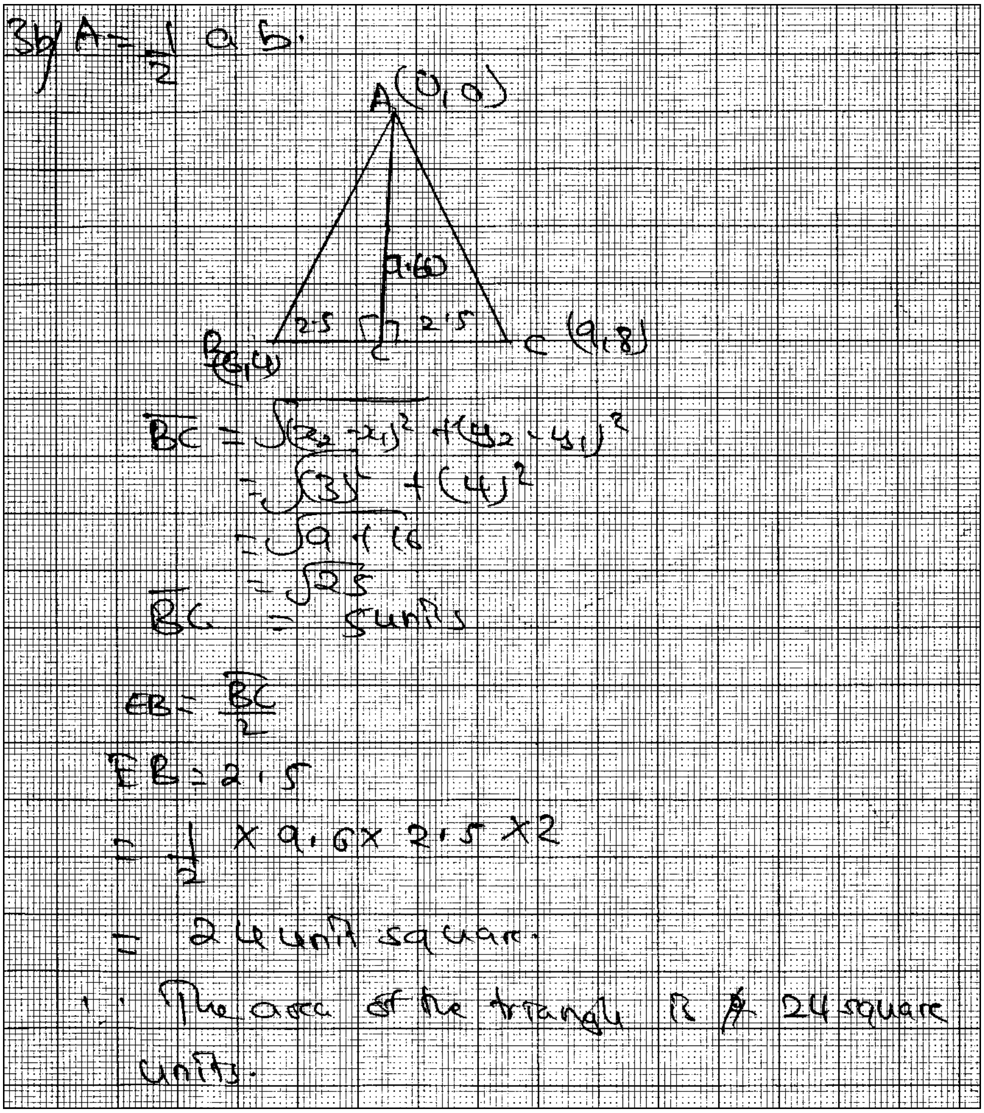
$= \sqrt{56.25+36}$

$= \sqrt{92.25}$

$= 9.60$ units.

3a/ Perpendicular distance from A to the segment BC is 9.60 units.

b/ Area of the triangle $= \frac{1}{2} ab \sin\theta$



3b) $A = \frac{1}{2} a b.$

$\overline{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$\overline{BC} = 5$ units

$EB = \frac{\overline{BC}}{2}$

$EB = 2.5$

$= \frac{1}{2} \times 9.6 \times 2.5 \times 2$

$= 24$ unit square.

$\therefore$ The area of the triangle is ~~a~~ 24 square units.

Extract 3.2: A sample of the candidate's incorrect responses to question 3

In Extract 3.2, in part (a), the candidate used irrelevant formula by applying the mid – point formula when finding the perpendicular distance. In part (b), the candidate failed to get the area of the triangle.

2.4 Question 4: Locus

The candidates were given that the point $P(x, y)$ moved in such a way that it is always 1 unit from the point $A(2,1)$ then they were required to;

(a) find the simplified equation of the locus p.

(b) sketch the graph of the locus on xy – plane.

As per the analysis, out of 394 candidates who attempted this question, 69 scored 0 to 1.5 marks, 41 candidates scored 2.0 to 3.5 marks, while 284 candidates scored 4.0 to 6.0 marks. Therefore, the candidates' performance on this question was good. Figure 5 illustrates the candidates' performance on this question.

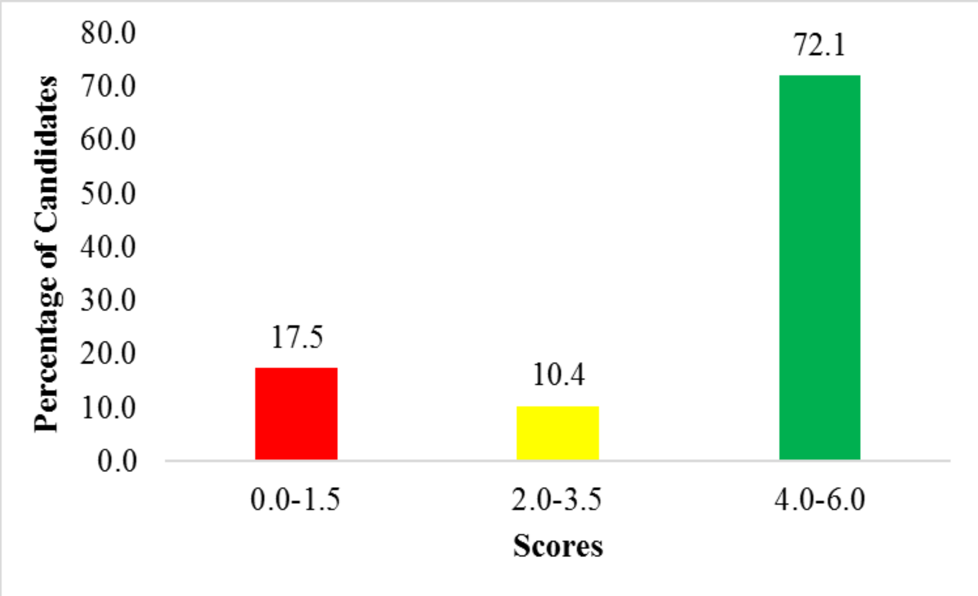


Figure 5: *Candidates' Performance on Question 4*

In part (a), the candidates were able to recall correctly the formula for the distance between the two points as $d=\sqrt{(x-x_1)^2+(y-y_1)^2}$. Then, they substituted the given distance, $d=1$unit and point $A(2,1)$ in the formula, which was $1=\sqrt{(x-2)^2+(y-1)^2}$. By squaring on both sides to remove the radical sign, they got $1^2=(x-2)^2+(y-1)^2$. Then, they expanded and simplified to get $x^2+y^2-4x-2y+4=0$ as the locus of points.

In part (b), since they responded correctly to part (a) and managed to get the equation $x^2+y^2-4x-2y+4=0$, the candidates recalled the general form of a circle as $(x-a)^2+(y-b)^2=r^2$, which implied $(x-2)^2+(y-1)^2=1^2$ and hence concluded $(2,1)$ and $r=1$ as the centre and radius of the circle respectively. Then, they applied their drawing skills and correctly sketched the circle. Extract 4.1 shows a sample of a response from one of the candidates who responded correctly to question 4.

4\. (a) $P(x, y)$. $A(2, 1)$.

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$1 \text{ unit} = (x-2)^2 + (y-1)^2$$

$$1 = (x-2)(x-2) + (y-1)(y-1)$$

$$1 = x^2 - 2x - 2x + 4 + y^2 - y - y + 1$$

$$1 = x^2 - 4x + 4 + y^2 - 2y + 1$$

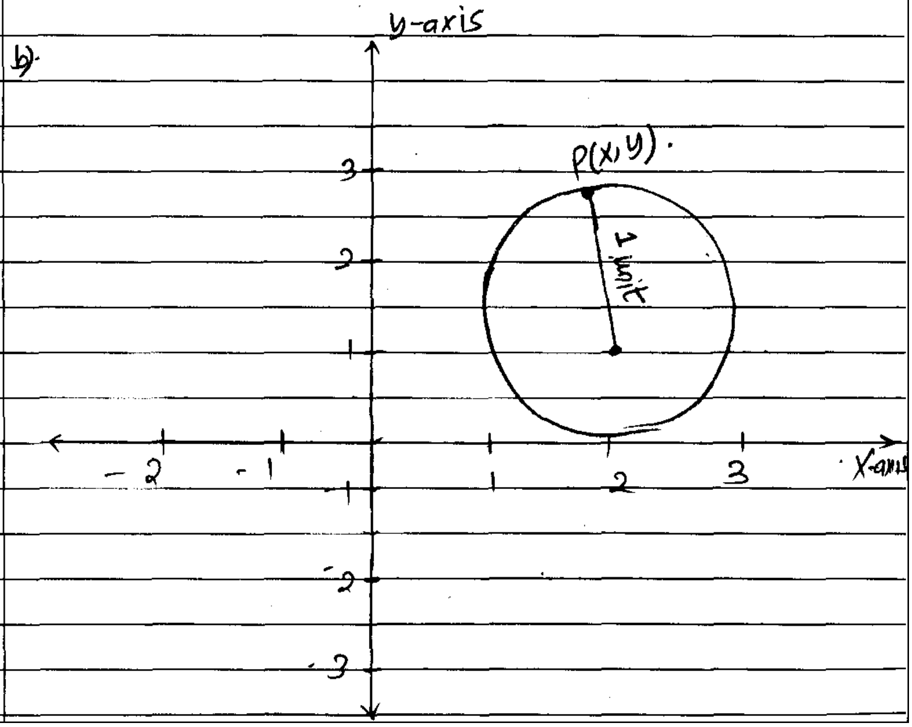
$$x^2 - 4x + 4 + y^2 - 2y + 1 - 1 = 0$$

$$x^2 - 4x + 4 + y^2 - 2y = 0$$

$$x^2 + y^2 - 4x - 2y + 4 = 0$$

$\therefore$ Equation: $x^2 + y^2 - 4x - 2y + 4 = 0$

(b).



Extract 4.1: A sample of the candidate's correct responses to question 4

In Extract 4.1, the candidate managed to get the simplified equation of the locus of point $P(x, y)$ using the distance formula in part (a). In part (b), the candidate correctly sketched the graph of the locus.

In spite of the good performance on this question, there were some candidates who faced difficulties while responding to this question. According to the analysis of data, those difficulties were due to inability to recall the appropriate formulae and misinterpretation of word problem in mathematical models.

In part (a), some candidates managed to recall the correct distance formula as $d = \sqrt{\left(x - x_1\right)^2 + \left(y - y_1\right)^2}$. However, they failed to interpret the condition of the locus as $\overline{AP} = d = 1$, instead they used $\overline{AP} = \overline{BP} \Rightarrow$ $d_1 = d_2$. For example, some candidates substituted $A\left(2,1\right)$ and $B\left(3,2\right)$ in the distance formula and got $d_1 = \sqrt{\left(x-2\right)^2 + \left(y-1\right)^2}$ and $d_2 = \sqrt{\left(x-3\right)^2 + \left(y-2\right)^2}$. Then, computed $d_1^2 = x^2 - 4x + 4 + y^2 - 2y + 1$ and $d_2^2 = x^2 - 6x + 9 + y^2 - 4y + 4$. Then, they equated the two equations as $x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 4y + 4$, and then simplified to get $x^2 + y^2 - 4x - 2y + 5$ as the equation of the locus P.

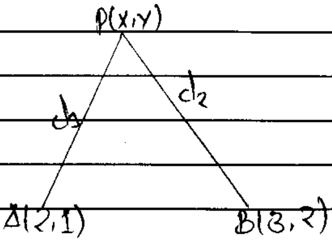
Further analysis reveals that, some of the candidates misinterpreted the question which led to incorrect responses. For instance, they wrote correctly the distance formula $d = \sqrt{\left(x - x_1\right)^2 + \left(y - y_1\right)^2}$. However, they substituted the coordinates of $A\left(2,1\right)$ without considering that $d = 1$. For example, they wrote $d = \sqrt{\left(x-2\right)^2 + \left(y-1\right)^2}$. Then, they squared on both sides as $d^2 = \left(x-2\right)^2 + \left(y-1\right)^2$, then expanded it to get $d^2 = x^2 + y^2 - 4x - 2y + 5$ as the locus of P instead of $x^2 + y^2 - 4x - 2y + 4 = 0$.

Additionally, some candidates used inappropriate formula, for example they used the formula for finding the perpendicular distance from a point to the line, $d = \left|\dfrac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$ and then substituted the point $A\left(2,1\right)$. Then, they simplified and got $4a^2 + c^2$ as a locus of points.

In part (b), some candidates perceived the given points $P\left(x, y\right)$ and $A\left(2\,,1\right)$ as the vertices of a right angled triangle. Then, they drew a triangle with the hypotenuse side which was equal to 1. Likewise, other candidates interpreted the locus by drawing a straight line passing through the two points. Extract 4.2 is a sample response of a wrong solution from one of the candidates who attempted this question.

4. (a) Given points

$A(2,1)$ and $B(3,2)$, $P(x,y)$



From

$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$

$d_1 = d_2$

Therefore $\sqrt{(x-x_0)^2+(y-y_0)^2} = \sqrt{(x-x_0)^2+(y-y_0)^2}$

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-3)^2 + (y-2)^2}$

$\sqrt{x^2-4x+4+y^2-2y+1} = \sqrt{x^2-6x+9+y^2-4y+4}$

Square on both sides

$\left(\sqrt{x^2-4x+4+y^2-2y+1}\right)^2 = \left(\sqrt{x^2-6x+9+y^2-4y+4}\right)^2$

$x^2-4x+4+y^2-2y+1 = x^2-6x+9+y^2-4y+4$

Take eqn $x^2-6x+9+y^2-4y+4$ to the right side.

$x^2-4x+4+y^2-2y+1-(x^2+6x+9+y^2-4y+4)=0$

$\cancel{x^2}-4x+4+y^2-2y+1-\cancel{x^2}-6x-9-y^2+4y-4=0$

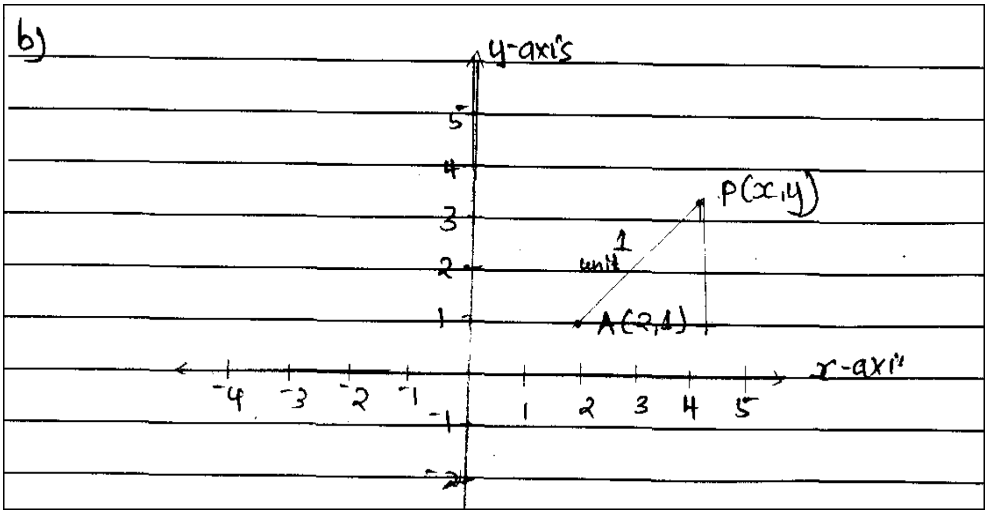
$-4x-6x-2y+4y+1-9-4=0$

$-10x+2y+1-18=0$

$-10x+2y-12=0$

$10x-2y+12=0$

$\therefore$ The equation of locus $10x-2y+12=0$



Extract 4.2: A sample of the candidate's incorrect responses to question 4

In Extract 4.2, part (a), the candidate failed to get the locus by comparing the distance between two points after introducing another point instead of using the given distance and points A and P. While in part (b), the candidate sketched incorrect locus which was in a triangular shape.

2.5 Question 5: Algebra

This question had parts (a) and (b). In part (a), the candidates were required to expand the expression $\left(a+\frac{1}{a}\right)^2$ and hence show that if $\left(a+\frac{1}{a}\right) \geq 4$ then $-\frac{1}{2}\left(a^2+\frac{1}{a^2}\right) \leq -7$. In part (b), they were given the question that $x-y=8$ and $x^2+y^2=160$, and instructed to evaluate the numerical value of xy without solving for x and y.

