THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATIONS (CSEE) 2021

ADDITIONAL MATHEMATICS

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042 ADDITIONAL MATHEMATICS

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TABLE OF CONTENTS

FOREWORD

The National Examinations Council of Tanzania is pleased to issue this report on Candidates' Item Response Analysis (CIRA) in Additional Mathematics for the Certificate of Secondary Education Examination (CSEE) 2021. The report is aimed at informing teachers, parents, policy makers and other educational stakeholders on how the candidates responded to the examination items. The report will enable the stakeholders to take appropriate measures to enhance students' performance and enable students to master the topics, which need more emphasis during teaching and learning.

The analysis of the candidates' responses was done in order to identify the strengths and weaknesses of the candidates on the deliverance of what they learnt. This will show what the education system has managed or failed to offer to learners in their four years of secondary education.

Good performance was contributed by the candidates' ability to solve mathematical problems on variations, algebraic equations, symmetric problems and being able to apply the theorems of probability and principle of permutation to solve problems. Other factors include the candidates' ability to perform basic operation on sets, rational function, linear transformation and dot product as well as determining the asymptotes. It also includes the ability to apply the methods and techniques of integration and differentiation in solving mathematical problems. The average performance was contributed by candidates' failure to apply the concepts of the double angles in solving related problems, construct truth tables as well as interpreting and applying the laws of algebra of propositions.

The National Examinations Council of Tanzania expects that, the education stakeholders will work on the challenges the candidates faced when attempting the examination questions so as to improve the performance in future examinations.

Finally, the National Examinations Council would like to thank the examination officers and all stakeholders who participated in the preparation of this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is a result of the analysis of candidates' items responses as examined in 042 Additional Mathematics for the Certificate of Secondary Education Examination (CSEE) 2021. The report is written for the purpose of providing feedback to stakeholders on the performance of the students who sat for the CSEE 2021. The paper was set according to the 2018 Examination format and the 2010 Additional Mathematics Syllabus for Secondary Schools. The report focuses on areas in which the candidates performed well as well as areas in which candidates faced challenges.

The examination paper consisted of two sections, A and B with a total of fourteen (14) questions. Section A comprised of 10 questions carrying 6 marks each while Section B had four (4) questions of 10 marks each. The candidates were required to answer all questions in both Sections.

In 2021, a total of 320 candidates sat for examination, of which 315 (98.75 %) candidates passed. However, the results of 1 candidate were withheld due to some reasons. In comparison, in 2020 a total of 339 candidates sat for the examination, whereby 319 (94.10%) candidates passed. Table 1 presents the summary of the candidates' performance in 2020 and 2021.

Table 1: Candidates' Performance in Additional Mathematics CSEE 2020 and 2021

Year	Students Sat	Passed		Grades				
		No.	%	A	B	C	D	F
2020	339	319	94.10	49	45	127	98	20
2021	320	315	98.75	72	63	126	54	04

Table 1 shows that the candidates' performance in 2021 had increased by 4.75 per cent compared to that of 2020. The percentages of candidates who passed in Additional Mathematics got different grades is shown in Figure 1.

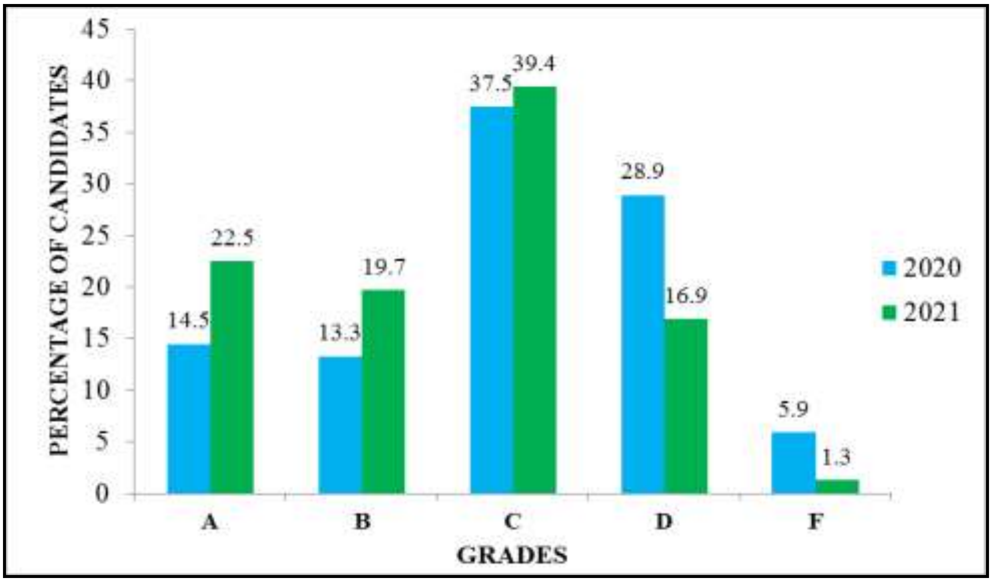


Figure 1: *Distribution of Grades A, B, C, D and F for the 2020 and 2021 Additional Mathematics results in CSEE*

In Figure 1, the trend shows that, the quality of performance in 2021 has risen compared to 2020. For example, the number of candidates who scored grade A in 2021 has increased by 8%, grade B has increased by 6.4% and grade C has increased by 1.9%. On the other hand, there is a decrease in number of candidates who scored grade F by 4.6%.

The analysis of candidates' performance in each questions is presented in section 2.0. It consists of descriptions of the requirements of the questions and responses by the candidates. It also includes extracts showing the strengths and weaknesses demonstrated by the candidates in answering each question.

The candidates' performance in each question is categorized by using percentage of candidates who scored 30 per cent or more of the total marks allocated to a particular question. The performance was categorized in three groups: 65 to 100 per cent for good performance; 30 to 64 per cent for average performance; and 0 to 29 per cent for weak performance. Furthermore, green, yellow and red colours were used to indicate good, average and weak performance respectively.

In section 3.0 the factors which contributed to weak performance in some topics are highlighted and recommendations for performance improvement in future examinations are provided in section 4.0. Additionally, the analysis of candidates' response per topic is presented.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE IN EACH QUESTION

This section presents the analysis of candidates' performance in each question. The National Examination results were based on the score intervals of 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 which are equivalent to excellent, very good, good, satisfactory and fail respectively. For the purpose of this report, the performance in each question is described into three categories; good, average or weak if the percentage of candidates who scored 30 per cent or more of the marks allocated for the question is in the intervals of 65 – 100, 30 – 64 and 0 – 29 respectively.

2.1 Question 1: Variations

The question had two parts, (a) and (b). In part (a), the candidates were required to write the variation equation expressing x in term of t. By using the information given that; an object falls a vertical distance x which varies directly as the square of the time t, if it falls 900 cm in 20 seconds. In part (b), they were required to find the value of y when x was 6. Given that y was inversely proportional to x^2, if $y = 4$ when $x = 3$.

The analysis shows that 320 (100%) candidates attempted the question out of which 4 (1.2%) candidates scored from 0.0 to 1.5 marks, while 20 (6.3%) candidates scored from 2.0 to 3.0 marks and 296 (92.5%) candidates scored 3.5 to 6.0 marks. The candidates' performance summary is presented in Figure 2.

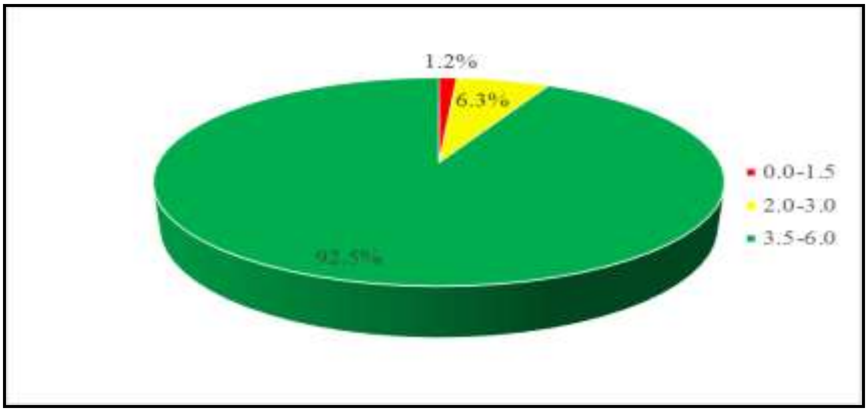


Figure 2: *Candidates' Performance in Question 1*

This result shows that, the candidates' performance in this question was good. In part (a), the candidates who performed well in this question managed to formulate the equation expressing x in term of t. That is, they correctly realized the symbol for direct variation and wrote $x \propto t^2$, then were able to substitute $x = 900\,\text{cm}$ and $t = 20\,\text{s}$ in the equation $x = kt^2$, finally, they correctly got $k = \frac{9}{4}$, thereafter substituted the value of k into the equation $x = kt^2$ to obtain $x = \frac{9}{4}t^2$ which was the required equation.

In part (b), the candidates managed to write the statement that, y is inversely proportional to x^2 as $y \propto \frac{1}{x^2}$, hence introduced a proportionality constant k to obtain the equation $y = k\frac{1}{x^2}$. They were able to substitute the values of $y = 4$ and $x = 3$ into the equation and get $k = 36$. Thereafter, they correctly substituted the obtained value of $k = 36$ and $x = 6$ into the equation $y = \frac{k}{x^2}$ to obtain $y = \frac{36}{6^2} = 1$ which was the required answer. Extract 1.1 is a sample response of one of the candidates who attempted the question correctly.

1\. a) $x \propto t^2$

$x = kt^2$

$k = x/t^2$

$k = 900/(20^2)$

$k = \frac{900}{400}$

$k = 9/4$

$\therefore\ x = (9/4)t^2$

$\therefore$ The variation equation is $x = (9/4)t^2$

b) $y \propto 1/x^2$

$y = k/x^2$

$k = x^2y$

$k = (3)^2 \times 4$

$k = 9 \times 4$

$k = 36$

$y = 36/x^2$

When $x = 6$

$y = 36/6^2$

$y = 36/36$

$y = 1$

$\therefore$ The value of y is 1 when x is 6

Extract 1.1: A sample of correct responses in question 1

In extract 1.1, the candidate managed to translate correctly the given problem into mathematical equation in part (a), and was thus able to get the required equation. In part (b), the candidate translated the given problem into mathematical equation and calculated correctly the value of y.

However, 1.2 per cent of the candidates who attempted this question scored low marks due to some challenges they faced. For example, in part (a), some candidates failed to interpret correctly the given problem, which led them to formulate incorrect mathematical equations such as $x = kt$ instead of $x = kt^2$. Then, after substituting the given values of $x = 900$ cm and $t = 20$ s into the equation $x = kt$ they got $k = 45$ instead of $k = \frac{9}{4}$. So they ended up producing incorrect equation $k = 45t$ instead of $x = \frac{9}{4}t^2$. Some other candidates wrote $x \propto \sqrt{t}$ instead of $x = kt^2$, and got the incorrect equation, $k = \frac{900}{\sqrt{2}}\sqrt{t}$.

In part (b), the analysis shows that some candidates failed to interpret the given variation and got the incorrect mathematical equation $y = kx^2$ instead of $y = \frac{k}{x^2}$ leading to obtaining the incorrect value of $y = 16$. Furthermore some candidates got the equation, $y = \frac{k}{x}$, hence obtained the incorrect value of $y = 6$ instead of $y = 1$. Extract 1.2 is a sample response selected from one of the candidates who faced challenges in attempting the question.

9 (a)

Given, distance $x = 900cm$.

time $t = 20$ sec.

But

$x \propto t$

$900x \propto 20t$

$\frac{900x}{900} = \frac{k \cdot 20t}{900}$

$\therefore x = \frac{20t \cdot k}{900}$

(b) $y \propto x^2$

$y = kx^2$

$4 = k(3)^2$

$\frac{4}{9} = \frac{k9}{9}$

$k = 4/9$

$y = kx^2$

$y = \frac{4}{9} \times (6)^2$

$y = \frac{4}{9} \times 36$

$y = 4 \times 4$

$y = 16$.

$\therefore$ The value of y when x is 6 is $y = 16$.

Extract 1.2: A sample of incorrect responses in question 1

In extract 1.2, it is observed that the candidate failed to interpret correctly the given problem in both parts (a) and (b). Consequently, provided incorrect responses.

2.2 Question 2: Statistics

The candidates were given the table, which showed yields of gold in tones produced by 100 traders at a certain mine in one day as the following table shows:

Gold in tons	15 – 20	21 – 26	27 – 32	33 – 38	39 – 44	45 – 50
Frequency	10	22	32	21	13	2

They were required to:

(a) Calculate the mean and mode if the assumed mean was 29.5 tones.

(b) Draw the cumulative frequency curve and from it calculate the semi – interquartile range.

The analysis reveals that 280 (87.6%) out of 320 (100%) candidates who attempted the question scored from 2.0 to 6.0 marks. The summary of candidates' performance in percentage for this question is presented in figure 3 showing the candidates who obtained good, average and low scores.

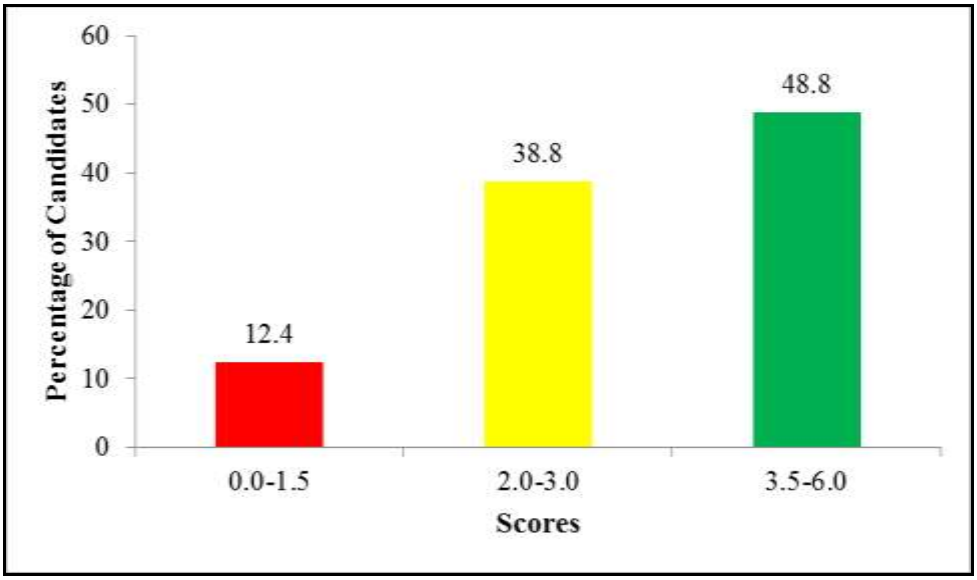


Figure 3: *Candidates' Performance in Question 2*

Figure 3 indicates the candidates' performance in this question was good.

The analysis depicts that, in part (a), the candidates who performed well in this question and manage to score all marks were able to construct correctly the frequency distribution table with columns of class boundaries, frequency, cumulative frequency, class marks (x), $d = x - A$ and fd. Then, they were able to recall correctly the formula for the mean as $\bar{x} = A + \frac{\sum fd}{N}$, hence identified that $A = 29.5$, $\sum fd = 66$ and $N = 100$.

Thereafter, they were able to substitute into the formula $\bar{x} = A + \frac{\sum fd}{N}$ to get $\bar{x} = 29.5 + \frac{66}{100}$, then computed and obtained $\bar{x} = 36.16$ tones which was the required answer. They also managed to recognize that the modal class is the class interval 27-32, hence they were able to recall correctly the formula for calculating mode $\hat{x} = L_1 + \left(\frac{t_1}{t_1 + t_2}\right) \times i$, Where $L_1 = 26.5$, $t_1 = 32 - 22 = 10$, $t_2 = 32 - 21 = 11$, $i = 6$. So they managed to substitute in the formula $\hat{x} = 26.5 + \left(\frac{10}{10 + 11}\right) \times 6$ to obtain $\hat{x} = 29.36$ tones which was the correct answer.

In part (b), the candidates drew correctly the cumulative frequency curve using cumulative frequency on a vertical line and the upper real limit on the horizontal line and later joined them correctly to obtain the curve hence correctly labelled as a cumulative frequency curve (Ogive).

Likewise, the candidates recalled correctly the position of lower quartile $q_1 \rightarrow \left(\frac{N}{4}\right)^{th}$ and upper quartile $q_3 \rightarrow \left(\frac{3N}{4}\right)^{th}$, then computed $q_1 \rightarrow \left(\frac{100}{4}\right)^{th} = 25^{th}$, $q_3 \rightarrow \left(\frac{3 \times 100}{4}\right) = 75^{th}$

From the graph $Q_1 = 24.59 \approx 24.6$ and $Q_3 = 35.64 \approx 35.6$, there after they calculated the semi interquartile range using a formula $\frac{Q_3 - Q_1}{2}$, substituted the values of Q_1 and Q_3 to obtain $Q_2 = 5.5$ which was the correct answer.

Extract 2.1 is a sample response of a candidate who attempted the question correctly.

2 a) solution:

Class interval	x	$d = x - A$	f	fd
15 − 20	17.5	−12	10	−120
21 − 26	23.5	−6	22	−132
27 − 32	29.5	0	32	000
33 − 38	35.5	6	21	126
39 − 44	41.5	12	13	156
45 − 50	47.5	18	2	36
			N = 100	$\Sigma fd = 66$

From

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 29.5 + \frac{66}{100}$$

$$= 29.5 + 0.66$$

$$= 30.16$$

$\therefore$ Mean = 30.16 tones.

From

$$\text{mode} = L + \left(\frac{t_1}{t_1 + t_2}\right)C$$

Modal class = 27 − 32

$L = 26.5$

$t_1 = 10$

$t_2 = 11$

$C = 23.5 - 17.5 = 6$

Now

$$\text{mode} = 26.5 + \left(\frac{10}{21}\right)6$$

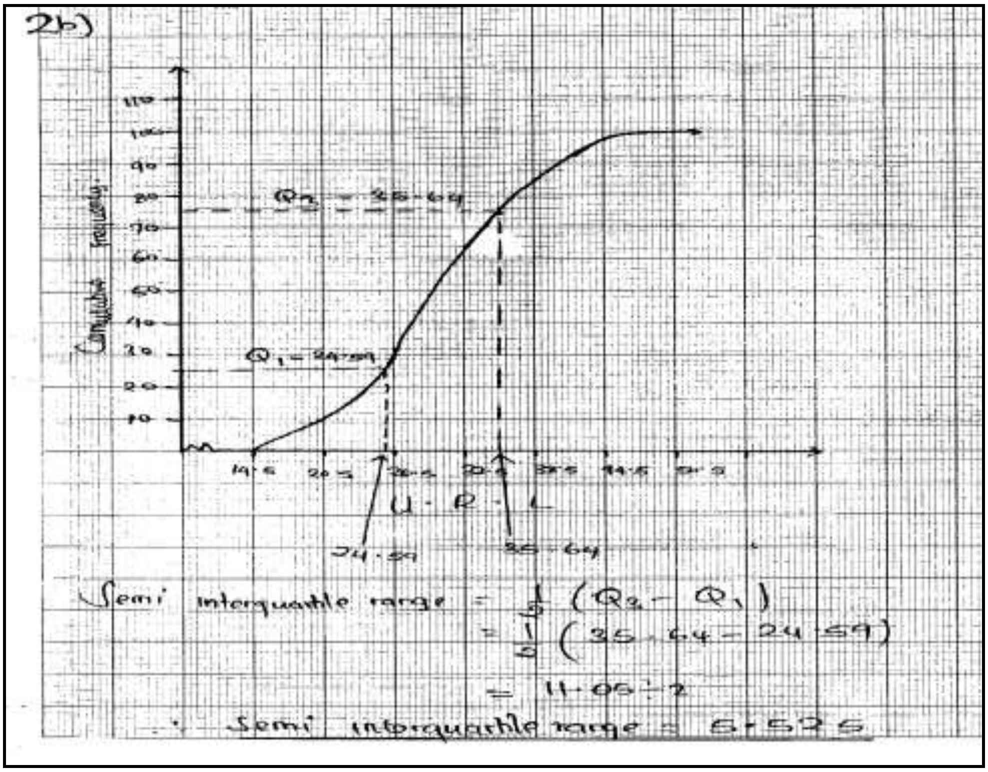
2a) Mode = 26.5 + 2.86

= 29.36

$\therefore$ Mode = 29.36 tones.

2b) SOLUTION:

Class interval	U.R.L	f	Cumulative frequency
15 − 20	20.5	10	10
21 − 26	26.5	22	32
27 − 32	32.5	32	64
33 − 38	38.5	21	85
39 − 44	44.5	13	98
45 − 50	50.5	02	100



Extract 2.1: A sample of correct responses in question 2

In extract 2.1, the candidate applied the appropriate formulas to calculate the mean and the mode in part (a). In part (b), the candidate demonstrated good understanding of the cumulative frequency curve by drawing using cumulative frequency against upper real limits. Thereafter, used it to determine the semi interquartile range.

Despite the good performance demonstrated by some candidates, there were 42 (13.0%) candidates who scored low marks.

