THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2020

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania has prepared the Candidates' Item Response Analysis Report (CIRA) showing the performance of the candidates who sat for the Certificate of Secondary Education Examination (CSEE) 2020 in 041 Basic Mathematics Examination. This report was purposely prepared in order to give feedback to students, teachers, policy makers and all other education stakeholders about the candidates' performance.

The examination paper in Basic Mathematics consisted of 14 compulsory questions, out of which 10 questions were in section A and 4 were in Section B. During the analysis, it was found that, none of the questions had good performance. The performance was average in questions 6, 7, 8 and 11 that were set from the topics of *Rates and variations*, *Accounts, Ratio, Profit and Loss*, *Sequence and Series* as well as *Statistics*. In contrast, the performance was weak in the remaining 10 questions with percentage of the candidates who scored at least 30 percent ranging from 25.6 to 6.3 percent as summarized in the Appendix.

The major factors that contributed to average and weak performance have been analyzed in this report basing on the candidates' competence in applying the correct formulae, rules, theorems, properties and procedures, formulating mathematical expressions, inequalities and equations from word problems, performing the mathematical operations, drawing diagrams or graphs as well as interpreting figures correctly.

The National Examinations Council of Tanzania is honored to express its sincere thanks to all the examiners, examination officers and all other individuals who were involved in preparing this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of the candidates' Item responses in 041 Basic Mathematics Examination for CSEE 2020. The analysis highlights the areas where the candidates had challenges when answering examination questions and areas in which they performed well.

The number of candidates who sat for the examination in CSEE 2020 was 435,345 out of which 87,582 (20.12%) candidates passed. A total of 422,332 candidates sat for the CSEE 2019, out of which 84,578 (20.03%) candidates passed. Therefore, the performance has increased by 0.09 percent.

The examination paper in Basic Mathematics subject had two sections, A and B. Each question in Section A weighs **six** (**06**) marks, whereas each question in Section B weighs **ten** (**10**) marks and the candidates were required to answer all questions from both sections.

The national examination results are based on the score intervals 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 which are equivalent to *excellent*, *very good*, *good*, *satisfactory* and *fail*, respectively. For the purpose of this report, the candidates' performance in each question is considered *good*, *average* or *weak* if the percentage of the candidates who scored at least 30 out of 100 percent is 65 – 100, 30 – 64 or 0 – 29, respectively as shown in the Figures in the analysis of each question and the Appendix at the end of this report.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

This section addresses the analysis of the candidates' performance in each question. The section briefly includes: descriptions of the requirements of the items, summary on how the candidates answered the items of each question, sample extracts showing the candidates' correct and incorrect responses and the reasons for the ability or failure to get the correct responses in each question. The description of data and charts was done using a criterion of the score intervals: 6.0 – 4.0, 3.5 – 2.0 and 1.5 – 0.0 out of 6 marks in Section A; and 10 – 6.5, 6.0 – 3.0 and 2.5 – 0 out of 10 marks in Section B for each question representing *good*, *average* and *weak* performance, respectively

2.1 Question 1: Numbers, Decimals and Percentages

This question consisted of parts (a) and (b). In part (a), the candidates were required to simplify the expression $\frac{0.25\times 8.85\times 25}{0.00625\times 0.5}$ without using mathematical tables, expressing the final answer correct to two significant figures. In part (b)(i), it was given that: "*Mr. Magani set an examination weighing a total of 96 marks with the following distribution: 20% of the marks were awarded for reading, 40% for writing, 15% for practical and the remaining percentage for spelling*". The candidates were required to find the marks that were awarded for spelling. In part (b)(ii), the question stated: "*Three airplanes arrived at Kilimanjaro International Airport (KIA) at the intervals of 30 minutes, 40 minutes and 55 minutes.*" The candidates were required to find the day and the time at which all three airplanes would arrive together again if they all arrived at KIA at 2:00 p.m. on Saturday.

This question was attempted by 367,598 (84.1%) candidates. The analysis shows that 48,913 (13.3%) candidates scored from 2 to 6 marks, among them 1,777 (0.5%) scored full marks. In contrast, out of 318,685 (86.7%) candidates who scored below average, 237,297 (64.6%) candidates scored 0 mark. This shows that the candidates' performance in this question was weak. The candidates' performance in this question is summarized in Figure 1.

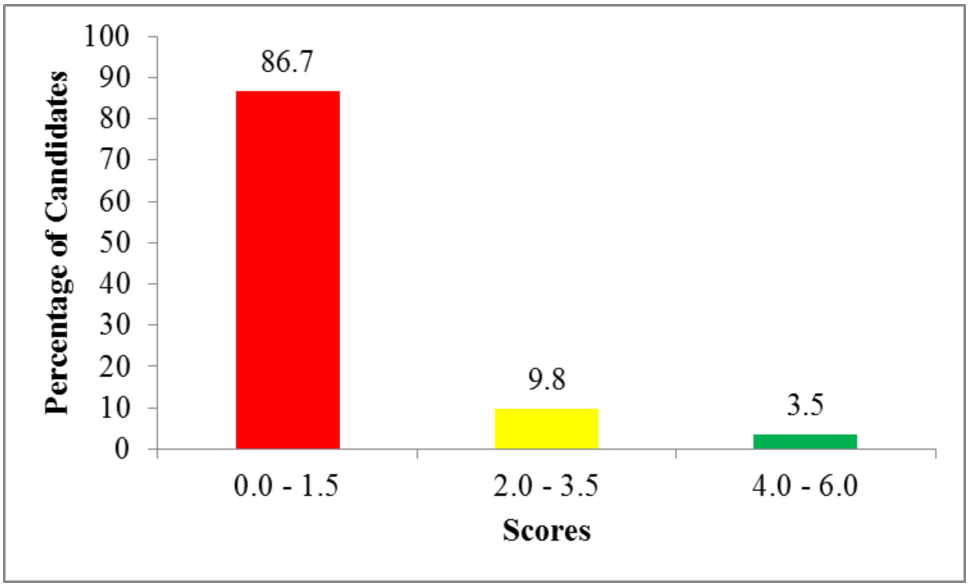


Figure 1: *Candidates' performance in question 1.*

The response analysis indicates that, the candidates failed to answer this question correctly due to various reasons: In part (a), they failed to simplify the given expression and carry out mathematical operations. Some of them were able to simplify the given expression into 1.77×10^4 but failed to express this result into two significant figures. Some of common errors committed include: 1.8×10^3, 17700, 2.0×10^4, 18 and 17000 instead of 18000 or 1.8×10^4 after rounding off 1.77×10^4 to two significant figures. There were also a few candidates who used mathematical tables or scientific calculator contrary to the given instructions. Furthermore, most of the candidates failed to deal with decimals in the expression when simplifying. For instance they wrote incorrect steps like $\frac{0.25\times8.85\times25}{0.00625\times0.5}=\frac{25\times10^2\times885\times10^2\times25}{625\times10^5\times5\times10^1}$ instead of writing it as $\frac{0.25\times8.85\times25}{0.00625\times0.5}=\frac{25\times10^{-2}\times885\times10^{-2}\times25}{625\times10^{-5}\times5\times10^{-1}}$. This shows that the candidates failed to recall that when shifting decimal points to the right, the exponent must be negative and vice versa.

In part (b)(i), some of the candidates subtracted 96 marks from 100 marks to get 4 marks and regarded this as the marks awarded for spelling which is incorrect. Others were able to get $100\%-(20\%+40\%+15\%)=25\%$ but did not proceed further to calculate 25% of 96 marks to get the answer. For example, they wrote incorrect expressions like $\frac{25}{96}\times 100$ instead of $\frac{25}{100}\times 96$. This indicates that those candidates lacked knowledge of application of percentages. In (b)(ii), some of the candidates were unable to realize that this question requires the application of the Lowest Common Multiples (LCM) and not the Greatest Common Factor (GCF). For example, they calculated the GCF of 30, 40 and 55 which is an incorrect step. In some cases, there were few candidates who correctly calculated the LCM and got 1320 minutes, converted it into 22 hours but failed to determine the day and the time at which the airplanes will arrive together again. Extract 1.1 illustrates a sample response of a candidate who failed to get the correct answers in this question.

1. a) $\frac{0.25 \times 8.85 \times 25}{0.00625 \times 0.5}$

$= \frac{\cancel{5}(0.5 \times 1.77 \times 5)}{\cancel{5}(0.0125 \times 0.1)}$

$= \frac{4.425}{0.00125}$

$= 3540$

In 2 significant figures:

$= 3500$

b) i) Soln.

96 marks total.

20% for reading

40% for writing.

15% for practical

x% for spelling.

$20\% + 40\% + 15\% + x\% = 96$

$75\% + x\% = 96$

$x\% = 96 - 75\%$

$x\% = 21\%$

∴ 21% was awarded for spelling.

(ii) Soln

Interval of 30 m, 40 m and 55 M

30 + 40 + 55

= 125 m

Extract 1.1: A sample of a candidate's incorrect response in question 1.

In Extract 1.1, the candidate simplified incorrectly the given expression in part (a). He/she also subtracted the sum of the given percentages from 96 marks in order to get the marks awarded for spelling that is $20\% + 40\% + 15\% + x = 96\%$ which is not correct in part (b)(i). Again, the candidate was unable to find the LCM that would give the marks awarded for spelling; instead, he/she added the given minutes in part (b)(ii).

Contrarily, the candidates who answered this question correctly were able to: (a) simplify the given expression giving the answer correct to two significant figures, (b)(i) find the marks awarded for spelling and (b)(ii) use appropriate procedures to find the time and day when all three airplanes will arrive together again. This indicates that the candidates had sufficient skills and knowledge of application of numbers and percentages in daily life. Extract 1.2 shows a response from one of the candidates who answered this question correctly.

01. a,

$$\frac{0.25 \times 8.85 \times 25}{0.00625 \times 0.5}$$

$$\frac{25 \times 10^{-2} \times 885 \times 10^{-2} \times 25}{625 \times 10^{-5} \times 5 \times 10^{-1}}$$

$$\frac{177 \times 10^{-4}}{1 \times 10^{-6}}$$

$177 \times 10^{2} = 17700 \approx \cancel{185}\ 18000$ (2.s.f)

∴ The answer is 18000 (2.s.f)

b, i, % spelling = 100% − (20% + 40% + 15%)

% spelling = 100% − ~~25%~~ 75%

% spelling = ~~75~~ 25%

spelling marks = $\frac{25}{100} \times 96$ = 24 marks

∴ 24 marks were awarded for spelling.

ii,

2	30	40	55
2	15	20	55
2	15	10	55
3	15	5	55
5	5	5	55
11	1	1	11
	1	1	1

LCM = $2^3 \times 3 \times 5 \times 11$

LCM = 1320 minutes

LCM = 22 hours

saturday, 2:00pm + 22 hours = sun, 12:00pm

∴ they will arrive together again on sunday at 12:00pm

Extract 1.2: A sample of a candidate's correct response in question 1.

In Extract 1.2, the candidate multiplied and divided the terms of the given expression correctly to get 17,700 and wrote it to two significant figures to obtain 18,000. The candidate also managed to formulate the equation $20\% + 40\% + 15\% + x = 100\%$, thus the value of x as the marks awarded for spelling. Lastly, he/she calculated the LCM of 30, 40 and 55 minutes to get 1320 minutes and hence the day and time at which all the three airplanes would arrive together again.

2.2 Question 2: Logarithms, Exponents and Radicals

This question had two parts, (a) and (b). In part (a), the candidates were required to find the values of a, b and c given that $\frac{\sqrt{3}}{2+\sqrt{3}} = a + b\sqrt{c}$. In part (b), they were asked to (i) solve the equation $\left(\frac{9}{\sqrt{3}}\right)^{2x} = \frac{1}{81}$ and (ii) find the value of $\log\left(2\frac{1}{4}\right)$ without using mathematical tables given that $\log 2 = 0.3010$ and $\log 3 = 0.4771$.

The analysis shows that, this question was attempted by 332,808 (76.1%) candidates. Furthermore, a total of 47,977 (14.4%) candidates scored from 2 to 6 marks and among them, 6,413 (0.9%) scored all 6 marks. Moreover, 284,831 (85.6%) candidates scored below 2 marks, out of whom 228,895 (68.8%) candidates scored 0 mark. This shows that the candidates' performance in this question was weak. The summary of the candidates' performance in this question is presented in Figure 2.