The analysis of data shows that 394 candidates attempted this question, out of whom 44.4 per cent of candidates scored 0 to 1.5 marks, 20.1 per cent of candidates scored 2.0 to 3.5 marks and 35.5 per cent candidates scored 4.0 to 6.0 marks. Figure 6 presents the candidates' performance summary on this question.

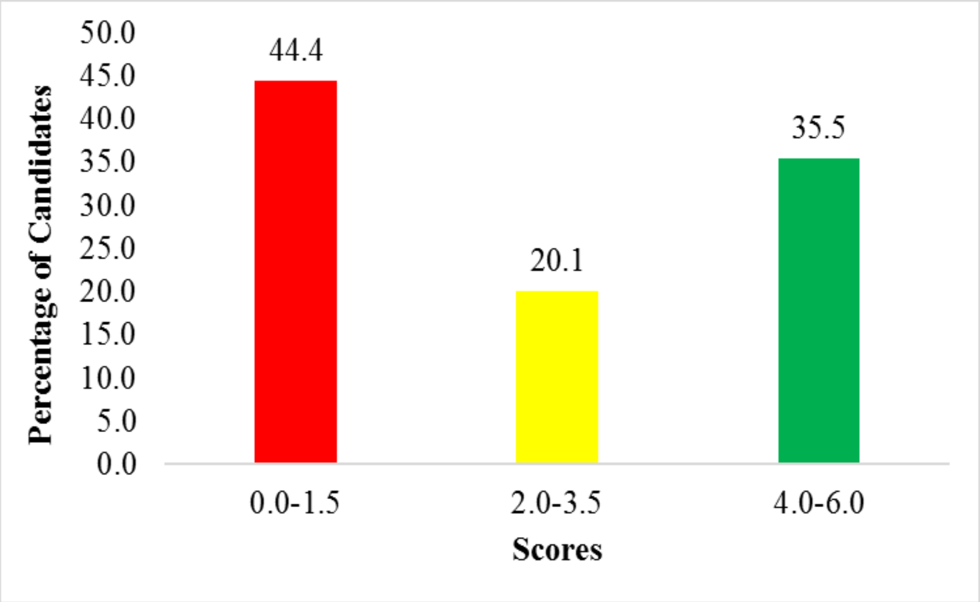


Figure 6: *Candidates' Performance on Question 5*

The analysis of data shows that, the candidates' performance on this question was on average. In part (a), the candidates correctly expanded the expression $\left(a+\frac{1}{a}\right)^2 = a^2 + 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2$ and simplified it to obtain $a^2 + \frac{1}{a^2} + 2$. Because the candidates were familiar with algebraic concepts. They squared the given expression $\left(a+\frac{1}{a}\right) \geq 4$ and got $\left(a+\frac{1}{a}\right)^2 \geq 4^2$. Then, they simplified to get $a^2 + \frac{1}{a^2} \geq 14$, and then multiplied throughout by $-\frac{1}{2}$ to obtain the inequality $-\frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) \leq -\frac{1}{2}(14)$ then, they got $-\frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) \leq -7$.

In part (b), the candidates demonstrated their algebraic skills by correctly relating a linear equation to a quadratic equation. They squared on both sides of the linear equation $x - y = 8$, to obtain $(x-y)^2 = 8^2$ then expanded to get $x^2 + y^2 - 2xy = 64$. Then, they substituted $x^2 + y^2 = 160$ to obtain

$160-2xy=64$. Finally, they solved for xy and managed to get the numerical value of $xy=48$. Extract 5.1 is a sample response from one of the candidates who correctly attempted the question.

5. a) Soln.

$$\left[a+\frac{1}{a}\right]^2$$

$$\left[a+\frac{1}{a}\right]\left[a+\frac{1}{a}\right]$$

$$=\left[a^2+1+1+\frac{1}{a^2}\right]$$

$$=\left[a^2+\frac{1}{a^2}+2\right]$$

$$\therefore \left(a+\frac{1}{a}\right)^2=\left[a^2+\frac{1}{a^2}+2\right]$$

from

$$\left(a+\frac{1}{a}\right)\geq 4$$

Square both sides

$$\left(a+\frac{1}{a}\right)^2\geq 4^2$$

$$\left(a^2+\frac{1}{a^2}+2\right)\geq 16$$

$$\left(a^2+\cancel{\frac{a}{a}}\ \frac{1}{a^2}\right)\geq 16-2$$

$$\left(a^2+\frac{1}{a^2}\right)\geq 14$$

5. a) Divide by -2 each side

$$\frac{\left(a^2 + \frac{1}{a^2}\right)}{-2} \geq \frac{14}{-2}$$

$$-\frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) \leq -7$$

Hence

$$-\frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) \leq -7 \quad \text{(shown)}$$

b) Soln.

$$x - y = 8$$
$$x^2 + y^2 = 160$$

$$(x-y)^2 = (x^2 - 2xy + y^2)$$

$$(x-y)^2 = x^2 + y^2 - 2xy$$

$$(8)^2 = (x^2 + y^2) - 2xy$$
$$64 = 160 - 2xy$$

$$-2xy = 64 - 160$$
$$\frac{-2xy}{-2} = \frac{-96}{-2}$$
$$xy = 48$$

$\therefore$ Value of xy is 48

Extract 5.1: A sample of the candidate's correct responses to question 5

In Extract 5.1, the candidate managed to verify the required expression correctly in part (a). Also, in part (b), the candidate demonstrated the algebraic skills by relating the linear equation to the quadratic equation and obtained the correct value of xy.

However, there were 175 candidates who performed poorly on this question and scored less than 2 marks. These candidates failed to respond according to the requirement of the questions and lacked computational skills.

In part (a), they managed to expand correctly the expression $\left(a+\frac{1}{a}\right)^2$ as $\left(a+\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}+2$ but failed to use the given information to show $-\frac{1}{2}\left(a^2+\frac{1}{a_2}\right)^2 \le -7$. For example, one candidate wrote $a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2$ and then substituted to $-\frac{1}{2}\left(a^2+\frac{1}{a_2}\right)^2 \le -7$ to obtain $-\frac{1}{2}\left[\left(a^2+\frac{1}{a_2}\right)^2-2\right]\le -7$, given that $a+\frac{1}{a}=4$. Then, wrote $-\frac{1}{2}\left(4^2-2\right)\le -7$, and simplified it to get $7 \le -7$. Another candidate expanded correctly the expression $\left(a+\frac{1}{a}\right)^2$ and equated it to zero to obtain $a^2+\frac{1}{a^2}+2=0$. Then, he/she tried to solve the equation $a^2+\frac{1}{a^2}=2$ by writing $\frac{a^4+1}{a^2}=2$. Finally, the candidate concluded wrongly that $\left(a+\frac{1}{a}\right)^2 \ge 4$ or $\left(a+\frac{1}{a}\right)^2=4$.

Moreover, some other candidates solved the inequality $\left(a+\frac{1}{a}\right)\ge 4$ instead of expanding the expression $\left(a+\frac{1}{a}\right)^2 \Rightarrow a^2+\frac{1}{a^2}+2$. For instance, they wrote $\left(a+\frac{1}{a}\right)\geq 4 \Rightarrow a \geq 4-\frac{1}{a}$ and then multiplied both sides by a to obtain the expression $a^2 \geq 4a-1$.

In part (b), some of the candidates wrote, $x^2 - y^2 = 160$ instead of $x^2 + y^2 = 160$. Then, they incorrectly wrote $x - y = 8$ as equation (i). Later, they substituted $x^2 - y^2 = 160$ in the equation $x^2 - y^2 - 2xy = 0$ to obtain $160 - 2xy = 0$ and solved it to get an incorrect value of $xy = -80$ instead of $xy = 48$.

Some other candidates responded incorrectly by applying the difference of two squares techniques then considered $x - y = 8$ as equation (i) and $x^2 + y^2 = 160$ as equation (ii) but misinterpreted the expression $x^2 + y^2 = (x+y)(x-y)$. Later, they substituted the value for $x - y = 8$ and $x^2 + y^2 = 160$ to obtain $(x+y)(x-y) = 160 \Rightarrow (x+y)\times 8 = 160$ which was simplified to $x + y = 20$ as equation (iii). Then, they solved equations (i) and (iii) simultaneously, which was $x - y = 8$ and $x + y = 20$ respectively and got incorrect values, $y = 6$ and $x = 14$. Finally, they calculated the product, $xy = 14 \times 6 = 84$.

Some candidates incorrectly solved the equations $x - y = 8$ and $x^2 + y^2 = 160$.They substituted the equation $x - y = 8$ in as $x^2 + y^2 = 160$ and got $(8+y)^2 + y^2 = 160$. Then, they wrote the equation as a product of factors as $(y-4)(y+12) = 0$ and solved to get $y = 4$ or -12, and finally concluded that $xy = 4 \times -12 = -48$. Extract 5.2 is a sample response from one of the candidates who responded incorrectly to this question.

5\. (b) Given that

$x - y = 8$ —(i)

$x^2 + y^2 = 160$ —(ii)

xy = Required.

~~$x = 8 - y$.~~

$x^2 + y^2 = 160$

$(x+y)^2 = 160$

$(x+y)(x-y) = 160$

$(x+y)(8) = 160$

$x + y = 20$ —(iii)

Combine Eqn (i) and (iii)

$$-\left|\begin{array}{l} x - y = 8 \\ x + y = 20 \end{array}\right.$$

$-2y = 8 - 20$

$\frac{-2y}{-2} = \frac{-12}{-2}$

$y = 6$

Subsitute to Eqn (i)

5. (b) $X - y = 8$

$X - 6 = 8$

$X = 14$

Numerical value of $Xy = 14 \times 6$

$\therefore \; Xy = 84$

Extract 5.2: A sample of the candidate's incorrect responses to question 5

In Extract 5.2, the candidate applied the techniques of difference of two square instead of expanding the given linear equation thus getting an incorrect value.

2.6 Question 6: Geometrical Constructions

The question had two parts, (a) and (b). In part (a), the candidates were given that; the size of an interior angle (I) of a regular polygon is five times the size of an exterior angle (E). Then, they were asked to find the number of sides of the polygon. In part (b), they were given that if $\overline{PQ} = 9$ cm and the point X is on the line segment PQ such that $\overline{PX} : XQ = 1:5$. They were required to use a ruler and pair of compass to draw the line segment PQ and indicate point X .

The analysis shows that out of 394 (100%) candidates who attempted this question, 39 (9.9%) candidates scored 0 to 1.5 marks, 324 (82.2%) candidates scored 2.0 to 3.5 marks and 31 (7.9%) candidates scored 4.0 to 6.0 marks. Generally, the candidates' performance on this question was good. The summary of the candidates' performance is presented in Figure 7.

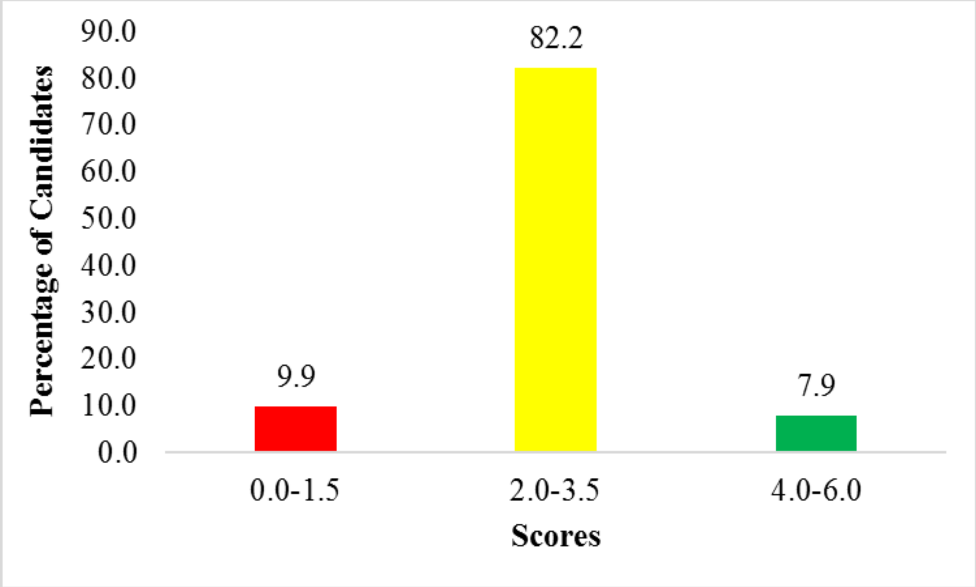


Figure 7: *Candidates' Performance on Question 6*

The analysis of data reveals that, 28(7.1%) candidates out of 355 (90.1%) candidates who scored from 2 to 6 marks demonstrated their skills on geometrical construction to answer all parts of this question.

In part (a), the candidates who managed to answer the question correctly were able to interpret the given word problem and correctly formulated mathematical equations $I = 5E$ and $I + E = 180^\circ$. Then, they were able to solve the two equations simultaneously and got an exterior angle, $E = 30^\circ$. Furthermore, the candidates correctly recalled the formula for exterior angle, $\text{Exterior angle} = \frac{360^\circ}{n}$. Then, they substituted the value of exterior angle as $30^\circ = \frac{360^\circ}{n}$ and obtained $n = 12$ which was the required number of sides.

In part (b), the candidates correctly applied knowledge on geometrical construction as they managed to use a ruler and a compass to determine each portion by dividing $\overline{PQ}$ by the sum of the ratios as $\frac{9\,\text{cm}}{6} = 1.5\,\text{cm}$. Finally, they constructed a line segment with portion of interval of $1.5\,\text{cm}$

and correctly indicated the point X . Extract 6.1 is a sample response from one of the candidates who correctly responded the question.

6(a) solution.

$I = 5E$.

$I - 5E = 0$ —— i) equation.

$I + E = 180°$ —— ii) equation.

solve simultaneously.

$I - 5E = 0$

$I + E = 180°$

$I = 180° - E$ substitute into i) equation.

$180° - E + 5E = 0$

$180° - 6E = 0$

$\frac{\cancel{6}E}{\cancel{6}} = \frac{180°}{6}$

$E = 30°$ equate into ii) equation.

$I + 30° = 180°$

$I = 180° - 30°$

$I = 150°$.

$\frac{360°}{n} = E$

$\frac{360°}{E} = n$

n = number of sides of polygon.

$\frac{360°}{30°} = n$

$n = 12$

∴ The polygon has 12 sides.

6b) Solution.

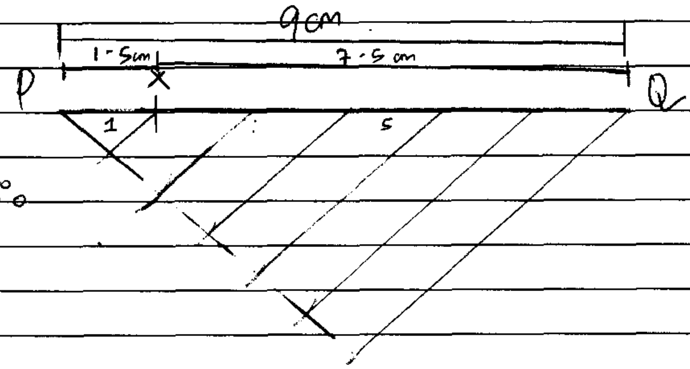
$\overline{PQ} = 9cm.$

$\frac{\overline{PX}}{\overline{XQ}} = \frac{1}{5}.$

$1:5 = 1+5 = 6.$

$\overline{PX} = 9cm \times \frac{1}{\cancel{5}} = 1.8cm.$

$\overline{XQ} = \cancel{9cm} - 1.8cm = 7.2cm.$



$\therefore$

$1:5 = 1+5 = 6.$

$\overline{PX} = \frac{1}{\cancel{6}\,2} \times \cancel{9}\,3 = 1.5cm.$

$\overline{XQ} = = \frac{5}{\cancel{6}\,2} \times \cancel{9}\,3 = \frac{15}{2} = 7.5cm.$

$\therefore \overline{PX} = 1.5cm \quad \overline{XQ} = 7.5cm.$

Extract 6.1: A sample of the candidate's correct responses to question 6

In Extract 6.1, the candidate demonstrated sufficient knowledge and skills on the tested concepts as he/she got the correct number of the sides in part (a). Also, in part (b), the candidate used geometrical instruments to draw a line segment with portion of 1.5 cm interval.

Nevertheless, some candidates encountered difficulties when responding to this question. Those difficulties were due to their in ability to recall the appropriate formulae and failure to interpret word problems.

In part (a), some candidates failed to recall the correct formula which defined the relationship between an interior angle and an exterior angle as $i+e=180^{\circ}$. Instead, they used $i+e=360^{\circ}$. Then, they interpreted correctly that $i=5e$, hence substituted as $5e+e=360^{\circ}$ to obtain an $\text{exterior angle}=60^{\circ}$ instead of 30°. Further analysis revealed that, some of the candidates correctly recalled the formula for the number of sides of the polygon as $n(\text{exterior angle})=360^{\circ}$ but substituted the wrong value of $e=60^{\circ}$ thus, resulting to an incorrect number of sides, $n=6$ instead of $n=12$.