In part (a), candidates constructed a frequency distribution table correctly, but made some computational errors when calculating $\sum fd$ which led to getting incorrect answer regarding the mean value. For example, one candidate obtained $\sum fd = 162$ instead of $\sum fd = 66$. Furthermore, some other candidates applied inappropriate formula for the mode instead of using $\hat{x} = L + \left(\frac{t_1}{t_1 + t_2}\right)i$, they applied the formula used to find the median, and wrote Mode $= L + \dfrac{\left(\frac{N}{2} - n_b\right)i}{n_w}$, then $L = 26.5$, $N = 100$, $n_b = 32$, $n_w = 32$, $i = 6$. They substituted in the formula as $26.5 + \left(\dfrac{\frac{100}{2} - 32}{32}\right) \times 6$, and computed wrongly to get the mode $= \dfrac{538}{16}$ instead of 29.36 tones. Some candidates were able to recall correctly the formula for mode as, $\hat{x} = L + \left(\dfrac{t_1}{t_1 + t_2}\right)c$ but failed to identify the correct values of t_1 and t_2. Consequently they got $t_1 = 85$ and $t_2 = 32$ instead of $t_1 = 10$ and $t_2 = 11$ hence got incorrect answer. This shows that some candidates had inadequate knowledge and skills on statistics especially on determining t_1 and t_2.

In part (b), few candidates constructed incorrect frequency distribution table which led them to get incorrect cumulative frequency curve as illustrated in extract 2.2. However, some other candidates mistook the cumulative frequency curve for a bar graph. In drawing they used frequency against class interval and drew bar graphs instead of cumulative frequency curve as seen in extract 2.2. Moreover, when calculating semi – interquartile range, some candidates failed to abide by the instructions as they applied the formula $Q_1 = L + \left(\dfrac{\frac{i \times N}{4} - n_b}{n_w}\right)c$ instead of using the graph. Extract 2.2 is a sample response from one of the candidates who faced challenges in attempting this question.

2

C-Interval	f	X	fx	d=x-A	fd
15-20	10	17.5		-12	-120
21-26	22	23.5		-6	-132
27-32	32	29.5		0	0
33-38	21	35.5		6	126
39-44	13	41.5		12	156
45-50	2	47.5		16	32

$\Sigma fd = 162$

$$\text{Mode} = L + \left(\frac{N/2 - nb}{nw}\right)i$$

$$= 26.5 + \left(\frac{100/2 - 32}{32}\right)6$$

$$= 26.5 + \left(\frac{50 - 32}{32}\right)6$$

$$= 26.5 + \left(\frac{18}{32}\right)6$$

$$= 26.5 + \left(\frac{9}{16}\right)6$$

$$= \frac{265}{10} + \frac{54}{16}$$

2 (a) $\frac{265}{10} + \frac{54}{16}$

$\frac{4840+540}{160}$

$= \frac{538\cancel{0}}{16\cancel{0}}$

$= \frac{538}{16}$

$\therefore$ Mode $= \frac{538}{16}$

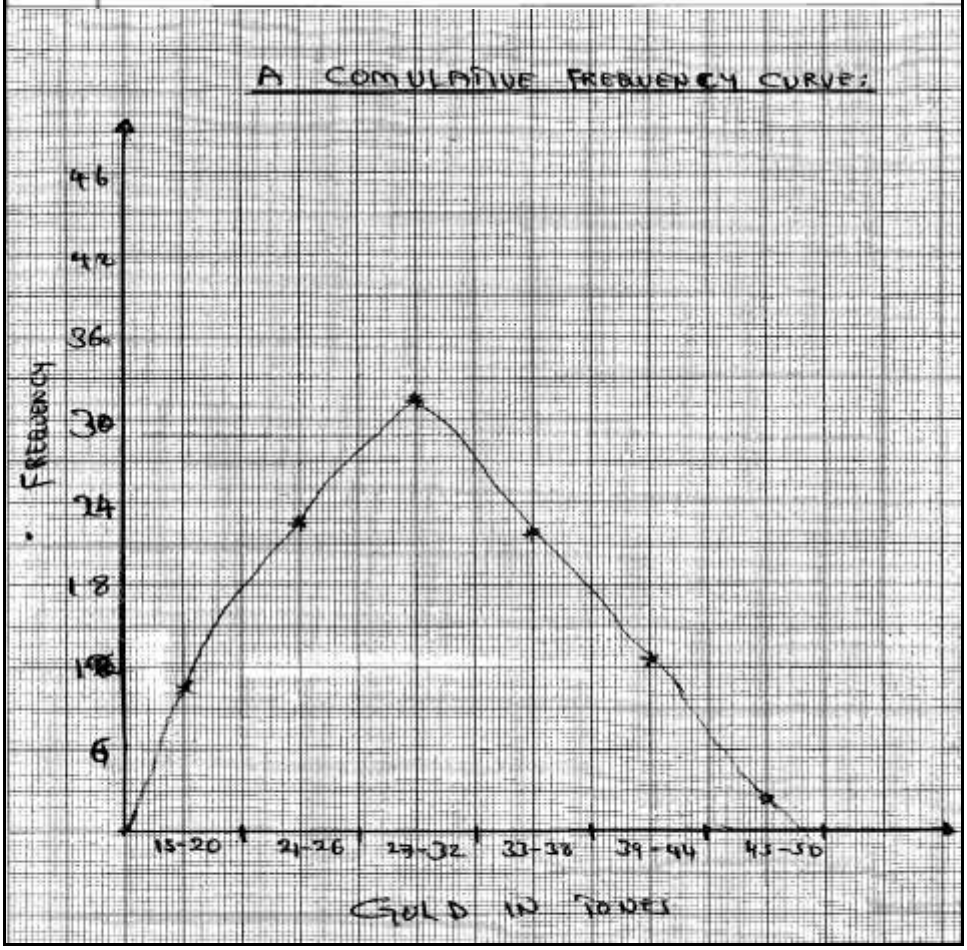
Mean $= A + \frac{\Sigma fd}{N}$

$= 29.5 + \frac{162}{100}$

$= 29.5 + 1.62$

$= 31.12$

$\therefore$ The Mean is 32.2



Extract 2.2: A sample of incorrect responses in question 2

Extract 2.2 is a sample of one of the incorrect responses, where in part (a), the candidate applied inappropriate formula for calculating the median instead of the mode. In part (b), the candidate drew a curve using the normal frequencies and class intervals instead of the cumulative frequencies and upper real limits.

2.3 Question 3: Coordinate Geometry

This question consisted of two parts, (a) and (b). In part (a), the candidates were required to determine the value of y, if the points $P(2,4)$, $Q(3,y)$ and $R(-3,4)$ were collinear. In part (b), they were required to determine the coordinates of the point dividing the line joining points $(2,3)$ and $(4,6)$ in the ratio 1:3; (i) internally (ii) externally.

As the analysis indicates, the performance of the candidates who scored the marks from 0.0 to 1.5 were 3.8 per cent, while 21.2 per cent scored 2.0 to 3.0 marks and 75 per cent scored 3.5 to 6.0 marks. Figure 4 presents the candidates' performance summary in this question.

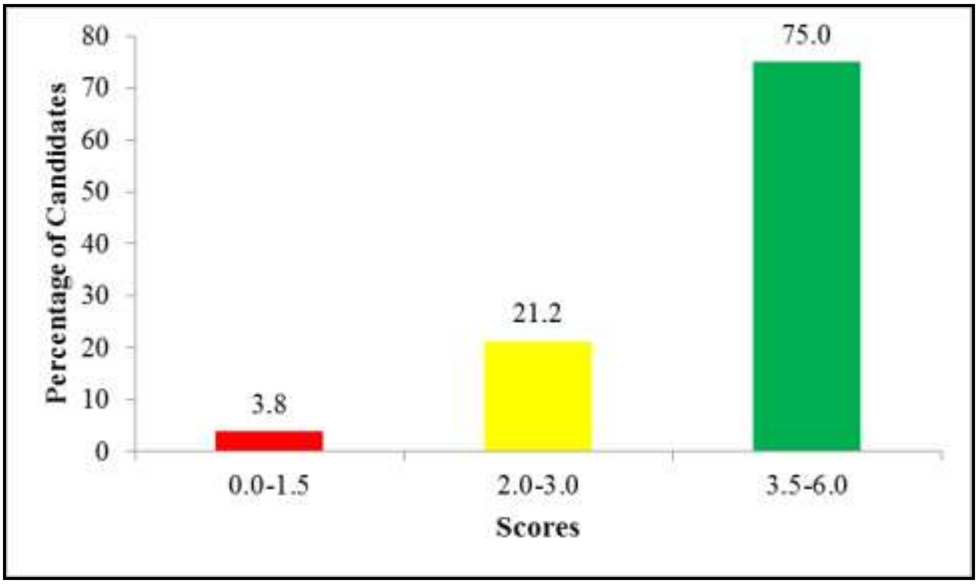


Figure 4: *Candidates' Performance in Question 3*

From Figure 4 it is observed that the candidates' performance in this question was good since 96.2 per cent of the candidates scored within a range of 2.0 to 6.0 marks.

In part (a), the candidates who responded correctly and managed to score all marks were familiar with the concept of collinear points. The points are collinear if they lie on the same straight line, hence have the same slope. Then they were able to recall correctly the formula for finding gradient and the condition for parallel lines as $M = \frac{y_2 - y_1}{x_2 - x_1}$ and $M_1 = M_2$ respectively. Thus, the gradient of $\overline{PQ}$ = gradient of $\overline{PR}$ = gradient of $\overline{QR}$. Thereafter, they computed the gradient of $\overline{PQ}$ as $M_1 = \frac{y-4}{3-2}$ and obtained $M_1 = y - 4$. Again they were able to compute the gradient of $\overline{PR}$ as $M_2 = \frac{4-4}{-3-3}$ to obtain $M_2 = 0$. Then, they equated the two gradients as $y - 4 = 0$ and got $y = 4$ which was the required answer.

In part (b) (i), the candidates were able to realize that the coordinate such as $A(x, y)$ divides the line internally in the ratio $\lambda : \mu$. Then, from the given ratio $1:3$, they managed to realize that $\lambda = 1$ and $\mu = 3$. Thereafter, the candidates were able to apply the correct formula for internal division as $A(x, y) = \left(\frac{\lambda x_2 + \mu x_1}{\lambda + \mu}, \frac{\lambda y_2 + \mu y_1}{\lambda + \mu} \right)$ and substituted the points in the formula as $A(x, y) = \left(\frac{(1 \times 4) + (3 \times 2)}{1+3}, \frac{(1 \times 6) + (3 \times 3)}{1+3} \right)$. Finally they simplified to get $A(x, y) = \left(\frac{5}{2}, \frac{15}{4} \right)$ which were the correct coordinates.

In part (b) (ii), the candidates correctly considered the coordinate $A(x, y)$ which divides the line externally in the ratio $1:3$, taking $\lambda : \mu = 1:3$. Then, they were able to apply the formula for external division, $A(x, y) = \left(\frac{\lambda x_2 - \mu x_1}{\lambda - \mu}, \frac{\lambda y_2 - \mu y_1}{\lambda - \mu} \right)$. Thereafter, they substituted the points in the formula as $A(x, y) = \left(\frac{(1 \times 4) - (3 \times 2)}{1-3}, \frac{(1 \times 6) - (3 \times 3)}{1-3} \right)$, then simplified it to get $A(x, y) = \left(\frac{4-6}{-2}, \frac{6-9}{-2} \right)$, and finally got, $A(x, y) = \left(1, \frac{3}{2} \right)$ which was the required answer. Extract 3.1 is a sample response from one of the candidates who attempted the question correctly.

3. a) Soln

For collinear points, $m_1 = m_2$

But,

$P(2,4)$ $Q(3,y)$ $R(-3,4)$

$x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$

So,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_1}{x_3 - x_1}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_1}{x_3 - x_1}$$

$$\frac{y - 4}{3 - 2} = \frac{4 - 4}{-3 - 2}$$

$$\frac{y - 4}{1} = \frac{0}{-5}$$

$$y - 4 = \frac{0}{-5}$$

$$y - 4 = 0$$

$$y = 4$$

$\therefore$ The value of y is 4.

3. b) i) internally = ? Soln

Points: $(2,3)$ and $(4,6)$ $m:n = 1:3$

$x_1\ y_1$ $x_2\ y_2$

From,

$$(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$(x,y) = \left(\frac{1\times4 + 3\times2}{1+3}, \frac{1\times6 + 3\times3}{1+3}\right)$$

$$(x,y) = \left(\frac{4+6}{4}, \frac{6+9}{4}\right)$$

3\. b) i) Soln

$(x, y) = \left(\frac{5}{2}, \frac{15}{4}\right)$

$\therefore$ The coordinates of the points are $\left(\frac{5}{2}, \frac{15}{4}\right)$.

3\. b) ii) externally = ? Soln

Points:

$(2, 3)$ and $(4, 6)$ $m:n = 1:3$

$x_1 \; y_1 \qquad x_2 \; y_2$

From,

$(x, y) = \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$

$(x, y) = \left(\frac{1 \times 4 - 3 \times 2}{1 - 3}, \frac{1 \times 6 - 3 \times 3}{1 - 3}\right)$

$(x, y) = \left(\frac{4 - 6}{-2}, \frac{6 - 9}{-2}\right)$

$(x, y) = \left(\frac{-2}{-2}, \frac{-3}{-2}\right)$

$(x, y) = (1, 3/2)$

$\therefore$ The coordinates of the points are $(1, 3/2)$.

Extract 3.1: A sample of correct responses in question 3

From extract 3.1, it is observed that in part (a), the candidate was able to identify the gradients of the two lines joining the collinear points which were equal. Then he/she was able to calculate the value of y after equating the gradients of $\overline{PQ}$ and $\overline{PR}$. In part (b), the candidate recalled correctly the formula for internal and external division of a line. Then, managed to calculate the coordinates of the point dividing the line internally and externally.

Despite the good performance in general, in this question 3.8 per cent of the candidates scored low marks. Those candidates performed weakly due to the following weaknesses;

In part (a), some of the candidates failed to understand the concept of collinear points. They added the points instead of equating the gradients of the lines joining the points. For example, one of the candidates responded by taking $P(2,4)+Q(3,y)+R(-3,4)$ and got $(2, y+8)$ and finally manipulated it to get $y=2+8=10$ instead of $y=4$. Some candidates applied the formula incorrectly to determine the gradient of the line as $m=\frac{x_2-x_1}{y_2-y_1}$ instead of $m=\frac{y_2-y_1}{x_2-x_1}$. For example, one of the candidates worked on the gradient $\overline{PQ}$ as $m_1=\frac{3-2}{y-4}=\frac{1}{y-4}$, and gradient $\overline{PR}$ as $m_2=\frac{-3-2}{4-4}=\frac{-5}{0}$. Finally equated the two gradients and obtained $\frac{1}{y-4}=\frac{-5}{0}$.

Moreover, some candidates interpreted the collinear points as points having equal distances between them, thus they considered the distance of $\overline{RQ}=$ distance of $\overline{PR}$. For, instance one of the candidates who failed to get it correctly, applied a distance formula , $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. Then substituted the points as $\sqrt{(3--3)^2+(y-4)^2}=\sqrt{(2+3)^2+(4-4)^2}$, to get $y^2-8y+27=0$.

In part (b) (i), the analysis depicts that some candidates applied incorrect formula $\left(\frac{mx_1-nx_2}{m+n}, \frac{my_1-ny_2}{m+n}\right)$ for finding internal division instead of $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$. While others used the formula $\left(\frac{mx_1+ny_1}{m+n}, \frac{mx_2+ny_2}{m+n}\right)$ instead of $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$. For example, one of the candidates used $\left(\frac{mx_1+ny_1}{m+n}, \frac{mx_2+ny_2}{m+n}\right)$, thereafter substituted the given points and computed it to obtain $\left(\frac{11}{4}, \frac{22}{4}\right)$ instead of $\left(\frac{5}{2}, \frac{15}{4}\right)$. Additionally, in part (b) (ii), some candidates applied inappropriate formula for external division of a line as $\left(\frac{mx_1+nx_2}{m+n}, \frac{my_1+ny_2}{m+n}\right)$ while others consider $\left(\frac{mx_1-ny_1}{m-n}, \frac{mx_2-ny_2}{m-n}\right)$ instead of $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$. For example, one of the candidates who used the formula $\left(\frac{mx_1-ny_1}{m-n}, \frac{mx_2-ny_2}{m-n}\right)$, substituted the given coordinates as $\left(\frac{(1\times 2)-(3\times 3)}{3-1}, \frac{(1\times 4)-(3\times 6)}{3-1}\right)$. Then simplified to obtain $\left(\frac{-7}{2}, -7\right)$ instead of $\left(1, \frac{3}{2}\right)$. This implies that some candidates had insufficient knowledge on the concept tested. Extract 3.2 is a sample response from one of the candidates who incorrectly responded to this question.

$$\overline{RQ} = DR$$

$$\sqrt{(3+3)^2 + (y-4)^2} = \sqrt{(2+3)^2 + (4-4)^2}$$

$$\sqrt{81 + y^2 - 8y + 16}^2 = (\sqrt{25})^2$$

$$y^2 - 8y + 97 = 25$$

$$y^2 - 8y + 97 - 25 = 0$$

$$y^2 - 8y + 72 = 0$$

$$\_\ \times\ \_ = 72$$

$$\_\ +\ \_ = -8$$

3 b) Given .

$x_1y_1 = (2,3)$

$x_2y_2 = (4,6)$

Division ratio $= 1:3 = m:n$

$\cancel{\text{i) Internally } \left(\frac{mx_1 - ny_1}{m+n}, mx_2 - n_v\right.}$

i) Internally

$$\left(\frac{mx_1 - nx_2}{m+n}, \frac{my_1 - ny_2}{m+n}\right)$$

$$= \left(\frac{1\times2 - 3\times4}{3+1}, \frac{1\times3 - 3\times6}{3+1}\right)$$

$$= \left(-\frac{5}{2}, -\frac{15}{4}\right)$$

$\therefore$ Coordinates of a point dividing the line internally are $\left(-\frac{5}{2}, -\frac{15}{4}\right)$

ii) Externally

$$\left(\frac{mx_1 + nx_2}{m+n}, \frac{my_1 + ny_2}{m+n}\right)$$

$$= \left(\frac{1\times2 + 3\times4}{3+1}, \frac{1\times3 + 3\times6}{3+1}\right)$$

$$= \left(\frac{7}{2}, \frac{21}{4}\right)$$

$\therefore$ Coordinates of a point dividing the line externally are $\left(\frac{7}{2}, \frac{21}{4}\right)$

Extract 3.2: A sample of incorrect responses in question 3

In extract 3.2, the candidate applied inappropriate formula used to find distance instead of using a formula for finding a gradient, hence failed to produce a correct response. In part (b), a candidate applied inappropriate formula for both (i) and (ii). Thereafter, obtained incorrect response after substituting the given points and the dividing ratio.

2.4 Question 4: Locus

This question had two parts, (a) and (b). In part (a), the candidates were required to define "Locus of a point" as applied in Mathematics. While in part (b), it was given that: the Cartesian coordinates of the points A and B are $(-3, 0)$ and $(3, 0)$ respectively. If point P moves so that $AP = 2PB$, then they were required to prove that its locus was the circle Z whose equation was $x^2 + y^2 - 10x + 9 = 0$.

The analysis shows that 320 (100%) candidates attempted this question, out of which 30.3 per cent scored 0.0 to 1.5 marks, while 9.7 per cent scored 2.0 to 3.0 marks. Furthermore, 60.0 per cent of the candidates scored 3.5 to 6.0 marks. Therefore, the overall performance in this question was good. Figure 5 illustrates the candidates' performance of the question.

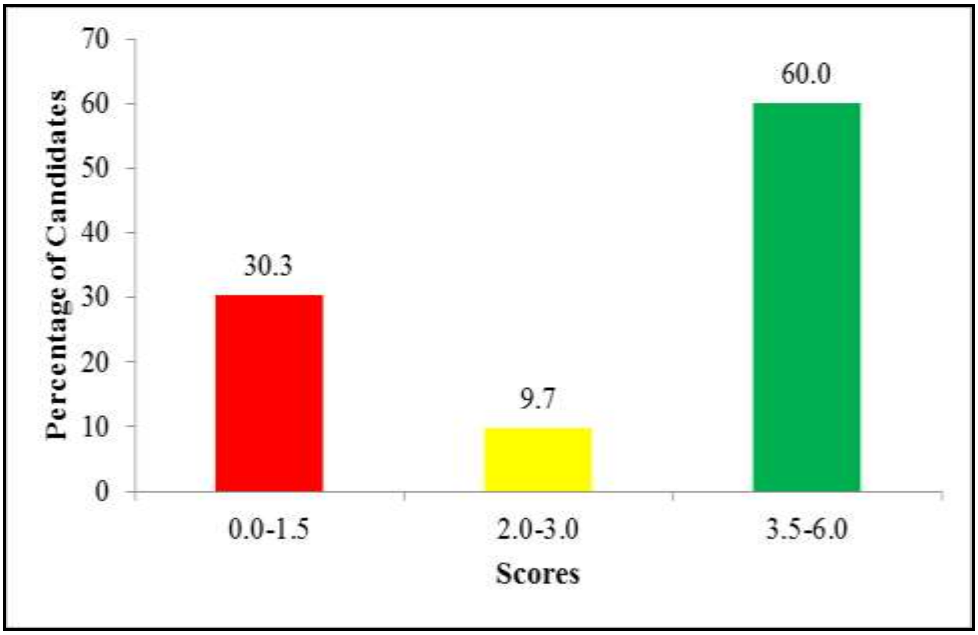


Figure 5: *Candidates' Performance in Question 4*

The analysis shows that, the candidates who performed well in this question and managed to score full marks had adequate knowledge and skills on locus. Thus, in part (a), they were able to define correctly the locus of a point as the "path traced out by the point as it moves under a certain condition". In part (b), the candidates let the coordinate of moving point be $P(x, y)$, then they correctly recalled the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Then, they were able to calculate the distance of point $P(x, y)$ from $A(-3, 0)$. That is $\overline{AP} = d_1 = \sqrt{(x+3)^2 + (y-0)^2}$. Then they took a square on both sides, and simplified it to get $d^2 = x^2 + y^2 + 6x + 9$ as equation (i). Likewise they calculated the distance of point $P(x, y)$ from $B(3, 0)$ as $\overline{BP} = d_2 = \sqrt{(x-3)^2 + (y-0)^2}$, then, took a square both sides, and simplified it to get $d^2 = x^2 + y^2 - 6x + 9$ as equation (ii). They were instructed that $\overline{AP} = 2\overline{BP}$ which is equivalent to $\left(\overline{AP}\right)^2 = 4\left(\overline{BP}\right)^2$. They substituted the two equations into the given relation as $x^2 + y^2 + 6x + 9 = 4(x^2 + y^2 - 6x + 9)$. After rearranging and collecting like terms, they got $3x^2 + 3y^2 - 3x + 27 = 0$, which was simplified further to obtain $x^2 + y^2 - 10x + 9 = 0$ which was the given equation. Extract 4.1 is a sample response from one of the candidates who attempted the question correctly.