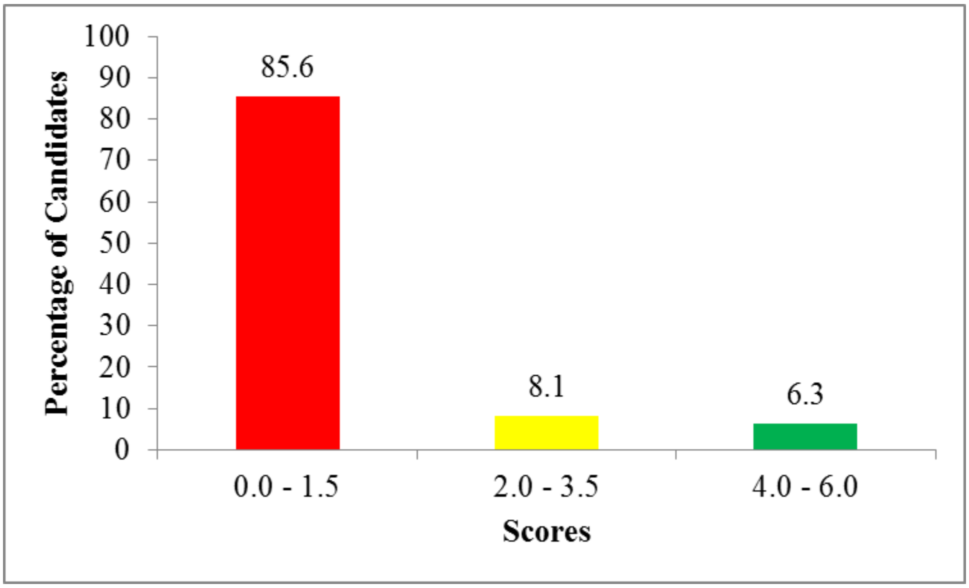


Figure 2: *Candidates' performance in question 2.*

The weak performance was contributed by the following factors: in part (a), most of them were unable to identify the correct rationalizing factor. They used incorrect rationalizing factors like $2+\sqrt{3}$, $\sqrt{3}$ and $\sqrt{3}-2$ instead of

$2-\sqrt{3}$, hence ended up with an incorrect answer. Others were able to identify the factor $2-\sqrt{3}$ but failed to multiply correctly. For instance some of them solved as follows: $\frac{\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{\sqrt{6}-\sqrt{9}}{4-3}=\sqrt{6}-3$ due to lack of knowledge in multiplying rational and irrational numbers. Besides, few candidates managed to rationalize the denominator of $\frac{\sqrt{3}}{2+\sqrt{3}}$ and got the correct answer $-3+2\sqrt{3}$ but failed to deduce the values of a, b and c by using the comparison $-3+2\sqrt{3}=a+b\sqrt{c}$. In part (b)(i), the candidates had inadequate knowledge of exponents since they failed to express $\frac{9}{\sqrt{3}}$ and $\frac{1}{81}$ in exponential form which was an important step in finding the value of x. As a result, they were unable to get the equation involving the exponents raised from the same base. Some of incorrect steps noted from their scripts include: $\left(\frac{3^2}{3}\right)^{2x}=\frac{1}{3^4}\Rightarrow 2(2x)=4$ and $(3)^{2x\times\frac{1}{2}}=3^4\Rightarrow x=4$. Instead, they were supposed to write $\left(\frac{3^2}{3^{\frac{1}{2}}}\right)^{2x}=\frac{1}{3^4}$ and then apply the laws of exponents or logarithm. In part (b)(ii), most of the candidates failed to apply the law of quotient of logarithm to get the value of $\log\left(2\frac{1}{4}\right)$ as they wrote incorrect steps like: $\log\frac{9}{4}=\log 9-2\log 4$, $\log\frac{9}{4}=\log(9-4)$, $\log\frac{3\times 3}{2\times 2}=\frac{\log 3+\log 3}{\log 2+\log 2}$ and $\log\frac{9}{4}=2\log 3-\log 2$ instead of $\log\left(2\frac{1}{4}\right)=\log\frac{9}{4}=\log 9-\log 4$ which could give $\log\frac{9}{4}=2\log 3-2\log 2$.

In addition, some candidates ignored the exponents as they solved as follows: $\log\left(\frac{3^2}{2^2}\right)=\log 3-\log 2=0.4771-0.3010$. Extract 2.1 illustrates the sample response of a candidate who answered the question incorrectly.

2 (a) $\sqrt{a}$ $\frac{\sqrt{3}}{2+\sqrt{3}} = a + b\sqrt{c}$

find the value a, b and c

soln

$$\frac{\sqrt{3}}{2+\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{2\sqrt{3} + \sqrt{9}}{4 \quad 2\sqrt{3} + 2\sqrt{3} + \sqrt{9}}$$

$$\frac{2\sqrt{3} + 3}{4 - 3}$$

$$\frac{2\sqrt{3} + 3}{1}$$

$\therefore$ $= 3 \cdot 2 + \sqrt{3}$

b) i) $\left(\frac{9}{\sqrt{3}}\right)^{2x} = \frac{1}{81}$

$$\left(\frac{9}{\sqrt{3}}\right)^{2x} = (81)^{-1}$$

$$\left(\frac{3^2}{(\sqrt{3})^2}\right)^{2x} = (3^4)^{-1}$$

$$\left(\left(\frac{3}{\sqrt{3}}\right)^2\right)^{2x} = 3^{-4}$$

$$3^{4x} = 3^{-4}$$

$$\frac{4x}{4} = \frac{-4}{4}$$

$$x = -1$$

$\therefore$ The value of x is -1.

2. b ii)
Data
log 2 = 0·3010
log 3 = 0·4771

$\log(2\frac{1}{4})$
soln.
$\log(\frac{9}{4})$
$\log 9 + \log 4$
$\log 3^2 + \log 2^2$
$2\log 3 + 2\log 2$
$2(0.4771) + 2(0.3010)$
$= 0.9542 + 0.3020$
1.2562
$\therefore$ The $\log(2\frac{1}{4}) = 1\cdot 2562$.

Extract 2.1: A sample of a candidate's incorrect response in question 2.

In Extract 2.1, the candidate multiplied the given radical by an incorrect fractional factor $\frac{2+\sqrt{3}}{2+\sqrt{3}}$ in part (a), whereas in part (b)(i), he/she failed to express the given exponential equation into an equation of the same base 3 to get the correct equation involving the exponents. In part (b)(ii), he/she applied the law of product instead of applying the law of quotient of logarithm when finding the value of $\log\left(\frac{9}{4}\right)$ using laws of logarithm.

Although the majority (85.6%) of candidates had weak performance, there were some candidates who answered this question correctly. They managed to rationalize the denominator of the expression $\frac{\sqrt{3}}{2+\sqrt{3}}$ by using the factor $2-\sqrt{3}$ and use the comparison of $-3+2\sqrt{3}=a+b\sqrt{c}$ to get the values of a, b and c in part (a). They also managed to get the value of x in part (b)(i) and the value of $\log\left(2\frac{1}{4}\right)$ in part (b)(ii) by using the appropriate laws of exponents and logarithms. Extract 2.2 represents the sample response of a candidate who answered this question correctly.

2. a) $\dfrac{\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$\dfrac{\sqrt{3}(2-\sqrt{3})}{(2)^2-(\sqrt{3})^2}$

$\dfrac{2\sqrt{3}-3}{4-3} = \dfrac{2\sqrt{3}-3}{1}$

$a+b\sqrt{c} = 2\sqrt{3}-3$

$b\sqrt{c}+a = 2\sqrt{3}+(-3)$

$\therefore a=-3,\ b=2$ and $c=3$.

b) i) $\left(\dfrac{9}{\sqrt{3}}\right)^{2x} = \dfrac{1}{81}$

$\left(\dfrac{3^2}{3^{1/2}}\right)^{2x} = \dfrac{1}{3^4}$

$(3^{3/2})^{2x} = 3^{-4}$

$3^{3x} = 3^{-4}$

$\dfrac{3x}{3} = \dfrac{-4}{3}$

$\therefore x = -4/3$

ii) $\log(2\tfrac{1}{4}) = \log(9/4)$

$\log(9/4) = \log 9 - \log 4$

$\log 3^2 - \log 2^2$

$2\log 3 - 2\log 2$

$2(0.4771) - 2(0.3010)$

$0.9542 - 0.6020 = 0.3522$

$\therefore \log(2\tfrac{1}{4}) = 0.3522$

Extract 2.2: A sample of a candidate's correct response in question 2.

In Extract 2.2, the candidate rationalized the denominator correctly and finally got $a=-3$, $a=2$ and $a=3$ in part (a). He/she was able to apply the laws of power and quotient to express the equation in common base 3, hence got the equation $3^{\frac{6}{2}x} = 3^{-4}$, then $\frac{6}{2}x=-4$ and finally $x=-\frac{4}{3}$ in part (b)(i). Again, the candidate was able to apply the laws of power and quotient to get $2(\log 3 - \log 2) = 2(0.4771 - 0.3010)$, which is an important step to get the correct answer in part (b)(ii).

2.3 Question 3: Sets and Probability

The question stated that: "*In a certain school, 40 students were asked about whether they like tennis or football or both. It was found that the number of students who like both tennis and football was three times the number of students who like tennis only. Furthermore, the number of students who like football only was 6 more than twice the number of students who like tennis only. However, 4 students like neither tennis nor football*". The candidates were required to: (a) represent this information in a Venn diagram, letting x be the number of students who like tennis only and, (b) use the results obtained in part (a) to determine the probability that a student selected at random likes; (i) football only and (ii) both football and tennis.

This question was attempted by 261,016 (59.7%) candidates. The analysis reveals that 21,359 (8.2%) candidates scored at least 2 out of 6 marks, among them 5,711 (2.2%) scored 6 marks. Nevertheless, out of 239,657 (91.8%) candidates who scored below the average, 181,204 (69.4%) candidates scored 0 mark. This indicates that the candidates' performance in this question was weak. The candidates' performance in this question is presented in Figure 3.

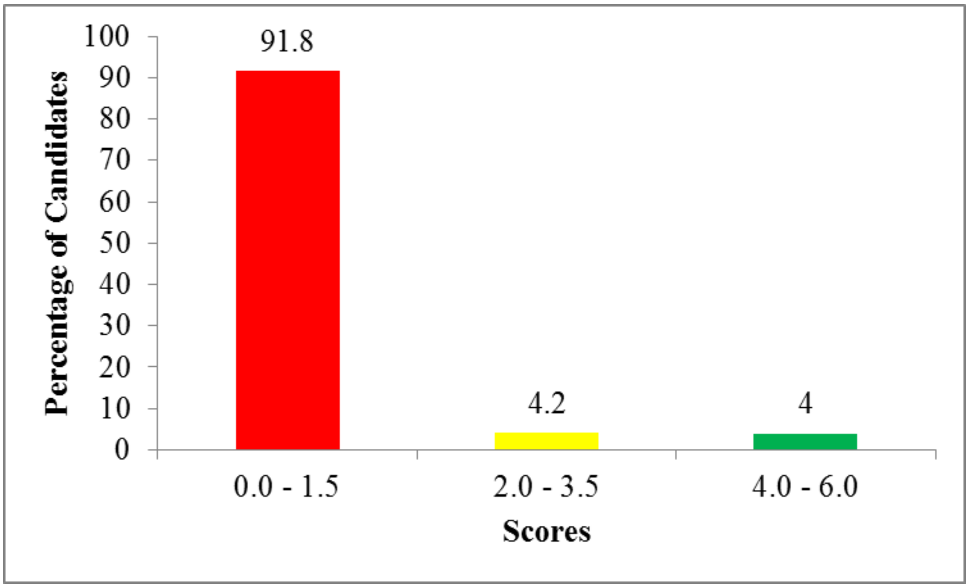
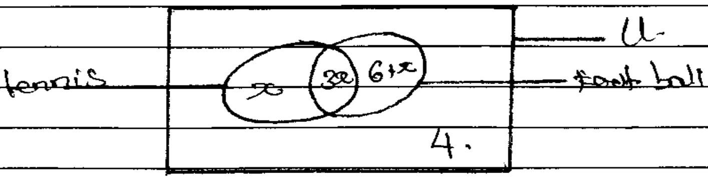


Figure 3: *Candidates' performance in question 3.*

The candidates' failure to get the correct response was caused by the following factors: in part (a), the majority of candidates failed to represent the given information correctly in the appropriate regions of a Venn diagram. For example, most candidates regarded 4 as the number of students who like both tennis and football. They were supposed to note that, 4 students like neither tennis nor football and $3x$ like both. Others formulated incorrect expression representing the number of students who like football and those who like tennis as $(2x+6)$ and x, respectively. The candidates failed to interpret the given information correctly and lacked sufficient knowledge in solving word problems using Venn diagrams.