On the other hand, some other candidates responded to the question by applying inappropriate formulae for the size of interior angle and exterior angle as $(n-2)\times 180^{\circ}$ and $\dfrac{(n-2)\times 180^{\circ}}{n}$ respectively. Then, they substituted the two formulae into a relation, $i=5e$ and obtained $(n-2)\times 180^{\circ}=5\left(\dfrac{(n-2)\times 180^{\circ}}{n}\right)$. Then, they multiplied it by n to obtain $(n^2-2n)\times 180^{\circ}=5(180n-360)$. They then simplified it to $n^2-7n+10=0$, and solved for n and got $n=2$ or $n=5$. They finally concluded that the polygon had 5 sides.

In part (b), some candidates calculated the appropriate lengths according to the given ratio but failed to draw the line segment. For example, they added the given ratios, $1+5=6$, then divided the length $\overline{PQ}$ by the sum of ratios and obtained $\dfrac{9}{6}=1.5\,\text{cm}$. They then stated that, point X is 1.5cm away from P and 7.5 cm away from Q instead of using ruler and compass to represent the obtained portions. Other candidates only used the ruler and ignored the compass while drawing. Furthermore, there were some other candidates who drew triangles. Extract 6.2 provides a sample of a response from one of the candidates who responded incorrectly to this question.

6. (a) From

interior angle(i) + Exterior angle(e) = 360°

$i + e = 360^\circ$ ... (i)

$i = 5e$ ... (ii)

substitute eqn (ii) into eqn (i)

$5e + e = 360^\circ$

6. (a) $\frac{6e}{6} = \frac{360^\circ}{6}$

$e = 60^\circ$

Then from

$n(\text{ext}) = 360^\circ$

$n(60^\circ) = 360^\circ$

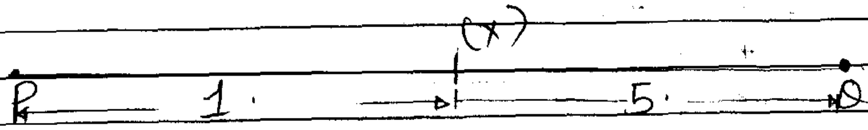
$\frac{60^\circ n}{60^\circ} = \frac{360^\circ}{60^\circ}$

$n = 6$

∴ Number of sides of a polygon = 6 sides.

6b) Solution:

$\overline{PQ} = 9$ cm, $\overline{PX} : XQ = 1:5$.



Extract 6.2: A sample of the candidate’s incorrect responses to question 6

In Extract 6.2, the candidate incorrectly expressed the relationship between the interior and the exterior angle which led to an incorrect number of sides in part (a). In part (b), the candidate drew a line with an incorrect interval.

2.7 Question 7: Trigonometry

This question had two parts, (a) and (b). In part (a), the candidates were given three angles named A, B and C. Then, they were required to show that $Cot\left(\frac{B+C}{2}\right)=\tan\left(\frac{A}{2}\right)$. In part (b), the candidates were given $4\cos 2\theta-2\cos\theta+3=0$ and were asked to find the values of θ for $0^{\circ}\leq\theta\leq 360^{\circ}$.

The analysis of data shows that 394 (100%) candidates attempted this question out of whom 49.2 per cent of the candidates scored 0 to 1.5 marks and 50.8 per cent of the candidates scored 2.0 to 6.0 marks. Therefore, the analysis indicates that the candidates' performance on this question was on average. The summary of candidates' performance on this question is shows in Figure 8.

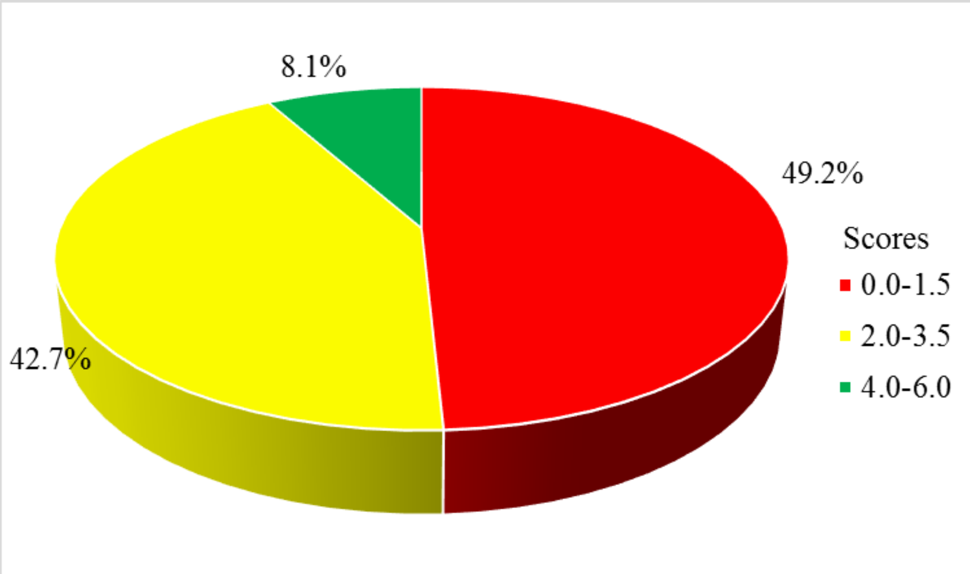


Figure 8: *Candidates' Performance on Question 7*

In part (a), according to the analysis, the candidates who scored the highest marks applied the concept correctly that, sum of angles in triangle equals to $A+B+C=180^{\circ}$. Then, they correctly convert it to a compound angle defined by $\frac{B+C}{2}=90^{\circ}-\frac{A}{2}$. Then, they introduced the cotangent(cot) on both sides to obtain $\cot\left(\frac{B+C}{2}\right)=\cot\left(90^{\circ}-\frac{A}{2}\right)$. They were also able to recall that $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, then responded as

$$\frac{\cos\left(90^\circ - \dfrac{A}{2}\right)}{\sin\left(90^\circ - \dfrac{A}{2}\right)} = \frac{\sin\left(\dfrac{A}{2}\right)}{\cos\left(\dfrac{A}{2}\right)} = \tan\left(\frac{A}{2}\right).$$

In part (b), the candidates recognized that the given interval $0^\circ \le \theta \le 360^\circ$ implies all 4 quadrants were involved. Then, they correctly recalled the trigonometric identity, $\cos 2\theta = 2\cos^2\theta - 1$. Then, they substituted it to $4\cos 2\theta - 2\cos\theta + 3 = 0$ and obtained $4\left(2\cos^2\theta - 1\right) - 2\cos\theta + 3 = 0$. Then, they simplified it and got $8\cos^2\theta - 2\cos\theta - 1 = 0$. Later, the candidates were able to factorize $(4\cos\theta + 1)(2\cos\theta - 1) = 0$ and solve for $\cos\theta$ to obtain $\cos\theta = \dfrac{-1}{4}$ and $\cos\theta = \dfrac{1}{2}$. Finally, they applied $\cos^{-1}$ to both sides of the equations and solved for θ and obtained 60°, 104.5°, 255.5° and 300°. Extract 7.1 is a sample response from one of the candidates who responded to this question correctly.

7(a) $\rightarrow \tan\left(\dfrac{A}{2}\right)$

$$A + B + C = 180^\circ$$

$$B + C = 180^\circ - A.$$

$$\cot\left(\frac{180 - A}{2}\right) = \cot\left(90 - \frac{A}{2}\right).$$

$$\cot\left(90 - \frac{A}{2}\right) = \frac{\cos\left(90 - A/2\right)}{\sin\left(90 - A/2\right)}$$

$$\frac{\cos 90 \cos(A/2) + \sin 90 \sin(A/2)}{\cos 90 \sin(A/2) - \cos(A/2)\sin 90}$$

$$= \frac{0 + \sin(A/2)}{\cos(A/2)}$$

$$= \tan\left(\frac{A}{2}\right)$$ hence shown.

7 (b). $4\cos 2\theta - 2\cos\theta + 3 = 0$.

$4[\cos^2\theta - \sin^2\theta] - 2\cos\theta + 3 = 0$

$4[\cos^2\theta - (1 - \cos^2\theta) - 2\cos\theta + 3 = 0$

$4[\cos^2\theta - 1 + \cos^2\theta] - 2\cos\theta + 3 = 0$

$4\cos^2\theta - 4 + 4\cos^2\theta - 2\cos\theta + 3 = 0$.

$8\cos^2\theta - 2\cos\theta - 1 = 0$.

Solve quadratically

$8\cos^2\theta - 2\cos\theta - 1 = 0$

$ac = {}^-8$

$b = {}^-2$.

$(8\cos^2\theta - 4\cos\theta) + (2\cos\theta - 1) = 0$

$4\cos\theta(2\cos\theta - 1) + 1(2\cos\theta - 1) = 0$

$(4\cos\theta + 1)(2\cos\theta - 1) = 0$.

$\cos\theta = 0.5$ or $-1/4$.

$\cos\theta = 0.5$

$\theta = \cos^{-1}(0.5)$

$\theta = 60^\circ$, 300°.

$\theta = \cos^{-1}(-1/4)$

$\theta = 104.5^\circ, 255.5^\circ$.

$\therefore \theta = 60^\circ, 104.5^\circ, 255.5^\circ$ and 300°.

Extract 7.1: A sample of the candidate's correct responses to question 7

In Extract 7.1, the candidate used the compound angle of sine and cosine to verify the given trigonometric equation in part (a). Also, in part (b), the candidate correctly applied the double angle formula to find the required angles.

On the other hand, there were 194 candidates who failed to answer the question accordingly and scored 1.5 marks or less. Such candidates faced the following challenges while responding to the question;

In part (a), some of the candidates applied incorrect relation between cotangent and tangent as $\cot\left(\frac{B+C}{2}\right)=\tan\left(\frac{180-A}{2}\right)$. Then, they incorrectly recalled the compound angle formula, $\tan(A-B)=\frac{\tan A-\tan B}{1-\tan A\tan B}$ and wrote $\tan(180^0-A)=\frac{\tan 180^0+\tan A}{1-\tan 180^0\tan A}$, which was simplified into $\frac{0+\tan A}{1-0\times\tan A}=\tan A$.

Moreover, some of the candidates failed to recall the correct trigonometric identities. For instance, they wrote $\cot\left(\frac{B+C}{2}\right)=\cot\left(\frac{B}{2}+\frac{C}{2}\right)$, then applied the formula, $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ implying that $\tan(A+B)=\frac{\cot\left(\frac{B}{2}+\frac{C}{2}\right)}{1-\cot\frac{B}{2}\cot\frac{C}{2}}$, then, they failed to proceed. In addition, some other candidates wrote $\cot\left(\frac{B+C}{2}\right)=\frac{\sin B+\cos C}{2}$, and then applied the incorrect formula, for $\cot\left(\frac{B+C}{2}\right)=\tan\frac{A}{2}$. Thereafter, they responded incorrectly as $\frac{\sin B+\cos C}{2}=\tan\left(\frac{A}{2}\right)$, and finally concluded that $\frac{\sin B+\sin C}{2}=\frac{\sin A}{2\cos A}$.

In part (b), some of the candidates considered the trigonometric equation as cartesian equation and responded incorrectly by dividing each term by 4. That is, $\frac{4}{4}\cos 2\theta-\frac{2}{4}\cos\theta+\frac{3}{4}=0$, later they applied the condition of the product of the root as $a\times c=1\times\frac{3}{4}=\frac{3}{4}$. Thereafter, instead of writing $\cos\theta=0.75$, the candidate wrote $\cos\theta=0.125$, $\theta=\cos^{-1}(0.125)$ and finally $\theta=0^0$ and $82^0 49'$. The candidates did not use the substitution of

$\cos 2\theta = \cos^2\theta - \sin^2\theta$, which could have resulted into the correct values of θ which were 60°, 104.5°, 255.5° and 300°.

Furthermore, some other candidates considered $\cos 2\theta$ as $(\cos\theta)^2$, instead of using $\cos 2\theta = \cos^2\theta - \sin^2\theta$. In their response, they wrote $4\cos 2\theta = \cos^2\theta$, then responded incorrectly as $4\cos 2\theta - 2\cos\theta + 3 = 0$ implying that $4(\cos\theta)^2 - 2\cos\theta + 3 = 0$. Lastly they set $\cos\theta = x$ and obtained $x = 0.25$, but $\cos\theta = 0.25$ resulting into incorrect values, $\theta = 75.5^\circ$ and 284.5° instead of $\theta = 60^\circ$, 104.5°, 255.5° and 300°.

However, some candidates were able to correctly recall that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin^2\theta = 1 - \cos^2\theta$. For example, one of the candidates substituted correctly the double angles as $4\left[\cos^2\theta - \left(1 - \cos^2\theta\right) - 2\cos\theta + 3 = 0\right]$. Then, they computed and obtained the incorrect equation $5\cos^2\theta - 2\cos\theta + 2 = 0$. Such candidate solved and got $\cos\theta = 0.25$ such that $\theta = \cos^{-1} 0.25$, that finally led to getting incorrect value, $\theta = 78.46^\circ$ and $\theta = 281.32^\circ$. Extract 7.2 is a sample response from one of the candidates who incorrectly responded to the question.

7 (a) $\cot\left(\frac{B+C}{2}\right) = \tan\left(\frac{A}{2}\right)$

from

$\cot\left(\frac{B+C}{2}\right) = \frac{\cos}{\sin}\left(\frac{B+C}{2}\right)$

Where but

$\cos\left(\frac{B+C}{2}\right) = \frac{\cos B\cos A}{2} - \frac{\sin A\sin B}{2}$

and

$\sin\left(\frac{B+C}{2}\right) = \frac{\sin A\cos B}{2} + \frac{\cos A\sin B}{2}$

divide them.

$\cos\left(\frac{B+C}{2}\right) = \left(\frac{\cos \cancel{B}\cos A}{\sin A\cos \cancel{B}} + \frac{\sin A\sin \cancel{B}}{\cos A\sin \cancel{B}}\right)\frac{1}{2}$

$= \left(\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)\frac{1}{2}$

$= \frac{\tan A}{2}$ hence shown

Soln

7(b) $4\cos 2\theta - 2\cos\theta + 3 = 0$

$4\cos 2\theta - 2\cos\theta = -3$

$2(2\cos 2\theta - \cos\theta = -3$

$2[2\cos^2\theta - \sin^2\theta - \cos\theta = -3$

$2[2\cos^2\theta - [1 - \cos^2\theta) - \cos\theta = -3$

$2[2\cos^2 + \cos^2\theta - 1 - \cos\theta = -3$

7(b)

$2[3\cos^2\theta - \cos\theta - 1 = -3$

Let $\cos\theta$ be X

$2[3x^2 - x - 1 = -3$

$3x^2 - x + 2 = 0$

~~by general formula~~

$\cancel{x = 1 \pm \sqrt{1 + 8}}$

$$2\begin{cases} 3x^2 - x + 2 = 0 \\ x(3x + 1) = 0 \\ x = 0 \text{ or} \\ 3x + 1 = 0 \\ \frac{3x}{3} = \frac{-1}{3} \\ x = -1/3 \end{cases}$$

$x = -\frac{2}{3}$ or 0 but $x = \cos\theta$

$-0.6 \simeq -1$.

$\cos\theta = -1$

$\theta = 0, 90°, 180°$ and $270°$

$\theta = 0, 90°, 180°$ and $270°$.

Extract 7.2: A sample of the candidate's incorrect responses to question 7

In Extract 7.2, the candidate recalled incorrectly the trigonometric identity of cotangent and compound angle formulae in part (a). In part (b), the candidate failed to recall the double angle formula to find the required angles.

2.8 Question 8: Numbers

The question consisted of parts (a) and (b). In part (a), the candidates were asked to find the first two terms of the sequence such that the sum of the first n terms (S_n) of a sequence is given by $S_n = \frac{n(n+1)(2n+1)}{6}$. In part (b), they were instructed to use the divisibility rule to show whether 20672 could be divisible by 6.

A total of 394 candidates attempted this question, out of whom 55 (14%) candidates scored 0 to 1.5 marks, 157 (39.8%) candidates scored 2.0 to 3.5 marks while 182 (46.2%) candidates scored 4.0 to 6.0 marks. Figure 9 summarises the candidates' performance on this question.