4. a) Locus of a point ~~is~~ Is a path or way taken by a point (x, y) under a certain condition. Example a point moving along circumference of a circle.

b) $AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x+3)^2 + (y-0)^2}$

$= \sqrt{x^2 + 6x + 9 + y^2}$

$AP = \sqrt{x^2 + y^2 + 6x + 9}$

Again;

$PB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x-3)^2 + (y-0)^2}$

$= \sqrt{x^2 - 6x + 9 + y^2}$

$= \sqrt{x^2 + y^2 - 6x + 9}$

But

$AP = 2PB$

$\sqrt{x^2 + y^2 + 6x + 9} = 2\sqrt{x^2 + y^2 - 6x + 9}$

$\left(\sqrt{x^2 + y^2 + 6x + 9}\right)^2 = \left(2\sqrt{x^2 + y^2 - 6x + 9}\right)^2$

$x^2 + y^2 + 6x + 9 = 4(x^2 + y^2 - 6x + 9)$

$x^2 + y^2 + 6x + 9 = 4x^2 + 4y^2 - 24x + 36$

$4x^2 - x^2 + 4y^2 - y^2 - 24x - 6x + 36 - 9 = 0$

$3x^2 + 3y^2 - 30x + 27 = 0$

$x^2 + y^2 - 10x + 9 = 0$

$\therefore$ The locus of the point is $x^2 + y^2 - 10x + 9 = 0$, Hence proved.

Extract 4.1: A sample of correct responses in question 4

In extract 4.1, the candidate defined the locus correctly in part (a), whereas in part (b), the candidate applied the appropriate formula used to calculate the distance between the two points to prove the given locus correctly.

However, there were 97 (30.3%) candidates who scored low marks. Those candidates encountered the following difficulties:

In part (a), some candidates failed to define correctly the term locus of a point. For example, some candidates defined locus as “a condition that supports all points within a given equation not a circle”, while others defined the locus of a point as “an equation found in a circle”. Furthermore, some other candidates defined the locus of the point as “the point in which it turns in various direction about the point and covering a certain distance”. Not only that, but also there were candidates who defined the locus as “a point where certain object moves its original centre”. Moreover, some candidates defined the locus of a point as “a branch of mathematics that deal with the position that can move from one point to another”.

In part (b), the analysis shows that, some candidates failed to abide by the given condition that is $AP = 2PB$ when required to prove the given locus. For instance, there were candidates who used the condition $AP = PB$ instead of $AP = 2PB$. While others applied the condition $P - A = 2(B - P)$ instead $AP = 2PB$. For example, one of the candidates recalled correctly the distance formula $d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$, but used the condition $AP = PB$, instead of $AP = 2PB$. Then, substituted the given coordinates $A(-3, 0)$ and $B(3, 0)$ into the formula as $\sqrt{\left(-3 - x\right)^2 + \left(0 - y\right)^2} = \sqrt{\left(3 - x\right)^2 + \left(0 - y\right)^2}$. After computation the candidate got $x^2 + 6x + 9 + y^2 = x^2 - 6x + 9 + y^2$, finally solved for x and obtained $x = 0$. Extract 4.2 is a sample response from one of the candidates who incorrectly responded to the question.

4 a) Locus of a point is a condition that supports all points within a given equation of a circle.

4. b) Data Given.

A $(-3, 0)$

B $(3, 0)$

P (x, y)

Cdn.

$$AP = 2PB$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(x - {}^-3)^2 + (y - 0)^2} = 2(\sqrt{(3 - x)^2 + (0 - y)^2}$$

$$\sqrt{(x + 3)^2 + (y - 0)^2} = 2(\sqrt{(3 - x)^2 + (0 - y)^2}$$

Squaring both sides.

$$(x+3)^2 + (y-0)^2 = 2((3-x)^2 + (0-y)^2)$$

$$(x+3)(x+3) + (y-0)(y-0) = 2((3-x)(3-x) + (0-y)(0-y))$$

$$x^2 + 3x + 3x + 9 + y^2 = 2((9 - 3x - 3x + x^2 + y^2)$$

$$x^2 + 6x + 9 + y^2 = 2(9 - 6x + x^2 + y^2)$$

$$x^2 + 6x + 9 + y^2 = 18 - 12x + 2x^2 + 2y^2$$

$$\cancel{x^2 + y^2 + 2x^2 + 2y}$$

$$x^2 - 2x^2 + y^2 - 2y^2 + 6x + 12x + 9 - 18 = 0$$

$$-x^2 - y^2 + 18x - 9 = 0$$

$$-(x^2 + y^2 - 18x + 9 = 0$$

$$x^2 + y^2 - 18x + 9 = 0$$

(hence not proved)

Extract 4.2: A sample of incorrect responses in question 4

In extract 4.2, part (a), the candidate failed to define correctly the term locus. In part (b), the candidate applied the correct distance formula but failed to apply the given condition as a result failed to prove the given locus.

2.5 Question 5: Algebra

This question consisted of parts (a) and (b). In part (a), the candidates were required to make t the subject of the formula $A=\left(\frac{1+t}{1-t}\right)^{\frac{1}{2}}$. In part (b), they were instructed to use the substitution method to solve the pair of simultaneous equations.

$$\begin{cases} x^2+y^2=18 \\ y-2x=-3 \end{cases}$$

This question was attempted by 320 (100%) candidates. The summary of candidates' performance in this question is presented in Figure 6.

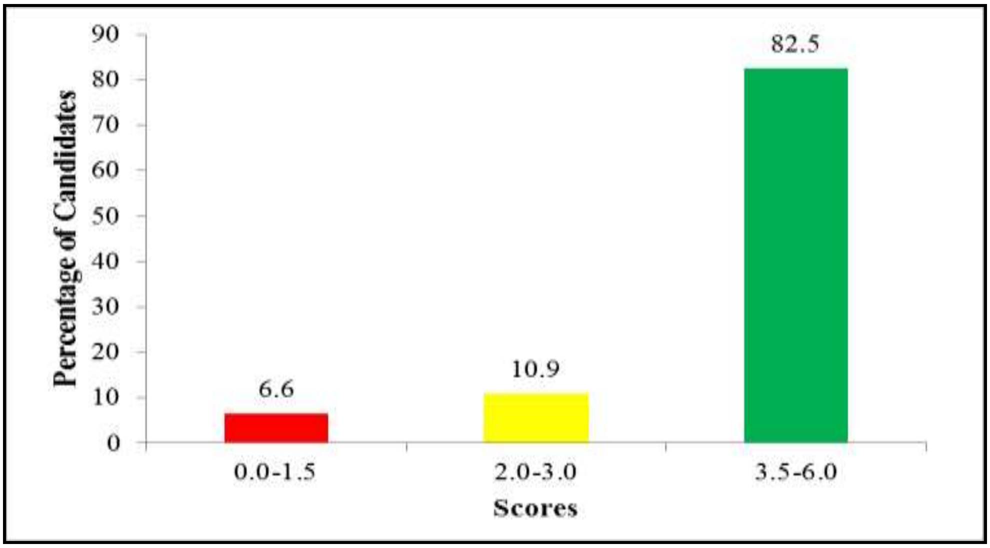


Figure 6: *Candidates' Performance in Question 5*

Among 320 (100%) candidates who attempted the question, 299 (93.4%) candidates scored 2.0 to 6.0 marks. Therefore, the candidates' performance in this question was good.

In part (a), the candidates who responded to the question correctly had adequate knowledge on the concept tested. They responded correctly to the equation $A=\left(\frac{1+t}{1-t}\right)^{\frac{1}{2}}$, by squaring both sides to obtain $A^2=\left(\frac{1+t}{1-t}\right)$.

Then, they were able to multiply by $(1-t)$ on both side and managed to get $A^2(1-t)=(1+t)$. Then, they further computed to get $A^2 - A^2t = 1+t$. Later, rearranged and collected like terms as $A^2 - 1 = t + A^2t$ which then resulted to $A^2 - 1 = t(A^2+1)$. Finally they divided by (A^2+1) on both sides, and got $t = \dfrac{A^2-1}{A^2+1}$ which was the correct answer.

In part (b), the candidates substituted equation (ii) into equation (i), as $x^2 + (2x-3)^2 = 18$, then expanded and managed to get $5x^2 - 12x - 9 = 0$. Thereafter, they were able to apply factorization method and factorized as $(5x+3)(x-3)=0$, then solved for x to obtain the value of $x = \dfrac{-3}{5}$ or $x = 3$. After getting the value of x, they substituted to equation (ii) to solve for y and obtained the value of $y = \dfrac{-21}{5}$ or $y = 3$. Hence the candidates got $(x, y) = (3, 3)$ or $\left(\dfrac{-3}{5}, \dfrac{-21}{5}\right)$ which was the correct answer. Extract 5.1 is a sample response of one of the candidates who attempted the question correctly.

5 a) solution:

$$A = \left(\frac{1+t}{1-t}\right)^{1/2}$$

Squaring both sides

$$A^2 = \left(\frac{1+t}{1-t}\right)^{1/2 \times 2}$$

$$A^2 = \frac{1+t}{1-t}$$

$$A^2(1-t) = 1+t \quad \text{(cross multiplication)}$$

$$A^2 - A^2t = 1+t$$

$$A^2 - 1 = A^2t + t$$

$$A^2 - 1 = t(1+A^2)$$

$$\frac{A^2-1}{1+A^2} = t \quad \text{(Dividing by } 1+A^2\text{)}$$

Then

$$t = \frac{A^2-1}{1+A^2}$$

5b) Solution

$$\begin{cases} x^2 + y^2 = 18 & \ldots (i) \\ y - 2x = -3 & \ldots (ii) \end{cases}$$

from

$y - 2x = -3$

$y = 2x - 3 \quad \ldots (iii)$

Substitute eqn iii in eqn (i)

$x^2 + y^2 = 18$

$x^2 + (2x-3)(2x-3) = 18$

$x^2 + 4x^2 - 12x + 9 = 18$

$5x^2 - 12x + 9 = 18$

$5x^2 - 12x + 9 - 18 = 0$

$5x^2 - 12x - 9 = 0$

By splitting middle term

$ac = -45$

$b = -12$

factors -15 and 3

Now

$5x^2 - 15x + 3x - 9 = 0$

$5x(x-3) + 3(x-3) = 0$

$(x-3)(5x+3) = 0$

Either

$x - 3 = 0$ or $5x + 3 = 0$

$x = 3 \qquad 5x = -3$

$x = -3/5$

from

$y = 2x - 3 \qquad$ for $x = -3/5$

for $x = 3$, $y = 2(3) - 3 \qquad y = 2(-3/5) - 3$

$= 6 - 3 \qquad = -6/5 - 3$

$= 3 \qquad = -21/5$

Solutions are $(3, 3)$ and $(-3/5, -21/5)$

Extract 5.1: A sample of correct responses in question 5

In extract 5.1 part (a), the candidate managed to make *t* the subject of the formula by first squaring both sides on the given formula and manipulated it to get the correct answer. In part (b), the candidate was able to solve properly the given simulteneous equation through substitution method and got the correct answers.

Despite the good performance of the candidates in this question, 6.6 per cent of the candidates had a weak performance. Those candidates who scored low marks faced the following challenges. In part (a), few candidates used to apply the square and square root interchangeably. For example, one candidate took a square root in each term on the r.h.s of the equation instead of squaring both sides. That is $A = \left(\frac{1+t}{1-t}\right)^{\frac{1}{2}} = \frac{1+\sqrt{t}}{1-\sqrt{t}}$, later multiplied by $1-\sqrt{t}$ both sides and obtained $A - A\sqrt{t} = 1+\sqrt{t}$, then factorized $\sqrt{t}$ and obtained $\sqrt{t} = \frac{A-1}{A+1}$ instead of $t = \frac{A^2-1}{A^2+1}$.

Other candidates were faced with a problem of computational error as they managed to square on both sides correctly to obtain $A^2 = \frac{1+t}{1-t}$, but failed to factorize correctly thus wrongly got $t = \frac{1-A^2}{A^2-1}$. In part (b), few candidates were unable to use substitution method, as they failed to substitute $y = 2x-3$ correctly, into $x^2 + y^2 = 18$. After substituting it as $x^2 - (-3+2x)^2 = 18$, they expanded it and got $-3x^2 + 12x - 27 = 0$, which when solved resulted into incorrect values of $x = 2$ and $y = 1$. Some other candidates failed to square the value of y, after substituting $y = 2x-3$ into $x^2 + y^2 = 18$ hence obtained $x^2 + 2\times -3 = 18$ after solving and got $x = 2.69$ or $x = -6.69$. Futhermore, there were candidates who responded to equation $x^2 + y^2 = 18$ as $\sqrt{(x+y)^2} = \sqrt{18}$, then substituted $y = -3+2x$ in the equation $\sqrt{x+(-3+2x)^2}$ and later obtained $x^2 - 2x - 1 = 0$ but finally they failed to solve for x. Extract 5.2 is a sample response from one of the candidates who incorrectly responded to the question.

5 a). Solution

$$A = \left(\frac{1+t}{1-t}\right)^{1/2}$$

$$\sqrt{A} = \sqrt{\left(\frac{1+t}{1-t}\right)^{\cancel{2}}}$$

$$\frac{\sqrt{A}}{1} = \frac{1+t}{1-t}$$

$$1+t = (1-t)\sqrt{A}$$

$$1+t = \sqrt{A} - t\sqrt{A}$$

$$t + t\sqrt{A} = \sqrt{A} - 1$$

$$t(1+\sqrt{A}) = \sqrt{A} - 1$$

5 a). $\frac{t(\cancel{1+\sqrt{A}})}{\cancel{1+\sqrt{A}}} = \frac{\sqrt{A}-1}{1+\sqrt{A}}$

$$t = \frac{\sqrt{A}-1}{1+\sqrt{A}}$$

$\therefore t = \frac{\sqrt{A}-1}{1+\sqrt{A}}$.

5 (b) Solution

$$\begin{cases} x^2 + y^2 = 18 \\ -2x + y = -3 \end{cases}$$

$$-2x + y = -3$$
$$y = -3 + 2x$$

$$x^2 - (-3+2x)^2 = 18$$
$$x^2 - (4x^2 - 12x + 9) = 18$$
$$x^2 - 4x^2 + 12x - 9 = 18$$
$$x^2 - 4x^2 + 12x - 9 - 18 = 0$$
$$-3x^2 + 12x - 27 = 0$$
$$\underline{x = 2 \text{ or } x = 2}$$

$$y = -3 + 2x$$
$$= -3 + 2(2)$$
$$= -3 + 4$$
$$\underline{= 1}$$

$$\underline{x = 2 \text{ and } y = 1}$$

Extract 5.2: A sample of incorrect responses in question 5

In extract 5.2, part (a), the candidate took a square root instead of squaring both sides. Then, the candidate further manipulated it to obtain t as a subject of the formula which was incorrect answer. In part (b), the candidate correctly made y as a subject from equation (ii) to obtain $y = -3 + 2x$. However, after substituting the obtained equation ($y = -3 + 2x$) to equation (i), the candidate made some computational error which led to getting incorrect answers.

2.6 Question 6: Symmetry

This question had two parts, (a) and (b). In part (a), it was stated that: a regular polygon is such that each interior angle is twice the exterior angle. The candidates were required to find the size of each interior angle and exterior angle. In part (b), they were required to (i) indicates all lines of symmetry on a diagram of a regular pentagon using dotted lines and (ii) state the order of rotational symmetry of the regular pentagon drawn in part b (i).

The analysis shows that 320 (100%) candidates attempted the question, out of which 205 (64.1%) scored 3.5 to 6.0 marks, while 84 (26.3%) candidates scored 2.0 to 3.0 marks and 31 (9.6%) candidates scored 0.0 to 1.5 marks. The candidates' performance summary is presented in Figure 7.

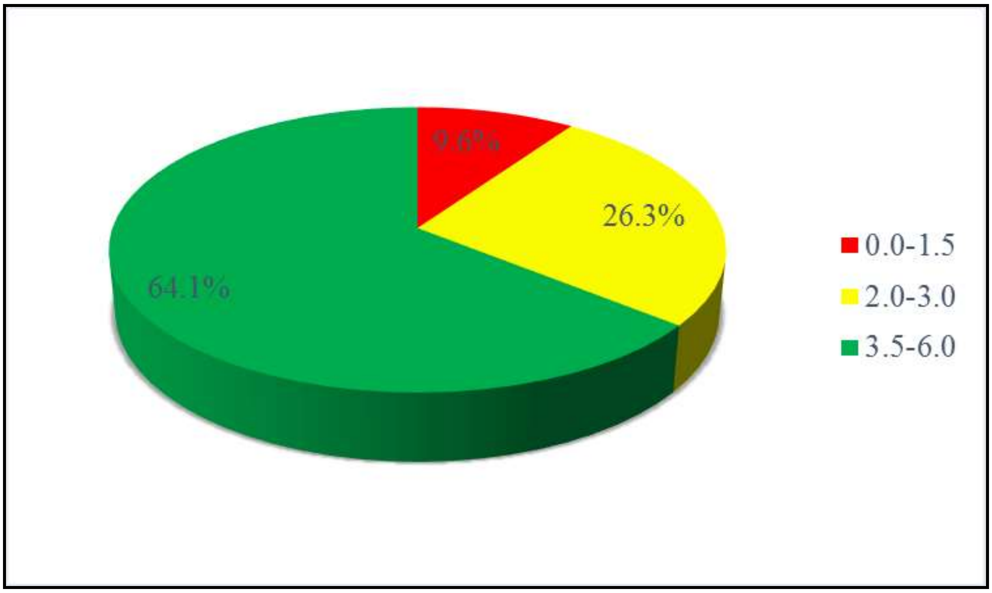


Figure 7: *Candidates' Performance in Question 6*

From Figure 7 it is observed that 9.6 per cent of the candidates had weak performance implying that, the overall performance in this question was good.

In part (a), the candidates who responded correctly had adequate knowledge and skills on the concept tested. They let α be the interior angle and β be the exterior angle, then interpreted correctly the mathematical statement which said; each interior angle is twice the exterior angle. The candidates formulated the mathematical relation, $\alpha = 2\beta$ as equation (i) likewise, recalled correctly the theorem that the sum of interior angle and its corresponding exterior angle is equal to 180°, that is $\alpha + \beta = 180^\circ$ as equation (ii). Then, they substituted equation (i) into (ii) to get $2\beta + \beta = 180^\circ$, which was simplified to obtain $\beta = 60^\circ$. Since $\alpha = 2\beta$ then,

$\alpha = 2 \times 60^\circ = 120^\circ$ hence each interior angle = 120° and each exterior angle is 60° which was the required answer. In part (b) (i), the candidates realized that a regular pentagon is a figure with five sides, and a line of symmetry is any line which divides a figure into two equal parts. Then, they were able to apply drawing skills and draw correctly regular pentagon with all its dotted lines of symmetry as indicated in extract 6.1.

The adequate knowledge and skills on symmetry enabled the candidates to state correctly the order of rotational symmetry of the regular pentagon drawn in part (b) (i) as 5. Extract 6.1 is a sample of correct response selected from one of the candidates who attempted the question correctly.

06 a) let x - interior angle

y - exterior angle

Given $x = 2y$ ______ i

but $x + y = 180^\circ$ ______ ii

From (i)

$2y + y = 180^\circ$

$3y = 180^\circ$

$y = 60^\circ$

From (ii) $x + 60^\circ = 180^\circ$

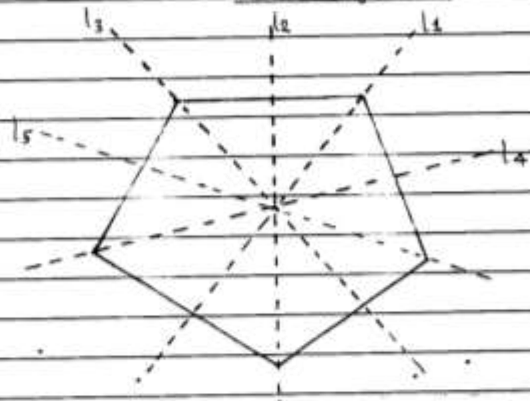
$x = 120^\circ$

$\therefore$ Interior angle = 120°

Exterior angle = 60°

b)

i.



5 lines of symmetry

Regular pentagon.