The candidates who failed to answer part (a) correctly were unable to get the correct response in part (b). This is because they could not get the correct number of students who like football only in part (b)(i) and the number of students who like both football and tennis which are necessary conditions for determining the required probabilities. There were the candidates who managed to get the correct results in part (a) but failed to determine the required probabilities because they failed to apply the correct formula. For instance, some of them wrote $P(E)=\frac{n(s)}{n(E)}=\frac{40}{16}=\frac{5}{2}$ instead of $P(E)=\frac{n(E)}{n(S)}=\frac{16}{40}=\frac{2}{5}$ in part (b)(i) and $P(F\cap T)=\frac{n(S)}{n(\text{Football only})}=\frac{40}{15}=\frac{8}{3}$ instead of $P(F\cap T)=\frac{n(\text{Football only})}{n(S)}=\frac{15}{40}=\frac{3}{8}$ in part (b)(ii). Moreover, there were candidates who failed to identify the correct number of sample space. For example, a certain candidate substituted $n(S)=36$ in the formula $P(E)=\frac{n(E)}{n(S)}$ and got $P(E)=\frac{16}{36}$ instead of substituting $n(S)=40$ to get $P(E)=\frac{16}{40}$. Others regarded the probability of selecting at random a student who likes football only as the number of students who like football only, that is 16; and the probability of selecting at random a student who likes both football and tennis as the number of students who like both football and tennis, that is 15. Extract 3.1 is a sample of an incorrect response.

3. a)



$x+3x+6+x+4=40$

$5x+10=40$

$5x=40-10$

$\frac{5x}{5}=\frac{30}{5}$

$x=6.$

∴ The number of students who like tennis only is 6.

b)

i) Football only $=6+x$

$=6+6.$

∴ Football only is 12.

ii) both football and tennis $=3x.$

$3\times6.=18.$

∴ both football and tennis is 18.

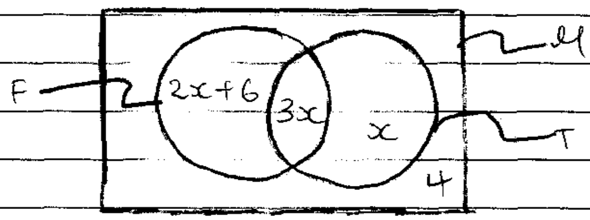
Extract 3.1: A sample of a candidate's incorrect response in question 3.

In Extract 3.1, the candidate wrote $(x+6)$ instead of $(2x+6)$ as the number of students who like football only in the region representing the set of students who like football only of the Venn diagram in part (a). Moreover, he/she lacked knowledge of the axioms and rules of probability in part (b).

Nevertheless, the general performance in this question was weak although some candidates were able to give the correct response. In part (a), they managed to: represent the given information correctly in a Venn diagram, formulate the equation from the Venn diagram and solve for x. In part (b), they were also able to use the results obtained in part (a) to find the required probabilities. Extract 3.2 shows a sample of a candidate's correct response in this question.

03. letting x be $n(T)$ only.

a, The venn diagram.



b, i, $n(\xi) = 40$

$n(T)\text{only} = x$

$n(T \cap F) = 3x$

$n(F)\text{only} = 2x+6$

$n(F \cup T)' = 4$

$40 = 4 + x + 3x + 2x + 6$

$40 = 10 + 6x$

$6x = 40 - 10$

$\frac{6x}{6} = \frac{30}{6}$

$x = 5$ students

$n(E) = n(F)\text{only} = 2x + 6$

$n(E) = 2(5)+6$

$n(E) = 16$ students

$n(P) = \frac{n(E)}{n(S)}$

$n(P) = \frac{16}{40} = \frac{2}{5}$

$\therefore$ the probability is $\frac{2}{5}$

ii, $n(E) = n(F \cap T) = 3(5) = 15$ students

$n(P) = \frac{15}{40} = \frac{3}{8}$

$\therefore$ the probability is $\frac{3}{8}$

Extract 3.2: A sample of a candidate's correct response in question 3.

In part (a) of Extract 3.2, the candidate managed to represent the given information in a Venn diagram, formulate the equation $x+3x+2x+6+4=40$ and hence $x=5$. In part (b)(i), he/she was able determine $n(E_1)=2x+6=2(5)+6=16$, hence found that $P(E_1)=\dfrac{n(E_1)}{n(S)}=\dfrac{16}{40}=0.4$. Similarly, in part (b)(ii), the candidate managed to write $n(E_2)=3x=3(5)=15$, hence got $P(E_2)=\dfrac{n(E_2)}{n(S)}=\dfrac{15}{40}=0.375$.

2.4 Question 4: Coordinate Geometry and Vectors

This question consisted of parts (a) and (b). In part (a)(i), the candidates were given the gradient and *x*-intercept of a line as $\dfrac{3}{2}$ and -3, respectively, and were asked to find the equation of the line in the form $y=mx+c$, where *m* and *c* are constants. In part (a)(ii), they were required to find the length of a line segment joining the points $(3,-2)$ and (15, 3). In part (b), they were informed that, a boat sails due North at a speed of 120 km/h and wind blows at a speed of 40 km/h due East and were asked to find the actual speed of the boat using the substitution $\sqrt{10}=3.16$.

This question was attempted by 256,682 (58.7%) candidates. The analysis shows that 43,994 (17.1%) candidates scored from 2 to 6 marks, among them 4,667 (1.8%) scored full marks. Again, out of 212,688 (82.9%) candidates who scored below 2 marks, 185,946 (72.4%) candidates scored 0 mark. This shows that the candidates' performance in this question was weak. The candidates' performance summary in this question is presented in Figure 4.

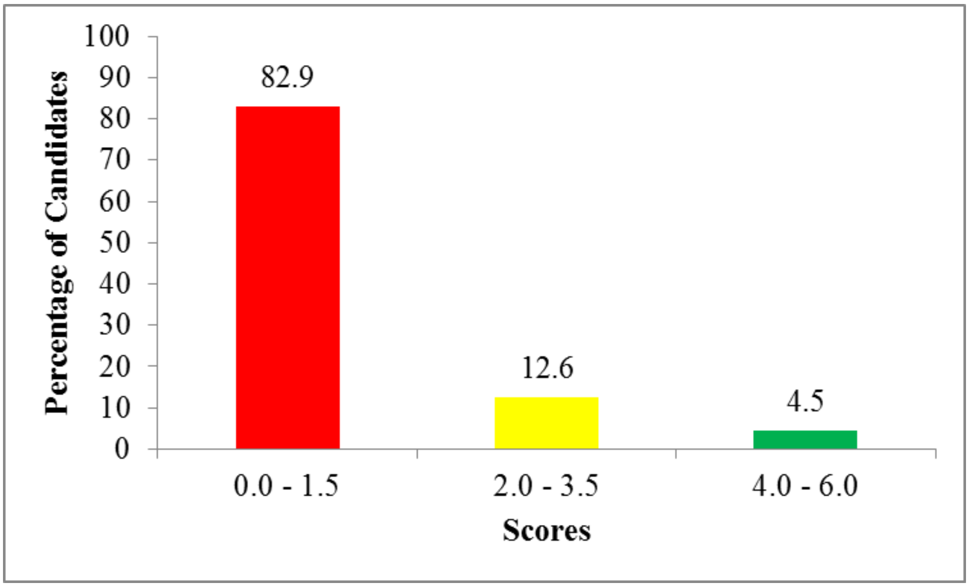


Figure 4: *Candidates' performance in question 4.*

According to response analysis, the majority of candidates (82.9%) scored low marks in this question due to various reasons. In part (a)(i), most of them treated the given x – intercept as the y–intercept, that is $c = -3$ and hence got the equation $y = \frac{3}{2}x - 3$ which is incorrect. Likewise, there were candidates who used the formula for finding the gradient but they substituted the values of $x_1 = 0$ and $y_1 = -3$ in $\frac{3}{2} = \frac{x - x_1}{y - y_1}$ incorrectly, that is $\frac{3}{2} = \frac{y - (-3)}{x - 0}$ instead of writing $\frac{3}{2} = \frac{y - 0}{x - (-3)}$ which is a necessary step to arrive at the correct equation. Moreover, others recalled the appropriate formula for slope but ignored the negative sign for the x – intercept as they wrote $\frac{3}{2} = \frac{y - 0}{x - 3}$ instead of $\frac{3}{2} = \frac{y - 0}{x - -3}$, as a result they ended up with incorrect equation.

In part (a)(ii), some candidates were unable to recall the appropriate formula for finding the length of a given line segment between two points. For example they wrote $d = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$, $d = \sqrt{(x_1 + x_2)^2 - (y_1 + y_2)^2}$, $d = \sqrt{(x_1 + x_2)^2 + (y_1 - y_2)^2}$ instead of

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, hence ended by getting incorrect values. On top of that, other candidates opted to locate the two points in the *xy* – plane but failed to measure accurately the distance using a ruler. Additionally, others confused the formula for finding the distance between two points with the distance for finding the midpoint between two points as some of them wrote that, $\text{distance}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ and substituted the coordinate values to get $\text{distance}=\left(\frac{3+15}{2},\frac{-2+3}{2}\right)$ contrary to the requirements of the question. The candidates failed to get the correct answers in part (b) because they lacked adequate knowledge of vectors and were unable to use the triangular law of addition of vectors to find the resultant velocity of the boat correctly. For example, some candidates wrote $40\underline{i}-120\underline{j}$ as a resultant velocity instead of $40\underline{i}+120\underline{j}$. Secondly, others confused between the speed (s) and velocity (v) as they wrote $s=40\underline{i}+120\underline{j}$ instead of finding its magnitude $|\underline{v}|$, that is $|\underline{v}|=\sqrt{(40)^2+(120)^2}$ implying that $|\underline{v}|=\sqrt{16000}=40\sqrt{10}=126.4$ in kilometres per hour. Other candidates calculated the resultant velocity and its magnitude but instead of simplifying $\sqrt{16000}$ to $40\sqrt{10}$ and use the substitution $\sqrt{10}=3.16$ to get 126.4, they evaluated $40\sqrt{10}$ by using the mathematical tables or scientific calculators, contrary to the instruction, to get 126.5 which is incorrect. Moreover, some candidates subtracted 40 km/h from 120 km/h to get 80km/h and concluded that it was the actual speed of the boat which is incorrect. Those candidates lacked knowledge of vectors and the application of Pythagoras theorem in solving problems. Extract 4.1 is a sample of an incorrect response from one of the candidates who answered this question.

4 a) i/

$m = \frac{3}{2}$

$x = -3$ $\qquad y = mx + c$

$y = \frac{3}{2}(-3) + c$

~~y = m c x~~

$y = -\frac{9}{2} + c$

ii) Length = distance

$d = \sqrt{(x_1 + x_2)^2 - (y_1 + y_2)^2}$

$(3, -2)$, $(15, 3)$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$d = \sqrt{(3+15)^2 - (-2 \times 3)^2}$

$d = \sqrt{18^2 - 36}$

$= \sqrt{324 - 36}$

$= 288$

$= 16.97\text{cm}$

4 (b). Speed of = 120km/h

Speed of 40 Km

actual speed = boat

actual speed = $\frac{\text{distance}}{\text{time}}$

actual = $\frac{120\text{Km/h}}{60\text{hr}}$

$\frac{12\cancel{0}^{2}}{\cancel{60}_{3}}$

The actual speed $\pi 10 = 3.16$

$\cancel{100} \times 2$

$= 6.32.$ $\qquad 6.32$

The actual speed of boat is 6.32 Km/h

Extract 4.1: A sample of a candidate's incorrect response in question 4.

In Extract 4.1, the candidate failed to (a) (i) find the equation of a straight line using the given gradient and *x*–intercept, (a)(ii) use the correct formula to find the length of a line segment and (b) use the concepts of either triangular laws of vector addition and magnitude or Pythagoras theorem to obtain the actual speed of the boat.