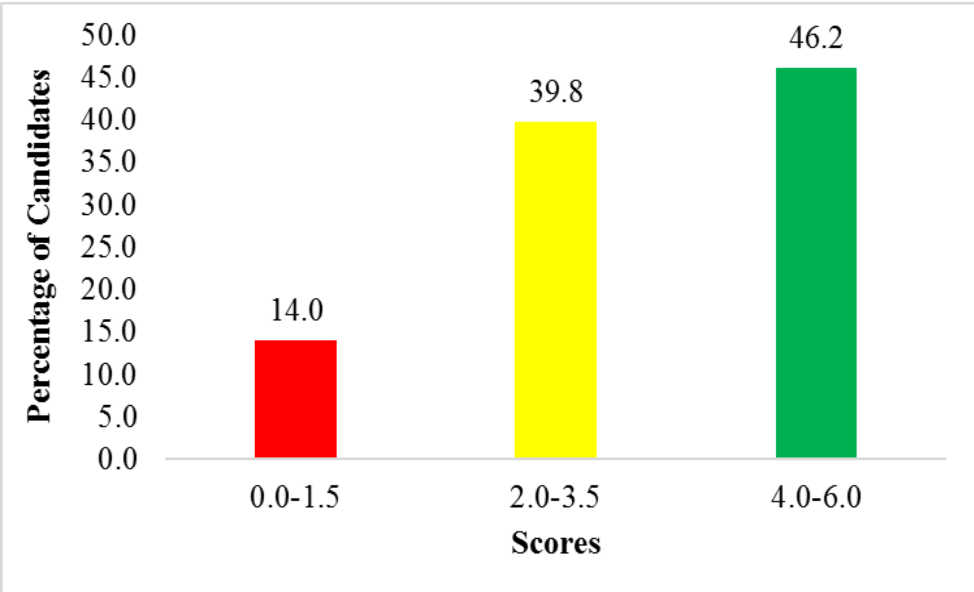


Figure 9: *Candidates' Performance on Question 8*

The analysis shows that, the performance of the candidates on this question was generally good. Thus, in part (a), the candidates who correctly responded to this question and scored all alloted marks were able to recognize that the sequence was arithmetic, where $A_1 = S_1$ and $A_1 + A_2 = S_2$.

Then, they applied correctly the formula for sum of the first n^{th} term, $S_n = \frac{n(n+1)(2n+1)}{6}$ and substituted the n^{th} values starting with $n=1$ to get S_1 which equaled to A_1 as $A_1 = S_1 = \frac{1(1+1)(2(1)+1)}{6} = 1$. Also, $S_2 = \frac{2(2+1)(2(2)+1)}{6} = 5$ but $A_1 + A_2 = S_2$ then, substituted the obtained value of $A_1 = 1$ into $A_1 + A_2 = 5 = S_2$ to get $1 + A_2 = 5$, which resulted to $A_2 = 4$. Then, they concluded that the first two terms were 1 and 4.

In part (b), the candidates seemed to be familiar with the divisibility rule of numbers. They correctly stated and applied the rule to check the divisibility of the given numbers. For instance, they were conversant with the fact that 20672 is divisible by 2 since digit 2 on ‘ones’ place value is an even number. On the other hand, the sum of the digits $2+0+6+7+2=17$ is not divisible by 3, hence 20672 is not divisible by 3. Therefore, 20672 is not divisible by 6. Extract 8.1 is a sample response from one of the candidates who correctly responded to this question.

8.	(a) $S_n = \frac{n(n+1)(2n+1)}{6}$
	$S_1 = \frac{1(1+1)(2+1)}{6}$
	$= \frac{1(2)(3)}{6}$
	$= 6/6$
	$S_1 = 1$, $a_1 = 1$
	$S_2 = \frac{2(2+1)(2(2)+1)}{6}$
	$= \frac{2(3)(4+1)}{6}$
	$= \frac{2\times 3\times 5}{6}$
	$S_2 = 5$
	$a_1 + a_2 = 5$
	$1 + a_2 = 5$
	$a_2 = 4$
	∴ The First two terms are 1 and 4.

8. (b) A number is divisible by 6 if it is divisible by
both 2 and 3

$\Rightarrow$ Divisibility by 2:

- Since the last digit of a number § 20672 is an
even number (2), then the number is divisible by 2

$\Rightarrow$ Divisibility by 3

- A number is divisible by 3 if the sum of its
digits is also divisible by 3

Then

sum $= 2 + 0 + 6 + 7 + 2$

$= 17/3 = 5.667$

since 20672 is not divisible by 3

$\therefore$ A number 20672 is not divisible by 6.

Extract 8.1: A sample of the candidate's correct responses to question 8

In Extract 8.1, the candidate realized that the sequence is arithmetic then applied the given formula to get the required two terms correctly in part (a). Also, in part (b), the candidate was familiar with the divisibility rule of number 6.

Nevertheless 55 (14%) candidates scored 1.5 marks or less, out of whom 10 (2.5%) candidates got zero. Those candidates faced various challenges including the following; In part (a), some of the candidates misinterpreted the given expression $S_n = \frac{n(n+1)(2n+1)}{6}$. For example, one of the candidates equated the given expression with the sum of arithmetic progression, $S_n = \frac{n}{2}(A_1 + n)$, which was $\frac{n(n+1)(2n+1)}{6} = \frac{n}{2}(A_1 + n)$. Then simplified to $2n(n+1)(2n+1) = 6n(A_1 + n)$, thereafter expanded to get $2n^3 + n^2 + 2n^2 + n = 6(nA_1 + n^2)$, which led to incorrect expression, $A_1 = \frac{2}{3}n^2 + \frac{1}{3}$ which was used to find the required two terms. The analysis also showed that some other candidates did wrong computations. For example, a candidate correctly identified that $n = 1$ and 2, then substituted in the formula $S_n = \frac{n(n+1)(2n+1)}{6}$ for $n=1$, $S_1 = \frac{1(1+1)(2(1)+1)}{6}$, simplified wrongly and got $S_1 = \frac{5}{6}$ instead of $S_1 = 1$. Then, they substituted $n=2$ and got $S_2 = \frac{2(2+1)(2(2)+1)}{6}$ which was simplified to incorrect value $S_2 = \frac{11}{6}$. Finally, they concluded that, the first two terms of the sequence were $\frac{5}{6}$ and $\frac{11}{6}$ instead of 1 and 4.

Some other candidates substituted $n=2$ in the formula $S_n = \frac{n(n+1)(2n+1)}{6}$, that is $S_2 = \frac{2(2+1)2\times 2+1}{6}$, then simplified and got 5. Then, they recalled the formula for sum of the first n^{th} term of an arithmetic progression as $S_n = \frac{n}{2}\left(2A_1 + (n-1)d\right)$ where $n=5$, for instance, $5 = \frac{n}{2}\left(2A_1 + n - 1)d\right)$. Then substituted $n=2$, which was $5 = \frac{2}{2}\left(2A_1 + 2 - 1)d\right)$ implying that $5 = 2A_1 + d$. Then, concluded that, the first two terms were obtained through $5 = 2A_1 + d$.

In part (b), the analysis reveals that, some of the candidates could not state the divisibility rule of 6 correctly. For instance, some candidates stated that, "if the last two digits are divisible by 6 the number also is divisible by 6". Some candidates stated "the number is divisible by 6 if and only if is divisible by 2". All these responses indicate that, the candidates lacked competence in the divisibility rules on numbers. They were required to state that, a number is divisible by 6 if it is even and it is divisible by both 2 and 3.

Further analysis shows that, there were few candidates who incorrectly responded as they stated that, "a number is not divisible by 6 because the sum of its last two digits is not divisible by 6". For example, $7+2=9$, so number 9 is not divisible by 6, therefore, 20672 is not divisible by 6.

Extract 8.2 is a sample response from one of the candidates who faced difficulty when responding to this question.

08 (a) given $A_n = n$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

A_1 and $A_2 = ?$

From

$$S_n = \frac{n}{2}[A_1 + A_n]$$

$$\frac{n(n+1)(2n+1)}{6} = \frac{n}{2}[A_1 + n]$$

08 (a)

$$\frac{n(n+1)(2n+1)}{6} = \frac{n}{2}(A_1 + n)$$

$$\frac{(n^2+n)(2n+1)}{6} = \frac{n}{2}(A_1 + n)$$

$$\frac{2n^3+n^2+2n^2+n}{6} = \frac{n \cdot A_1 + n^2}{2}$$

$$\frac{2n^3+3n^2+n}{6} = \frac{n \cdot A_1 + n^2}{2}$$

$$4n^3+6n^2+2n = 6(n \cdot A_1 + n^2)$$

$$4n^3+6n^2+2n = 6n(A_1+n)$$

$$\frac{4n^3+6n^2+2n}{6n} = \frac{6n(A_1+n)}{6n}$$

$$A_1 + n = \tfrac{2}{3}n^2 + n + \tfrac{1}{3}$$

$$A_1 = \tfrac{2}{3}n^2 + n - n + \tfrac{1}{3}$$

$$A_1 = \tfrac{2}{3}n^2 + \tfrac{1}{3}$$

Solve for n will be that

$n = 0.71$ or $n = -0.71$

$$A_1 = \tfrac{2}{3}(0.71)^2 + \tfrac{1}{3} \text{ or } \tfrac{2}{3}(-0.71)^2 + \tfrac{1}{3}$$

$$A_1 = \frac{3347}{5000} \text{ or } \frac{3347}{5000}$$

(b) solution:
20672 To test first
The Divisibility rule of 6 states "that if the last
Two digits are divisible by 6 the number
are also divisible by 6.

∴ The number is divisible by 6.

Extract 8.2: A sample of the candidate's incorrect responses to question 8

In Extract 8.2, the candidate misinterpreted the given expression as sum of arithmetic progression in part (a). While in part (b), the candidate used inappropriate rule of divisibility to show that 20672 is divisible by 6.

2.9 Question 9: Logic

The candidates were instructed to use the truth table to test the validity of the argument: $p \rightarrow \sim q, r \rightarrow q, r \therefore \sim p$.

The analysis shows that out of 394 (100%) candidates who attempted this question, 142 (36.0%) candidates scored 0 to 1.5 marks. On the other hand, 126 (32.0%) candidates scored 2.0 to 3.5 marks while 126 (32.0%) candidates scored 4.0 to 6.0 marks. Therefore, the candidates' performance on this question was average. The summary of the candidates' performance on this question is presented in Figure 10.

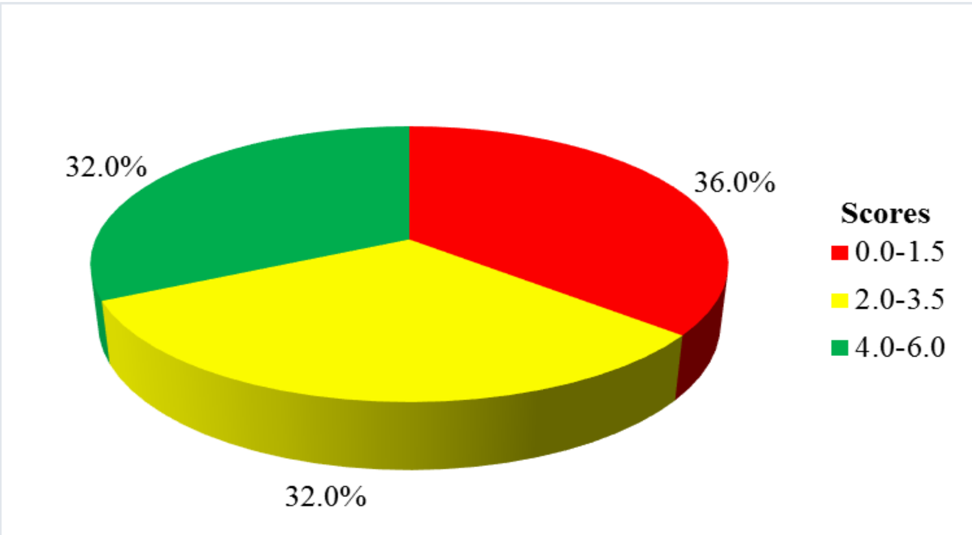


Figure 10: *Candidates' Performance on Question 9*

According to the analysis of data, the candidates who attempted this question and scored all marks were able to identify the required truth table of 9 columns and 9 rows. Thereafter, they constructed the truth table with correct entries (truth values) for p, q, r, $p \to \sim q$, $r \to q$, $r \therefore \sim p$ and found the last column containing all T for truth values, implying that the argument was valid. Extract 9.1 is a sample response from one of the candidates who correctly constructed the truth table.

9. Soln.

$\left[(p \to \sim q) \wedge (r \to q) \wedge r\right] \therefore \sim p.$

$\Rightarrow$

$\left[(p \to \sim q) \wedge (r \to q) \wedge r\right] \to \sim p$

p	q	r	$\sim q$	$\sim p$	$p \to \sim q$	$r \to q$	$(p\to\sim q)\wedge(r\to q)$	$(p\to\sim q)\wedge(r\to q)\wedge r$	$a \to \sim p$
T	T	T	F	F	F	T	F	F	T
T	T	F	F	F	F	T	F	F	T
T	F	T	T	F	T	F	F	F	T
T	F	F	T	F	T	T	T	F	T
F	T	T	F	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	T
F	F	T	T	T	T	F	F	F	T
F	F	F	T	T	T	T	T	F	T

provided $a = (p \to \sim q) \wedge (r \to q) \wedge r$

Since all values of $\left[(p \to \sim q) \wedge (r \to q) \wedge r\right] \to \sim p$ are true

$\therefore$ It is Valid

$\therefore$ The statement $p \to \sim q$, $r \to q$, $r \therefore \sim p$ is Valid

Extract 9.1: A sample of the candidate's correct responses to question 9

In Extract 9.1, the candidate identified the required truth table with correct entries.

On contrary, 36 per cent of candidates scored less than 2 marks. These candidates faced the following difficulties when attempting the question. Some of the candidates did not recognize that, they were required to construct a truth table combining $p \rightarrow \sim q$, $r \rightarrow q$ $r \therefore \sim p$. They tested the validity of the argument separately and could not meet the demand of the question, which was to combine all proposition in one table.

Moreover, other candidates faced challenges in determining the correct number of columns and rows required in their responses. They constructed the truth table with 8 columns and 8 rows. All truth tables constructed were unrelated to the given proposition.

The analysis also revealed that, some other candidates failed to enter the correct truth values in the truth table and testing the validity of the argument. They constructed the truth tables containing 8 columns and 8 rows and another truth table containing 7 columns and 6 rows. Hence, the incorrect truth tables led to failure to testify the validity of the argument, since the last column was found to contains all the F, implying that the validity is all F (Fallacy) instead of all T (Tautology). Extract 9.2 shows a sample of a response from one of the candidates who incorrectly responded to this question.

9 (a) $p \rightarrow \sim q$

	p	q	~q	p→~q	
	T	T	F	F	
	T	F	T	T	
	F	T	F	T	
	F	F	T	T	

9 (a) $p \rightarrow \sim q$

The argument is not validity

(b) $r \rightarrow q$

r	q	$r \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$\therefore$ The argument is not validity

(c) $r \therefore \sim p$

r	p	$\sim p$	$r \therefore \sim p$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

$\therefore$ The argument is not validity

Extract 9.2: A sample of the candidate's incorrect responses to question 9

In Extract 9.2, the candidate constructed separate truth tables to test the validity for each proposition hence failed to get the validity of the argument given.

2.10 Question 10: Sets

This question had two parts, (a) and (b). In part (a), the candidates were required to write the statement "$A=B$" in words such that A and B denote the sets of "boys" and "football players" respectively for a class containing both boys and girls. In part (b), they were given two joint sets A and B which were the subsets of the universal set μ. They were required to shade the regions in Venn diagrams which represent; (i) $A^C \cap B^C$ and (ii) $(A \cap B)^C$.

The analysis shows that, out of 394 candidates who attempted this question, 50 (12.7%) candidates scored 0 to 1.5 marks while 344 (87.3%) candidates scored 2.0 to 6.0 marks. Therefore, the general candidates' performance on this question was good. Figure 11 shows a summary of the candidates' performance on this question.

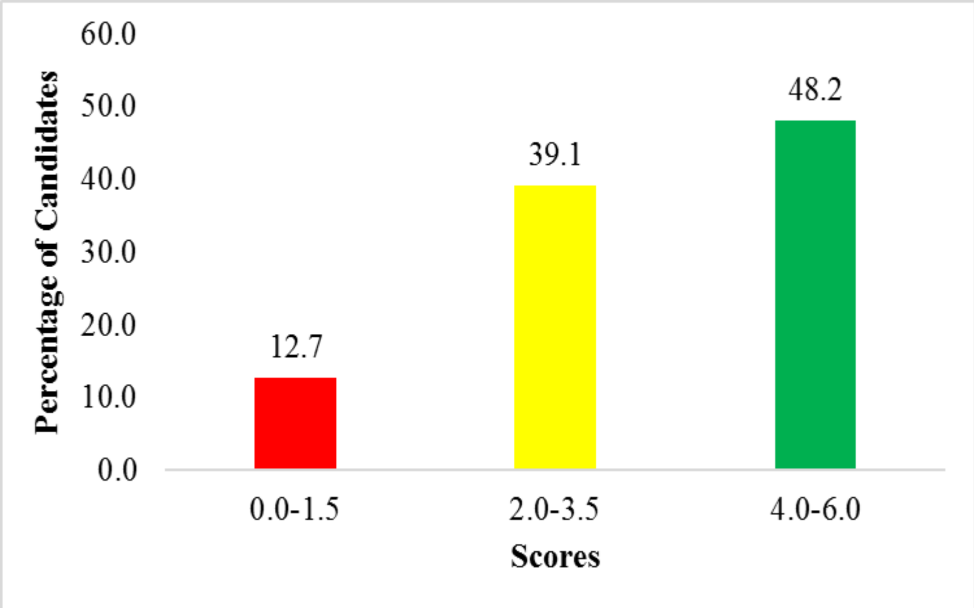


Figure 11: *Candidates' Performance on Question 10*

The analysis shows that a larger number of the candidates who attempted the question responded correctly and managed to score high marks. In part (a), the candidates correctly interpreted the word problem in which A and B denoted the sets of "boys" and "football players" respectively, and then correctly transformed the mathematical symbol "$A=B$" into statement as "All boys are football players and all football players are boys".