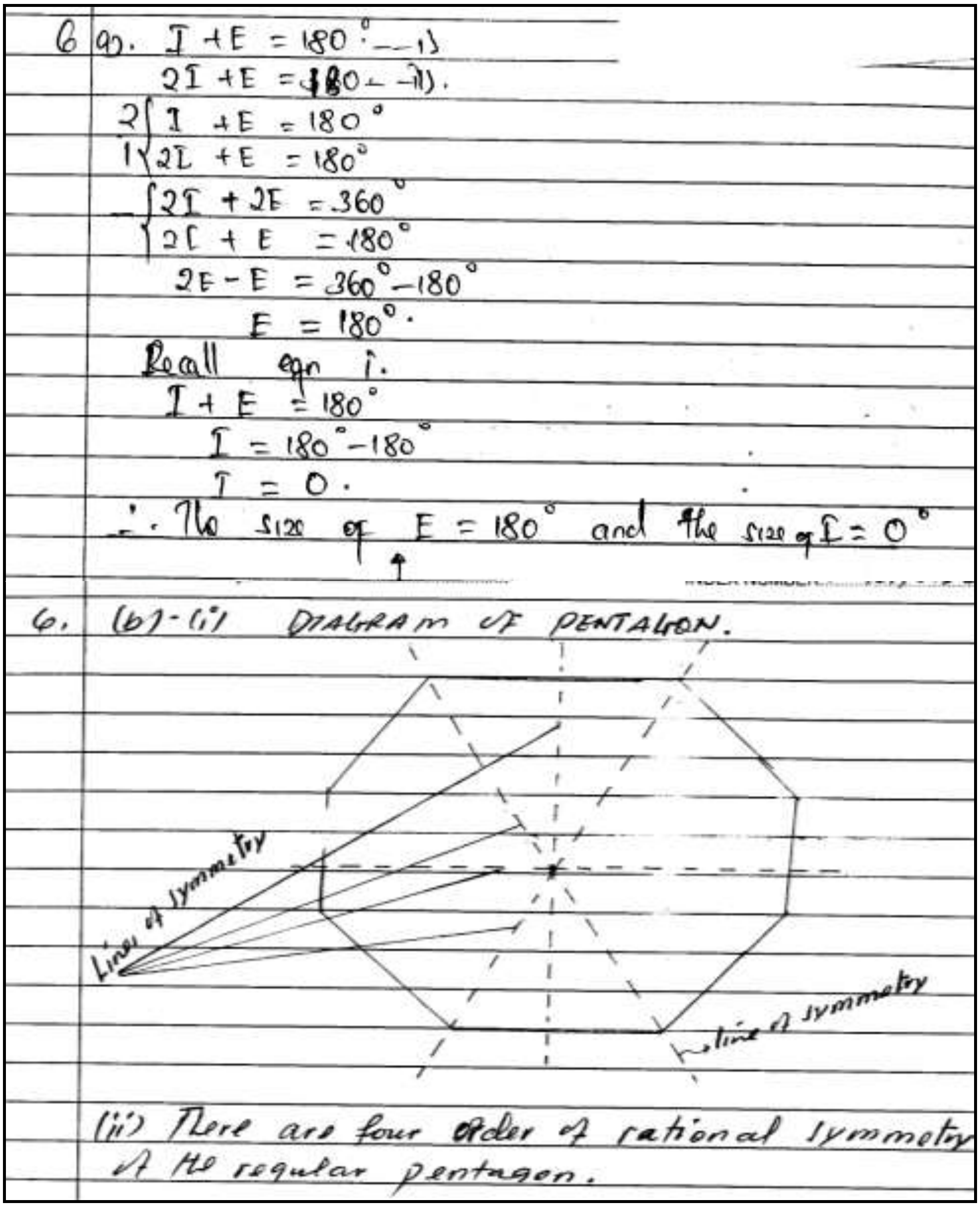
ii There are 5 orders of symmetry on a regular pentagon, that is, its order of symmetry is of order 5.

Extract 6.1: A sample of correct responses in question 6

In extract 6.1, the candidate showed good understanding of the concept asked as in part (a), the candidate determined correctly internal and external angles asked. In part (b), the candidate correctly drew the polygon required and identified clearly the lines of symmetry and stated the order of rotational symmetry.

Despite the good performance by the candidates in this question, there were 9.6% of candidates who scored low marks. Those candidates encountered with the following difficulties. In part (a), few candidates used inappropriate formula while making a comparison of the relation between the interior and exterior angles according to the instruction of the question. For example, one candidate responded as follows; interior angle = 2 exterior angle, there after wrote $(n-2)180^\circ = 2\left(\frac{360^\circ}{n}\right)$, and got $n^2 - 2n = 4$ after simplification. Further analysis detect that, some candidates incorrectly applied the concept not required as they responded by taking exterior angle $(n) = \frac{360^\circ}{x}$, and let $x = 60^\circ$, that resulted to $n = 6$. Then, substituted the value of $n = 6$ to the formula $(n-2)180^\circ$ and obtained 720°. Furthermore, other candidates incorrectly responded to the question by forming the system of simultaneous equations like $\begin{cases} I + E = 180^0 \\ 2I + E = 180^0 \end{cases}$. After solving, they obtained $E = 180^\circ$, $I = 0^\circ$ instead of the interior angle as 120° and each exterior angle as 60°. These candidates lacked knowledge on geometrical construction.

In part (b) (i), some candidates failed to realize that a regular pentagon is a five sided figure. There were some candidates who drew figures with six sides and other figures with four sides instead of pentagon as indicated in extract 6.2. Moreover, other candidates managed to draw correctly a regular pentagon, but failed to indicate the required lines of symmetry. So they indicated four and others six lines of symmetry. Furthermore, there were some other candidates who had misconceptions on the line of symmetry and elevations. They drew a pentagon correctly, they represented a figure by showing plan, front and side elevation. In part (b) (ii), due to difficultness faced in drawing regular pentagon from (b) (i), this led to some candidates to state wrongly the order of rotational symmetry. For instance, some candidates stated that, there are four order of rotational symmetry of a polygon, other candidates stated that, there is only one line of rotational symmetry. Extract 6.2 is a sample response from one of the candidates who responded incorrectly to this question.


6 (a). $I + E = 180°$ ---i)

$2I + E = 180°$ ---ii).

$2\{ I + E = 180°$

$1\{ 2I + E = 180°$

$\{ 2I + 2E = 360°$

$\{ 2I + E = 180°$

$2E - E = 360° - 180°$

$E = 180°$.

Recall eqn i.

$I + E = 180°$

$I = 180° - 180°$

$I = 0$.

$\therefore$ The size of $E = 180°$ and the size of $I = 0°$

6. (b)-(i) DIAGRAM OF PENTAGON.

(ii) There are four order of rational symmetry of the regular pentagon.

Extract 6.2: A sample of incorrect responses in question 6

In extract 6.2, part (a), the candidate solved simulteneous equations to find the interior and exterior angles after formulating two equations relating the sum of the interior angle and exterior angle. The approach was wrong as it led to incorrect answer. Whereas in part (b), the candidate drew a figure of 8 sides instead of a pentagon which has 5 sides and stated incorrectly the order of rotational symmetry.

2.7 Question 7: Trigonometry

The question consists of parts (a) and (b). In part (a), the candidates were required to show that; $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\tan\theta$. While in part (b), they were required to derive the trigonometric identity; $\cos^2\theta+\sin^2\theta=1$.

Out of 320 (100%) candidates who attempted this question, 114 (35.6%) candidates scored 0.0 to 1.5 marks, 60 (18.8%) candidates scored 2.0 to 3.0 marks while 146 (45.6%) candidates scored 3.5 to 6.0 marks. The summary of candidates' performance in this question is presented in Figure 8.

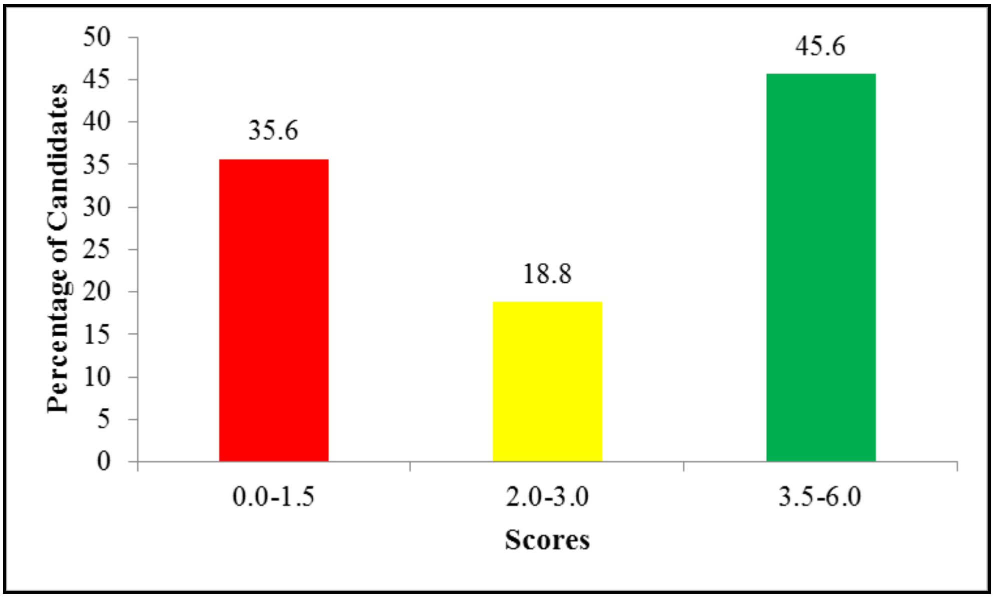


Figure 8: *Candidates' Performance in Question 7*

From Figure 8 it is observed that, the total percentage of candidates for yellow and green is less than 65 per cent implying that, the overall candidates' performance in this question was average.

In part (a), the candidates who performed well and managed to score full marks, had recalled correctly the double angle formula. That is; $\sin 2\theta=2\sin\theta\cos\theta$ and $\cos 2\theta=\cos^2\theta-\sin^2\theta$, also trigonometry identity $\cos^2\theta+\sin^2\theta=1$, they were able to substitute in the LHS of the expression as, $\dfrac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\dfrac{\sin\theta+2\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta+\cos\theta+\cos^2\theta-\sin^2\theta}$. Thereafter simplified to obtain $\dfrac{\sin\theta+2\sin\theta\cos\theta}{2\cos^2\theta+\cos\theta}$, then factorize it as $\dfrac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}$ and finally got $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$. Which justify that LHS = RHS.

In part (b), the candidates demonstrated a good understanding of Pythagoras theorem. They managed to construct a right angled triangle with adjacent side, opposite side and hypotenuse side been labelled as x, y and r respectively. Then they were able to recall correctly the Pythagoras theorem, that is $(\text{Hypotenuse})^2=(\text{Adjacent})^2+(\text{Opposite})^2$. They also managed to substitute the sides to obtain $r^2=x^2+y^2$. Then they were able to realize that $\cos\theta=\dfrac{x}{r}\ \Rightarrow x=r\cos\theta$, likewise $\sin\theta=\dfrac{y}{r}\ \Rightarrow y=r\sin\theta$. They managed to substitute the values of x and y in the equation $r^2=x^2+y^2$, that is $(r\cos\theta)^2+(r\sin\theta)^2=r^2$, which resulted to $r^2\cos^2\theta+r^2\sin^2\theta=r^2$, then they factorized r^2 and obtained $r^2(\cos^2\theta+\sin^2\theta)=r^2$, finally they divided by r^2 both sides to get $\cos^2\theta+\sin^2\theta=1$. Extract 7.1 is a sample response from one of the candidates who attempted the question correctly.

7a) $\dfrac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$

Consider LHS

$\dfrac{\sin\theta + 2\sin\theta\cos\theta}{1+\cos\theta + 2\cos^2\theta - 1}$

$\dfrac{\sin\theta\,(\cancel{1+2\cos\theta})}{\cos\theta\,(\cancel{1+2\cos\theta})}$

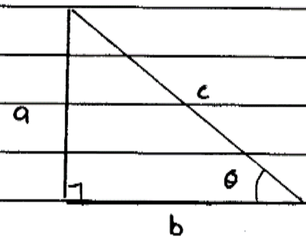
$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

Since LHS = RHS

$\therefore \dfrac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$ - - - proven

7b) Consider the triangle ABC below



$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

$\sin\theta = \dfrac{a}{c}$

$\sin^2\theta = \left(\dfrac{a}{c}\right)^2$

$$\sin^2\theta = \frac{a^2}{c^2}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{b}{c}$$

$$\cos^2\theta = \left(\frac{b}{c}\right)^2$$

$$\cos^2\theta = \frac{b^2}{c^2}$$

$$\sin^2\theta + \cos^2\theta = \frac{a^2}{c^2} + \frac{b^2}{c^2}$$

$$\sin^2\theta + \cos^2\theta = \frac{a^2+b^2}{c^2}$$

According to pythagorus theorem, $a^2+b^2=c^2$

Hence, $\sin^2\theta + \cos^2\theta = \frac{c^2}{c^2}$

$$\sin^2\theta + \cos^2\theta = 1$$

$\therefore \sin^2\theta + \cos^2\theta = 1$ - - - proven

Extract 7.1: A sample of correct responses in question 7

Extract 7.1, shows the correct response from one of the candidates who responded correctly. In part (a), the candidate applied a double angle formula on Left Hand Side (LHS), then simplified to obtain the result equal to that of the Right Hand Side (RHS). While in part (b), the candidate drew a right angled triangle labelled on the sides, then determined trigonometrical ratios of $\cos\theta$ and $\sin\theta$. Thereafter, the candidate was able to apply the Pythagoras theorem and substituted the respective ratios, to prove the given trigonometric identity.

On the other hand, 35.6 per cent of the candidates got weak performance when responding to this question due to some weaknesses. In part (a), some candidates failed to relate double angle formula and trigonometrical ratios of sine and cosine. For example, few candidates were unable to realized that $\sin 2\theta$ and $\cos 2\theta$ are double angles. When responding to the question, they added the identities on L.H.S as $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\frac{\sin 3\theta}{1+\cos 3\theta}$ then divided by 3 and got $\frac{\sin\theta}{1+\cos\theta}$. Further analysis shows that other candidates separated the expression $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$ to obtain $\frac{\sin\theta}{1+\cos\theta+\cos 2\theta}+\frac{\sin 2\theta}{1+\cos\theta+\cos 2\theta}$, then simplified it wrongly to get $\frac{\sin\theta}{1+\cos\theta}$ and equated to $\frac{\sin\theta}{\cos\theta}$. In this category, another candidate responded by factorizing the question as $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\frac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\cos\theta)}$, then considered that $\cos\theta=\sin\theta$ and simplified it more to get $\frac{\sin\theta}{\cos\theta}$ which led to $\tan\theta$.

In part (b), the analysis showed that, some candidates correctly constructed the right angled triangle but then incorrectly wrote $\sin=\frac{a}{b}$, $\cos=\frac{b}{a}$ without indicating the angle of reference, later in consideration of Pythagoras theorem, $a^2+b^2=c^2$ they substituted $\left(\frac{a}{b}\right)^2+\left(\frac{b}{c}\right)^2=\left(\frac{a}{c}\right)^2$, instead of the sides in terms of trigonometrical ratios and concluded to have proved but contrary to what was required.

Moreover, few candidates responded to the question by expanding the terms in the identity $\cos^2\theta+\sin^2\theta=1$ as $(\cos\theta)(\cos\theta)+(\sin\theta)(\sin\theta)$, then divided by $\cos\theta$ and $\sin\theta$ on each term respectively as $\frac{(\cos\theta)(\cos\theta)}{\cos\theta}+\frac{(\sin\theta)(\sin\theta)}{\sin\theta}$, to obtain $\cos\theta+\sin\theta=1$ then they concluded that, it had been proved. Some other candidates applied the concept of perfect square like this,

$\left(\cos\theta+\sin\theta\right)^2 = \left(\cos\theta+\sin\theta\right)\left(\cos\theta+\sin\theta\right)$, and at last wrote $\cos\theta^2 = 2\cos\theta\sin\theta+\sin 2\theta = 1$. This implies that, these candidates had inadequate knowledge about trigonometry. Extract 7.2 is a sample response from one of the candidates who incorrectly answered the question.

7 (a) $\frac{\sin\varphi + \sin 2\varphi}{1+\cos\varphi+\cos 2\varphi}$.

from:

$\sin 2\varphi = 2\sin\varphi\cos\varphi$

$\cos 2\varphi = \cos^2\varphi + \sin^2\varphi$.

$\frac{\sin\varphi + 2\sin\varphi\cos\varphi}{1+\cos\varphi+\cos^2\varphi+\sin^2\varphi}$

from:

$\cos^2\varphi+\sin^2\varphi = 1$.

$\frac{\sin\varphi+2\sin\varphi\cos\varphi}{1+\cos\varphi+1}$.

$\frac{\sin\varphi+2\sin\varphi\cos\varphi}{2+\cos\varphi}$

$\frac{\frac{\sin\varphi}{\cos\varphi}+\frac{2\sin\varphi\cos\varphi}{\cos\varphi}}{\frac{2+\cos\varphi}{\cos\varphi}}$.

$\frac{\tan\varphi + 2\sin\varphi}{2\sin\varphi}$.

$= \tan\varphi$

$\therefore \frac{\sin\varphi+\sin 2\varphi}{1+\cos\varphi+\cos 2\varphi} = \tan\varphi$ Hence shown.

7 b) $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta$

$\cos\theta\cos\theta + \sin\theta\sin\theta = 1$

$\cos\theta\cos\theta + \sin\theta\sin\theta = 1$.

$\cos 2A = 1-\sin^2\theta$

$\frac{\cos 2A}{\sin 2A} = 1$

$\tan 2A = 1$

Extract 7.2: A sample of incorrect responses in question 7

Extract 7.2 shows that in part (a), the candidate failed to recall correctly the expansion of $\cos 2\theta$ which led to obtaining incorrect answer. In part (b), the candidate expanded the terms of the identity unnecessarily and lastly failed to prove the given identity.

2.8 Question 8: Numbers

The question was composed of two parts, (a) and (b). In part (a), (i) the candidates were required to state the divisibility rule of any number by 9. (ii) the candidates were required to show whether 1091524 is divisible by 9.

In part (b), they were given a table which showed the pattern of coefficients in Pascal's triangle as,

Power	**Coefficients**						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1
7							

Then, they were required to show how could the entry 20 in the sixth line be obtained and write the entries in the seventh line.

The analysis shows that out of the 320 (100%) candidates who attempted the question, 18 (5.6%) candidates scored marks from 0.0 to 1.5, 53 (16.6%) candidates scored marks from 2.0 to 3.0 and 249 (77.8%) candidates scored marks from 3.5 to 6.0. Figure 9 presents the summary of candidates' performance.

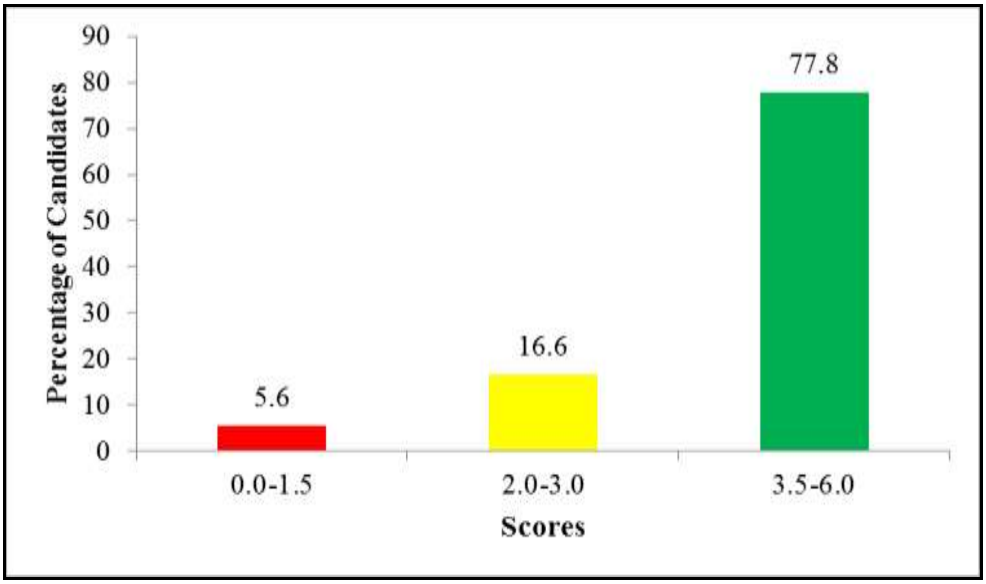


Figure 9: *Candidates' Performance in Question 8*

From the candidates' performance summary presented in Figure 9, there were a small number of candidates (5.6%) who scored less than 2.0 marks, implying that the candidates' overall performance in this question was good.

In part (a) (i), the candidates who responded correctly to this question were able to state clearly the divisibility rule of any number by 9 as "a number is divisible by 9 if the sum of its digits is also divisible by 9". In part (a) (ii), they were able to apply the divisibility rule of 9 by summing all the digits as $1+0+9+1+5+2+4=22$, then commented that since the sum of digits was 22 which is not divisible by 9, then 1091524 is not divisible by 9.

In part (b), the candidates who responded correctly had adequate knowledge on how to develop patterns of coefficients by using Pascal's triangle. They were able to consider the fact that: "*each new number between two numbers and below then is the sum of two numbers above it*". Then they applied it to determine the number 20 in the 6^{th} row by adding two numbers above it in the 5^{th} row, that is 10 + 10 = 20. Thereafter, the candidates were able to use the fact stated to generate correctly the coefficients in 7^{th} row as 1 7 21 35 35 21 7 1. Extract 8.1 is a sample response from one of the candidates who attempted this question correctly.

8 (a) i. Solution

The rule is any number divisible by 9 ~~is~~ means that the sum of individual digits of the whole number is ~~td~~ 9 or divisible by 9.

Example: 72, 2+7 = 9 ~~the~~ (Divisible by 9)

69, 6+9 = 15 (not divisible).