On the other hand, some candidates who answered part (a)(i) correctly were able to find the equation of a line in form $y = mx + c$ and used the formula for finding the gradient, that is $m = \frac{\text{change in } y}{\text{change in } x}$ and the concept that at the x–intercept, $y = 0$, then substituted the slope $\frac{3}{2}$ to get $\frac{3}{2} = \frac{y-0}{x+3}$ and hence $y = \frac{3}{2}x + \frac{9}{2}$ as the equation of the line. Alternatively, others substituted $x = -3,\ y = 0$ and $m = \frac{3}{2}$ in $y = mx + c$ to get $c = \frac{9}{2}$ and later substituted the values of c and m in $y = mx + c$ to get the required equation. In part (a)(ii), they applied the distance formula correctly to get the required distance between the two points. In answering part (b), the candidates were able to use the triangular law of vector addition to obtain the resultant velocity of the boat, then determine its magnitude using the formula $|v| = \sqrt{x^2 + y^2}$ to get $\sqrt{40^2 + 120^2}$ and finally simplify the answer to $40\sqrt{10}$ and substitute $\sqrt{10} = 3.16$ to obtain 126.4km/h as the required speed. Extract 4.2 illustrates a sample response of a candidate who responded correctly to the question.

04 a, i, given, $m = \frac{3}{2}$

x-intercept, $(x_1, y_1) = (-3, 0)$

$y = m(x - x_1) + y_1$

$y = \frac{3}{2}(x - (-3)) + 0$

$y = \frac{3}{2}(x + 3)$

$y = \frac{3}{2}x + \frac{9}{2}$

$\therefore$ the equation is $y = \frac{3}{2}x + \frac{9}{2}$

ii, length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

given, $(x_1, y_1) = (3, -2)$

$(x_2\ y_2) = (15, 3)$

$L = \sqrt{(15-3)^2 + (3-(-2))^2}$

$L = \sqrt{(12)^2 + (5)^2}$

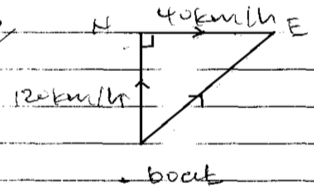
$L = \sqrt{144 + 25}$

$L = \sqrt{169}$

$L = 13$units

$\therefore$ the length is 13units.

b,



$c^2 = a^2 + b^2$

$c^2 = (120)^2 + (40)^2$

$c^2 = 14400 + 1600$

$\sqrt{c^2} = \sqrt{16000}$

$c = \sqrt{1600} \times \sqrt{10}$

$c = 40 \times 3.16$

$c = 126.4$km/h

$\therefore$ the actual speed is 126.4km/h.

Extract 4.2: A sample of a candidate's correct response in question 4.

In Extract 4.2, the candidate substituted $m = \frac{3}{2}$, $x_1 = -3$ and $y_1 = 0$ in the general form $y = m(x - x_1) + y_1$ to obtain the required equation. He/she was also able to apply the distance formula to find the length of the line segment and the Pythagoras theorem to get the actual speed of the boat.

2.5 Question 5: Area and Perimeter

This question consisted of parts (a) and (b). In part (a), the candidates were required to find the area of the triangle *ABC* given that $\overline{AB} = 8\,\text{cm}$, $\overline{BC} = 11.3\,\text{cm}$ and $A\hat{B}C = 30^{\circ}$. In part (b)(i), they were required to find the perimeter of the regular hexagon inscribed in a circle whose radius is $100\,\text{m}$. In part (b)(ii), the question stated that, if $\frac{\overline{AB}}{\overline{\overline{KL}}} = \frac{\overline{BT}}{\overline{\overline{LC}}} = \frac{\overline{TA}}{\overline{\overline{CK}}} = 3$, where $\overline{AB}$, $\overline{BT}$ and $\overline{TA}$ are the sides of the triangle *ABT* and $\overline{KL}$, $\overline{LC}$ and $\overline{CK}$ are the sides of the triangle *KLC*. The candidates were required to give the implication of this information.

This question was attempted by 278,170 (63.6%) candidates. The analysis indicates that 48,016 (17.3%) candidates scored from 2 to 6 marks, among them 8,671 (3.1%) scored full marks. Apart from that, out of 230,154 (82.7%) candidates who scored below the average, 211,773 (76.1%) candidates scored 0 mark. This shows that the candidates' performance in this question was weak. The summary of the candidates' performance in this question is presented in Figure 5.

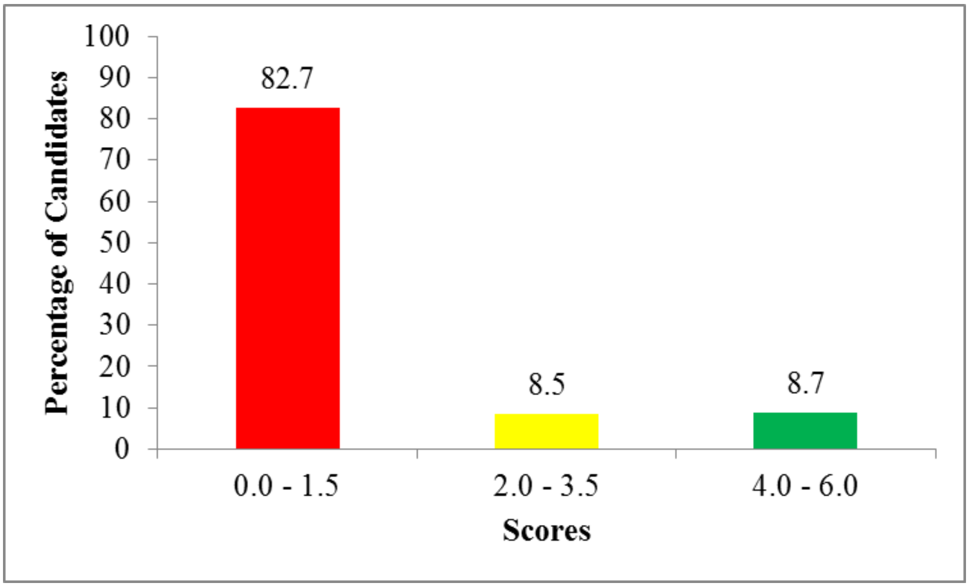


Figure 5: *Candidates' performance in question 5.*

The weak performance was contributed by several factors as follows: in part (a), the candidates applied incorrect formulae when finding the area

(A) of a triangle with two given sides and included angle such as $A = \frac{1}{2} bh \cos\theta$, $A = S_1 \times S_2 \times \sin\theta$ and $A = \frac{1}{2} \times \overline{AB} \times \overline{BC}$. For example, those who used $A = \frac{1}{2} bh \cos\theta$ made mistakes in finding h as they wrote $h = 8\cos 30^\circ$ and $A = \frac{1}{2} \times 11.3 \times 8\cos 30^\circ$ which are incorrect steps. In some cases, few candidates applied the concept of the cosine rule to find the area, that is $a^2 + b^2 - 2abCos\theta = 8^2 + 11.3^2 - 2(8)(11.3)\cos(30^\circ)$ and considered it the area of the triangle. Instead, they were supposed to write $\text{Area} = \frac{1}{2} \times \text{b} \times \text{h}$, where $\text{h} = (8\,\text{cm}) \times \sin 30^\circ$ and $\text{b} = 11.3\,\text{cm}$ and then proceed with calculations to get the required area.

In part (b)(i), the candidates made mistakes in writing the formulae for the perimeter of a regular hexagon. They wrote incorrect formulae like: $\text{P} = \frac{1}{2} \pi r^2 \sin\left(\frac{180^\circ}{n}\right)$ and $\text{P} = \frac{1}{2} \pi r^2 \sin\left(\frac{360^\circ}{n}\right)$ instead of $P = 2nr\sin\left(\frac{180^\circ}{n}\right)$, where n is the number of sides of the polygon and r is the radius of the circle. Some candidates also regarded that the given hexagon is a seven sided polygon and therefore substituted $n = 7$ in the formula instead of taking $n = 6$ which is the correct number of sides of a regular hexagon. Moreover, others calculated the perimeter of a circle as they applied the formula $\text{P} = 2\pi r$ instead of using the formula for perimeter of a regular polygon inscribed in a circle. In part (b)(ii), the candidates confused the concepts of congruence with that of similarity. For example, some of them wrote that the triangle *ABT* is congruent to triangle *KLC*. In other cases, a few of them stated that the triangles *ABT* and *KLC* are right angled triangles. Extract 5.1 shows one of the candidate's incorrect responses in this question.

	from · Cosine rule.
5	a). $b^2 = a^2 + c^2 - 2ac\cos B$
	$b^2 = a^2 + c^2 - 2ac\cos B$
	$b^2 = 8^2 + 11.3^2 - 2(8 \times 11.3)\cos 30°$
	$b^2 = 64 + 127.69 - 2(90.4)\cos 30°$
	$b^2 = 64 + 127.69 - 180.8\cos 30°$
	$b^2 = 191.69 - 180.8\cos 30°$
	$b^2 = 191.69 - 180.8(0.8660)$
	$b^2 = 191.69 - 156.6$
	$\sqrt{b^2} = \sqrt{39}$
	$b = 6.25$ cm
	from the triangle above
	area = $a \times b \times c$
	area = abc
	area = $8 \times 11.3 \times 6.25$
	area = $90.4\ cm^2$
	The area of a triangle = $90.4\ cm^2$

b/ i/ P = $nd \sin\left(\frac{360°}{n}\right)$

$d = 2 \times r$

$= 100 \times 2$

$= 200$

hexagon = 6 sides; $n = 6$

$= 6 \times 200 \times \sin\left(\frac{360}{6}\right)$

$= 6 \times 200 \times \sin 60$

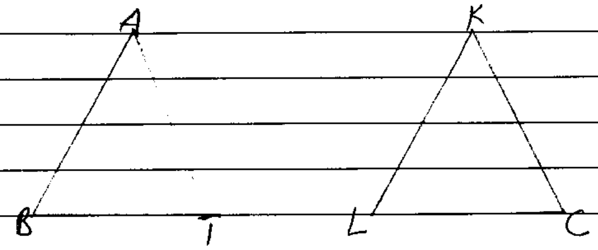
$= 6 \times 200 \times 0.8660$

$= 1039.2$ m

$\therefore$ The perimeter is 1039.2m

(ii) Soln.

$\frac{\overline{AB}}{KL} = \frac{\overline{BT}}{LC} = \frac{\overline{TA}}{CK} = 3$



$\frac{\overline{AB}}{KL} = \frac{\overline{BT}}{LC} = \frac{\overline{AT}}{KC} = 3$

$\therefore$ This implies that the sides of this triangles are equal due to s.s.s theorem.

Extract 5.1: A sample of a candidate's incorrect response in question 5.

In Extract 5.1, the candidate used the cosine rule to get the length of side $\overline{AC}$ of the triangle *ABC* and then multiplied the lengths of all sides, that is $\overline{AB} \times \overline{BC} \times \overline{AC}$ and considered it the area in part (a). He/she also used incorrect formula $P = nd \sin\left(\frac{360°}{n}\right)$ when finding the perimeter of the hexagon inscribed in a circle in part (b)(i). In part (b)(ii), the candidate also stated that the given triangles are equal due to lack of adequate knowledge of the concepts of congruence and similarity.

Despite the poor performance in this question, there were few candidates (17.3%) who demonstrated a good understanding in answering the question. In part (a), they were able to write correctly the formula for the area of the triangle with two sides and the included angle by using the correct formula $A = \frac{1}{2} \times \overline{AB} \times \overline{BC} \times Sin\,(A\hat{B}C)$ to get the required area. In part (b)(i), they were able to identify the number of sides of regular hexagon as 6, radius of the circle as $r = 100\,cm$ and apply the formula $\mathrm{P} = \frac{1}{2}\pi r \sin\left(\frac{180^\circ}{n}\right)$ to get $\mathrm{P} = 600\,\mathrm{cm}$ as the perimeter of the given regular hexagon. In part (b)(ii), they realized that the ratio is the same; the corresponding sides are proportional and concluded that triangle ABT is similar to triangle KLC. Extract 5.2 is a sample of response from one of the candidates who answered this question correctly.