In part (b) (i), the candidates were familiar with correct definition of complement in sets A and B as the elements that are not found in A and B but are in universal set. Then, they correctly applied their competencies in drawing and shading the regions representing $A^C \cap B^C$.

In part (b) (ii), the candidates correctly recalled the definition of $(A \cap B)$ as the set having the common elements between A and B. Then, they applied the complements to the intersection set to shade the region representing

$(A \cap B)^C$. Extract 10.1 provides a sample of a response from one of the candidates who correctly responded to this question.

10 (a) Soln

Given

A = ~~se~~ boys

B = football players

$A = B$ in word's = ?

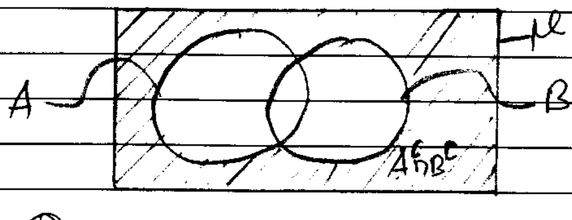
$\Rightarrow$ "All boys are football players and all football players are boys"

(b) (i) Soln

Given

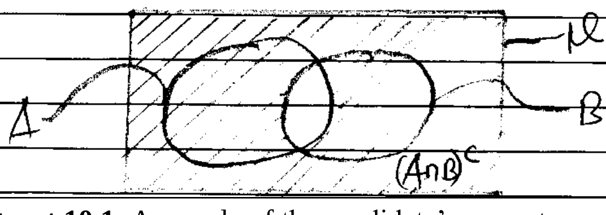
$A^C \cap B^C = A' \cap B'$

but; $A' \cap B' = (A \cup B)'$



(ii) Soln

Given; $(A \cap B)^C = (A \cap B)'$



Extract 10.1: A sample of the candidate's correct responses to question 10

In Extract 10.1, the candidate interpreted correctly the word problem in part (a), while in part (b), the candidate correctly applied the definition of complement in sets and clearly shaded the required regions.

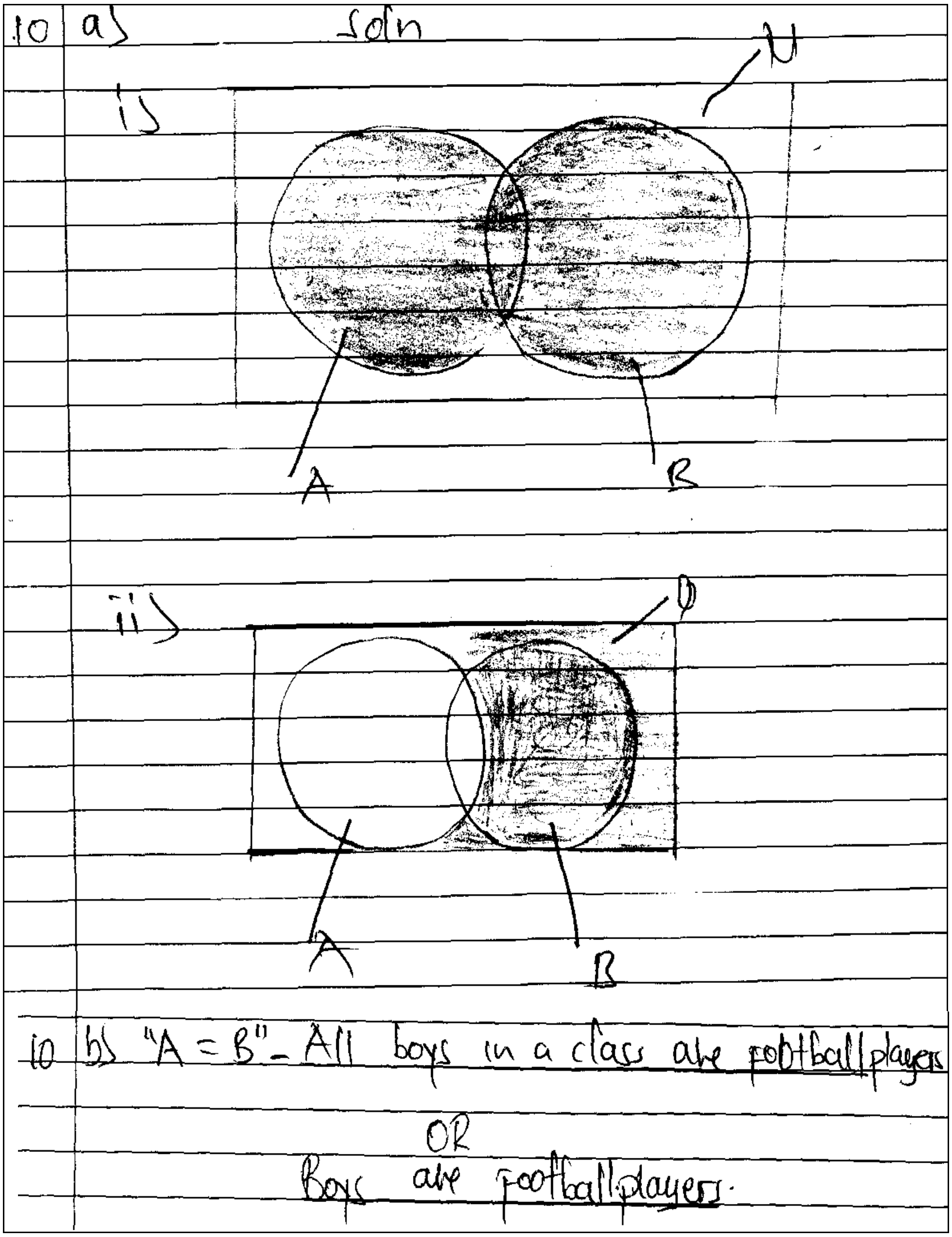
Despite the strengths demonstrated by most of the candidates, there were a few candidates who faced some difficulties when attempting the question;

For example in part (a), some candidates were not able to correctly interpret "*A*=*B*" from the statement that; A and B denote the set of "boys" and "football players" respectively thus, they responded incorrectly. For instance, some responded that; both boys and girls in a class are B. Others wrote "*A*=*B*" in symbol as $A \cap B$ (implying boys and football players).

Also, there were responses of $A = B$, which meant that the number of boys is equal to that of football players. Moreover, other candidates wrote "Boys are playing football if and only if are not football players". Some other candidates wrote; "in a class containing boys and girls who both play football". All these responses imply that the candidates lacked competence in the tested concepts because they were supposed to respond as "all boys are football players and all football players are boys".

In part (b) (i), the candidates misinterpreted the tested concepts of $A^C \cap B^C$ in their responses, in which they were required to shade the region outside sets A and B. However, they shaded the region of $A \cup B$, and ignored the law of complement. Likewise, other candidates perceived $A^C \cap B^C$ as $A \cap B$, in the responses as they shaded the region of $A \cap B$ instead of $A^C \cap B^C$. In addition, few candidates were not able to distinguish between joint sets and disjoint sets. In drawing Venn diagrams the ovals did not overlap each other while were supposed to.

In part (b) (ii), some of the candidates misinterpreted $(A \cap B)^C$ as $A^C \cap B^C$. They shaded the region outside sets A and set B. Furthermore, some of the candidates, after drawing Venn diagrams, they shaded only the region of set B. Extract 10.2 is a sample response from one of the candidates who were not able to attempt the question correctly.



Extract 10.2: A sample of the candidate's incorrect responses to question 10

In Extract 10.2, the candidate failed to interpret the given word problem hence wrote incorrect statement of "A=B" in part (a). In part (b), the candidate failed to shade the required regions as in (i), the candidate shaded the region of $A \cup B$. In (ii), the candidate shaded the region of $A^C \cap B$ instead of $(A \cap B)^C$.

2.11 Question 11: Functions

The question comprised parts (a) and (b). In part (a), the candidates were required to sketch the graph of $f(x)=\frac{1}{x^2-1}$ while in part (b), they were to state the domain and range of $f(x)$.

The analysis shows that 394 candidates attempted this question, out of whom 61 (15.4%) scored 0 to 2.5 marks, 148 (37.6%) scored 3.0 to 6.0 marks, while 185 (47%) scored 6.5 to 10 marks. According to the analysis, the candidates' performance on this question was generally good. Figure 12 illustrates the candidates' performance summary on this question.

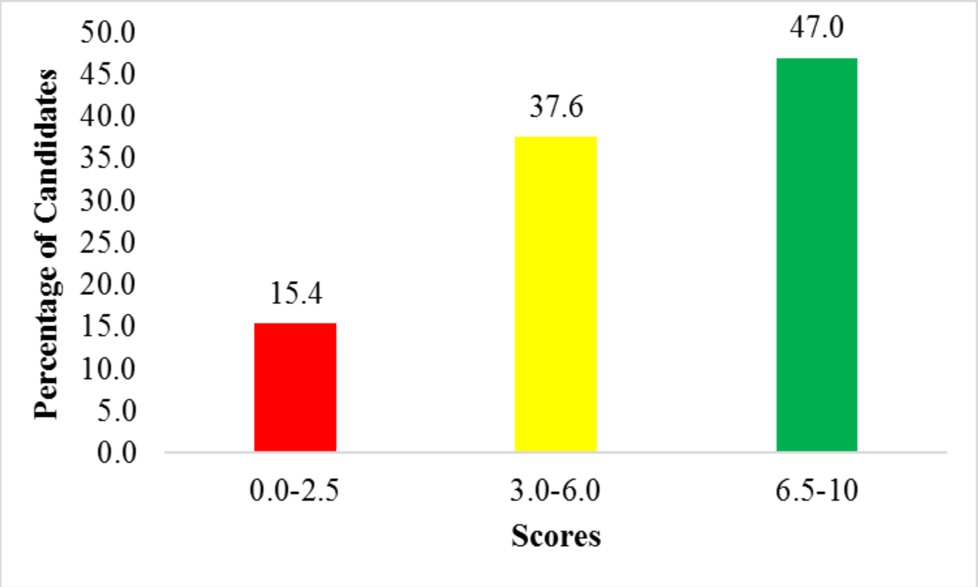


Figure 12: *Candidates' Performance on Question 11*

In part (a), the analysis of data depicts that, the candidates who responded correctly to this question were able to identify the type of function as a "rational function". Due to that, the vertical asymptote was correctly determined as $x^2-1=0$, and obtained $x=\pm1$ from the difference of the two squares $(x-1)(x+1)=0$.

On the other hand, the candidates calculated the horizontal asymptote by considering the highest degree of variable x which was 2. Then, they assumed that, x approaches to infinity that is, $f(x)=\frac{1}{x^2-1}$ implying that

$f(x)=\dfrac{\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}$ then simplified to $f(x)=\dfrac{\frac{1}{x^2}}{1-\frac{1}{x^2}}$. As $x\to\infty$ they finally, obtained the horizontal asymptote $y=0$. Furthermore, the candidates correctly determined the x and y intercepts as ∞ and -1 respectively. Finally, they worked on the turning point of the function and got $(x, y)=(0,-1)$ that helped them to sketch the correct graph as illustrated in Extract 11.1.

In part (b), the candidates who were able to attempt this question correctly were able to determine the domain and range by excluding the vertical asymptote and horizontal asymptote respectively. Finally, they correctly obtained the domain and range as domain $=\{\text{All real numbers, } x: x\neq -1 \text{ and } 1\}$ and range $=\{\text{All real numbers, } y: -1\leq y\leq 0\}$. Extract 11.1 is a sample response from one of the candidates who correctly answered the question.

11. a) Vertical asymptote

$x^2-1=0$

$\sqrt{x^2}=\sqrt{1}$

$x=1$ or $x=-1$

Horizontal asymptote

$y=\dfrac{1/x^2}{x^2/x^2-1/x^2}$

$y=\dfrac{0}{1-0}$ $(0,0)$

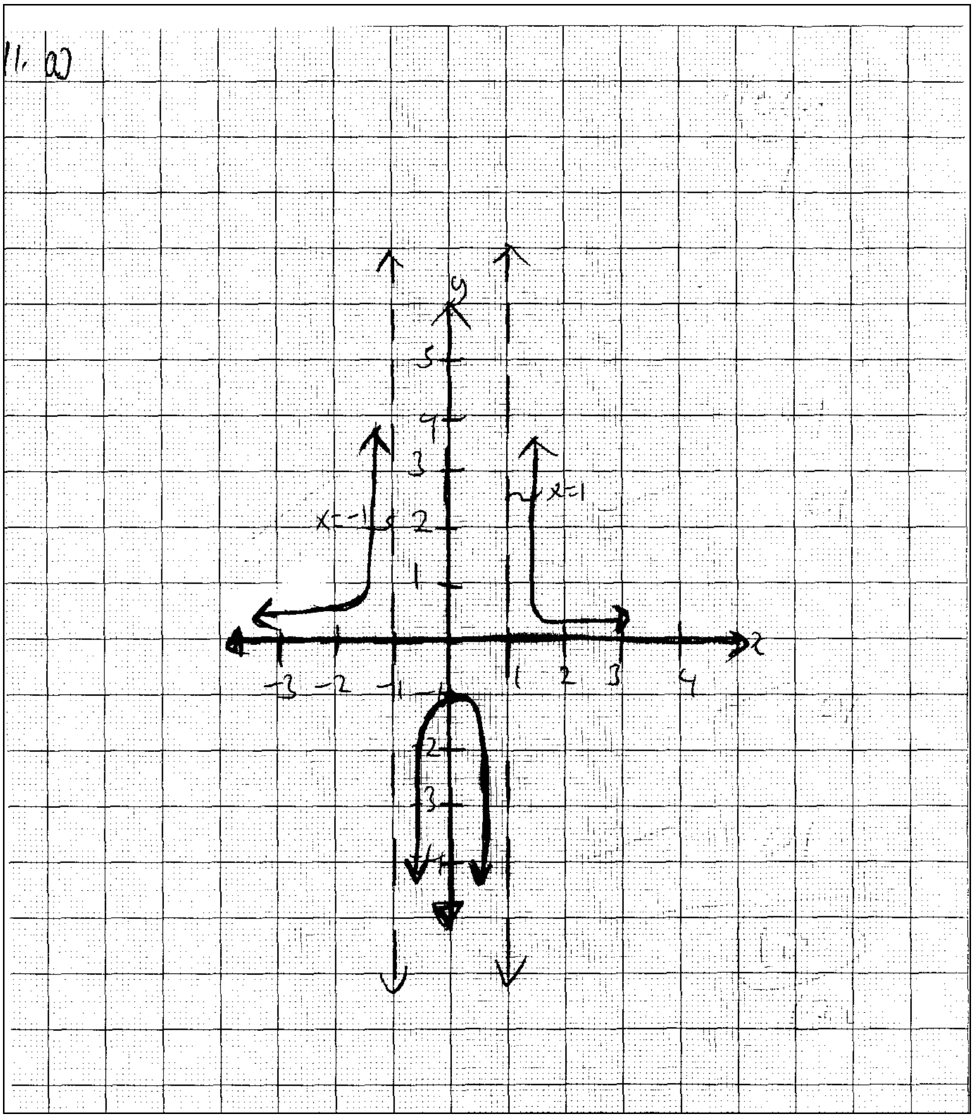
$y=0$

Table of values

	$x<-1$	$-1<x<1$	$x>1$
$0x+1$	+ve	+ve	+ve
x^2-1	+ve	−ve	+ve
$\frac{1}{x^2-1}$	+ve	−ve	+ve

b) Domain $=\{x\neq 1$ and $x\neq -1\}$

Range $=\{y\leq -1$ and $y>0\}$.



Extract 11.1: A sample of the candidate's correct responses to question 11

In Extract 11.1, part (a), the candidate managed to identify the type of function as rational function, as well as both asymptotes and turning point that were used to sketch a graph. In part (b), the candidate correctly stated the domain and range of the function.

Despite the good performance of a larger number of candidates, 15.4 per cent of the candidates' who attempted the question were unable to answer the question correctly as they scored 2.5 marks or less. Some candidates had problems when responding to the question as they had limited knowledge on the tested concepts.

In part (a), the analysis shows that, some candidates were unable to find intercepts or asymptotes of the given function. For example, one of the candidates when determining the vertical asymptotes, correctly equated the function's denominator as $x^2-1=0$, then solved for x, and obtained only $x=1$ instead of $x=\pm 1$. In additional, other candidates were confused on how to find horizontal asymptotes and vertical asymptotes, as they considered the denominator and highest degree of the function respectively. For instance, for the horizontal asymptotes, they considered the denominator as $x^2-1=0$, solved and got $x=1$ while for the vertical asymptote, they responded as $y=\dfrac{\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}$. They then let $x=\infty$, to get $y=\dfrac{0}{1-0}$ implying that $y=0$.