(a) ii. Solution

1091524

Sum of individual digits = 1+0+9+1+5+2+4

= 22.

The number 22 is not divisible by 9.

$$\begin{array}{r} 2 \\ 9 \overline{)22} \\ -18 \\ \hline 4 \end{array}$$

∴ ~~A~~ 1091524 is not divisible by 9.

(b). Solution.

Entry 20 is obtained by adding the two entries above entry 20. Example

7	10 10
6.	20.

10+10 = 20.

ii. Entries in the seventh line.

Power	Coefficients.							
7	1	7	21	35	35	21	7.	1.

Extract 8.1: A sample of correct responses in question 8

Extract 8.1 shows the correct response from one of the candidates who responded correctly. That is, in part (a) (i), the candidate stated clearly the rule governing the divisibility of number 9. While in part (a) (ii), the candidate showed clearly how the given number is not divisible by number 9 by using divisibility rule. In part (b), the candidate showed how the entry 20 in the 6th row was obtained and determined correctly the entries in the 7th row.

Despite the good performance in this question, there were some candidates who had weak performance. Those candidates showed the following weakness; In part (a) (i), the analysis showed that some candidates failed to recall correctly the rule governing the divisibility of any number by 9. For example, one candidate stated that "*a number is divisible by 9 if the sum of the first and last digit is divisible by 9*". Another candidate stated that "*the rule governing the divisibility of any number by 9 is the last number times 7 and the rest number minus the obtained number*". Likewise, another candidate stated that "*if the last two digits is divisible by 3, then the number will also be divisible by 9*". Furthermore, another candidate interchanged the rule governing a number to be divisible by 4 with the rule governing a number to be divisible by 9. That candidate stated that, "*A number is divisible by 9 if the number formed by its last digit is divisible by 9*". In part (a) (ii), few candidates applied incorrect rule to show whether 1091524 is divisible by 9 or not. For example, one candidate took $4 \times 7 = 28$, then subtracted the result from the remaining number, that is $109152 - 28 = 109124$. Then the candidate failed to go further. Another candidate used the approach of omitting the last digit of the given number 1091524 as follows;

1091524

109152 – 4 = 109152

10915 – 2 = 10915

1091 – 5 = 1091

109 – 1 = 109

10 – 9 = 10

10 – 0 = 10

Thereafter, the candidate failed to conclude whether the given number was divisible by 9 or not. This indicates that these candidates were not familiar with the divisibility rule.

In part (b), few candidates failed to apply Pascal's triangle to determine the coefficients of the 7^{th} line. For example, one candidate obtained incorrect coefficients in the 7^{th} row as 1 7 21 21 14 7 1 instead of 1 7 21

35 35 21 7 1. Extract 8.2 is a sample of incorrect responses from one of the candidates who attempted this question.

8. (a) (i) Since the sum of first and last digit is divisible by 9. for example.
81 → 8+1 = 9 so the sum of digit number is divisible by 9.

(ii) 1091524
Solution.
109152 − 4 = 109152
10915 − 2 = 10915
1091 − 5 = 1091
109 − 1 = 109
10 − 9 = 10
10 − 0 = 10
→ Does not divisible by 9 since the last digit number is 4 (four).

8 b)

Power	Coefficient
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 21 14 7 1

Extract 8.2: A sample of incorrect responses in question 8

In extract 8.2 part (a) (i), the candidate failed to state the correct rule governing divisibility of any number by 9. Moreover, in part (a) (ii), the candidate used inappropriate rule when required to show whether 1091524 is divisible by 9 or not. In part (b), the candidate failed to determine the correct coefficients of the 7^{th} row of the given Pascal's triangle.

2.9 Question 9: Logic

This question had two parts (a) and (b), from which the candidates were (a) required to prepare the truth table for compound statement $\left[(\sim p \vee \sim q) \rightarrow \sim (p \wedge q)\right] \vee \left[(p \vee q) \rightarrow (\sim p \wedge \sim q)\right]$, and in part (b), the candidates were asked to show whether the statement $p \wedge q$ logically implies $p \leftrightarrow q$ by using the laws of algebra of propositions.

The analysis shows that, 320 (100%) candidates attempted this question out of these, 138 (43.1%) candidates scored 0.0 to 1.5 marks. While 153 (47.8%) candidates scored 2.0 to 3.0 marks and 29 (9.1%) candidates scored 3.5 to 6.0 marks. The summary of the candidates' performance in this question is presented in Figure 10.

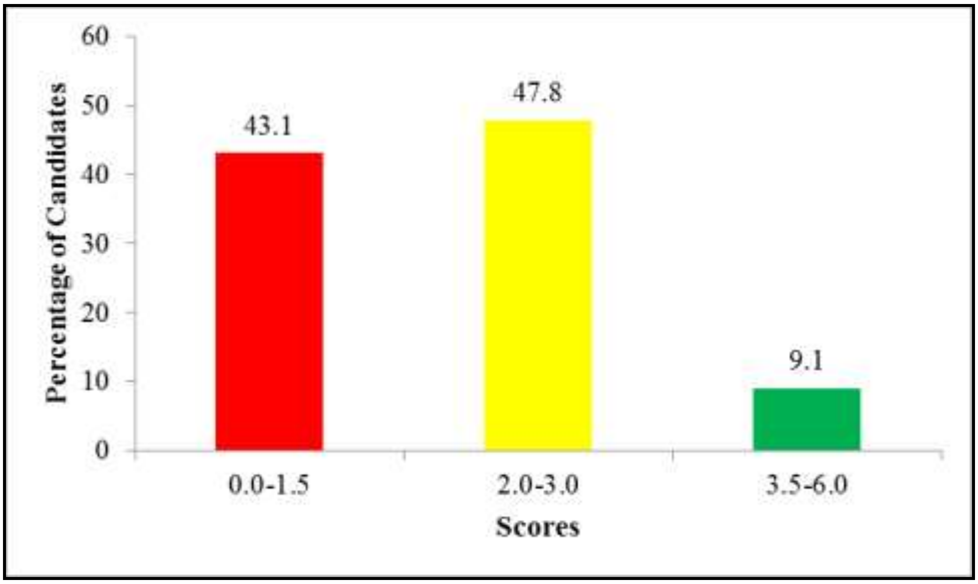


Figure 10: *Candidates' Performance in Question 9*

Figure 10 indicates that, the candidates' performance in this question was of average. The candidates who responded correctly to this question showed their competence about the knowledge and skills in *Logic*. In part (a), these candidates constructed correctly a truth table with columns p , q, $\sim p$,

$\sim q$, $(\sim p \vee \sim q)$, $\sim(p \wedge q)$, $(p \vee q)$ and $\sim p \wedge \sim q$. Then they were able to apply the connective $\rightarrow$ appropriately as seen in extract 9.1.

In part (b), the candidates were able to apply appropriately the laws of algebra of prepositions and managed to show that the statement $p \wedge q$ logically implies $p \leftrightarrow q$. Extract 9.1 is a sample response from one of the candidates who attempted the question correctly.

9. (a) Soln

Truth table.

p	q	$\sim p$	$\sim q$	$\overset{a}{\sim p \vee \sim q}$	$\overset{b}{p \wedge q}$	$\overset{c}{p \vee q}$	$\overset{d}{\sim p \wedge \sim q}$	$\overset{e}{\sim b}$	$\overset{f}{a \rightarrow \sim b}$	$\overset{g}{c \rightarrow d}$	$f \vee g$
T	T	F	F	F	T	T	F	F	T	F	T
T	F	F	T	T	F	T	F	T	T	F	T
F	T	T	F	T	F	T	F	T	T	F	T
F	F	T	T	T	F	F	T	T	T	T	T

(b) Soln.

$p \wedge q \rightarrow (p \leftrightarrow q)$ .... Given.

$(p \wedge q) \rightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$ ... Double implication law

$(p \wedge q) \rightarrow (\sim p \vee q) \wedge (\sim q \vee p)$ .... Implication law

$\sim(p \wedge q) \vee (\sim p \vee q) \wedge (\sim q \vee p)$ ... Conditional law

$(\sim p \vee \sim q) \vee [(\sim p \vee q) \wedge (\sim q \vee p)]$ .. De Morgan's law

$(\sim p \vee \sim q \vee (\sim p \vee q)) \wedge (\sim p \vee \sim q \vee (\sim q \vee p))$ ... Distributive law

$(\sim p \vee \sim q \vee q) \wedge (\sim p \vee \sim q \vee p)$ ..... Idempotent law

$(\sim p \vee T) \wedge (T \vee \sim q)$ ..... Complement law

$T \wedge T$ ...... Identity law

T ..... Idempotent law

$\therefore$ $p \wedge q$ implies to $p \leftrightarrow q$ (It is tautology)

Extract 9.1: A sample of correct responses in question 9

Extract 9.1 part (a), shows the candidate's competence in constructing correctly the required truth table. In part (b), the candidate applied the appropriate laws of algebra of prepositions and managed to show that the statement $p \wedge q$ logically implies $p \leftrightarrow q$.

Inspite of the average performance of the candidates in this question, 43.1 per cent of the candidates who attempted this question scored low marks as they encountered the following problems;

In part (a), some candidates managed to construct correctly the truth table with eleven columns and four rows, but failed to observe the truth-value in columns and rows. The analysis also shows that, other candidates incorrectly constructed the truth table with eight columns, instead of eleven columns and failed to recognize the truth-values in each column as shown in extract 9.2. In part (b), a number of candidates failed to respond to the question due to insufficient knowledge about the laws of algebra of prepositions, as they responded incorrectly without stating the laws. Meanwhile, some other candidates stated the laws but were unable to state the appropriate names of the laws particularly, the distributive, De Morgan and identity laws. Moreover, there were candidates who failed to adhere to the instruction of using the laws of algebra of proposition, instead they constructed the truth table. Extract 9.2 presents one of the incorrect responses from one of the candidates who attempted this question.

9. a)	p	q	~p	~q	a (~p v ~q)	b ~(p ∧ q)	c (p v q)	d (~p ∧ ~q)	e. b v c
	T	T	F	F	F	F	T	F	T
	T	F	F	T	T	T	T	F	T
	F	T	T	F	T	T	T	F	T
	F	F	T	T	T	T	F.	T	T

f. a → e.	f → d.
T	F
T	F
T	F
T	T.

b. $p \wedge q = p \leftrightarrow q$

By using thruth table.

$p \wedge q = p \leftrightarrow q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \leftrightarrow q$
T	T	~~F~~ T
T	F	F
F	T	F
F	F	T

∴ ~~$p \wedge q$~~ The statement $p \wedge q$ is not logically implies $p \leftrightarrow q$.

Extract 9.2: A sample of incorrect responses in question 9

Extract 9.2 part (a), shows that a candidate constructed a correct truth table but failed to observe the correct truth values required in the 9th column. In part (b), the candidate constructed a truth table instead of using the laws of algebra of prepositions.

2.10 Question 10: Sets

This question comprised of parts (a) and (b). In part (a), the candidates were given the universal sets $\mu = \{1, 2, 3, ...12\}$ and its subset $A = \{1, 3, 5, 7\}$, $B = \{2, 3, 4, 5, 6, 8\}$ and $C = \{2, 3, 7, 10\}$, then were required to find the elements of $(A \cap B)' \cup C$. In part (b), the candidates were required to find $n(B \cap C)$ given that A, B and C are any three sets such that $n(A) = 8$, $n(\mathrm{B}) = 12$, $n(\mathrm{C}) = 16$, $n(A \cap B) = 5$, $n(A \cap C) = 4$, $n(A \cup B \cup C) = 20$ and $n(A \cap B \cap C) = 2$.

This question was attempted by 320 (100%) candidates, out of these 65 (20.3%) candidates scored marks from 0.0 to 1.5. Therefore, the general candidates' performance in this question was good as shown in Figure 11.

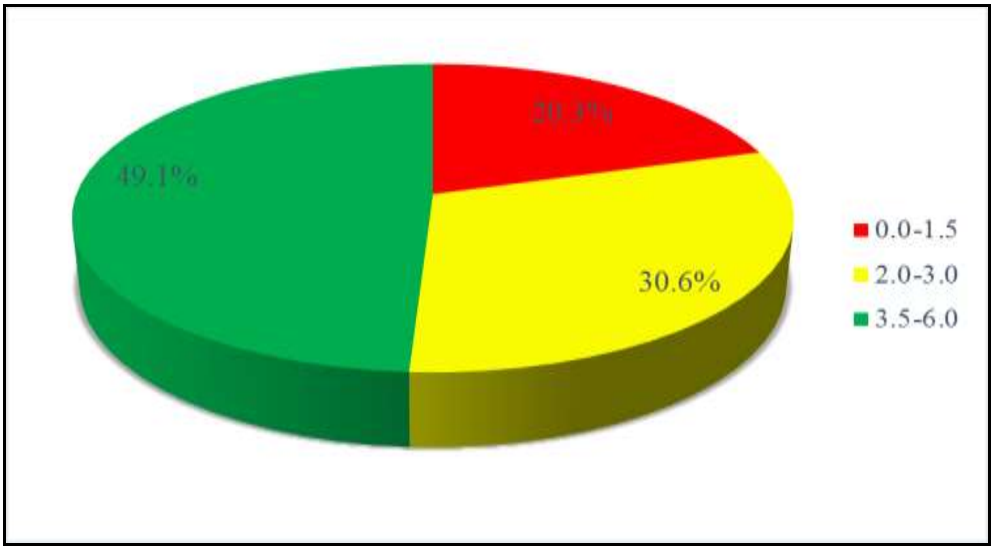


Figure 11: *Candidates' Performance in Question 10*

As Figure 11 shows, 255 (79.7%) candidates scored marks from 2.0 to 6.0. This implies that, most of the candidates had adequate knowledge and skills in Operations on Sets and Number of Members in a Set. In part (a), the analysis showed that, the candidates who responded correctly to this question were able to find $(A \cap B) = \{3, 5\}$, then were able to recognize that $(A \cap B)'$ means complement. Thereafter they managed to identify $(A \cap B)' = \{1, 2, 4, 6, 7, 8, 9, 10, 11, 12\}$, then they were able to find $(A \cap B)' \cup C = \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12\}$ which was the correct answer. In part (b), the candidates applied the appropriate formula for $n(A \cup B \cup C)$, that is

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

Then, were able to substitute all the given value of the sets as $20 = 8 + 12 + 16 - 5 - 4 - n(B \cap C) + 2$. Thereafter, simplified it to get $n(B \cap C) = 9$ which was the correct answer as shown in Extract 10.1.

10a) SOLUTION

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{1, 3, 5, 7\}$

$B = \{2, 3, 4, 5, 6, 8\}$

$C = \{2, 3, 7, 10, 10\}$

$A \cap B = \{3, 5\}$

$(A \cap B)' = \{1, 2, 4, 6, 7, 8, 9, 10, 11, 12\}$

$\underline{(A \cap B)' \cup C = \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12\}}$

10b) SOLUTION

$n(A) = 8, \quad n(B) = 12, \quad n(C) = 16, \quad n(A \cap B) = 5$

$n(A \cap C) = 4 \quad n(A \cup B \cup C) = 20 \quad n(A \cap B \cap C) = 2$

From

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$20 = 8 + 12 + 16 - 5 - 4 - n(B \cap C) + 2$$

$$20 = 20 + 16 - 9 + 2 - n(B \cap C)$$

$$20 = 36 - 9 + 2 - n(B \cap C)$$

$$20 = 36 - 7 - n(B \cap C)$$

$$20 = 29 - n(B \cap C)$$

$$n(B \cap C) = 29 - 20$$

$$n(B \cap C) = 9.$$

$$\therefore \underline{n(B \cap C) = 9.}$$

Extract 10.1: A sample of correct responses in question 10

In extract 10.1, part (a), the candidate was able to determine $(A \cap B)$, thereafter identified correctly $(A \cap B)'$. Then, the candidate was able to obtain $(A \cap B)' \cup C$. In part (b), the candidate applied correctly the formula for finding $n(A \cup B \cup C)$, and obtained the correct answer to $n(B \cap C)$.

In spite of good performance by candidates in the question, 20.3 per cent of them had weak performance due to some weaknesses they showed. In part (a), some candidates were not able to find $(A \cap B) = \{3, 5\}$ first, which could had enabled them to get $(A \cap B)'$. Instead, they responded directly to the question to find $(A \cap B)' \cup C = \{1, 2, 3, 7, 10, 10\}$ which led them to get incorrect answers. Other candidates, considered $(A \cap B)'$ as $A \cap B'$, then they worked out for $A \cap B'$ and finally got $(A \cap B') \cup C$ instead of $(A \cap B)' \cup C$. This implies that they had inadequate knowledge about complement of sets. Furthermore, there were candidates who applied Venn diagram and entered incorrect data because they failed to use intersection $(\cap)$ and union $(\cup)$. This approach was contrary to the demand of the question. In part (b), some candidates incorrectly recalled the formula for $n(A \cup B \cup C)$ as $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) - n(A \cap B \cap C)$ Then, substituted the given values of sets as $20 = 8 + 12 + 16 - 5 - 4 - n(B \cap C) - 2$. Then after simplifying this, they got $n(B \cap C) = 5$ instead of 9. Other candidates applied a formula for union of two sets like $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, then substituted $n(A) = 8$, $n(B) = 12$, $n(A \cap B) = 5$. Then, they determined $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 8 + 12 - 5$, which was simplified to get $n(A \cup B) = 15$ instead of $n(B \cap C) = 9$. Extract 10.2 is a sample response from one of the candidates who attempted the question incorrectly.

10. (a). Find $(A \cap B)' \cup C$.

Now;

$(A \cap B)' \cup C$

$(A \cap B') \cup C$.

$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

$B = \{2, 3, 4, 5, 6, 8\}$.

$B' = \{1, 2, 7, 9, 10, 11, 12\}$.

$A = \{1, 3, 5, 7\}$.

$A \cap B' = \{1, 7\}$.

$(A \cap B') \cup C = \{1, 2, 3, 7, 10, 10\}$.

$\therefore (A \cap B)' \cup C = \{1, 2, 3, 7, 10, 10\}$.

(b) Solution.

$n(A) = 8 - 2 = 6$, $n(A) = 6 - 5 = 1$

$n(B) = 12 - 2 = 10$, $n(B) = 10 - 5 = 5$

$n(C) = 16 - 2 = 14$, $n(C) = 14 - 4 = 10$

Again

$n(A) = 4 - 1 = 3$.

Now; $n(A) + n(B) + n(C) + 4 = 20$.

$n(A) + n(B) + n(C) + n = 20$.

$3 + 5 + 10 + n = 20$

$18 + n = 20$

$n = 20 - 18$

$n = 2$.

$\therefore n(B \cap C) = 2$.

$\therefore n(B \cap C) = 2$.

Extract 10.2: A sample of incorrect responses in question 10

In extract 10.2, part (a), the candidate determined $A \cap B' = \{1, 7\}$ instead of $(A \cap B)' = \{3, 5\}$, thus failed to get the correct answer. In part (b), the candidate applied incorrect formula when required to determine $n(A \cup B \cup C)$, which resulted to get a wrong answer.

2.11 Question 11: Function and Remainder Theorem

This question had three parts (a), (b) and (c). In part (a), the candidates were given that, the roots of a quadratic equation $ax^2+bx+c=0$ are such that, the first root is three times the second root. They were required to show that $3b^2=16ac$. In part (b), the candidates were required to use remainder theorem to compute the value of k when the function $f(x)=2x^4+kx^3-11x^2+4x+12$ is divided by $x-3$, the remainder is 60. In part (c), the candidates were required to sketch the graph of $f(x)=\frac{x+2}{x^2-9}$.

The analysis depicts that 40.0 per cent of 320 candidates who attempted the question scored 6.5 to 10 marks, while 48.4 per cent of the candidates scored 3.0 to 6.0 marks. This indicates that, the overall candidates' performance in this question was good. Figure 12 illustrates the candidates' performance in this question.

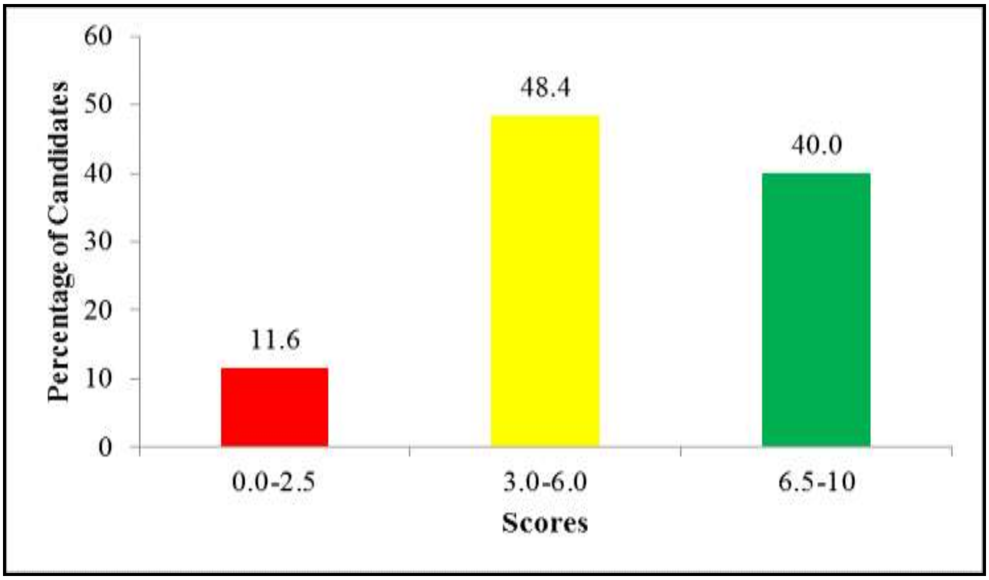


Figure 12: *Candidates' Performance in Question 11*

In part (a), the candidates who responded correctly to this question, the analysis reveals that they let the second root of $ax^2+bx+c=0$ to be α. Since the first root is three times the second, means 3α then they recalled correctly the condition for the sum of roots of a quadratic equation

$\alpha + 3\alpha = \frac{-b}{a} \Rightarrow 4\alpha = \frac{-b}{a}$ as equation (i). They also recalled that, the products of roots $3\alpha^2 = \frac{c}{a}$ as equation (ii). Now from equation (i) they made α the subject of the formula and got $\alpha = \frac{-b}{4a}$. Then, they substituted the value of α into equation (ii) and got $3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$. Later they manipulated it and ended up with $3b^2 = 16ac$ which was the correct answer.