5 a) Soln.

$$\text{Area} = \frac{1}{2} ab \sin\theta$$

$$= \frac{1}{2} \times 8 \times 11.3 \times \sin 30$$

$$= 4 \times 11.3 \times \sin 30$$

$$= 45.2 \times 0.5$$

$$= 22.6$$

∴ the area of the triangle is $22.6 cm^2$.

b) i) Soln.

hexagon = 6 sides

$n = 6.$

$r = 100m.$

$$\text{perimeter} = 2rn \sin(180/n)$$

$$= 2 \times 100m \times 6 \times \sin(180/6)$$

$$= 200 \times 6 \times 0.5$$

$$= 600 m^{\#}.$$

∴ The perimeter of the hexagon is 600m.

ii) Soln

$$\frac{\overline{AB}}{\overline{KL}} = \frac{\overline{BT}}{\overline{LC}} = \frac{\overline{TA}}{\overline{CK}} = 3.$$

∴ the information imply that the two triangles namely triangle ABT and triangle KLC are similar since the ratio of the corresponding sides is equal.

Extract 5.2: A sample of a candidate's correct response in question 5.

In Extract 5.2, the candidate managed to find the area of the triangle and the perimeter of a regular hexagon inscribed in a circle using appropriate formula.

He/she was able to identify that ΔABT is similar to ΔKLC since $\frac{\overline{AB}}{\overline{KL}} = \frac{\overline{BT}}{\overline{LC}} = \frac{\overline{TA}}{\overline{CK}} = 3$, therefore the corresponding sides are proportional.

2.6 Question 6: Rates and Variations

This question consisted of parts (a) and (b). In part (a), the question read, "*The variables t and z in the following table are related by the formula $z = at^n$ where a is a constant and n is a positive integer*".

t	1	2	3	4	5
z	0.5	4	13.5		

The candidates were required to:

(i) determine the values of a and n from the table.

(ii) use the values of a and n obtained in part (a)(i) to complete the given table.

In part (b), they were required to find the value of v when $x = 5$ and $y = 4$ given that v varies directly as the square of x and inversely as $\sqrt{y}$ when $v = 18$, $x = 3$ and $y = 16$.

This question was attempted by 210,924 (48.2%) candidates. The analysis shows that, 66,493 (31.5%) candidates scored from 2 to 6 marks, among them 1,864 (0.9%) scored 6 marks. This shows that the candidates' performance in this question was average. Although the performance was average, a total of 144,431 (68.5%) candidates scored from 1.5 to 0 mark and among them, 126,993 (60.2%) candidates scored 0 mark. The candidates' performance in this question is summarized in Figure 6.

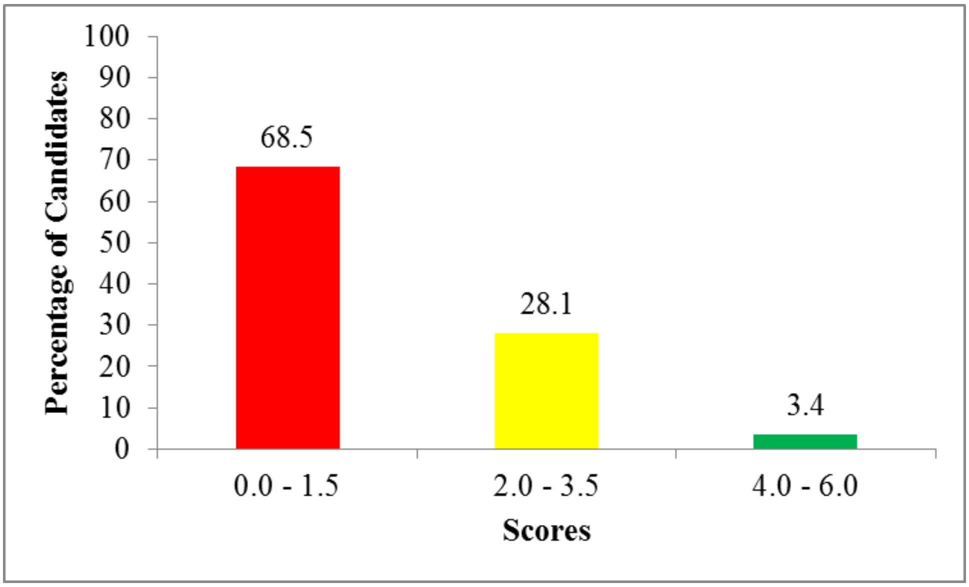


Figure 6: *Candidates' performance in question 6.*

Nearly one third of the candidates (31.5%) scored high marks in this question. In part (a)(ii), they managed to formulate the equations $a(2)^n = 4$ and $a(3)^n = 13.5$ or $a(1)^n = 0.5$ by using the values given in the table, hence obtained correctly the values, that is $a = \frac{1}{2}$ and $n = 3$. By using these values, they were able to complete the table accordingly.

In part (b), they were able to transform the given word problem into the forms $v \alpha x^2$ and $v \alpha \frac{1}{\sqrt{y}}$ and combined them to obtain $v = \frac{kx^2}{\sqrt{y}}$, where k is a constant. Lastly, they substituted the given values of x, y and v to obtain $k = 8$ which was an important step to get the correct value of v. Extract 6.1 shows a sample of a correct response from one of the candidates.

06. a) i) $z = at^n$

when $z = 0.5$, $t = 1$

$0.5 = a \times 1^n$

$a = \frac{0.5}{1^n} \ldots\ldots (i)$

when $z = 4$, $t = 2$

$a = \frac{4}{2^n} \ldots\ldots (ii)$

$\frac{0.5}{1^n} = \frac{4}{2^n}$

$\frac{4 \times 1^n}{4 \times 2^n} = \frac{0.5 \times 2^n}{4 \times 2^n}$

$\left(\frac{1}{2}\right)^n = \frac{1}{8}$

$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^3$

thus $n = 3$

From eqn (i): $a = \frac{0.5}{1^n}$

$a = 0.5$

$\therefore$ $a = 0.5$ and $n = 3$

ii) from $z = at^n$

when $t = 4$

$z = \frac{1}{2} \times 4^3$

$z = \frac{1}{2} \times 64$

$z = 32$

when $t = 5$

$z = \frac{1}{2} \times 5^3$

$z = \frac{1}{2} \times 125$

$z = 62.5$

the table

t	1	2	3	4	5
z	0.5	4	13.5	32	62.5

06. b) $v \propto \frac{x^2}{\sqrt{y}}$

$v = \frac{kx^2}{\sqrt{y}}$

$k = \frac{v\sqrt{y}}{x^2}$

$v = 18$ when $x = 3$ and $y = 16$

$k = \frac{18 \times \sqrt{16}}{3^2}$

$k = \frac{18 \times 4}{9}$

$k = 8$

when $x = 5$ and $y = 4$

$v = \frac{8 \times 5^2}{\sqrt{4}}$

$v = \frac{8 \times 25}{2}$

$\therefore v = 100$

Extract 6.1: A sample of a candidate's correct response in question 6.

In Extract 6.1, the candidate formulated and solved the equations $a(2)^n = 4$ and $a(1)^n = 0.5$ simultaneously and got the correct values of a and n which were used to complete the table given in part (a). He/she also formulated the correct equation and solved for the value of v in part (b). This indicates that, the candidate had adequate knowledge of the concepts of variations.

Despite the fact that the performance was average, further analysis of the candidates' responses shows that, many candidates (60.2%) failed to get the correct answers in this question. In part (a)(i), some of them interchanged the values of t and z when substituting in the equation $z = a(t)^n$. For example, they wrote the equations like $2 = a(4)^n$ and $1 = a(0.5)^n$ which were the incorrect steps. Others were able to formulate the equations $a(2)^n = 4$ and $a(1)^n = 0.5$ but they simplified them incorrectly, as they got incorrect simplified forms like $a^n = 2$ and $a^n = \frac{1}{2}$. In part (a)(ii), there were some candidates who interchanged the values of t and z and hence failed to complete the table using the correct values of z. Other candidates were able to solve for the values of a and n correctly but they failed to use the substitution $t = 4$ and $t = 5$ to get the corresponding values of z in the given table. In part (b), the candidates were unable to formulate the required equation. For example, some of them wrote incorrect equations like: $v = kx^2\sqrt{y}$, $v = \sqrt{x}$, $V = \frac{k\sqrt{x}}{\sqrt{y}}$, $v = \frac{k}{x^2\sqrt{y}}$ instead of $v = \frac{kx^2}{\sqrt{y}}$; as a result they ended up getting incorrect values of k and v. There were also some candidates who formulated the correct equation but were unable to get the values of v due to incorrect substitution of the given values and wrong manipulations. This indicates that the candidates lacked adequate knowledge of simple arithmetic and variations. Extract 6.2 is a sample response from one of the candidates who failed to answer this question correctly.

6	a) Soln
	$Z = at^n$
	$4 = a2^n$ i
	$Z = at^n$
	$\frac{1}{2} = a^n$ ii

6 a) i)

$$4 = a2^n \;\ldots\; i$$
$$\tfrac{1}{2} = a^n \;\ldots\; ii$$

Take eqn i/

$$\frac{4}{2^n} = \frac{a2^n}{2^n}$$

$$\frac{4}{2^n} = a$$

Substitute a into eqn ii

$$\frac{1}{2} = a \times 1^n$$

$$\frac{1}{2} = \frac{4}{2^n} \times 1^n$$

$$\frac{1}{2} = \frac{4^n}{2^n}$$

$$\cancel{\frac{1}{2} = \left(\frac{2^2}{2}\right)^n} \qquad \frac{1}{2} = \left(\frac{2}{1}\right)^n$$

Reciprocate

$$\frac{1}{2} = \left(\frac{1}{2}\right)^n$$

$$n = 1$$

Substitute n into eqn i

$$4 = a2^n$$

$$\frac{4}{2} = \frac{a2}{2}$$

$$2 = a$$

∴ the values of a and n are 2 and 1 respectively

6 a) ii) When, $t = 4$

$$z = at^n$$
$$z = 2 \times 4^1$$
$$z = 2 \times 4$$
$$z = 8$$

$\therefore$ When $t = 4$ then, $z = 8$

when $t = 5$

$$z = at^n$$
$$z = 2 \times 5^1$$
$$z = 10$$

$\therefore$ When $t = 5$ then, $z = 10$

b) Soln

$$v \propto x^2 \propto \frac{1}{\sqrt{y}}$$

$$v \propto \frac{x^2}{\sqrt{y}}$$

$$v = \frac{kx^2}{\sqrt{y}}$$
$$18 = \frac{k(3)^2}{\sqrt{16}}$$

6(b) then

$$v \propto \frac{x}{\sqrt{y}}$$

$$v = k\left(\frac{x}{\sqrt{y}}\right)$$

$$\sqrt{y} \times v = \frac{kx}{\cancel{\sqrt{y}}} \times \cancel{\sqrt{y}}$$

$$\frac{k\cancel{x}}{\cancel{x}} = \frac{\sqrt{y}\,v}{x}$$

$$k = \frac{\sqrt{y} \times v}{x}$$

Then substitute:

$$k = \frac{\sqrt{16} \times 18}{3}$$

$$= \frac{4 \times \cancel{18}\,6}{\cancel{3}\,1}$$

$$k = 24$$

To find the value of v is:

$$k = \frac{\sqrt{y} \times v}{x}$$

$$5 \times 24 = \frac{\sqrt{4} \times v \times \cancel{5}}{\cancel{5}}$$

$$120 = 2 \times v$$

$$\frac{\cancel{120}\,60}{\cancel{2}\,1} = \frac{\cancel{2}v}{\cancel{2}}$$

$$v = 60$$

∴ The value of v is 60

Extract 6.2: A sample of a candidate's incorrect response in question 6.

In Extract 6.2, the candidate was unable to formulate the required equations using the information from the given table and failed to obtain the correct values of *a* and *n*, hence was unable to complete the table accordingly. He/she also formulated an incorrect equation $v = \frac{kx}{\sqrt{y}}$ that led to incorrect values of *k* and *v*

2.7 Question 7: Accounts, Ratio, Profit and Loss

The question had parts (a) and (b). Part (a)(i) stated that: "*A school has 2,000 students, of whom 1,500 are boys*". The candidates were required to write the ratio of boys to girls in the school. Part (a)(ii) stated that: "*Matiku bought a book for Tshs. 120,000. A year later, he sold the book at profit of 20%"*. The candidates were required to find the selling price of the book.