Further analysis reveals that, some candidates were unable to find the $x-\text{intercept}$ correctly, setting $y=0$, then responded incorrectly as $y=\left(x^2-1\right)x=1 \Rightarrow y=-x^2+1$. Then, substituted $y=0$ to get $x=1$ while $x-\text{intercept}$ which did not exist because the horizontal asymptote was on the $x-\text{axis}$. Following that, they responded for $y-\text{intercept}$ by setting $x=0$ and worked on it as $y=-x^2+1$ but $x=0$, $y=0+1 \Rightarrow y=1$. This notable error affected the sketching of the graph of $f(x)$. The majority of the candidates failed to find even the turning point of the graph which was $\left(x\,,\mathrm{y}\right)=\left(0\,,\text{-}1\right)$

In part (b), majority of the candidates stated incorrectly the domain and range. The analysis shows that the responses were domain = {All real numbers} and range = {All real numbers}. Also, some candidates wrote $\text{domain}=\left\{x:x>-1 \text{ and } x<\text{-}1\right\}$ and $\text{range}=\left\{y:y>0\right\}$. Some other candidates stated the domain as $\text{domain }=\left\{x\!:x>1 \text{ or } x<1\right\}$ rather than $\text{domain }=\left\{x:x\neq\pm 1\right\}$. For the range, they correctly considered the highest degree of x as 2, then recalled that range has relation with horizontal asymptotes, hence responded as, $y = \frac{1}{x^2 - 1} \Rightarrow \frac{\frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$, then let x approach to infinity, $y = \frac{0}{1-0} = 0$. Based on these responses, the candidates lacked understanding on the concept of domain and range. Extract 11.2 provides a sample of a response from one of the candidates who incorrectly responded to the question.

11a) $F(x) = \frac{1}{x^2 - 1}$

$H \cdot A = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denom}}$

leading coef of numerator = 1

" " " denominator = 1

$H \cdot A = 1$

$V \cdot A = x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

$x = 1$

b) Domain $x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

$x = 1$

$\therefore$ Domain: All reall numbers except 1

Range $F(x) = \frac{1}{x^2 - 1}$

$F(x) = y = \frac{1}{x^2 - 1}$

$y = \frac{1}{x^2 - 1}$

$y(x^2 - 1) = 1$

$yx^2 - y = 1$

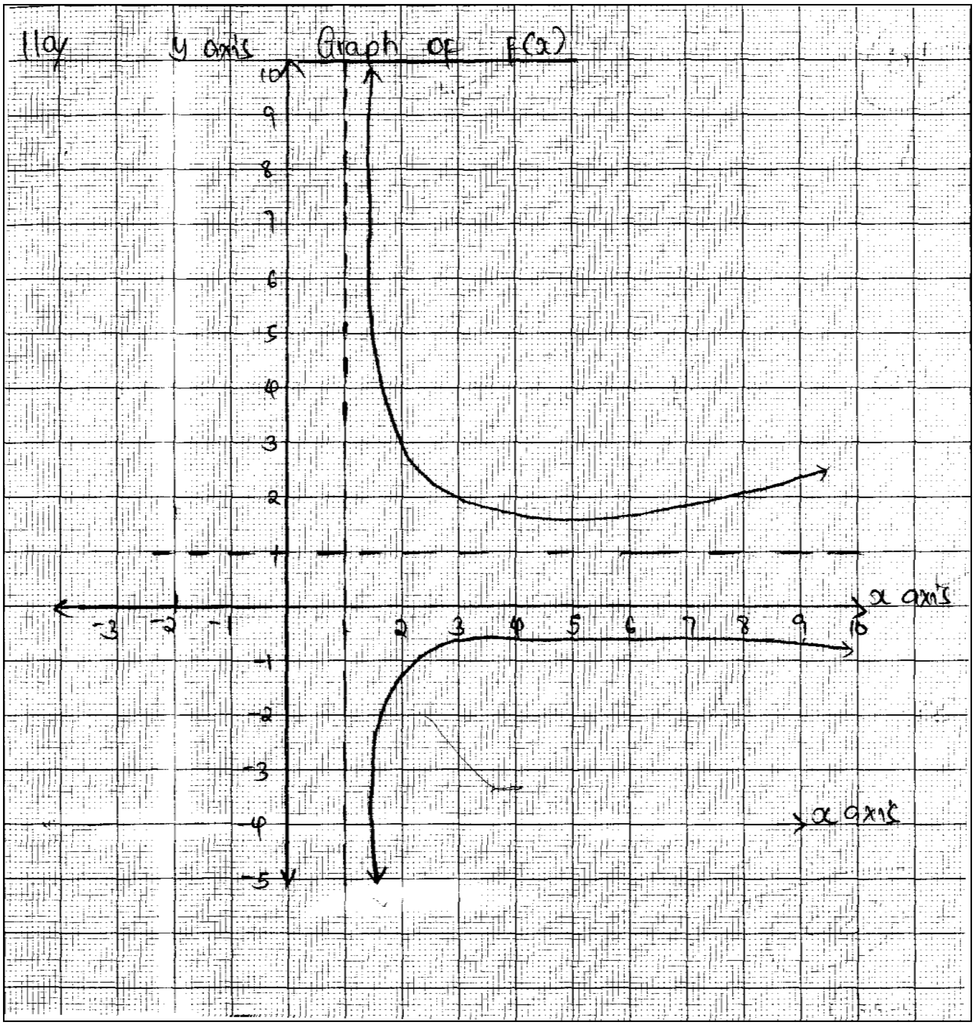
$\cancel{y(x^2-1)}\ \frac{yx^2}{y} = \frac{1+y}{y}$

$\sqrt{x^2} = \sqrt{\frac{1+y}{y}}$

$x = \sqrt{\frac{1+y}{y}}$

$\sqrt{y} = \cdot$

$y = 1$



Extract 11.2: A sample of the candidate's incorrect responses to question 11

In Extract 11.2, part (a), the candidate failed to draw the correct graph due to the failure of obtaining the correct asymptotes. In part (b), the candidate failed to obtain the domain and range of the function due to the lack of adequate knowledge.

2.12 Question 12: Integration and Differentiation

The question had two parts, in part (a), the candidates were required to evaluate $\int_0^{2\sqrt{2}} 4x\left(2x^2+9\right)^2 dx$ correct to two decimal places. While in part (b), candidates were given the statement that; spherical balloon is blown up in such a way that its volume is increasing at the constant rate of 138.6

cm^3/s. If the volume of the balloon is 4851cm^3, then the candidates were required to find the rate of the increase of the radius $\left(\text{Use } \pi = \frac{22}{7}\right)$.

The analysis shows that 120 (30.5%) candidates scored 6.5 to 10 marks out of 394 candidates who attempted this question. Also 162 (41.1%) candidates scored 0 to 2.5 marks, while 112 (28.4%) candidates scored 3.0 to 6.0 marks. The candidates' performance summary on this question is presented in Figure 13.

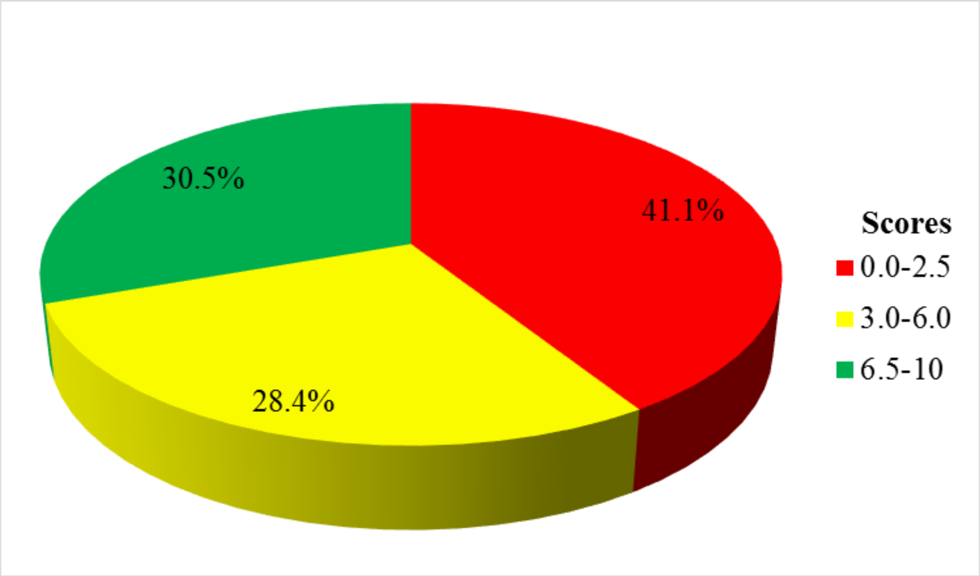


Figure 13: *Candidates' Performance on Question 12*

According to the analysis, the candidates' performance on this question was on average. Thus, the candidates who were able to respond correctly and scored high marks to this question, were competent enough on integration and differentiation.

In part (a), the candidates correctly identified the techniques to be used, thus applied the substitution technique by letting $u = 2x^2 + 9$ then differentiated u with respect to x as $\frac{du}{dx} = 4x$ and later obtained $du = 4xdx$. Then, they substituted correctly the lower and upper limits 0 and $2\sqrt{2}$ into $u = 2x^2 + 9$ to obtain the new limits 9 and 25 respectively.

Thereafter, they wrote $\int_0^{2\sqrt{2}} 4x\left(2x^2+9\right)^2 dx = \int_9^{25} u^2 du$, then correctly integrated it to obtain 4965.33.

In part (b), the candidates correctly recalled the formula for the volume of a spherical balloon as $V = \frac{4}{3}\pi r^3$, then substituted $V = 4851\,\text{cm}^3$ in the formula as $4851 = \frac{4}{3} \times \frac{22}{7} \times r^3$ and simplified to obtain $r = 10.5\,\text{cm}$. Then, they recognized that the increasing rate of radius had relation with differentiation, which is $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$. Finally, they applied the differentiation technique in $V = \frac{4}{3}\pi r^3$ as $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ since $\frac{dV}{dr} = 4\pi r^2$ and substituted $\frac{dV}{dt} = 138.6\,\text{cm}^3/\text{s}$, which is $138.6 = 4 \times \frac{22}{7} \times (10.5)^2 \times \frac{dr}{dt}$. Then simplified to get the rate of increase of radius, $\frac{dv}{dt} = 0.1\,\text{cm/s}$. Extract 12.1 is a sample response from one of the candidates who correctly responded to the question.

12 (a) soln:

$\int_0^{2\sqrt{2}} 4x\,(2x^2+9)^2 dx$

Letting $2x^2+9 = u$.

Now $du/dx = 4x$.

$dx = \frac{du}{4x}$.

$= \int_0^{2\sqrt{2}} 4x \; u^2 \frac{du}{4x}$

$= \int_0^{2\sqrt{2}} u^2 du$

$= \left[\frac{u^3}{3}\right]_0^{2\sqrt{2}}$ but $u = 2x^2+9$

$= \left[\frac{(2x^2+9)^3}{3}\right]_0^{2\sqrt{2}}$

12	(a)
	$= \left(\frac{2 \times (2\sqrt{2})^2 + 9}{3}\right)^3 - \left(\frac{2(0^2)+9}{3}\right)^3$
	$= \left(\frac{(2 \times 4 \times 2) + 9}{3}\right)^3 - \left(\frac{9^3}{3}\right)$
	$= \frac{(16+9)^3}{3} - \frac{729}{3}$
	$= \frac{25^3}{3} - \frac{729}{3}$
	$= \frac{15625}{3} - \frac{729}{3}$
	$= \left(\frac{15625 - 729}{3}\right)$
	$= \frac{14,896}{3}$
	$= 4,965 \cdot 33$
	$\therefore \int_0^{2\sqrt{2}} \left(4x\,(2x^2+9)^2\,dx\right) = 4,965 \cdot 33.$

12 (b) soln:

given

$$\frac{dv}{dt} = 138.6 \text{ cm}^3/\text{s}$$

volume, $v = 4851 \text{ cm}^3$

but volume, $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dr} = 4\pi r^2$$

And from

$$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}.$$

but $v = 4851 = \frac{4}{3}\pi r^3$

$$\frac{4\pi r^3}{4} = \frac{3 \times 4851}{4}$$

$$\frac{22}{7} r^3 = \frac{14553}{4} \times \frac{7}{22}$$

$$r^3 = 1157.625$$

$$r = \sqrt[3]{1157.625}$$

$$r = 10.5 \text{ cm}$$

12 (b) From

$$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \left(\frac{dv}{dt} \div \frac{dv}{dr}\right)$$

$$= \left(138.6\,cm^3/s\right) \div \frac{dv}{dr}.$$

$$= \frac{138.6\,cm^3/s}{4\pi(r^2)}$$

$$\frac{dr}{dt} = \frac{138.6\,cm^3/s}{4 \times 22/7 \times 10.5^2\,cm^2}$$

$$= \frac{138.6\,cm^3/s}{1386\,cm^2}$$

$$= 0.1\,cm/s.$$

$$\therefore \frac{dr}{dt} = 0.1\,cm/s$$

$\therefore$ The rate of increase of radius is 0.1 cm/s.

Extract 12.1: A sample of the candidate’s correct responses to question 12

In Extract 12.1, part (a), the candidate integrated the given function correctly using the appropriate technique and obtained the required value. In part (b), the candidate recalled the correct formula for volume of the sphere and managed to differentiated it to obtain the rate of the increase of radius.

In spite of the good performance, there were some candidates who failed to answer the question accordingly as they lacked knowledge on integration and differentiation. In part (a), some of the candidates were not able to recognize the techniques required to evaluate a definite integral. Instead of applying substitution technique, they applied the difference of two the squares that was contrary to the requirement of the question. For instance, $\int_0^{2\sqrt{2}} 4x\left(2x^2+9\right)^2 dx = \int_0^{2\sqrt{2}} 4x(2x+3).(2x-3)dx$, which was wrongly integrated and obtained as $\left[4x(2x+3)(2x-3)\right]_0^{2\sqrt{2}}$. Then, they substituted the highest limit as $4\left(2\sqrt{2}\right)\left(2\left(2\sqrt{2}\right)+3\right)\left(2\left(2\sqrt{2}\right)-3\right)$, simplified and got incorrect value $5.6(0.2\times 5.8)=6.496$ instead of 4965.33.

Also, other candidates responded to this question by substituting the limits without integrating the definite integral. For instance $\int_0^{2\sqrt{2}} 4x\left(2x^2+9\right)^2 dx$ $=\left[4x\left(2x^2+9\right)\right]^2 - 4x\left(2x^2\right) = 4\left(2\sqrt{2}\right)\left[2\left(2\sqrt{2}\right)+9\right]-4(0)\left[2(0)+9\right]$, then simplified it and managed to get $8\sqrt{2}(25)\approx 7071.068$ instead of ≈ 4965.33.

Furthermore, other candidates integrated separately the definite integral. For instance, $\int_0^{2\sqrt{2}} 4x dx + \int_0^{2\sqrt{2}} 2x^2 dx + \int_0^{2\sqrt{2}} 9dx$. Thereafter, it was wrongly integrated to obtain $\frac{4}{2}x^2 + \frac{2}{3}x^3 + 9$. Then, they failed to substitute the limits. All these wrong responses were due to lack of competence on integration techniques.

In part (b), some of the candidates misinterpreted the question and perceived it as the volume of a cross section, instead of applying the differentiation techniques. They applied the incorrect formula, $\text{Volume} = \frac{4}{3}\pi\left(R^3 - r^3\right)$ and wrongly identified R = 4851 and r = 138.6. Then, they substituted the values in the incorrect formula, $V = \frac{4}{3}\times\frac{22}{7}(4851)^3 - (138.6)^3$, computed and finally obtained 4.78351425, hence concluded that, the rate of increase was 4.78351425×10^{11} cm/s.

The analysis also shows that, some other candidates were not able to recall correctly the formula for volume, as $V = \frac{4}{3}\pi r^3$. Instead, they recalled $V = \pi r^3$ then differentiated as $\frac{dV}{dr} = 3\pi r^2$ where $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dV}{dr}$ implying that $(138.6\,\text{cm}^3/s) \times 3\pi r^2$, which was incorrectly simplified to get $138.6\,\text{cm}^3/\text{s} \times 4851$, and finally obtained 472,348.6 instead of $0.1\,\text{cm/s}$.

In addition, some of the candidates were unable to recall correctly the formula for volume (V) of a balloon. They incorrectly applied the formula for the area of a circle, which is $\text{V} = \pi\,\text{r}^2$. Following that, they responded incorrectly as $\text{V} = k\pi\,\text{r}^2$, but $\text{V} = 4801\text{cm}^3$ and $k = 138.6$, then substituted in the incorrect formula and got $4801\,\text{cm}^3 = \frac{22}{7} \times r^2 \times 138.6$, hence computed to obtain $r = 3.34\,\text{cm/s}$. Moreover, other candidates incorrectly responded to the question as they wrote $\text{Volume} = \frac{dr}{dv} \times \frac{dt}{dr}$, then $r = \frac{4851 \times 7}{22 \times 138.6}$ and after simplifying they got incorrect value of the rate of increase as $11.14\,cm/s$. Extract 12.2 is a sample of a response from one of the candidates who faced difficulties when attempting the question.