In part (b), the candidates equated the divisor $x-3=0$ and solved for x to get $x=3$. By using the remainder theorem they had $f(x)=60$, then substituted $x=3$ into the equation $f(x)=2x^4+kx-11x^2+4x+12$, and managed to obtain $27k+87=60$ then simplified it to get the value of $k=-1$.

In part (c), the candidates computed $x-$intercept and $y-$ intercept, then substituted them in the function $f(x)=\frac{x+2}{x^2-9}$. They managed to get x and y intercepts as -2 and $-\frac{2}{9}$ respectively. For vertical asymptotes, they considered the denominator and equated it equal to zero, that is $x^2-9=0$ then solved it to get $x=\pm 3$. As for horizontal asymptotes, they considered the highest degree at the function which was 2, then divided each term by x^2, that is; $y=\frac{x+2}{x^2-9} \Rightarrow \frac{\frac{x}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}-\frac{9}{x}}$. Thereafter, they evaluated it to get $y=\frac{\frac{1}{x}+\frac{2}{x^2}}{1-\frac{9}{x}}$, then, they considered the fact that as x is approaching to infinity, finally got $y=0$ as horizontal asymptotes. All these enabled the candidates to sketch a correct graph as illustrated in extract 11.1.

11. a) let the roots be α and β

$\alpha+\beta = -\frac{b}{a}$

$\alpha\beta = c/a$

But

$\beta = 3\alpha$

$\alpha + 3\alpha = -\frac{b}{a}$

$4\alpha = -b/a \Rightarrow \alpha = -b/4a$ — ①

Again

$\beta = 3\alpha$

$\alpha(3\alpha) = c/a$

$3\alpha^2 = c/a$

Substitute the value of α ~~obtained~~ from ①

$3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$

$\frac{3b^2}{16a^2} = \frac{c}{a}$

$3b^2 = \frac{c \times 16a^2}{a}$

$3b^2 = 16ac$

$3b^2 = 16ac$, Hence Shown.

b) $f(x) = 2x^4 + kx^3 - 11x^2 + 4x + 12$

$x - 3 = 0$

$x = 3$

$f(3) = 2(3)^4 + k(3)^3 - 11(3)^2 + 4(3) + 12$

$= 2 \times 81 + 27k - 99 + 12 + 12$

$= 162 + 27k - 99 + 24$

$f(3) = 27k + 87$

But

$f(3) = \text{Remainder} = 60$

$60 = 27k + 87$

$27k = 60 - 87$

$27k = -27$

$k = -1$

$\therefore$ The value of k is -1

c) $f(x) = \dfrac{x+2}{x^2-9}$

x-intercept, $y=0$

$0 = x+2$

$x = -2$

y-intercept, $x=0$

$y = \dfrac{0+2}{0-9}$

$y = -2/9$

Vertical asymptote, V.A:

$x^2 - 9 = 0$

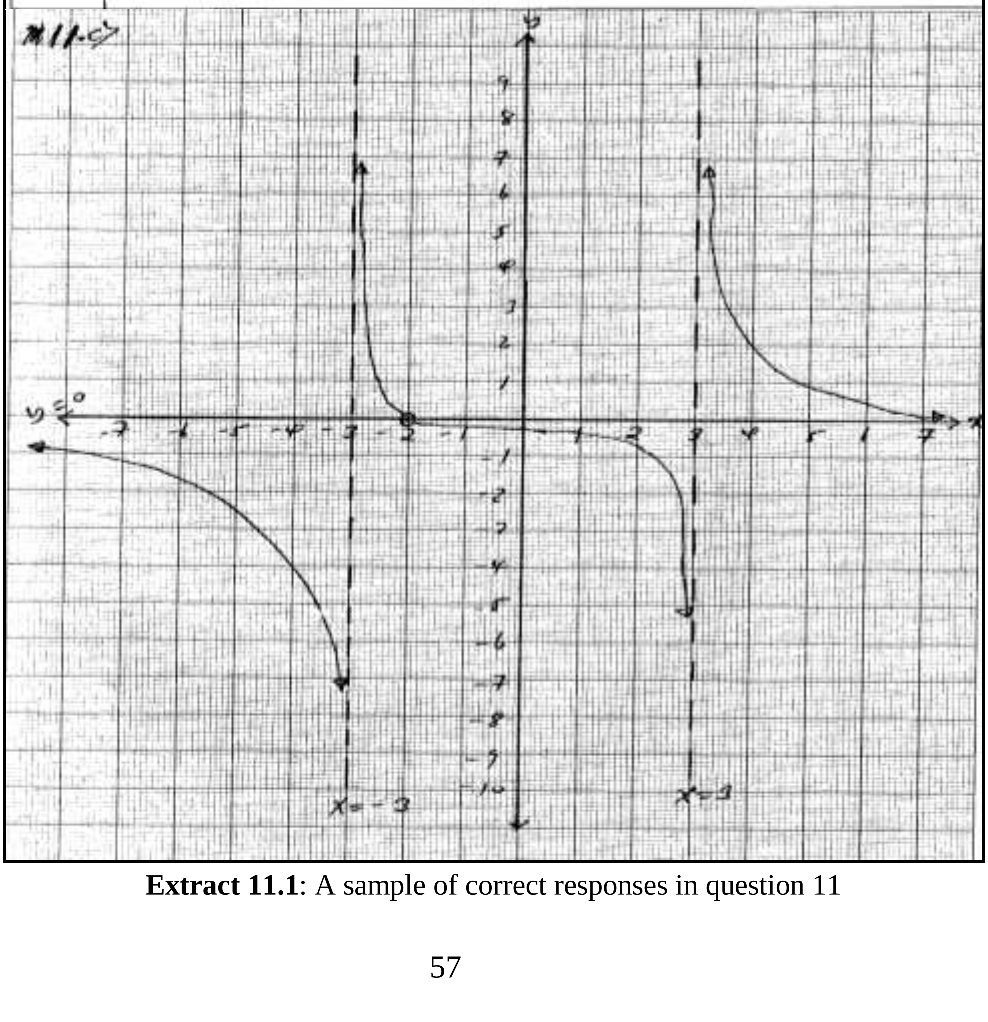
$(x+3)(x-3) = 0$

$x+3=0$ or $x-3=0$

$x=-3$ or $x=3$

Horizontal asymptote, H.A:

$y = 0$



Extract 11.1: A sample of correct responses in question 11

As extract 11.1 shows in part (a), the candidate correctly determined the roots which made it possible to show that $3b^2 = 16ac$. While in part (b), the candidate applied correctly the remainder theorem to obtain the required value of k. In part (c), the candidate managed to sketch the graph of the given function by using x and y intercepts and asymptotes.

However, 11.6 per cent of the candidates who attempted the question scored low marks, 0.0 to 2.5. Those candidates had the following weaknesses: In part (a), some of them failed to connect the relation between the roots, that is the first root as three times the second root that led them to obtain incorrect response in this question by giving $3\alpha = 16\beta$, but $\alpha\beta = \frac{c}{a}$, then $c = a\alpha\beta$. Then, they wrongly obtained $\alpha = \frac{16a}{3}(\alpha\beta)$ instead of showing that $3b^2 = 16ac$. Further analysis reveals that, some candidates misinterpreted the question as they were unable to recognize the root as three times another. This led them to apply concept incorrectly. For example, one candidate responded as $(x+3a)(x-a) = ax^2 + bx + c$. Then, the candidate wrote $x(x+a) + 3a(x+a) = 0$, and finally got $48a^2 = 48a^2$. Also other candidates responded to the question by deriving the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, instead of showing that $3b^2 = 16ac$.

In part (b), the analysis reveals that some candidates committed computational errors. Although they correctly let the divisor, $x - 3 = 0$ and managed to solve for x to get $x = 3$, but they failed to obtain a correct value of k after substituting into the given function. For example, one candidate when substituted the value in the function $f(x) = 2x^4 + kx^3 - 11x^2 + 4x + 12$, obtained $60 = 87 + 3k$, then got the value of $k = -9$ instead of $k = -1$. Other candidates applied long division method by dividing $2x^4 + kx^3 - 11x^2 + 4x + 12$ with $x - 3$ but failed to get the correct answer due to manipulation error. For example, one candidate incorrectly divided it and got $kx^3 = 6x^3$, after simplification $k = 6$ was obtained instead of $k = -1$.

In part (c), some candidates could neither find intercepts nor asymptotes for sketching the graph. For instance, in the vertical asymptote, some of them responded as $x^2 \neq 9$ instead of $x^2 = 9$ as illustrated in extract 11.2.

11. (a) Solution

Let the Second Root $= \cancel{x}\ a$

$(x+3a)(x+a) = ax^2+bx+c$

$x(x+a)+3a(x+a)=0$

$x^2+xa+3xa+3a^2$

$x^2+4ax+3a^2$

$a=1$

$b=4a$

$c=3a^2$

$3b^2=16ac$

$3((4a)^2)=16(1\times 3a^2)$

$3(16a^2)=16(3a^2)$

$48a^2=48a^2$

$\therefore\ 3b^2=16ac$ (Proved)

11. b) Solution

$f(x) = 2x^4+kx^3-11x^2+4x+12 \div x-3$ remainder 66

$$\begin{array}{r} 2x^3+6x^2+8x+28 \\ x-3\overline{)2x^4+kx^3-11x^2+4x+12} \\ -\ \ 2x^4-6x^3 \\ kx^3+6x^3-11x^2 \\ -\ \ 6x^3-18x^2 \\ 8x^2+4x \\ -\ \ 8x^2-24x \\ 28x+12 \\ -\ \ 28x+84 \\ 60 \end{array}$$

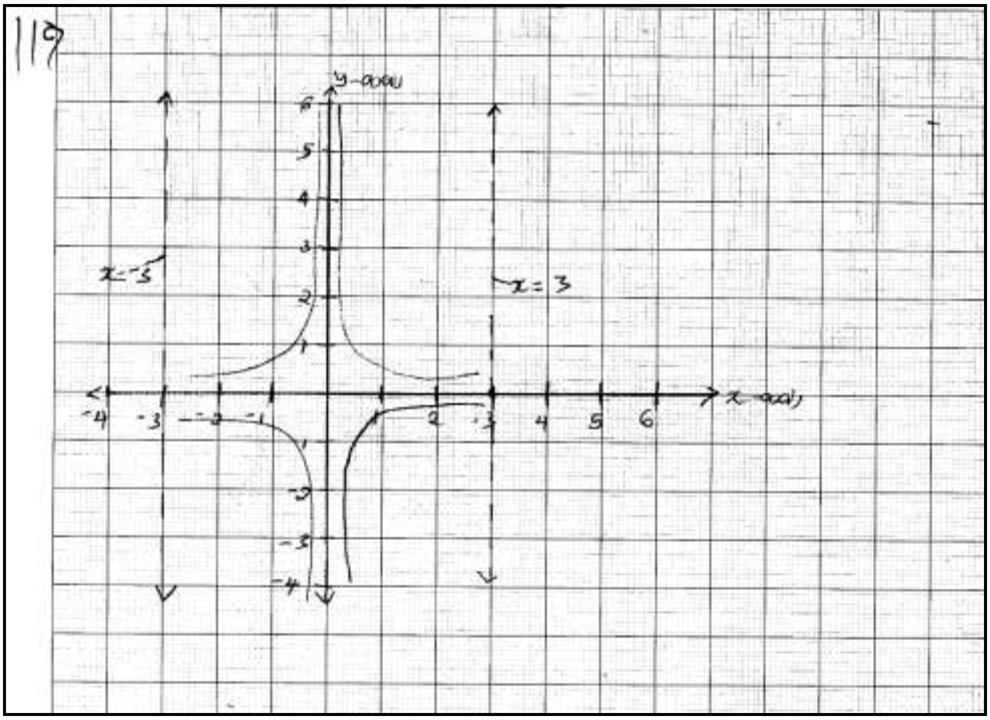
Now

$kx^3 = 6x^3$

$\frac{kx^3}{x^3} = \frac{6x^3}{x^3}$

$k = 6$

$\therefore k$ is 6



Extract 11.2: A sample of incorrect responses in question 11

In extract 11.2, part (a), the candidate failed to show that $3b^2 = 16ac$ since the given relation of the two roots was misinterpreted. While in part (b), the candidate used long division method, but failed to divide correctly leading to failure to obtain the correct answer. In part (c), the candidate failed to identify the asymptotes hence got a wrong sketched graph.

2.12 Question 12: Integration and Differentiation

This question consisted of three parts (a), (b) and (c). In part (a), the candidates were instructed to use the quotient rule to differentiate $\left(\frac{1+x}{2+x}\right)^2$ with respect to x. In part (b), they were given the curve $y = 2x^3 - 3x^2 - 36x + 3$ and required to: (i) find the minimum value of y, (ii) determine the value of x at the point of inflexion. In part (c), the candidate were required to compute the area enclosed by the curve $y = x^2 - 4$ and the $x-$axis.

The analysis shows that 320 (100%) candidates attempted the question whereby 106 (33.1%) candidates scored 0.0 to 2.5 marks, while 120

(37.5%) candidates scored 3.0 to 6.0 marks and 94 (29.4%) candidates scored 6.5 to 10 marks. The candidates' performance summary in this question is presented in Figure 13.

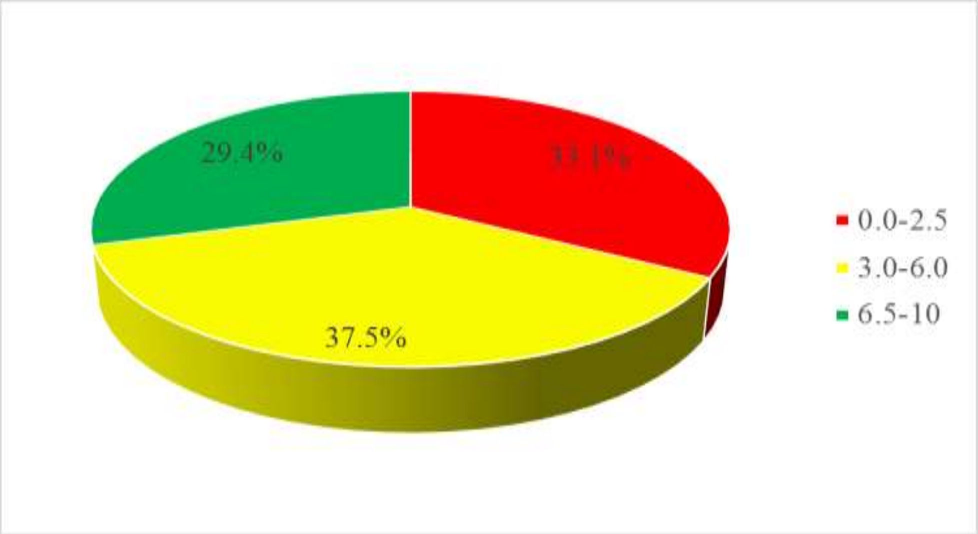


Figure 13: *Candidates' Performance in Question 12*

According to figure 13, the candidates' performance in this question was good. The analysis shows that, candidates who responded correctly to this question and managed to get full marks had adequate knowledge and skills on the concept tested. In part (a), the candidates applied the quotient rule appropriately by first writing the function $y=\left(\frac{1+x}{2+x}\right)^2$ as $y=\frac{(1+x)^2}{(2+x)^2}$,

Then, let $u=(1+x)^2 \Rightarrow \frac{du}{dx}=2(1+x)$ as equation (i) and $v=(2+x)^2$ $\Rightarrow \frac{dv}{dx}=2(2+x)$ as equation (ii). They wrote the given equation as $y=\frac{u}{v}$ so as to apply the quotient rule. Then the candidates recalled correctly the quotient rule of differentiation as $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$. Then they substituted as $\frac{dy}{dx}=\frac{(2+x)^2\times 2(1+x)-(1+x)\times 2(1+x)}{(2+x)^4}$, and managed to compute it to get $\frac{dy}{dx}=\frac{2(1+x)}{(2+x)^3}$.

In part (b) (i), the candidates correctly determined the stationary points of the curve $y = 2x^3 - 3x^2 - 36x + 3$ by setting $\frac{dy}{dx} = 0$ and evaluated $\frac{dy}{dx} = 6x^2 - 6x - 36$ then changed it to $x^2 - x - 6 = 0$. Then, they applied a factorization method on it to obtain $(x+2)(x-3) = 0$, then solved for x and managed to get $x = -2$ or $x = 3$. They were also able to work out for y, when $x = -2$, obtained $y = 47$ and when $x = 3$, they got $y = -78$. Then, they found the nature of stationary point (x, y) by computing the second derivative when $x = -2$ and $x = 3$. That is, $\left.\frac{d^2y}{dx^2}\right|_{(x=-2)} = 12x - 6 \Rightarrow 12(-2) - 6 = -30 < 0$, Likewise when $x = 3$ $\left.\frac{d^2y}{dx^2}\right|_{(x=3)} = 12x - 6 \Rightarrow 12(3) - 6 = 30 > 0$. Since $\frac{d^2y}{dx^2}$ is greater than zero when $x = 3$, hence they concluded that, the curve has minimum value which is $y = -78$.

In part (b) (ii), the candidates realized that the value of x at the point of inflexion is determined when $\frac{d^2y}{dx^2} = 0$. Therefore, from $y = 2x^3 - 3x^2 - 36x + 3$, they differentiated it and managed to get $\frac{d^2y}{dx^2} = 12x - 6 \Rightarrow 12x - 6 = 0$, then simplified it to get $x = \frac{1}{2}$.

In part (c), the candidates applied correctly the graph method and drew the graph to identify the limits which are from -2 to 2. Others determined the limits using the $x-$intercepts of the graph of $y = x^2 - 4$ after setting $y = 0$. Then, they found the area by using the formula $\int_a^b y\,dx$ as follows $\int_a^b y\,dx = \int_{-2}^{2}\left(x^2 - 4\right)dx = \left[\frac{x^3}{3} - 4x\right]_{-2}^{2}$. Then, they computed $\left[\frac{x^3}{3} - 4x\right]_{-2}^{2}$ to get the area enclosed to be equals to 10.67 square units. Extract 12.1 is a sample response from one of the candidates who attempted the question correctly.

12a) $\left(\dfrac{1+x}{2+x}\right)^2 = \dfrac{(1+x)^2}{(2+x)^2}$

Let $v = (2+x)^2$

$\dfrac{dv}{dx} = 2(2+x) \cdot \dfrac{d(2+x)}{dx}$

$\dfrac{dV}{dx} = 2(2+x) \cdot 1$

$\dfrac{dV}{dx} = 2(x+2)$

Let $u = (1+x)^2$

$\dfrac{dy}{dx} = 2(1+x) \cdot \dfrac{d(1+x)}{dx}$

$\dfrac{dy}{dx} = 2(1+x) \cdot 1$

$\dfrac{dy}{dx} = 2(1+x)$

$\dfrac{dy}{dx} = \dfrac{V\dfrac{dy}{dx} - U\dfrac{dv}{dx}}{V^2}$

$\dfrac{dy}{dx} = \dfrac{(2+x)^2 \cdot 2(1+x) - (1+x)^2 \cdot 2(2+x)}{((2+x)^2)^2}$

$\dfrac{dy}{dx} = \dfrac{2(1+x)(x+2)^2 - 2(x+2)(x+1)^2}{(x+2)^4}$

$\dfrac{dy}{dx} = \dfrac{2(1+x)(x+2)\left[\cancel{(x+2) - (x+1)}\right]}{(x+2)^4}$

$\dfrac{dy}{dx} = \dfrac{2(1+x)(x+2)[x+2-x-1]}{(x+2)^4}$

$\dfrac{dy}{dx} = \dfrac{2(1+x)[1]}{(x+2)^3}$

$\dfrac{dy}{dx} = \dfrac{2(1+x)}{(x+2)^3}$

$\therefore \dfrac{dy}{dx} = \dfrac{2(1+x)}{(x+2)^3}$

12b) i) $y = 2x^3 - 3x^2 - 36x + 3$

$\frac{dy}{dx} = \cancel{30}\ 6x^2 - 6x - 36$ but at min, $\frac{dy}{dx} = 0$

$\frac{6x^2}{6} - \frac{6x}{6} - \frac{36}{6} = \frac{0}{6}$

$x^2 - x - 6 = 0$

$\cancel{x^2 - 5x - x - 6 = 0}$ $\cancel{x^2 + 5x - 6x - 6 = 0}$

$\cancel{x(x-5)-}$

$x^2 + 2x - 3x - 6 = 0$

$x(x+2) - 3(x+2) = 0$

$(x-3)(x+2) = 0$

Case 01: $x - 3 = 0$

$x = 3$

$y = 2x^3 - 3x^2 - 36x + 3$

$y = 2(3)^3 - 3(3)^2 - 36(3) + 3$

$y = -78 \longrightarrow (3, -78)$

Case 02: $x + 2 = 0$

$x = -2$

$y = 2x^3 - 3x^2 - 36x + 3$

$y = 2(-2)^3 - 3(-2)^2 - 36(-2) + 3$

$y = 47 \longrightarrow (-2, 47)$

$\frac{d^2y}{dx^2} = 12x - 6$

Case 01: $(3, -78)$

$\frac{d^2y}{dx^2} = 12x - 6$

$= 12(3) - 6$

$= 36 - 6$

$= 30$

i)	$\therefore$ The minimum value of $y = (-78)$
	Case 02: $(-2, 47)$
	$\frac{d^2y}{dx^2} = 12x - 6$
	$= 12(-2) - 6$
	$= -24 - 6$
	$= -30$
b) ii)	$\frac{d^2y}{dx^2} = 12x - 6$
	At point of inflexion, $\frac{d^2y}{dx^2} = 0$
	$12x - 6 = 0$
	$12x = 6$
	$\frac{12x}{12} = \frac{6}{12}$
	$x = \frac{1}{2}$
	$\therefore$ $x = \frac{1}{2}$ at the point of inflexion
12 c)	$y = x^2 - 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

$$A = \int_{-2}^{2} (x^2 - 4)\, dx$$

$$A = \int_{-2}^{2} x^2\, dx - 4\int_{-2}^{2} dx$$

$$A = \left. \frac{x^3}{3} - 4x \right|_{-2}^{2}$$

12c) Area enclosed $= \left| \left(\frac{(2)^3}{3} - 4(2)\right) - \left(\frac{(-2)^3}{3} - 4(-2)\right) \right|$

$= \left| \left(\frac{8}{3} - 8\right) - \left(\frac{-8}{3} + 8\right) \right|$

$= \left| \frac{-16}{3} - \left(\frac{16}{3}\right) \right|$

$= \left| \frac{-16}{3} - \frac{16}{3} \right|$

$= \left| \frac{-32}{3} \right|$

$= \left| -10.667 \right|$

$\therefore$ The area enclosed btn the curve $y = x^2 - 4$ and x axis is 10.667 sq units

Extract 12.1: A sample of correct responses in question 12

In extract 12.1, the candidate responded correctly according to the demand of the question and managed in part (a), to differentiate the given function using the quotient rule. In part (b) (i), the candidate determined the minimum value of the given function at $x = 3$ as well as in (ii), the candidate calculated the point of inflexion correctly. In part (c), the candidate applied the integration techniques to get the correct area enclosed by the curve and the $x-$axis.