Part (b) stated that: "*Halima started a business on 1st September, 2018 with a capital of Tshs. 25,000/= in cash.*

September	*2,*	*bought goods for cash 15,000/=*
	3,	*sold goods for cash 3,000/=*
	5,	*sold goods for cash 5,000/=*
	6,	*paid carriage on goods 500/=*
	9,	*sold goods for cash 14,000/*
	15,	*bought goods for cash 1,000/=*
	19,	*paid rent 2,000/=*
	20,	*purchased goods 6,000/=*
	27,	*paid wages 5,000/=*
	28,	*sold goods on credit 1,000/="*

The candidates were required to prepare the cash account by using these transactions.

The analysis shows that, this question was attempted by 349,822 (80.0%) candidates. Further analysis shows that, 138,670 (39.9%) candidates scored from 2 to 6 marks, among them 2,023 (0.6%) scored full marks. This shows that the candidates' performance in this question was average. Although the performance was average, a total of 211,152 (60.4%) candidates scored below 2 marks, out of which 108,482 (31.0%) candidates scored 0 mark. The candidates' performance in this question is summarized in Figure 7.

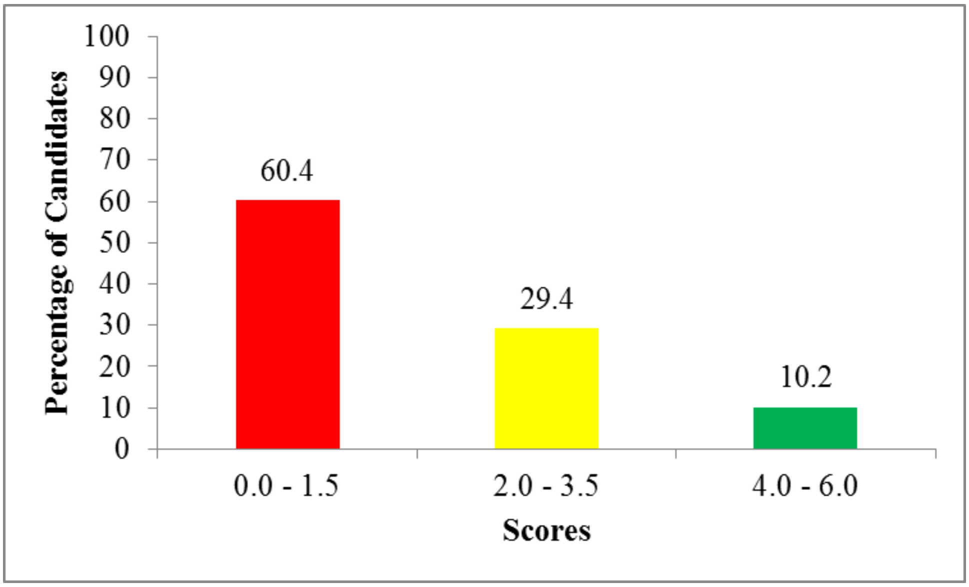


Figure 7: *Candidates' performance in question 7.*

According to response analysis, the candidates had an average performance in this question because of the following reasons: in part (a)(i), the candidates were able to (i) find the ratio of boys to girls in the school, firstly by subtracting the number of boys from the total number of students in the school, that is $2000-1500=500$ and then used the result to evaluate the ratio of boys to girls, that is $1500:500$ or 3:1. In part (a)(ii), they were capable of calculating the selling price of the book by applying the appropriate formula and procedures to get the correct selling price of the book.

In part (b), the candidates were able to prepare the cash account by using the given transactions. For example, they were able post the capital and sales in the Debit (DR) column; and the purchases, expenses, rent and wages in the Credit (CR) column of the cash account. Extract 7.1 illustrates a sample response of a candidate who answered this question correctly.

7. a) i) 1st find the no of girls

Girls = 2000 − 1500

= 500

Ratio of Boys to girls

1500 : 500

$= \frac{\cancel{1500}^{3}}{\cancel{500}_{1}}$

$= 3/1$

$= 3 : 1$

∴ The Ratio is 3 : 1

ii) Buying Price = 120,000

% Profit = 20%

7. a) ii) $\% \text{Profit} = \frac{\text{Profit}}{\text{Buying Price}} \times 100\%$

$20\% = \frac{\text{Profit}(x)}{120,000} \times 100\%$

$\frac{2,400,0\cancel{00}}{1\cancel{00}} = \frac{\cancel{100}x}{\cancel{100}}$

$\text{Profit}(x) = 24,000/=$

Selling Price = Buying Price + Profit

= Tshs 120,000 + 24,000 Tsh

= Tshs 144,000

∴ The selling price was Tshs 144,000

b) DR HALIMA'S CASH ACCOUNT (01) CR

DATE	DETAILS	FOLIO	AMOUNT	DATE	DETAILS	FOLIO	AMOUNT
1/09/2018	Capital	02	25,000	2/09/2018	Purchases	03	15,000
3/09/2018	Sales	04	3,000	6/09/2018	Carriage	05	500
5/09/2018	Sales	04	5,000	15/09/2018	Purchases	03	1,000
9/09/2018	Sales	04	14,000	19/09/2018	Rent	06	2,000
				20/09/2018	Purchases	03	6,000
				27/09/2018	Wages	07	5,000
				30/09/2018	Balance	c/d	17,500
			47,000				47,000
1/10/2018	Balance	b/d	17,500				

Extract 7.1: A sample of a candidate's correct response in question 7.

In Extract 7.1, the candidate applied the concepts of ratio of quantities of the same kind to get the ratio of boys to girls. He/she also managed to apply the formula for percentage profit to calculate the selling price of the book in part (a). In part (b), the candidate used the correct format of a ledger to prepare the cash account by using the given transactions.

Despite the average performance in this question, some candidates attempted it incorrectly due to the following reasons: in part (a)(i), some candidates were able to find the number of girls correctly but computed the ratio of the number of girls to the total number of students as $\frac{500}{2000}=\frac{1}{4}$ contrary to the given instructions. By the same token, others calculated the ratio of the number of boys to the total number of students, that is $\frac{1500}{2000}=\frac{3}{4}$. In part (a)(ii), some candidates evaluated 20% of 120,000 and got Tshs. 24,000 as the selling price of the book which is incorrect. Others applied incorrect formulae when determining the selling price, like: $\text{Selling price} = \text{Buying price} \times \text{Percentage profit}$, $I=\frac{PRT}{100}$ which is the formula for calculating the simple interest, $\text{Percentage profit} = \frac{\text{Buying price}}{\text{Selling price}} \times 100$ instead of $P=\left(\frac{S-B}{B}\right)\times 100$, where S, B and P stand for selling price, buying price and percentage profit, respectively.

In part (b), most candidates posted the given transactions in the wrong sides of the cash account as they failed to differentiate between the DR side and CR side. Others failed to determine the total amounts in the DR and CR columns, hence were unable to correctly get the balance carried/brought down. There were also some candidates who were unable to present the right format for cash account while few of them named the columns for debit side as CR and credit side as DR, which is incorrect. Furthermore, most candidates recorded sales on credit in the DR column which was not supposed to be posted in the cash account, hence got the incorrect total amount of Tshs. 48,000/= instead of Tshs. 47,000/=. Extract 7.2 illustrates an incorrect response from one of the candidates who failed to answer this question correctly.

7: (a) Soln

Data given.

2,000 student, 1500 are boys

2,000 − 1500

500

The ratio of ~~g~~ boys to girls is 500.

ii/ Soln

Data given

$\frac{IPT}{100} = \frac{120,000}{100} - 20\% \times ?$

$120,00 = 20\% \times 100$

$\frac{120,000}{20} = \frac{20}{20}$

$60,00$

∴ The Selling prime of the book is 6000.

(b) Solution

Data given.

Date	particular	folio	amount	Date	particular	folio	amount
2	bought	1	15,000	3	Salies	2	3,000
6	paid	3	500	5	Salies	2	5,000
15	bought	4	1,000	9	Salies	2	14,000
20	purchase	6	6,000	19	rent	5	2,000
~~28~~			20,000	27	wages	7	5,000
			22500	28	Salies	2	1,000
			=7,5000				30,000
							7,500

Extract 7.2: A sample of a candidate's incorrect response in question 7.

In extract 7.2, the candidate lacked the knowledge of ratios as he/she considered the given number of girls as the required ratio. In finding the selling price, the candidate failed to apply the appropriate procedures to calculate the selling price of the book. Lastly, he/she was unable to prepare the required cash account by using the right format and, yet made wrong posting of the given transactions.

2.8 Question 8: Sequence and Series

This question had two parts (a) and (b). In part (a), candidates were required to find the first term and the common difference of an arithmetic progression whose 5^{th} term and 8^{th} term are 21 and 30, respectively. In part (b), they were required to find the 10^{th} term of a sequence whose first three consecutive terms are 5, 15 and 45, leaving the answer in exponent form.

The analysis shows that, this question was attempted by 187,477 (42.9%) candidates. Further analysis shows that, 74,792 (39.9%) candidates scored from 2 to 6 marks, among them 19,604 (10.5%) scored 6 marks. This shows that the candidates' performance in this question was average. On the other hand, out of 112,685 (60.1%) candidates who scored below average, 106,734 (56.9%) candidates scored 0 mark. The candidates' performance in this question is summarized in Figure 8.

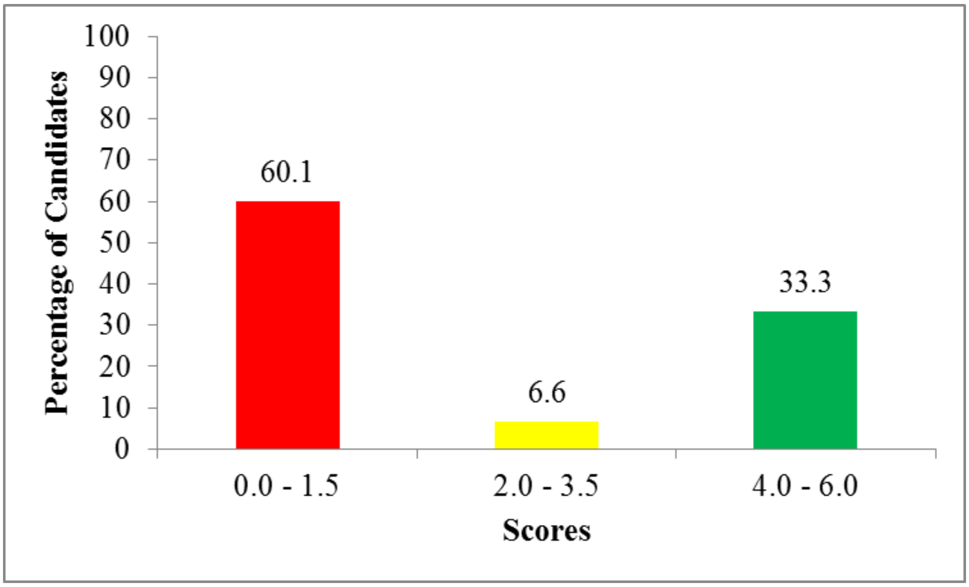


Figure 8: *Candidates' performance in question 8.*

The candidates who performed well in part (a) were able to apply the general equation for the n^{th} term of an arithmetic progression to formulate the equations $A_1 + 7d = 30$ and $A_1 + 4d = 21$ which were solved simultaneously to obtain the first term and common difference correctly as 9 and 3, respectively. Moreover, in part (b), they were able to identify the given sequence as a geometric progression with common ratio 3 and later used the general formula $G_n = G_1 r^{n-1}$, where n is the number of terms, G_1 is the first term and r is the common ratio, to obtain the 10$^{\text{th}}$ term as $5(3^9)$ in exponent form. Extract 8.1 illustrates this case.

08.	a) Soln
	$A_5 = 21$
	& $A_8 = 30$.
	But
	$A_5 = A_1 + 4d$.
	$A_8 = A_1 + 7d$.
	$21 = A_1 + 4d$ . . . (i)

08. $30 = A_1 + 7d \cdots$ (ii)

Solve eqn (i) and (ii) simultaneously

$$-\begin{cases} A_1 + 4d = 21 \\ A_1 + 7d = 30 \end{cases}$$

$4d - 7d = 21 - 30$.

$-3d = -9$

$d = 3$.

$A_1 + 4d = 21$

$A_1 + 4(3) = 21$

$A_1 + 12 = 21$

$A_1 = 21 - 12$

$A_1 = 9$

∴ The value of first term is 9 and the common difference is 3.

8 b Soln.

$5 + 15 + 45 \ldots$

$r = \frac{15}{5} = \frac{45}{15} = 3$

(It is a geometric progression).