12. (a)

$$\int_0^{2\sqrt{2}} 4x(2x^2+9)^2\,dx$$

Solution

$$\int_0^{2\sqrt{2}} 4x(2x^2+9)^2\,dx$$

$$\int_0^{2\sqrt{2}} 4x\,dx + \int_0^{2\sqrt{2}} 2x^2 + \int_0^{2\sqrt{2}} 9\,dx$$

$$4\int_0^{2\sqrt{2}} \frac{x^{1+1}}{1+1} + 2\int_0^{2\sqrt{2}} \frac{x^{2+1}}{2+1} + \int_0^{2\sqrt{2}} 9\,dx$$

$$\frac{4x^2}{2} + \frac{2}{3}x^3 + 9 + \cancel{c}.$$

$$\frac{1}{2}x^2 + \frac{2}{3}x^3 + 9$$

b) Volume $= \frac{\pi R}{100}\ \frac{4\pi}{3}(R^3 - r^3)$.

Volume $= \frac{4}{3} \times \frac{22}{7}\left((4851)^3 - (138.6)^3\right)$.

Volume $= \frac{88}{21}\left(1.141547071 \times 10^{11} - 2662500.456\right)$

Volume $= 4.78351425 \times 10^{11}$

Volume $= 4.78 \times 10^{11}$.

$\therefore$ The rate of increase of the radius is $4.78 \times 10^{11}\ cm^3$.

Extract 12.2: A sample of the candidate's incorrect responses to question 12

In Extract 12.2, part (a), the candidate integrated separately the definite integral. In part (b), the candidate used a wrong formula to calculate the volume of a sphere.

2.13 Question 13: Probability

This question consisted of parts (a) and (b). In part (a), the candidates were given that "If A and B are mutually inclusive events such that $P(A) = 0.4$ and $P(B) = 0.3$" and were asked to find (i) $P(A \text{ and } B)$ (ii) $P(A \text{ or } B)$. While in part (b), the candidates were given that, a family has two children, assuming that boys and girls are equally likely, then they were required to use tree diagram to determine the probability that the family had;

(i) one boy and one girl given that the first child is a boy.

(ii) at least one girl given that the younger one is a girl.

The analysis shows that, 394 candidates attempted this question. 41 (10.4%) candidates scored 0 to 2.5 marks, 140 (35.5%) candidates scored 3.0 to 6.0 marks while 213 (54.1%) candidates scored 6.5 to 10 marks. The candidates' performance on this question is summarized in Figure 14.

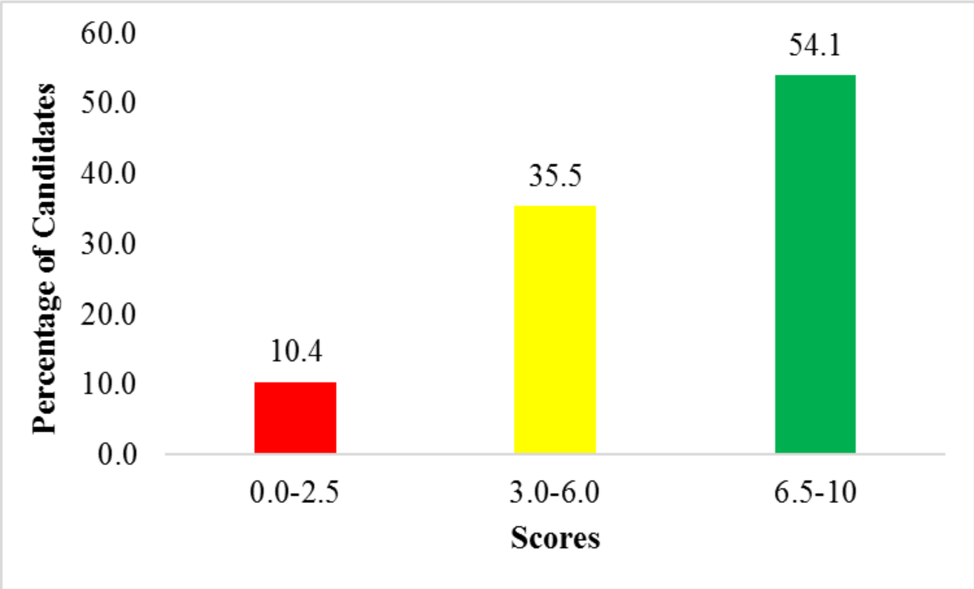


Figure 14: *Candidates' Performance on Question 13*

The analysis shows that, the performance of the candidates on this question was good as a total of 80 candidates managed to attempt the question as required and scored all alloted marks.

In part (a), the candidates demonstrated their competence on mutually inclusive events as they managed to correctly translate and transform the given probability statements into mathematical symbols as $P(A \text{ and } B) \to P(A \cap B)$ and $P(A \text{ or } B) \to P(A \cup B)$. Then, they substituted and computed the given probabilities to obtain the correct solution as in (i) $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$, given $P(A) = 0.4$ and $P(B) = 0.3$ implying that $P(A \cap B) = 0.4 \times 0.3 = 0.12$. Similarly, in (ii), the candidates applied the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, to compute $P(A \cup B) = 0.4 + 0.3 - 0.12$. Finally, they obtained $P(A \text{ or } B) = 0.58$ as the required value.

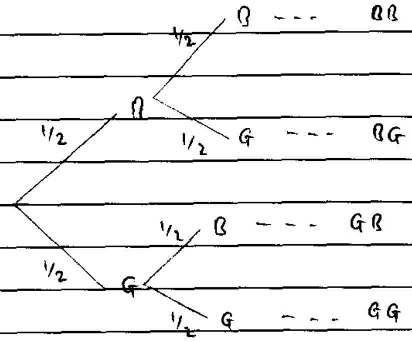
In part (b), the candidates demonstrated good understanding on probability as they correctly presented the given information on a tree diagram with two branches and four ends. Thereafter, they managed to identify the correct sample space as well as the number of sample space as $S = \{BB, BG, GB, GG\}$ and $n(s) = 4$ respectively.

In part (b) (i), the candidates determined the required event as $E = \{BG\}$ and $n(E) = 1$. Then, they recalled and applied the formula $P(E) = \frac{n(E)}{n(S)}$ and computed by substituting the given values as $P(E) = \frac{1}{4}$ which was the required probability.

In part (b) (ii), the candidates identified the correct event as $E = \{BG, GG\}$ implying that $n(E) = 2$; then, they applied the same formula, $P(E) = \frac{n(E)}{n(S)}$ that resulted to $P(E) = \frac{2}{4} = \frac{1}{2}$. Extract 13.1 shows a sample of a response from one of the candidates who responded to this question correctly.

13	(a) (i) $P(A \cap B)$
	From
	$P(A \cap B) = P(A) \times P(B)$
	$= 0.4 \times 0.3$
	$P(A \cap B) = 0.12$
	$\therefore P(A \text{ and } B) = 0.12$
	(ii) $P(A \cup B)$
	From
	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	$= 0.4 + 0.3 - 0.12$
	$= 0.7 - 0.12$
	$P(A \cup B) = 0.58$
	$\therefore P(A \text{ or } B) = 0.58.$
13.	(b) let
	Girl child $= G$
	Boy child $= B$
	Then

13. (b) In Tree diagram



(i) $P(BG) = \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

$\therefore$ Probability $= \frac{1}{4}$

(ii) $P(BG) + P(GG)$

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1+1}{4}$

$= \frac{2}{4}$

$\therefore$ Probability $= \frac{1}{2}$

Extract 13.1: A sample of the candidate's correct responses to question 13

In Extract 13.1, part (a), the candidate correctly translated and transformed the statement then applied the correct formula to find the required probabilities. While in part (b), the candidate drew the correct tree diagram containing all the necessary information thus getting the correct probabilities.

However, some of the candidates performed this question poorly, as they scored 2.5 marks or less. These candidates did not provide the relevant solution in accordance with the demand of the question.

In part (a) (i), some of the candidates considered $P(A \text{ and } B)$ as $P(A \text{ or } B)$. For instance, one candidate incorrectly responded as $P(A \cap B) = P(A) + P(B)$, then substituted the given values of $P(A) = 0.4$ and $P(B) = 0.3$ in an incorrect formula, $P(A) + P(B) = 0.4 + 0.3$ and concluded that $P(A \text{ and } B) = 0.7$. Likewise, another candidate incorrectly remembered $P(A \cap B) = P(A) + P(B) - P(A \cup B)$, and responded as $P(A \cap B) = 0.4 + 0.3 - 0.7$, then obtained $P(A \cap B) = 0$ instead of 0.12. Furthermore, the analysis also shows that, the responses of some other candidates were in coordinate form, so $P(A \cap B) = (\ 0.4\ ,\ 0.3\ )$ instead of multiplying the probabilities.

In part (a) (ii), the candidates mistook one concept for another concepts by finding the $P(A \text{ and } B)$ instead of $P(A \text{ or } B)$ by using the formula of $P(A \cap B) = P(A) \times P(B)$, substituting $P(A) = 0.4$ and $P(B) = 0.3$ in the formula as $P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.3$ and got $P(A \cap B) = 0.12$ instead of using $P(A \text{ or } B)\ = P(A) + P(B) - P(A \cap B)$, which could result to getting a correct probability of 0.58.

In part (b), some candidates presented a tree diagram with incorrect probabilities and outcomes. For instance, they indicated probabilities as 0.3, 0.2, $\frac{1}{3}$ and $\frac{2}{3}$ on the first and second branches of the tree diagram, instead of $\frac{1}{2}$ and $\frac{1}{2}$ which produced the incorrect probability in part (b) (i) and (ii).

Further analysis shows that, some candidates presented a tree diagram with three instead of two branches. This mistake led to a sample space, $S = \{BBB, BBG, BGB, BGG, GBB, GBG,\ GGB, GGG\}$, instead of $S = \{BB, BG,\ GB, GG\}$. The noted mistake affected the result of part (b) (i) and (ii). Extract 13.2 is a sample response from one of the candidates who incorrectly answered the question.

13 a) soln:

$P(A) = 0.4$ and $P(B) = 0.3$

i) $P(A \text{ and } B) = P(0.4 + 0.3)$

$= P(0.7)$

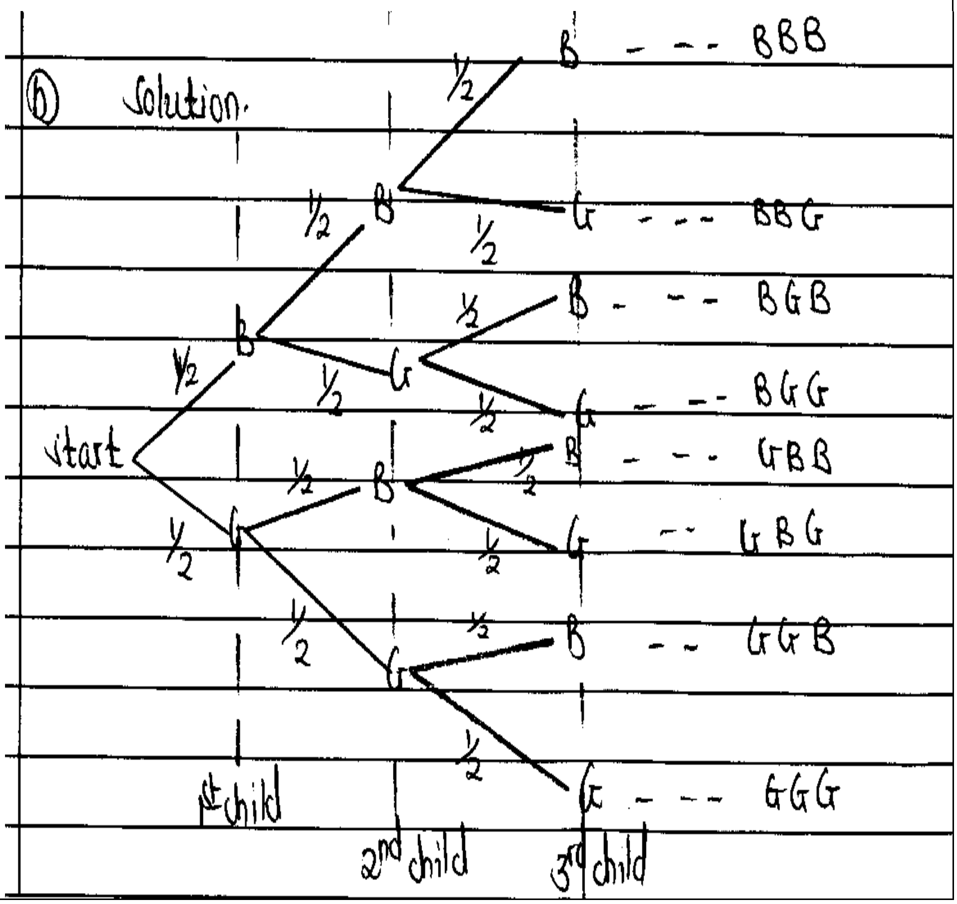
$\therefore P(A \text{ and } B) = 0.7$

ii] $P(A \text{ or } B) = P(0.4 \times 0.3)$

$= P(0.12)$

$\therefore P(A \text{ or } B) = 0.12$

(b) Solution.



13 b) i. One boy and one girl given that the first child is a boy.

$P(BBG \text{ or } BGB \text{ or } BGG) = P(BBB) + P(BGB) + P(BGG)$

$= (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= 3/8$

Thus the probability that the family has one boy and one girl given that the first child is a boy is 3/8

ii. at least one girl given that the younger one is a girl.

$P(BBG \text{ or } BGG \text{ or } GBG \text{ or } GGG) = P(BBG) + P(BGG) + P(GBG) + P(GGG)$

$= (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= 4/8 = \frac{1}{2}$

Thus the probability of the family having at least one girl and the youngest being a girl is $\frac{1}{2}$.

Extract 13.2: A sample of the candidate's incorrect responses to question 13

In Extract 13.2, part (a), the candidate used a wrong formula to find the $P(A \cap B)$ in part (a) (i) while in part (a) (ii), the candidate failed to get $P(A \text{ or } B)$ due to the use of inappropriate formula of $P(A \cap B)$. In part (b), the candidate presented irrelevant tree diagram with three instead of two branches.

2.14 Question 14: Vectors and Transformations

This question had three parts (a), (b) and (c). In part (a), the question was that; if $\underset{\sim}{a} = 5i - j - 3k$ and $\underset{\sim}{b} = i + 3j - 5k$, then the candidates were required to show that $\underset{\sim}{a} + \underset{\sim}{b}$ and $\underset{\sim}{a} - \underset{\sim}{b}$ are perpendicular. While in part (b), the candidates were asked to calculate the area of a triangle, given that, the position vectors of the two adjacent sides of a triangle are $\underset{\sim}{a} = i - 3j + k$ and $\underset{\sim}{b} = -i + 3j - 2k$ respectively. In part (c), the candidates were instructed to find $T[8v - 7u]$, given that $T[u] = [4, 1]$ and $T[v] = [5, 2]$.

The analysis reveals that 394 candidates attempted this question. Among them, 9.1 per cent of candidates scored 0 to 2.5 marks, 47.7 per cent of candidates scored 3.0 to 6.0 marks while 43.2 per cent of candidates scored 6.5 to 10 marks. Therefore, the candidates' performance on this question was generally good. Figure 15 shows the candidates' performance summary on this question.

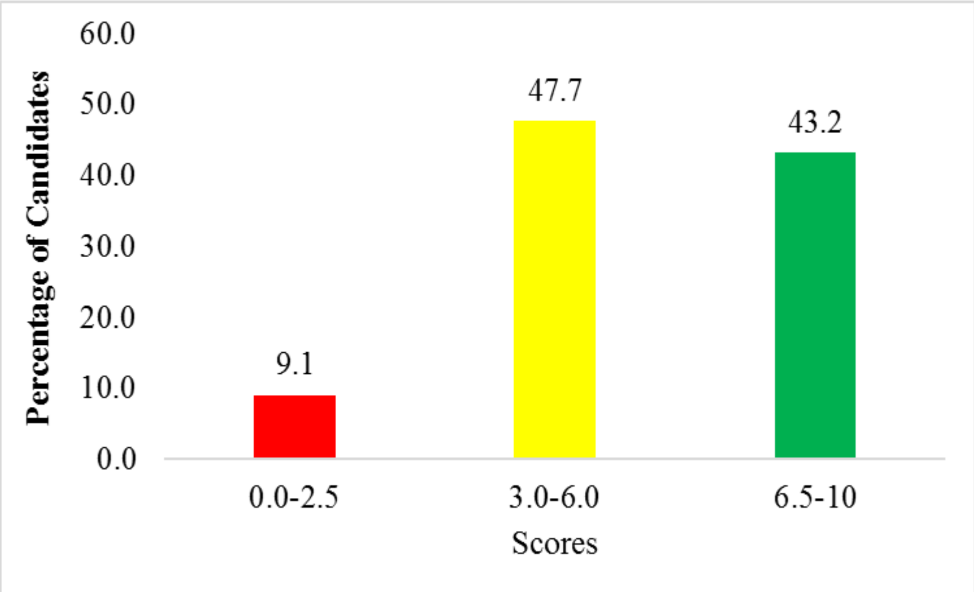


Figure 15: *Candidates' Performance on Question 14*

The analysis shows that 17 per cent of candidates who correctly responded to this question managed to score all alloted marks.