On the other hand, 106 (33.1%) candidates scored low marks due to facing some difficulties while attempting this question. In part (a), some candidates failed to recall the correct formula for quotient rule, for example one candidate used the formula $\frac{dy}{dx} = v\frac{du}{dx} - u\frac{dv}{dx}$ instead of $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$. Thereafter, wrote $\frac{dy}{dx} = (2+x)^2 \times 1 - (x+1)^2 \times 1$, then computed wrongly and got the answer = 6. Other candidates let $u = \left(\frac{1+x}{2+x}\right)$, $\frac{du}{dx} = 2u$ and recalled incorrectly the quotient rule as $2\left(\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right)$. Also some candidates recalled correctly the quotient rule as $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ but failed to apply it appropriately, since they responded as $\frac{dy}{dx}=\frac{(2+x)\times 1-(1+x)}{(2+x)^2}$. Then, they simplified incorrectly to obtain $\frac{dy}{dx}=\frac{1}{x^2-8x+24x^2+16}$ instead of $\frac{dy}{dx}=\frac{2(1+x)}{(2+x)^3}$.

In part (b), where it was required to determine the value of x at the point of inflexion, some candidates used inappropriate formula for example, they applied the formula for finding the line of symmetry of quadratic, $\left(\frac{-b}{2a}\right)$ to find a point of inflexion. They identified $a=-3$, $b=-36$ then substituted in the formula as $\frac{36}{2\times -3}=\frac{36}{-6}$, which was simplified to -6. This implies that these candidates had inadequate knowledge on differentiation concepts.

In part (c), some candidates applied inappropriate formula like $A=\pi\int_b^a y^2dx$ which is used to find the volume of solid of revolution but not the area of the curve. Furthermore, there were candidates who applied the formula $A=\int_b^a \pi y^2dx=\int_a^b \pi\left(x^2-y\right)^2dx$. Then, wrote $\pi\int_b^a x^2dx-\int_a^b 16dx$. Finally, they got $\pi\left[\frac{x^5}{5}dx\right]$ and simplified it to $\frac{\pi x^5}{5}$ square units. These candidates failed to meet the requirement of the question. Extract 12.2 is a sample response from one of the candidates who attempted this question incorrectly.

12\. a) $\left(\frac{1+x}{2+x}\right)^2$

Soln.

$$\left(\frac{1+x}{2+x}\right)^2$$

$$\left(\frac{1+x^2}{4+x^2}\right)$$

let, $u = 1+x^2$ $\quad v = 4+x^2$

$u' = 2x$ $\quad v' = 2x$

By Q.R

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

12\. a)

$$\frac{dy}{dx} = \frac{(4+x^2)(2x) - (1+x^2)(2x)}{(4+x^2)^2}$$

$$= \frac{(4+x^2)(2x) - (1+x^2)(2x)}{(4+x^2)(4+x^2)}$$

$$= \frac{2x - 2x + 2x^3}{4+x^2}$$

$$= \frac{2x^3}{4+x^2}$$

$$\therefore \frac{dy}{dx} = \frac{2x^3}{4+x^2}$$

b)(i) $2x^3 - 3x^2 - 36x + 3$

$= \frac{b^2 - 4ac}{4a}$

$= \frac{(-3)^2 - 4(2)(-36)}{4(2)}$

$= \frac{9 + 288}{8}$

$= 297/8$

$= 37.1$

∴ The minimum value of y is 37.1

ii). $x = -b/2a$

$x = \frac{-(-36)}{2(2)}$

$x = \frac{36}{4}$

$x = 9.$

∴ Value of $x = 9.$

c) Given.

$y = x^2 - 4$

x - axis

From,

$A = \int_a^b \pi y^2 \, dx$

$= \int_a^b \pi (x^2 - 4)^2 \, dx$

$= \int_a^b \pi (x^4 - 16) \, dx$

$= \pi \int_a^b x^4 \, dx - \int_a^b 16 \, dx$

$= \pi \left[\frac{x^5}{5} dx \right]$

$= \pi \frac{x^5}{5}$ square units

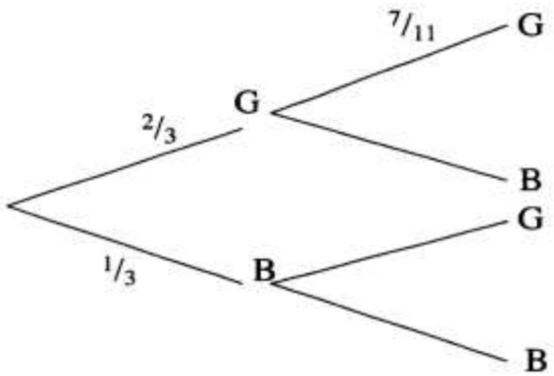
$A = \frac{\pi x^5}{5}$ square units.

Extract 12.2: A sample of incorrect responses in question 12

In extract 12.2 part (a), the candidate incorrectly expanded the numerator and denominator of the given function leading to obtaining incorrect answer. In part (b), inappropriate formula was used instead of using differentiation to determine the minimum values of y and x at the point of inflexion. In part (c), the candidate applied the formula used to find the volume of a solid of revolution while it was required to find the area enclosed by the curve.

2.13 Question 13: Probability

The question had three parts (a), (b) and (c). Where in part (a), the question said that: A bag contains 8 green discs (G) and 4 blue discs (B). A disc is drawn and not replaced. A second disc is drawn. Then, the candidates were required to copy and compute a tree diagram.



Then they were to find the probability that:

(i) Both discs are green.
(ii) Both discs are blue.
(iii) One disc is green and one disc is blue.

While in part (b), the candidates were required to find $P(A \cup B)$ and $P(B \cap A')$ if A and B are dependent events where by $P(A) = \frac{1}{5}$, $P(\text{B}) = \frac{3}{10}$ and $P\left(\frac{A}{B}\right) = \frac{1}{10}$. In part (c), the candidates were asked to find the number of ways in which 11 people could be selected on a bench if only 6 seats are available.

The analysis shows that, 320 (100%) candidates attempted this question out of which 71(22.2%) scored 0.0 to 2.5 marks, 104 (32.5%) candidates scored 3.0 to 6.0 marks and 145 (45.3%) candidates scored 6.5 to 10.0 marks. The candidates' performance summary in this question is presented in Figure 14.

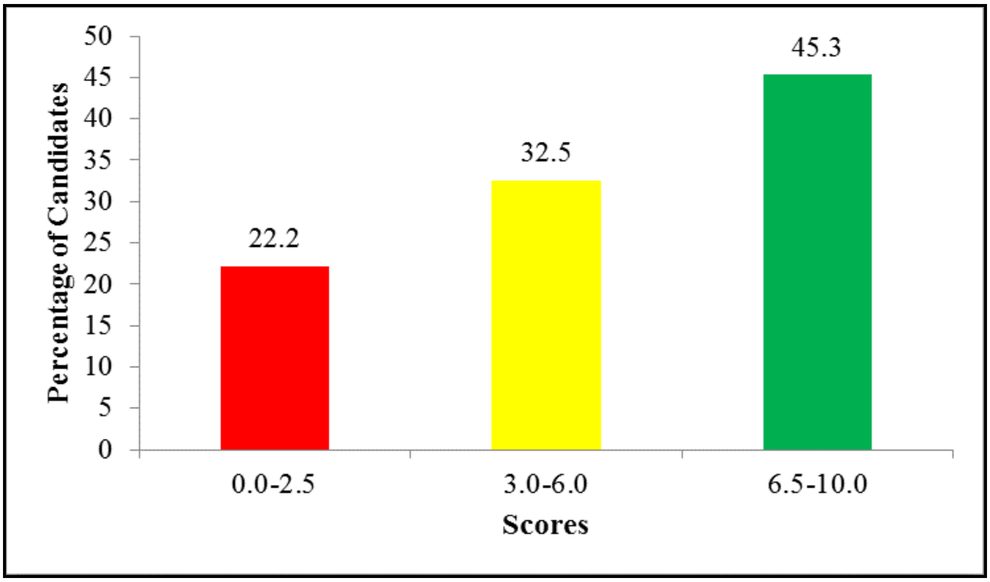


Figure 14: *Candidates' Performance in Question 13*

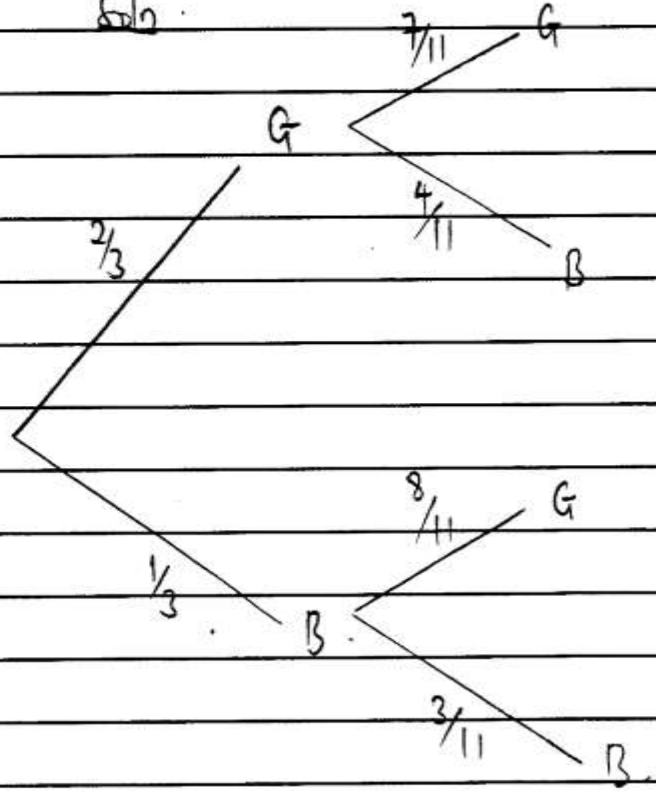
The summary of candidates' performance presented in Figure 14 indicates that, the performance of candidates in this question was good. The candidates who responded correctly in part (a), were familiar with construction of the tree diagram. The analysis shows that, they managed to draw a required tree diagram and its probabilities as indicated in extract 13.1. In (i), the candidates realized that the probability for both discs to be green is determined by $P(GG) = P(G_1) \times P(G_2)$, but from the tree diagram $P(\mathrm{G}_1) = \frac{2}{3}$ and $P(\mathrm{G}_2) = \frac{7}{11}$ then substituted to $P(\mathrm{GG}) = \frac{2}{3} \times \frac{7}{11}$ which resulted to $P(\mathrm{GG}) = \frac{14}{33}$. In (ii), the candidates realized that, the probability for both discs to be blue is determined by $P(BB) = P(B_1) \times P(B_2)$ where $P(B_1) = \frac{1}{3}$ and $P(B_2) = \frac{3}{11}$. Then, they substituted the identified value in $P(BB) = \frac{1}{3} \times \frac{3}{11}$ and computed to get $\frac{1}{11}$. In (iii), the candidates recognized that, the probability of one disc to be green and one disc be blue is determined by taking $P(GB)$ or $P(BG)$. Thereafter they wrote correctly the formula $P(G)\times P(B)+P(B)\times P(G)$. Then, they identified the probability of green disc and blue disc and substituted as $\frac{2}{3}\times\frac{4}{11}+\frac{1}{3}\times\frac{8}{11}$. After simplification, they managed to get $\frac{16}{33}$ which was the correct answer.

In part (b) (i), the candidates recalled correctly the appropriate formula for $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ but $P(A\cap B)=P(B)\times P\left(A/B\right)$ and substituted the probabilities, that is $P(A\cap B)=\frac{3}{10}\times\frac{1}{10}$ then simplified to get $P(A\cap B)=\frac{3}{100}$. Then, they found $P(A\cup B)=\frac{1}{5}+\frac{3}{10}-\frac{3}{100}$, and managed to get $P(A\cup B)=\frac{47}{100}$. While in part (b) (ii), they realized that $P(B\cap A')=P(B)-P(\mathrm{A}\cap\mathrm{B})$ then, they substituted correctly the $P(B)=\frac{3}{10}$ and $P(A\cap B)=\frac{3}{100}$, thus $P(B\cap A')=\frac{3}{10}-\frac{3}{100}$ and obtained $\frac{27}{100}$.

In part (c), the candidates had adequate knowledge on arrangement. They correctly realized that the concept tested is permutation. Thus they recalled the formula for permutation as ${}^{n}P_{r}=\frac{n!}{(n-r)!}$, but $n=11$ and $r=6$, so that ${}^{n}P_{r}={}^{n}P_{r}=\frac{11!}{(11-6)!}=\frac{11!}{5!}$. After applying correctly the permutation they expanded $\frac{11!}{5!}$ as $\frac{11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1}$. Finally, they managed to get 332640 ways, which is correct answer. Extract 13.1 is a sample response from one of the candidates who attempted this question correctly.

13.

(a) Soln.



(i) Soln.

$P(G_1 \cap G_2) = \frac{2}{3} \times \frac{7}{11} = \frac{14}{33}$

$\therefore$ The probability is $14/33$.

(ii). $P(B_1 \cap B_2)$

$P(B_1 \cap B_2) = P(B_1) \times P(B_2)$

$= \frac{1}{3} \times \frac{3}{11}$

$= \frac{1}{11}$.

∴ The probability is $1/11$

(iii).

$P((G_1 \cap B_2) \text{ or } (B_1 \cap G_2))$

$= \left(\frac{2}{3} \times \frac{4}{11}\right) + \left(\frac{1}{3} \times \frac{8}{11}\right)$

$= \frac{8}{33} + \frac{8}{33}$

$= \frac{16}{33}$.

∴ The probability is $16/33$.

13. (b)

Soln.

Given, $P(A) = 1/5$ $P(A/B) = 1/10$.

$P(B) = 3/10$. $P(A \cup B) = ?$

$P(B \cap A') = ?$

From,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{1}{10} = \frac{P(A \cap B)}{3/10}.$$

$$P(A \cap B) = \frac{1}{10} \times \frac{3}{10}$$

$$P(A \cap B) = \frac{3}{100}.$$

From,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{5} + \frac{3}{10} - \frac{3}{100}$$

$$= \left(\frac{2+3}{10}\right) - \left(\frac{3}{100}\right)$$

$$= \left(\frac{5}{10}\right) - \left(\frac{3}{100}\right)$$

$$= \frac{50-3}{100}$$

$$= \frac{47}{100}.$$

$$\therefore P(A \cup B) = 47/100.$$

From,

$P(B \cap A') = P(B) - P(A \cap B)$

$= \frac{3}{10} - \frac{3}{100}$

$= \frac{30-3}{100}$

$= \frac{27}{100}$

$\therefore P(B \cap A') = \frac{27}{100}$

(c) Soln

The operation is permutation, ^nP_r

where,

$n = 11$

$r = 6$

From,

$^nP_r = \frac{n!}{(n-r)!}$

$= \frac{11!}{(11-6)!}$

$= \frac{11!}{5!}$

$= 332,640$

$\therefore$ There are 332,640 ways.

Extract 13.1: A sample of correct responses in question 13

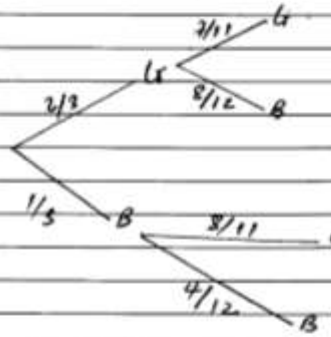
As shown in extract 13.1, part (a), the candidate copied and completed the tree diagram correctly then used it to determine the required probabilities. In part (b), the candidate recalled the correct formula which was used to find $P(A \cup B)$ and $P(B \cap A')$ thus getting the required answer. In part (c), the candidate determined correctly the number of ways in which 11 people could seat by using appropriate formula of permutation.

Nevertheless 22.2 per cent of the candidates scored low marks. Those candidates faced some challenges while attempting this question. In part (a), few candidates failed to copy and complete the given tree diagram correctly as shown in extract 13.2. Also some candidates misinterpreted the type of probability where by instead of considering the given probability as independent, they treated it as a normal probability. For example, one of the candidates responded as follows; in (i) when both discs were green, the candidate determined $n(S)=8$ and $n(E)=1$. Then, applied the formula $P(E)=\frac{n(E)}{n(S)}=\frac{1}{8}$ instead of $P(GG)=P(G)\times P(G)=\frac{2}{3}\times\frac{7}{11}=\frac{14}{33}$. While in (ii), when both discs were blue the candidate applied $n(S)=8$ and $n(E)=1$ then computed it as $P(E)=\frac{n(E)}{n(S)}=\frac{1}{8}$. However, the correct approach was $P(BB)=P(B)\times P(B)=\frac{1}{3}\times\frac{3}{11}=\frac{3}{33}=\frac{1}{11}$. In (iii), when one disc is green and one disc is blue, the candidate identified $n(S)=8$ and $n(E)=6$ then applied the formula $P(E)=\frac{n(E)}{n(S)}=\frac{6}{8}=\frac{3}{4}$, instead of using the formula $P(GB)=P(G)\times P(B)+P(B)\times P(G)$ to compute $P(GB)$, which could lead to $P(GB)=\frac{16}{33}$.

In part (b), few candidates failed to translate the probability joined by the symbol union or intersection due to insufficient knowledge about probability of dependent events. They considered the symbol union and intersection as addition. For example, one candidate wrote $P(A\cup B)=P(A)+P(B)$ then computed as $P(A\cup B)=\frac{1}{5}+\frac{3}{10}=\frac{1}{2}$. Then, in (ii) the candidate calculated $P(B\cap A')$ by using a formula $P(B\cap A')=P(B)+P\left(A/B\right)=\frac{3}{10}+\frac{1}{10}=\frac{2}{5}$.

While in part (c), the candidates applied incorrect formula for permutation to determine the number of arrangements of 11 people. For example, one candidate responded as ${}^{11}C_6=\frac{11!}{6!}=\frac{11\times10\times9\times8\times7\times6!}{6!}$, that led to getting an incorrect response on arrangements. Extract 13.2 is a sample of an incorrect response.

13. (a) Solution.



(i) $P(G) = 7/11 + 8/11 \quad \frac{7+11}{11} = 18/11$

$P(G) = 1\,7/11.$

(ii) $P(B) = 8/12 + 4/12 \quad \frac{8+4}{12} = 14/12$

13 (b) Find $P(A \cup B)$

Solution:

$\frac{1}{5} + 3/10 = \frac{2+3}{10} = 5/10 = 1/2 = 0.5$

$1/2 + 1/10 \quad \frac{5+1}{10} = 6/10 \quad 3/5$

$= 0.6$ or $3/5$.

$\therefore$ The value of $P(A \cup B)$ is $3/5$.