$G_1 = G_1$

$G_2 = G_1 r$

$G_n = G_1 r^{n-1}$

$G_{10} = 5(3)^{10-1}$

$G_{10} = 5 \times 3^9$

$G_{10} = \cancel{15^9}$,

∴ The 10^{th} term is $5(3)^9$

Extract 8.1: A sample of a candidate's correct response in question 8.

In Extract 8.1, the candidate was able to use concepts of arithmetic progression to formulate and solve two simultaneous equations to obtain correct values of the first term and common difference. In part (b), he/she identified the given sequence as a geometric progression and hence applied the correct formula to get the 10th term.

Despite the average performance in this question, more than half of the candidates (56.9%) failed to give correct responses because of the following reasons. In part (a), the candidates were unable to recall the formula for finding the n^{th} term of an arithmetic progression. They applied incorrect formulae like $S_n = \frac{n}{2}\left[2A_1 + (n-1)d\right]$ which is the formula for finding the sum of the first n terms of an arithmetic progression, $A_n = A_1 + (n+1)d$, $A_n = A_1(n-1)d$ and $A_n = A_1 - (n+1)d$ and hence got incorrect equations like $30 + 29d = A_8$ and $21 + 4d = A_4$. Instead, they were supposed to use the correct formula $A_n = A_1 + (n-1)d$ in formulating the equations from the given problem. Others were able to recall the formula and apply it to formulate the equations correctly but failed to solve them simultaneously. In addition, some candidates used the formula for geometrical progression, that is $G_n = G_1 r^{n-1}$, and got $G_5 = G_1 r^4 = 21$, $G_8 = G_1 r^7 = 30$ as they confused between the arithmetic and geometric progressions. In some other instances, few candidates assumed the 5th and 8th terms as the first and second terms of an arithmetic progression respectively, that is $A_1 = 21$ and $A_2 = 30$ and hence calculated the common difference by subtracting, that is $30 - 21 = 9$.

Those who gave incorrect responses in part (b) failed to recognize that the given consecutive terms belong to a geometric progression with common ratio 3. Instead, they treated the sequence as an arithmetic progression and hence applied the formula $A_n = A_1 + (n-1)d$ that could not give the intended results. Moreover, others considered that, 5, 15 and 45 are the first, second and third terms of the arithmetic progression, respectively. As a result, they calculated the difference. Lastly, they subsequently wrote incorrect steps like $A_{10} = A_1 + 9d$, leading to $A_{10} = A_1 + 9(10)$ which is incorrect.

However, some candidates identified the sequence as a geometric progression but failed to recall the correct formula to get the 10th term. For example, some of them wrote the formula for the sum of the first *n* terms of a geometric progression, that is $G_n = \frac{G_1 r^{n-1}}{r-1}$, instead of using the formula for finding the *n*th term, that is $G_n = G_1 r^{n-1}$. Moreover, a few of them did not leave the answer in exponent form. Instead of writing the answer in the form $G_{10} = 5(3^9)$, they wrote $G_{10} = 98{,}415$ contrary to the given instruction. A sample solution in Extract 8.2 illustrates this scenario.

8a

$a_5 = a_1 + 20d$ --(i)

$a_8 = a_1 + 29d$ --(ii)

$a_1 = ?$

$d = ?$

the formular

$a_n = a_1 + (n-1)d$

$a_1 + 20d = 21$

$a_1 + 29d = 30$

$$a_{21} = a_1 + 20d$$
$$a_{30} = a_1 + 29d$$

$$20d - 29d = 21 - 30$$
$$\frac{-9d}{-9} = \frac{-9}{-9}$$

$$d = 1$$

$$a_1 = ?$$

~~from formular~~

$$a_5 = a_1 + (5-1)1$$

$$a_{20} = a_1 + 4$$
$$20 - 4 = a_1$$

$$16 = a_1$$

$\therefore$ the first term is 16 and common difference is 1

8b, 5, 15, 45, . . .

common ratio = 3 , $G_1 = 5$, $n = 10$

$$G_n = G_1\left(\frac{r^n - 1}{r - 1}\right)$$

$$\cancel{G_{10} = 5(3}$$

$$G_{10} = 5\left(\frac{3^{10} - 1}{3 - 1}\right)$$

$$G_{10} = 5\left(\frac{59049 - 1}{2}\right)$$

$$G_{10} = 5\left(\frac{59048}{2}\right)$$

$$G_{10} = 5 \times 29524$$

$$G_{10} = 147620$$

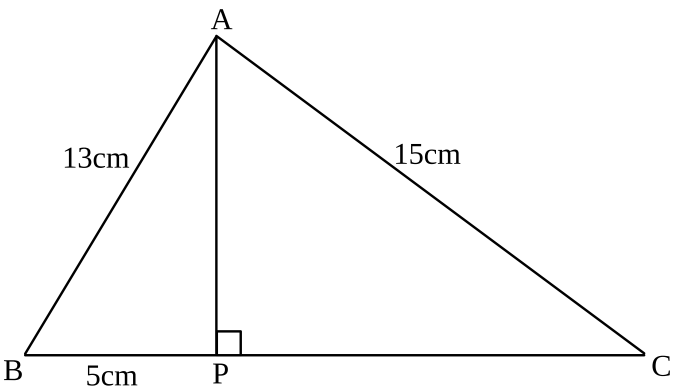
$\therefore$ The 10^{th} term of the sequence is 147620.

Extract 8.2: A sample of a candidate's incorrect response in question 8.

Extract 8.2 illustrates a sample solution of a candidate who failed to recall and use appropriate formulae from arithmetic and geometric progressions to find the required values due to lack of adequate knowledge in sequence and series.

2.9 Question 9: Pythagoras Theorem and Trigonometry

The question consisted of parts (a) and (b). In part (a), the candidates were required to calculate the lengths of $\overline{AP}$ and $\overline{CP}$ in the following figure, given that $\overline{AP}$ is perpendicular to $\overline{BC}$, $\overline{AB} = 13\,\text{cm}$, $\overline{BP} = 5\,\text{cm}$ and $\overline{AC}$ =15cm.



In part (b), the candidates were given the following information: "*from the top of a building 75 m high, John sees a lorry and a minibus along the road, both being on one side of the building at the angles of depression of* 30° *and* 60° *respectively*" Then, they were required to: (i) sketch a diagram representing this information and, (ii) determine the distance between the cars, leaving the answer in surd form.

The data analysis shows that, this question was attempted by 264,939 (60.6%) candidates. Further analysis shows that, 67,935 (25.6%) candidates scored from 2 to 6 marks, among them 969 (0.4%) scored full marks. On the other hand, a total of 197,004 (74.4%) candidates scored below average, among them 163,166 (66.6%) candidates scored 0 mark. This indicates that the candidates had weak performance in this question. The candidates' performance summary in this question is presented in Figure 9.

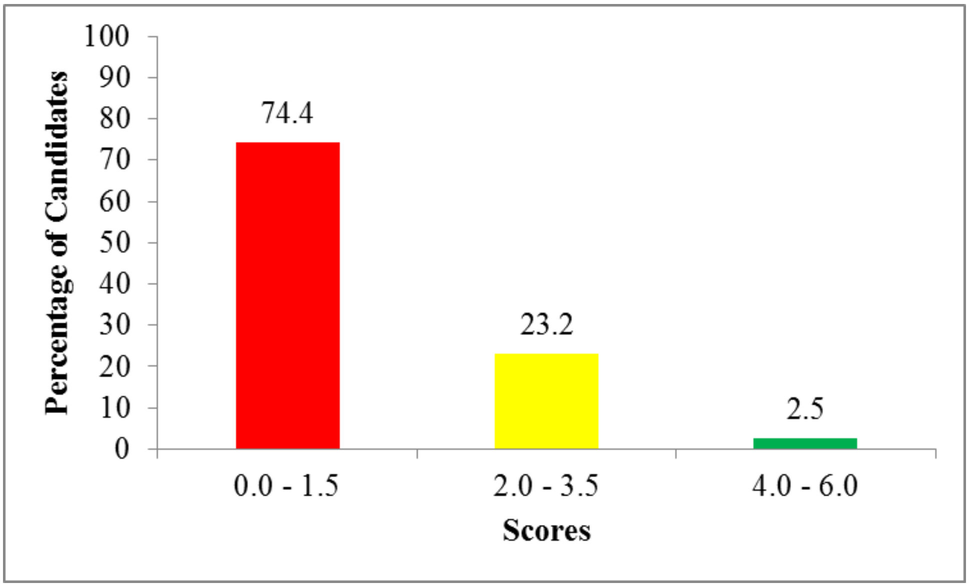
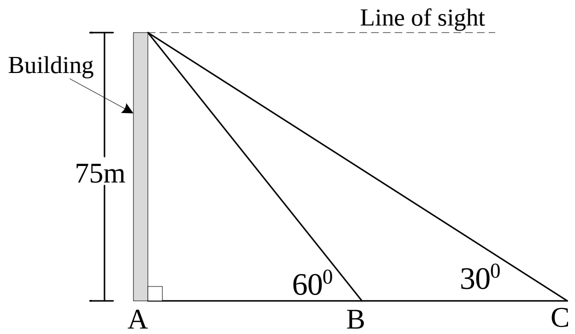


Figure 9: *Candidates' performance in question 9.*

In part (a), candidates who scored low marks were unable to apply the Pythagoras theorem. For example, some candidates calculated the lengths of $\overline{AP}$ and $\overline{CP}$ by using the formula for finding the area of a triangle such as $A = \frac{1}{2} \times \overline{CP} \times height$ and $A = \frac{1}{2} \times \overline{AP} \times base$, which is a wrong approach. However, most candidates failed to apply the Pythagoras theorem to get the correct relations. For example, they wrote incorrect relations like: $\overline{BP}^2 + \overline{AB}^2 = \overline{AP}^2$ instead of $\overline{BP}^2 + \overline{AP}^2 = \overline{AB}^2$; and $\overline{AP}^2 + \overline{AC}^2 = \overline{PC}^2$ instead of $\overline{AP}^2 + \overline{PC}^2 = \overline{AC}^2$ because they were unable to identify the opposite, adjacent and hypotenuse sides of the right angled triangles *ABP* and *ACP*. As a result, they failed to get the correct lengths of $\overline{AP}$ and $\overline{CP}$.

In part (b)(i), the majority were unable to sketch a diagram representing the given information which was an important step in obtaining the required solution for part (b)(ii). They were supposed to draw the diagram like the following, where the points B and C are the positions of the cars.



Instead, most of them wrote the angles at B and C as 30° and 60°, respectively. Others regarded that, $\overline{AC} = 75\,\text{m}$ because they confused the distances of the cars from the base of the building with the height of the building. Also, others sketched the correct diagram but were not able to use the concepts of trigonometric ratios of tangent of the given angles to determine the distance between the cars in surd form. For example, they wrote incorrect trigonometric ratios like: $\cos 60^\circ = \frac{x}{75}$, $\sin 60^\circ = \frac{75}{x}$, and $\cos 60^\circ = \frac{x+y}{75}$ instead of $\tan 60^\circ = \frac{75}{\overline{AB}}$ and $\tan 30^\circ = \frac{75}{\overline{AC}}$ so that $\overline{BC} = \overline{AC} - \overline{AB}$. Extract 9.1 is a sample response of a candidate who failed to answer this question correctly.

9 a) $\frac{AB}{BP} = \frac{XC}{PC}$

$\frac{13}{5} = \frac{15}{x}$

$13x = 75$ $\therefore$ Value of CP = 5.7cm

$\frac{\cancel{13}x}{\cancel{13}} = \frac{75}{13}$

$PC = 5.7$

for length XP

$\frac{AP}{5} = \frac{AP}{PC}$

$\frac{AP}{3} = \frac{AP}{5}$

$\frac{\cancel{2}XP}{\cancel{2}} = \frac{25}{2}$

$XP\ 12.5$

$\therefore$ length of XP = 12.5cm.

9\. b) ii) The distance of the car from the building is given by.

$\tan 30° = \frac{x \text{ (opposite)}}{75\text{m (Adjacent)}}$

$75 \times \tan 30° = x$.

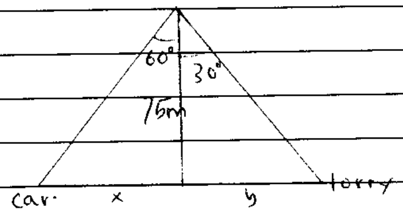
$x = 129.90$ m.