In part (a), the analysis shows that the candidates were familiar with the basic operations of vectors as they managed to find the sum and difference of vectors as $\underset{\sim}{a} + \underset{\sim}{b} = 6i + 2j - 8k$ and $\underset{\sim}{a} - \underset{\sim}{b} = 4i - 4j + 2k$ respectively.

Then, they recalled the condition for two vectors to be perpendicular, that is their dot product must be zero. Then, computed the dot product as $(\underline{a}+\underline{b})\bullet(\underline{a}-\underline{b})=\left(6i+2j-8k\right)\bullet\left(4i-4j+2k\right)=24-8-16=0$. Finally, they concluded that, since $\left(\underline{a}+\underline{b}\right)\bullet\left(\underline{a}-\underline{b}\right)=0$, therefore $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$ are perpendicular.

In part (b), the candidates were able to recall and apply correctly the formula for the area of a triangle as $A=\frac{1}{2}\left|\underline{a}\times\underline{b}\right|$ and computed as $A=\frac{1}{2}\begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ -1 & 3 & -2 \end{vmatrix}$. Thereafter, using cross product concepts, they ended with $A=\frac{\sqrt{10}}{2}$ square units.

In part (c), the candidates recalled the correct properties of linear transformation as $T\left[tu \text{ -tv}\right]=tT\left[u\right]-tT\left[v\right]$, then applied it to the given transformation $T\left[8v-7u\right]=8T\left[v\right]-7T\left[u\right]$, substituted the given value of $T\left[u\right]=\left(4,1\right)$ and $T\left[v\right]=\left(5,2\right)$ to get $8(5,1)-7(4,1)$ which resulted to $T[8v-7u]=\left(40,16\right)-\left(28,7\right)=\left(12,9\right)$. Extract 14.1 is a sample response from one of the candidates who responded to the question correctly.

14\. (a) solution:

$\underline{a} + \underline{b} = (5\underline{i} - \underline{j}\ 3\underline{k}) + (\underline{i} + 3\underline{j} - 5\underline{k})$

$\underline{a} + \underline{b} = 6\underline{i} + 2\underline{j} - 8\underline{k}$

$\underline{a} - \underline{b} = (5i - j - 3k) - (i + 3j + \cancel{0} - 5k)$

$= (5i - i) - j - 3j - 3k + 5k.$

$= +4i - 4j + 2k.$

$\underline{a} - \underline{b} = 4\underline{i} - 4\underline{j} + 2k$

For perpendicular vectors say a and b.

$a \cdot b = |a||b| \cos\theta = 0$

since $\cos 90 = 0$

$a \cdot b = 0.$

$(\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b}) \Rightarrow 0.$

$$= \begin{pmatrix} 6 \\ 2 \\ -8 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$$

$= (6 \times 4) + (2 \times -4) + (-8 \times 2)$

$= 24 - 8 - 16.$

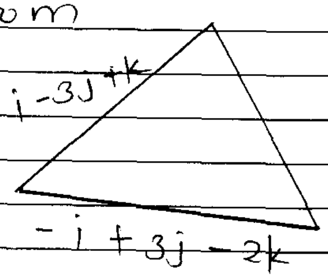
$= 24 - 24$

$= 0.$

$\therefore$ since $(\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b}) = 0$, then $(\underline{a} + \underline{b})$ and $(\underline{a} - \underline{b})$ are perpendicular vector.

14 (b) soln:

From



$$\text{Area} = \frac{1}{2}\left|a \times b\right|$$

$$= \frac{1}{2}\begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ -1 & 3 & -2 \end{vmatrix}$$

$$= \frac{1}{2}\left| i\begin{vmatrix} -3 & 1 \\ 3 & -2 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} + k\begin{vmatrix} 1 & -3 \\ -1 & 3 \end{vmatrix} \right|$$

$$= \frac{1}{2}\left| i(6-3) - j(-2++1) + k(3-3) \right|$$

$$= \frac{1}{2}\left| 3i + j + 0k \right|$$

$$= \frac{1}{2}\left|\sqrt{3^2 + 1^2 + 0^2}\right|$$

$$= \frac{1}{2} \times \left|\sqrt{9 + 1 + 0}\right|$$

14 (b) Area, $A = \left(\frac{1}{2}\sqrt{10}\right)$ square units

$= 1\cdot58$ square units

$\therefore$ The area of the triangle is $1\cdot58$ square units

(c) soln:

$T[u] = (4,1)$ and $T(v) = (5,2)$

$T[8v - 7u]$

from $T[u - v] = T[u] - T[v]$

$T[8v - 7u] = T[8v] - T[7u]$

and $Tku = kTu$

$T[8v - 7u] = 8T(v) - 7T[u]$

$= 8(5,2) - 7(4,1)$

$= (40,16) + (28,-7)$

$= (12,9)$

$\therefore T[8v - 7u] = (12,9)$

Extract 14.1: A sample of the candidate's correct responses to question 14

In Extract 14.1, parts (a) and (b), the candidate correctly performed the dot product and cross product on vectors respectively. In part (c), the candidate correctly applied the linear transformation properties.

Despite the strengths demonstrated by most of the candidates who responded to this question, other candidates who performed poorly encountered the following challenges ;

In part (a), some of the candidates recalled and applied inappropriate approach in verifying that, $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$ are perpendicular to each other. The analysis shows that, those candidates applied the cross product instead of the dot product formulae.

For example one candidate wrote $\underline{a}\bullet\underline{b}=\begin{vmatrix} 5 & 1 \\ -1 & 3 \\ -3 & -5 \end{vmatrix}$ simplified it, and got $\underline{a}\bullet\underline{b}=5+(-1\times 3)+(-3\times -5)$, then further simplified it to get $\underline{a}\bullet\underline{b}=17$. Furthermore, other candidates recalled the formula for calculating the angle between two vectors as their approach to determine whether the given vectors are perpendicular to each other but made some computational errors while working on it. For example, one candidate wrote $a\bullet b=(-1\times 1)+(3\times -3)+(-2\times 1)$ and obtained -12 after simplifying; then worked on $|a|=\sqrt{x^2+y^2+z^2}=\sqrt{1^1+(-3)^2+1^2}$ and simplified to obtain $\sqrt{11}$. Similarly, for $|b|=\sqrt{x^2+y^2+z^2}=\sqrt{(-1)^1+(-3)^2+(-2)^2}$, they simplified to get $\sqrt{14}$. Based on perpendicular vectors, they used the formula $\cos\theta=\dfrac{a\bullet b}{|a||b|}$ and obtained $\cos\theta=\dfrac{-12}{\sqrt{11}\sqrt{14}}$, which was incorrectly computed as $\cos\theta=0.97$. According to the analysis, all these responses indicate limited knowledge on vectors.

In part (b), the analysis shows that, some of the candidates applied incorrect formula to determine the area of a triangle. For example, one candidate responded as;

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & \mathrm{y}_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{vmatrix}$$

instead of $\text{Area}=\dfrac{1}{2}\begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$, thereafter, substituted the given values, as

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 1 \\ -3 & 3 \\ 1 & -2 \\ 1 & 1 \end{vmatrix}$$

and later simplified by adding the product of the leading elements as follows; $\text{Area} = \frac{1}{2}\left|(1\times 3)+(-3\times -2)+(1\times 1)+(-3\times 1)+(1\times 3)+(1\times -2)\right|$. After simplification, the candidate ended up with $\text{Area} = 2\,\text{cm}^2$ instead of $\text{Area} = \frac{\sqrt{10}}{2}$ square unit. Further analysis reveals that, some candidates recalled the vector techniques of the area of a triangle, but incorrectly represented the vectors in matrix, for instance, $\underline{a} = i - 3j + k$ and $\underline{b} = -i + 3j - 2k$ were arranged as $\begin{vmatrix} 1 & -1 \\ -3 & 3 \\ 1 & -2 \end{vmatrix}$ instead of $\begin{vmatrix} i & k & j \\ 1 & -3 & 1 \\ -1 & 3 & -2 \end{vmatrix}$. Then, they responded wrongly as $A = \frac{1}{2}[(1\times -1)+(-3\times 3)+(-2\times 1)]$, then simplified to obtain 86. Thereafter, they applied the square root as $\sqrt{86} = 9.3$ and incorrectly worked on area of the triangle as $\frac{1}{2}\times 9.3$ and simplified to get incorrect value of 4.65 instead of $\frac{\sqrt{10}}{2}$ square units.

In addition, other candidates in presenting the vectors in matrix, used 4 rows and 2 columns as $A = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -3 & 3 \\ 1 & 2 \\ 1 & 1 \end{vmatrix}$ incorrectly and then responded as $A = \frac{1}{2}(1\times 3 + -3\times -2 + 1\times 1 + 1\times 3 + -3\times -1)$, which was simplified to obtain $\frac{1}{2}(10-4) = 3\,\text{cm}^2$ instead of $\frac{\sqrt{10}}{2}$ square units.

In part (c), some candidates seemed to have no idea about the linear transformation. For instance, some candidates responded by considering that, $T[u]=(4\,,1)\rightarrow 4\times 1=4$ and $T[v]=(5\,,2)\rightarrow 5\times 2=10$. Thereafter, they applied the linear transformation property $T[8v-7u]\rightarrow 8T[v]-7T[u]$. Then, they substituted the obtained value of $T[u]=4$ and $T[v]=10$ to obtain $8(10)-7(4)$ and ended up with 52 instead of $(12\,,9)$.

Furthermore, the analysis also shows that, some candidates responded to the question as $T[8\underline{u}-7\underline{v}]=8(4\,,\,8)-7(5\,,\,2)$, simplified and obtained $32i+64j-35i+14j=-3i+50j$, then found the modulus as $\sqrt{(-3)^2+(50)^2}$, then computed it to get incorrect value of 50.08 instead of $(12, 9)$.

Also, some candidates applied inappropriate formula while responding to the question. For example one candidate used the gradient formula resulting to incorrect response, which is, slope $(\mathrm{M})=\dfrac{\mathrm{y}_2-\mathrm{y}_1}{x_2-x_1}$, then substituted the value of $T[u]=(4,1)$ and $T[v]=(4,2)$, and hence simplified it and obtained 1. Then, they used the slope obtained to determine the values of x incorrectly as $1=\dfrac{y-2}{x-5}$, also $1=\dfrac{y-1}{x-4}$, and got $x_1=-1.3$ and $x_2=2.3$. Then, they considered it as $v=-1.3$ and $u=2.3$, thus they substituted in $T[8\underline{u}-7\underline{v}]$ as $T[8(-1.3)-7(2.3)]$ and finally obtained $T(5, 7)$ instead of $(12, 9)$. Based on the analysis, these responses were due to lack of competence in the tested concepts. Extract 14.2 is a sample response of an incorrect answer from one of the candidates.

14. b) To calculate area of triangle

soln

we have $a = i - 3j + k$

$b = i + 3j - 2k$

from

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 1 \\ -3 & 3 \\ 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$\text{Area} = \frac{1}{2}(1\times3) + (-3\times-2) + (1\times1) + (-3\times1) + (1\times3) + (1\times-2)$

$\text{Area} = \frac{1}{2}(3 + -6 + 1) + (-3 + 3 + -2)$

$\text{Area} = \frac{1}{2}(4 - 6) + (-5 + 3)$

$\text{Area} = \frac{1}{2}(-2 + (-2))$

$\text{Area} = \frac{1}{2}(-2 + 2)$

$\text{Area} = \frac{1}{2}(0) = \frac{0}{2}$ $\quad$ $\text{Area} = -\frac{4}{2}$

$\text{Area} = +2\ cm^2$

$\therefore$ The area $= 2\ cm^2$

14. c) Given that $T(u) = 4, 1$

soln

we have $T(u) = (4, 1)$

$T(v) = (5, 2)$

$T[8v - 7u]$

$T(u) = 4i + j$

$T(v) = 5i + 2j$

$T[8v - 7u] = 8(4i + j) - 7(5i + 2j)$

$T[8v - 7u] = 32i + 8j - 35i - 14j$

$T[8v - 7u] = 32i - 35i + 8j - 14j$

$T[8v - 7u] = -3i - 6j$

$\therefore$ The value of $T(8v - 7u) = -3i - 6j$

Extract 14.2: A sample of the candidate's incorrect responses to question 14

In Extract 14.2, part (b), the candidate failed to use the correct formula of cross product of vectors to calculate the area of the triangle. In part (c), the candidate recalled incorrect formula for linear transformation properties.

3.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH TOPIC

The paper for Additional Mathematics in CSEE 2022 consisted of two sections A and B with a total of 14 questions from 14 topics. The tested topics were *Numbers*, *Variations*, *Statistics*, *Locus*, *Coordinate geometry*, *Symmetry*, *Logic*, *Sets*, *Algebra*, *Trigonometry*, *Function*, *Integration and Differentiation*, *Probability* and *Vectors* as well as *Linear Transformation*.

The analysis indicates that the candidates' performance was good in nine (9) topics. Those topics were: *Variations* (99.7%), *Statistics* (98.5%), *Vectors and Transformation* (90.9%), *Geometrical Constructions* (90.1%), *Probability* (89.6%), *Sets* (87.3%), *Numbers* (86.0), *Functions* (84.6%) and *Locus* (82.5%). The candidates' good performance in these topics was due to their ability to translate word problems into mathematical models, competence in the tested concepts, ability to identify the requirements of the questions and recall appropriate formula and laws.

The analysis shows further that five (5) topics were performed on average. The topics include: *Logic* (64%), *Integration and Differentiation* (58.9%), *Algebra* (55.6%), *Trigonometry* (50.8%) and *Coordinate Geometry* (47%). The candidates' average performance in these topics were due to lack of competences in the tested concepts, misinterpretation of word problems into mathematical models, inability to recall appropriate formula, errors in doing basic operations and failure to adhere to the requirement of the questions. The analysis of the candidates' performance on each topic is presented in Appendix I of the report.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The prepared candidates' Item Response Analysis (CIRA) report had the purpose of creating awereness to students, teachers and other educational stakeholders about the responses of the candidates in Additional Mathematics for CSEE in the year 2022. Therefore, the main objective of the analysis was to identify the strengths and weaknesses of the candidates' responses to different items on each question.

The analysis shows that 13 topics were examined for the two consecutive years. However, *Geometrical Constructions* and *Symmetry* topics were not examined for 2021 and 2022 respectively. It is revealed that, among the examined topics, seven (7) had an increase in performance while six (6) topics showed a decrease in performance compared to that of 2021. The comparison of the candidates' performance on each topic for the two consecutive years is shown in Appendix II.

Basically, the report has shown the key areas where the candidates performed well and on average. It is expected that the recommendations made in this report will be implemented in order to improve the performance on the Additional Mathematics examination in future.

4.2 Recommendations

In order to improve the candidates' performance in this subject, it is recommended that;

(a) Students should be encouraged to participate fully in the mathematics clubs' discussions in order to improve their competence in Additional Mathematics.

(b) Teachers should ensure that the real life related practices are incorporated when instructing the students during teaching and learning process, including the use of various teaching aids to improve students' competences.

(c) The candidates should make sure that, they read and understand the requirement of the questions so that they can provide the related solutions.

APPENDIX I: Analysis of Candidates' Performance on Each Topic

S/N	Topics	Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks
1.	Variations	1	99.7	Good
2.	Statistics	2	98.5	Good
3.	Vectors and Transformations	14	90.9	Good
4.	Geometrical Constructions	6	90.1	Good
5.	Probability	13	89.6	Good
6.	Sets	10	87.3	Good
7.	Numbers	8	86.0	Good
8.	Functions	11	84.6	Good
9.	Locus	4	82.5	Good
10.	Logic	9	64.0	Average
11.	Integration and Differentiation	12	58.9	Average
12.	Algebra	5	55.6	Average
13.	Trigonometry	7	50.8	Average
14.	Coordinate Geometry	3	47.0	Average

APPENDIX II: Comparison of Candidates' Performance on Each Topic in 2021 and 2022

S/N	Topics	2021			2022		
		Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks
1.	Variations	1.	98.8	Good	1.	99.7	Good
2.	Statistics	2	87.6	Good	2.	98.5	Good
3.	Vectors and Transformations	14.	72.5	Good	14.	90.9	Good
4.	Geometrical Constructions				6.	90.1	Good
5.	Probability	13.	77.8	Good	13.	89.6	Good
6.	Sets	10.	79.7	Good	10.	87.3	Good
7.	Numbers	8.	94.4	Good	8.	86.0	Good
8.	Functions	11.	88.4	Good	11.	84.6	Good
9.	Locus	4.	69.7	Good	4.	82.5	Good
10.	Logic	9.	56.9	Average	9.	64.0	Average
11.	Integration and Differentiation	12.	66.9	Good	12.	58.9	Average
12.	Algebra	5.	93.4	Good	5.	55.6	Average
13.	Trigonometry	7.	64.4	Average	7.	50.8	Average
14.	Coordinate Geometry	3.	96.2	Good	3.	47.0	Average
15.	Symmetry	6.	90.4	Good			