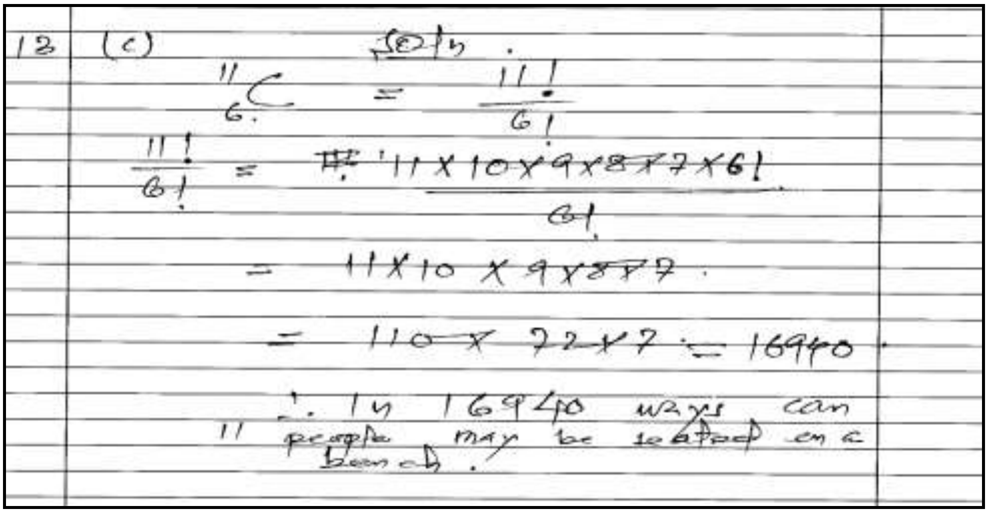
Find

$P(B \cap A')$

$= 3/10 + 1/10 = \frac{3+1}{10} = 4/10$

$= 4/10 = 2/5 = 0.4.$

$\therefore$ The value of $P(B \cap A')$ is $2/5$.



13 (c)

Soln.

$^{11}_{6}C = \frac{11!}{6!}$

$\frac{11!}{6!} = \frac{11\times10\times9\times8\times7\times6!}{6!}$

$= 11\times10\times9\times8\times7$

$= 110\times72\times7 = 16940$

∴ In 16940 ways can 11 people may be seated on a bench.

Extract 13.2: A sample of incorrect responses in question 13

Extract 13.2 shows that, in part (a), the candidate copied and completed the tree diagram with wrong entries then used inappropriate formula to determine the asked probabilities which resulted to an incorrect answer. In part (b), the candidate applied incorrect formula which resulted to incorrect answer. In part (c), the candidate used inappropriate formula thus getting incorrect response.

2.14 Question 14: Vectors and Transformations

The question consisted of three parts (a), (b) and (c). In part (a), the candidates were required to find the work done when a force given by $\underline{F} = 4\underline{i} - 3\underline{j} + 6\underline{k}$ displaces an object from $A(0, 4, 5)$ to $B(3, 12, 10)$. In part (b), they were given that, vector of points A and B are $\underline{a} = 5\underline{i} - \underline{j} - 3\underline{k}$ and $\underline{b} = \underline{i} + 3\underline{j} - 5\underline{k}$ respectively. Then, were required to show that vector $\underline{a} + \underline{b}$ is perpendicular to the vector $\underline{a} - \underline{b}$. In part (c), they were required to determine the image of $(3, -8)$ under a reflection in the line $x + y = 0$ followed by a rotation of -90° clock wise about the origin.

The analysis depicts that, a total of 320 (100%) candidates attempted this question, out of which 88 (27.5%) candidates scored 0.0 to 2.5 marks, 119 (37.2%) candidates scored 3.0 to 6.0 marks and 113 (35.3%) candidates scored 6.5 to 10.0 marks. The candidates' performance summary for this question is presented in Figure 15.

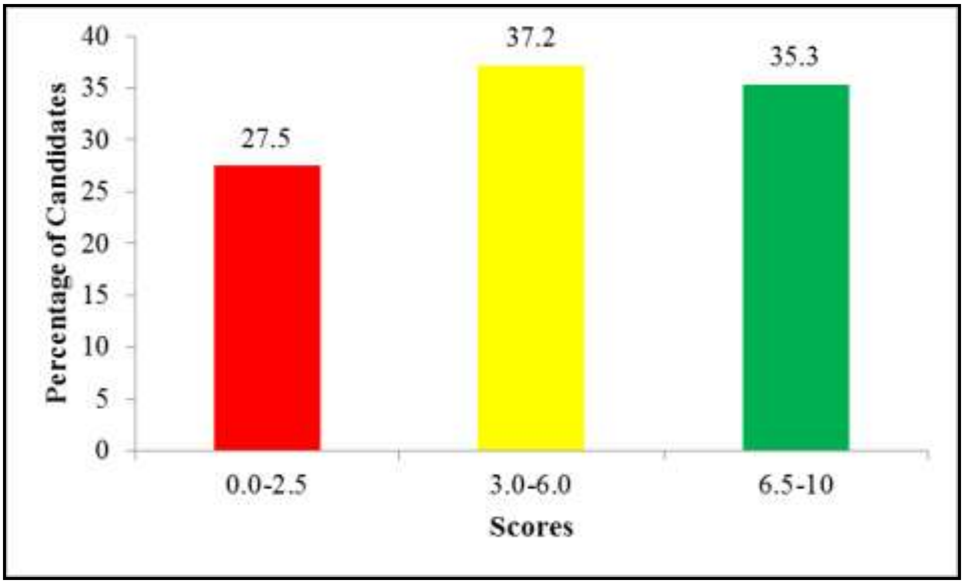


Figure 15: *Candidates' Performance in Question 14*

Further analysis shows that, the overall candidates' performance in this question was good.

In part (a), the analysis showed that the candidates who responded correctly managed to recognized that, the displacement vector is given by $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$. Then, they computed it and got $\overrightarrow{AB} = 3\underset{\sim}{i} + 8\underset{\sim}{j} + 5\underset{\sim}{k}$. Furthermore the candidates recalled and applied correctly the formula for determining the work done as $W = \overline{F}.\overrightarrow{AB}$. Then, substituted and computed it correctly to get $W = (4\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}).(3\underset{\sim}{i} + 8\underset{\sim}{j} + 5\underset{\sim}{k})$ and obtained 18 Joules.

In part (b), the candidates managed to determine vectors $\underset{\sim}{a} - \underset{\sim}{b}$ and $\underset{\sim}{a} + \underset{\sim}{b}$ such that $\underset{\sim}{a} - \underset{\sim}{b} = (5\underset{\sim}{i} - \underset{\sim}{j} - 3\underset{\sim}{k}) - (\underset{\sim}{i} + 3\underset{\sim}{j} - 5\underset{\sim}{k})$ and obtained $4\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$. Then calculated $\underset{\sim}{a} + \underset{\sim}{b} = (5\underset{\sim}{i} - \underset{\sim}{j} - 3\underset{\sim}{k}) + (\underset{\sim}{i} + 3\underset{\sim}{j} - 5\underset{\sim}{k})$ and obtained $6\underset{\sim}{i} + 2\underset{\sim}{j} - 8\underset{\sim}{k}$. Then, they recalled the condition for vectors to be perpendicular as the dot product should be zero (0). Thereafter, they applied and computed it as $(6\underset{\sim}{i} + 2\underset{\sim}{j} - 8\underset{\sim}{k})\bullet(4\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k})$ which resulted to 0. Finally they concluded that, since $(\underset{\sim}{a} + \underset{\sim}{b})\bullet(\underset{\sim}{a} - \underset{\sim}{b}) = 0$ then the two vectors are perpendicular.

In part (c), the candidates recalled and applied correctly the transformation formula, that is; $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, and substituted correctly the given entries $(x, y) = (3, -8)$ and $\alpha = 135^0$ to the transformation formula as $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2(135^0) & \sin 2(135^0) \\ \sin 2(135^0) & -\cos 2(135^0) \end{pmatrix} \begin{pmatrix} 3 \\ -8 \end{pmatrix}$ then computed it to obtain $(x', y') = (8, -3)$. Then, they recalled correctly the rotation formula to be $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$, and substituted the correct values of the entries $(x', y') = (8, -3)$, $\theta = 270^0$ $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos 270^0 & -\sin 270^0 \\ \sin 270^0 & \cos 270^0 \end{pmatrix} \begin{pmatrix} 8 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos(-90^0) & -\sin(-90^0) \\ \sin(-90^0) & \cos(-90^0) \end{pmatrix} \begin{pmatrix} 8 \\ -3 \end{pmatrix}$ to get $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix}$. Therefore, the image for the double transformation was $(-3, -8)$. Extract 14.1 is a sample response from one of the candidates who attempted the question correctly.

14 a)	$\overrightarrow{AB} = (B - A)$
	$\overrightarrow{AB} = (3, 12, 10) - (0, 4, 5)$
	$\overrightarrow{AB} = (3, 8, 5)$
	$\overrightarrow{AB} = 3\underline{i} + 8\underline{j} + 5\underline{k}$
	Work done $= \underline{f} \cdot \underline{d}$
	$= \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} \begin{pmatrix} 3 \\ 8 \\ 5 \end{pmatrix}$
	$= (4 \times 3) + (-3 \times 8) + (6 \times 5)$
	$= 12 - 24 + 30$
	$= 18\,J$

14b) $(\underline{a}+\underline{b})\cdot(\underline{a}-\underline{b}) = 0$

$(\underline{a}+\underline{b}) = 5\underline{i} - \underline{j} - 3\underline{k} + \underline{i} + 3\underline{j} - 5\underline{k}$

$(\underline{a}+\underline{b}) = 6\underline{i} + 2\underline{j} - 8\underline{k}$

$(\underline{a}-\underline{b}) = (5\underline{i} - \underline{j} - 3\underline{k}) - (\underline{i} + 3\underline{j} - 5\underline{k})$

$= 5\underline{i} - \underline{j} - 3\underline{k} - \underline{i} - 3\underline{j} + 5\underline{k}$

$(\underline{a}-\underline{b}) = 4i - 4\underline{j} + 2\underline{k}$

$$(\underline{a}+\underline{b})\cdot(\underline{a}-\underline{b}) = \begin{pmatrix} 6 \\ 2 \\ -8 \end{pmatrix}\begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$$

$= 6\times4 + 2\times-4 - 8\times2$

$24 - 8 - 16$

$= 24 - 24$

$= 0$

$\therefore$ $(\underline{a}+\underline{b})$ and $(\underline{a}-\underline{b})$ are perpendicular to each other

14c) $y = -x$

$M = -1$

$\theta = 135^\circ$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(2\times135^\circ) & \sin(2\times135^\circ) \\ \sin(2\times135^\circ) & -\cos(2\times135^\circ) \end{pmatrix}\begin{pmatrix} 3 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 270^\circ & \sin 270^\circ \\ \sin 270^\circ & -\cos 270^\circ \end{pmatrix}\begin{pmatrix} 3 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

$\theta = -90^\circ$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos -90^\circ & -\sin(-90^\circ) \\ \sin(-90^\circ) & \cos(-90^\circ) \end{pmatrix}\begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix}$$

$\therefore$ The final image is $\begin{pmatrix} -3 \\ -8 \end{pmatrix}$

Extract 14.1: A sample of correct responses in question 14

Extract 14.1 shows that, in part (a), the candidate calculated the work done correctly after determining a displacement $\overrightarrow{AB}$ and multiplied with the given force. In part (b), the candidate determined vectors $\underset{\sim}{a}+\underset{\sim}{b}$ and $\underset{\sim}{a}-\underset{\sim}{b}$ then multiplied it and obtained a zero which proved the two vectors to be perpendicular. In part (c), the candidate applied the correct formulas for determining the double transformation.

However, 27.5 per cent of the candidates scored low marks as they faced the following difficulties; in part (a), few candidates failed to determine the displacement because they applied inappropriate formula for calculating the distance $\overrightarrow{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ and substituted the given entries as $\overrightarrow{AB}=\sqrt{(3-0)^2+(12-4)^2+(10-5)^2}$ and got $\overrightarrow{AB}=9.9$ units instead of determined the displacement vector as $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$. Then, they calculated the work done by using a formula $WD=\text{Force}\times\text{distance}$, and obtained work done, $WD=69.3\,\text{Joules}$. Other candidates applied incorrect techniques for finding the work done, whereby they took $\underset{\sim}{F}=4\underset{\sim}{i}-3\underset{\sim}{j}+6\underset{\sim}{k}$, and considered $d_1=0\underset{\sim}{i}+4\underset{\sim}{j}+5\underset{\sim}{k}$ and $d_2=3\underset{\sim}{i}+12\underset{\sim}{j}+10\underset{\sim}{k}$, thereafter they found a total displacement, $d_1+d_2=3\underset{\sim}{i}+16\underset{\sim}{j}+15\underset{\sim}{k}$. Then applied the distance formula to find $d=\sqrt{x^2+y^2+k^2}=\sqrt{490}=22.13$ units and $F=\sqrt{x^2+y^2+k^2}=\sqrt{61}=7.81$. Lastly they recalled the formula for work done and calculated it as $WD=\text{Force}\times\text{displacement}=22.13\times7.81$ and ended up with W.D = 172.8 units. This indicates that the candidates had inadequate knowledge and skills about vectors.

In part (b), there were few candidates who were not conversant with vectors as they applied incorrect formula for determining whether $\underline{a}+\underline{b}$ is perpendicular to $\underline{a}-\underline{b}$. Instead of using $(\underline{a}+\underline{b})\bullet(\underline{a}-\underline{b})=0$, they applied $|a|\bullet|b|=0$. Moreover the candidates determined $|a|=\sqrt{35}$ and $|b|=\sqrt{35}$, $|\underline{a}+\underline{b}|=\sqrt{104}$ and $|\underline{a}-\underline{b}|=\sqrt{34}$. Then, they were unable to go further to meet the demand of the question.

In part (c), few candidates failed to realize the correct value of θ, for example, when determining a reflection the candidate used $\theta=90^0$ instead of 135^0 and substituted to the formula as $\begin{pmatrix} x' \\ y' \end{pmatrix}=\begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & -cos 90 \end{pmatrix}\begin{pmatrix} 3 \\ -8 \end{pmatrix}$ and computed it to obtain $(x',\mathrm{y}')=(-8,3)$ instead of the correct value $(x',\mathrm{y}')=(8,-3)$. Extract 14.2 is a sample of incorrect response from one of the candidates.

14 a) workdone = force × displacement.

$F_1 = 4i - 3j + 6k.$

$d_1 = 0i + 4j + 5k.$

$d_2 = 3i + 12j + 10k.$

total displacement $= d_1 + d_2.$

$= 0i + 4j + 5k + 3i + 12j + 10k.$

$= 3i + 16j + 15k.$

$d = \sqrt{x^2 + y^2 + k^2}$

$= \sqrt{3^2 + 16^2 + 15^2}$

$= \sqrt{9 + 256 + 225}$

$= \sqrt{490}$

displacement $= 22.13$ units.

$F = \sqrt{x^2 + y^2 + k^2}$

$= \sqrt{4^2 + (-3)^2 + (6)^2}$

$= \sqrt{16 + 9 + 36}$

$= \sqrt{61}$

$= 7.81$ units.

Workdone = displacement × force.

$= 22.13 \times 7.81$

$= 172.8$ units.

$\therefore$ The required workdone is 172.8 units.

(b).

$|\underset{\sim}{a}|.|\underset{\sim}{b}| = 0$

$|a| = \sqrt{5^2+1^2+3^2}$

$|\underset{\sim}{a}| = \sqrt{35}$

$|\underset{\sim}{b}| = \sqrt{1^2+3^2+5^2}$

$|b| = \sqrt{35}$

$\therefore \ \underset{\sim}{a} + \underset{\sim}{b} = 5i - j - 3k + (i + 3j - 5k)$

$= 5i - j - 3k + i + 3j - 5k$

$\underset{\sim}{a} + \underset{\sim}{b} = 6i + 2j - 8k$

$\underset{\sim}{a} - \underset{\sim}{b} = 5i - j - 3k - i - 3j + 3k$

$= 4i - 4j + 2k$

$\sqrt{6^2+2^2+8^2} = |\underset{\sim}{a} + \underset{\sim}{b}|$

$\sqrt{104} = |\underset{\sim}{a} + \underset{\sim}{b}|$

$\sqrt{34.} = |\underset{\sim}{a} - \underset{\sim}{b}|$

14(c)

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & -\cos 90 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -8 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -8 \\ 3 \end{pmatrix}$

$\therefore \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} +\cos 90 & +\sin 90 \\ -\sin 90 & -\cos 90 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Extract 14.2: A sample of incorrect responses in question 14

Extract 14.2 details a sample of incorrect responses as in part (a), the candidate failed to determine the displacement $\overrightarrow{AB}$ which led to obtaining the incorrect work done. In part (b), instead of using the dot product to show that the two vectors are perpendicular, the candidate calculated the absolute values of $\underset{\sim}{a}+\underset{\sim}{b}$ and $\underset{\sim}{a}-\underset{\sim}{b}$. In part (c), the candidate failed to realize the correct angle of reflection because, instead of using $\theta = 135^{\circ}$, $\theta = 90^{\circ}$ was used.

3.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE IN EACH TOPIC

The Additional Mathematics paper comprised of fourteen (14) questions in sections A and B that were composed from fourteen (14) topics. The topics include; *Numbers*, *Variations*, *Statistics*, *Locus*, *Coordinate geometry*, *Symmetry*, *Logic*, *Sets*, *Algebra*, *Trigonometry*, *Function and Remainder Theorem*, *Integration and Differentiation*, *Probability* and *Vectors and Linear Transformation*. The analysis of candidates' performance per topic in 2021 shows that, candidates performed well in twelve (12) topics while in two (2) topics the performance was of average (Appendix I).

The analysis depicts further that, the topics in which the performance was good were *Variations* (98.8%), *Coordinate geometry* (96.2%), *Numbers* (94.4%), *Algebra* (93.4%), *Symmetry* (90.4%), *Function and Remainder Theorem (88.4%), Statistics (87.6%), Sets (79.7%), Probability (77.8%), Vectors and Linear Transformations (72.5%), Locus (69.7%), Integration and Differentiation (66.9%).*

The good performance of the candidates in these topics was attributed to adequate knowledge and skills about the topics, ability to use appropriate concepts in responding to the questions as well as correct interpretation of the requirements of the questions.

On the other hand, the topics of *Trigonometry* and *Logic* had average performance of 64.4 per cent and 56.9 per cent respectively. The average performance in these topics was mainly attributed to candidates' moderate knowledge on the tested concepts. The summary of the candidates' performance in each topic is presented in Appendix I.

The analysis of candidates' performance topic wise in 2020 shows that, ten (10) topics were well performed, while three (3) topics were averagely performed. The performance was weak in one (01) topic. Primarily, the analysis of candidates' performance per topic reveals that for the year 2021 there is a decrease in performance in *Trigonometry* topic compared to the year 2020 by 5.7 per cent as shown in Appendix II.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The Candidates' Item Response Analysis (CIRA) report for CSEE 2021 was prepared to reveal the strengths and weaknesses of the candidates' responses on the items of each question in the topics tested. The analysis revealed that, the general performance in 2021 was good since 85.7 per cent of the tested topics had good performance. There was no topic in which the performance was weak.

The report identified the areas in which some of the candidates demonstrated some weaknesses when responding to the questions. Those weaknesses include the candidates' failure to apply the concepts of the double angles in solving related problems, construct truth tables as well as interpreting and applying the laws of algebra of propositions.

It is expected that this report will help teachers and students to improve the teaching and learning strategies in Additional Mathematics. The recommendations suggested in this report will be helpful in improving the performance in Additional Mathematics subject in future.

4.2 Recommendations

In order to improve the candidates' performance in this subject, it is recommended that teachers should:

(a) lead students to discuss;

- (i) the applications of double angle formulae in solving trigonometric problems.
- (ii) about laws of algebra of prepositions and its applications in solving problems in logic.
- (iii) the applications of *Differentiation and Integration* in solving problems such as finding turning points, points of inflexion of curves, finding area under a curve.
- (iv) on how to solve pairs of simultaneous equations involving linear and quadratic equations.

(b) encourage students to establish subjects' clubs aiming at discussing various mathematics concepts to build their competences.

(c) provide adequate number of exercises to enable students to develop skills of solving questions involving graphs.

APPENDIX I

Analysis of Candidates' Performance in Each Topic

S/N	Topics	Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks
1.	Variations	1	98.8	Good
2.	Coordinate Geometry	3	96.2	Good
3.	Numbers	8	94.4	Good
4.	Algebra	5	93.4	Good
5.	Symmetry	6	90.4	Good
6.	Functions and Remainder Theorem	11	88.4	Good
7.	Statistics	2	87.6	Good
8.	Sets	10	79.7	Good
9.	Probability	13	77.8	Good
10.	Vectors and Transformations	14	72.5	Good
11.	Locus	4	69.7	Good
12.	Integration and Differentiation	12	66.9	Good
13.	Trigonometry	7	64.4	Average
14.	Logic	9	56.9	Average

APPENDIX II

Comparison of Candidates' Performance in Each Topic for the Year 2020 and 2021

S/N	Topics	2020			2021		
		Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks
1.	Variations	1.	91.4	Good	1.	98.8	Good
2.	Coordinate Geometry	3.	62.4	Average	3.	96.2	Good
3.	Numbers	8.	51.9	Average	8.	94.4	Good
4.	Algebra	5.	93.8	Good	5.	93.4	Good
5.	Symmetry				6.	90.4	Good
6.	Functions and Remainder Theorem	11.	89.4	Good	11.	88.4	Good
7.	Statistics	2.	69.6	Good	2	87.6	Good
8.	Sets	10.	98.8	Good	10.	79.7	Good
9.	Probability	13.	77.2	Good	13.	77.8	Good
10.	Vectors and Transformations	14.	45.2	Average	14.	72.5	Good
11.	Locus	4.	68.6	Good	4.	69.7	Good
12.	Integration and Differentiation	12.	65.1	Good	12.	66.9	Good
13.	Trigonometry	7.	70.1	Good	7.	64.4	Average
14.	Logic	9.	26.7	Weak	9.	56.9	Average
15.	Plan and Elevation ; and Geometrical Constructions	6.	93.5	Good			