The distance of the lorry from the building is given by

$\tan 30° = \frac{x+y}{75}$

$x+y = 75 \tan 60°$

9\. b) ii)



Distance between the car and the building

$\tan 60° = \frac{x \text{ (opposite)}}{75\text{m (Adjacent)}}$

$75 \tan 60° = x$

$x = 129.90$m.

Distance between the lorry and the building.

$\tan 30° = \frac{y}{75}$

$75 \tan 30° = y$

$y = 43.3$m.

$129.9 + 43.3 = 173.2$m.

$\therefore$ The distance between the two cars is 173.2 m.

Extract 9.1: A sample of a candidate's incorrect response in question 9.

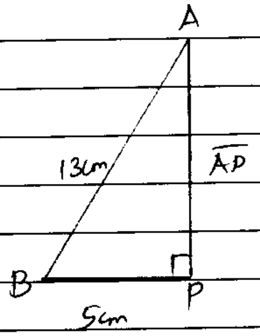
In part (a) of Extract 9.1, the candidate used the ratio theorem in finding the lengths of $\overline{AP}$ and $\overline{CP}$, that is $\frac{AB}{BP} = \frac{AC}{PC}$ implying that $\frac{13}{5} = \frac{15}{x}$ which is a wrong approach indicating lack of knowledge of the application of

Pythagoras theorem. In part (b), he/she failed to sketch the correct diagram that could give the correct ratio of the tangent of the given angles, which is an important step towards getting the distance between the cars.

Even though almost two thirds of the candidates performed below the average, there were few candidates (0.4%) who managed to answer this question correctly. In part (a), these candidates were able to calculate the required lengths $\overline{AP} = 12\,\text{cm}$ and $\overline{CP} = 9\,\text{cm}$ by applying the Pythagoras theorem correctly. In part (b)(i), the candidates were able to sketch a diagram representing the given information correctly and hence use the diagram to determine the distance between the cars in part (b)(ii). The candidates had sufficient knowledge in applying the concepts of trigonometric ratios to solve real life problems. Extract 9.2 shows a sample solution of a candidate who performed well in this question.

9. a) Soln: -

Consider $\triangle APB$



Apply pythagoras theorem.

$$a^2 + b^2 = c^2$$

$$(\overline{AP})^2 + (5cm)^2 = (13cm)^2$$

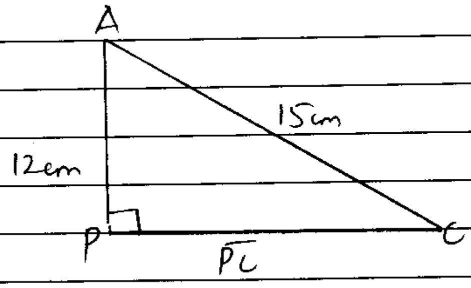
$$(\overline{AP})^2 + 25cm^2 = 169\,cm^2$$

$$(\overline{AP})^2 = 169\,cm^2 - 25cm^2$$

$$\sqrt{(\overline{AP})^2} = \sqrt{144\,cm^2}$$

$$\overline{AP} = 12cm$$

Then consider $\triangle APC$



Apply pythagoras theorem.

$$a^2 + b^2 = c^2$$

$$(12cm)^2 + (\overline{PC})^2 = (15cm)^2$$

$$144\,cm^2 + (\overline{PC})^2 = 225cm^2$$

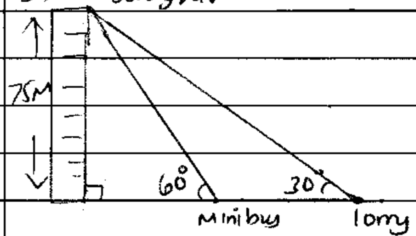
$$(\overline{PC})^2 = 225cm^2 - 144cm^2$$

$$\sqrt{(\overline{PC})^2} = \sqrt{81\,cm^2}$$

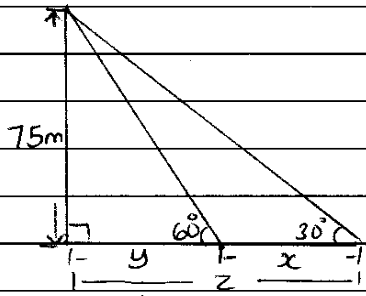
$$\overline{PC} = 9cm = \overline{CP}$$

$\therefore$ The length of $\overline{AP}$ and $\overline{CP}$ is 12cm and 9 respectively.

b) i) diagram



Mathematically



Required to find x (distance between the cars)

9	b) ii) 1st determine y
	$\frac{\text{Tan } 60°}{1} = \frac{75M}{y}$
	$\frac{y \text{ Tan } 60°}{\text{Tan } 60°} = \frac{75M}{\text{Tan } 60°}$
	$y = 75M/\sqrt{3}$
	Then determine Z
	$\frac{\text{Tan } 30°}{1} = \frac{75M}{Z}$
	$\frac{Z \text{ Tan } 30°}{\text{Tan } 30°} = \frac{75M}{\text{Tan } 30°}$
	$Z = \sqrt{3} \times 75M$
	But to get x
	$Z = x + y$
	$75\sqrt{3} = x + 75/\sqrt{3}$
	$\frac{75\sqrt{3}}{1} - \frac{75}{\sqrt{3}} = x$
	$\frac{75 \times 3 - 75}{\sqrt{3}} = x$
	$\frac{150}{\sqrt{3}} = x = \frac{\sqrt{3} \times \cancel{150}\, 50}{\cancel{3}_1}$
	$x = 50\sqrt{3}$
	The distance between two cars in surd form is $50\sqrt{3}$ M

Extract 9.2: A sample of a candidate's correct response in question 9.

In Extract 9.2, the candidate calculated the required lengths of $\overline{AP}$ and $\overline{CP}$ by applying the Pythagoras theorem correctly in part (a), sketched a diagram representing the given information in part (b)(i) and determined the distance between the cars correctly, leaving the answer in surd form in part (b)(ii). This indicates that, the candidate had adequate knowledge in applying Pythagoras theorem and trigonometric ratios to solve related problems.

2.10 Question 10: Quadratic Equations and Algebra

This question comprised of parts (a) and (b). In part (a), the candidates were given that: "*Rachel is three years older than her brother John. Three years to come, the product of their ages will be 130 years*". Then, they were required to formulate a quadratic equation representing this information, hence find their present age by using a quadratic formula and taking x as the present age of Rachel. In part (b), the candidates were required to find two consecutive positive numbers whose sum of their squares is 61.

The data analysis shows that, this question was attempted by 153,723 (35.2%) candidates. Further analysis shows that, 9,650 (6.3%) candidates scored from 2 to 6 marks, among them 1,724 (1.1%) scored full marks. On the other hand, a total of 144,073 (93.7%) candidates scored below the average, among them 122,974 (80.0%) candidates scored 0 mark. Generally, the candidates' performance in this question was weak. The candidates' performance in this question is summarized in Figure 10.

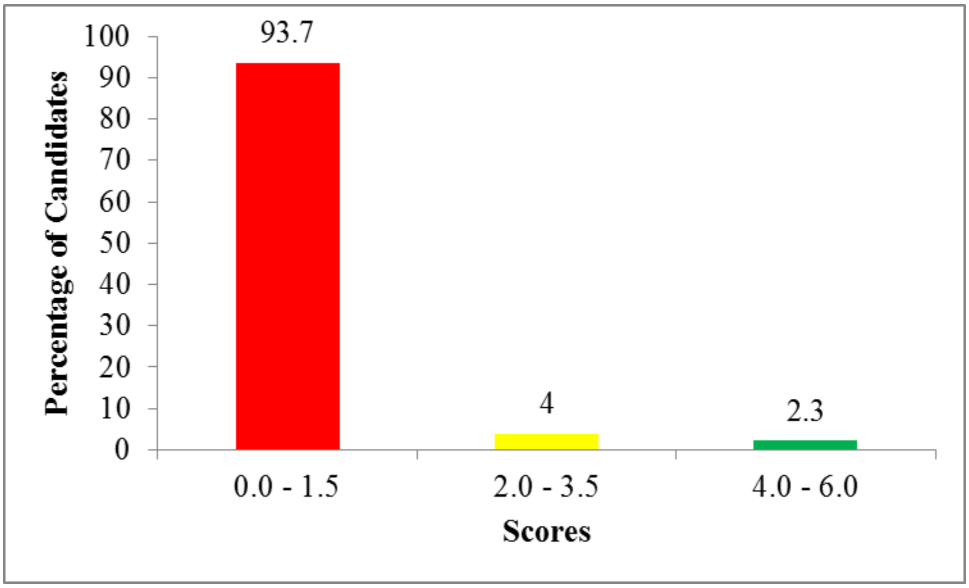


Figure 10: *Candidates' performance in question 10.*

The response analysis shows that, the candidates failed to answer the question correctly due to the following reasons: In part (a), the candidates failed to identify that, if John is x years old, then Rachel will be $(x+3)$ years old. Instead, some candidates wrote the Rachel's age as $(x-3)$ or $3x$ years if John's age is x years. There were candidates who managed to write correctly that, John's age is x years and Rachel's age is $(x+3)$ years but they did not add 3 years to each of their ages before multiplying. Most of them wrote $(x)(x+3)=130$ instead of $(x+3)(x+6)=130$. Others added the terms representing their ages contrary to the given instruction, that is $(x+3)+(x+6)=130$. Also, the majority of candidates failed to multiply correctly the terms $(x+3)$ and $(x+6)$ in the equation $(x+3)(x+6)=130$ to get $x^2+9x-112=0$. In solving the equation $x^2+9x-112=0$, there were some candidates who applied incorrect formulae like: $x=-b\pm\frac{\sqrt{b^2-4(a)(c)}}{2(a)}$ and $x=\frac{b\pm\sqrt{b^2-4(a)(c)}}{2(a)}$ instead of using the correct formula, $x=\frac{-b\pm\sqrt{b^2-4(a)(c)}}{2(a)}$. Others failed to identify that $a=1,\ b=9,\ c=-112$. As a result, they made substitution of incorrect values like: $a=3,\ b=-9$ and $c=112$ due to lack of sufficient knowledge in solving word problems related to quadratic equations.

In part (b), the majority of candidates were unable to translate the given word problem correctly. Most of them failed to realize that, if n is a small number in a pair of two consecutive positive numbers, then the succeeding number should be $n+1$, so they got incorrect equations like $n+n=61$, $n+m=61$ and $n^2+n^2=61$ instead of $n^2+(n+1)^2=61$ that could be solved by using any of the factorization methods or quadratic formula. Some candidates managed to formulate the correct equation but failed to solve it correctly. Extract 10.1 shows a sample response from a candidate who was unable to answer this question correctly.

10 a)

Rachel John

$(3x+3)(x+3) = 130$

$3x^2+9x+3x+9 = 130$

$3x^2+12x+9 = 130$

$3x^2+12x = 130-9$

$3x^2+12x = 121$

$3x^2+12x-121 = 0$

Solve by general formula.

$\cancel{x \pm}\ x = \frac{-b \pm \sqrt{b^2+4ac}}{2a}$

$x = \frac{-12 \pm \sqrt{12^2+4\times3\times-121}}{2\times3}$

$= \frac{-12 \pm \sqrt{144-1452}}{6}$

$= \frac{-12 \pm \sqrt{1308}}{6}$

$x = -2 \pm \sqrt{1308}$

10 b) let the first number be x

second number be $x+1$

$x + x+1 = 61$

$2x+1 = 61$

$2x = 61-1$

$\frac{2x}{2} = \frac{60}{2}$

$x = 30$

2nd number $= x+1$

$= 30+1$

$= 61$

$\therefore$ the numbers are 30 and 31

Extract 10.1: A sample of a candidate's incorrect response in question 10.

In Extract 10.1, the candidate failed to formulate the correct equation in part (a). For example, he/she wrote that, Rachel's age is $3x+3$ years, instead of $x+6$. Also, the candidate applied the incorrect formula, that is $x = \dfrac{-b \pm \sqrt{b^2 + 4(a)(c)}}{2(a)}$. In part (b), he/she did not square the terms x and $x+1$ before adding. He/she wrote $x + x + 1 = 61$ instead of $x^2 + (x+1)^2 = 61$ as instructed. The candidate lacked adequate knowledge in solving word problems related to quadratic equations.

In spite of the weak performance, there were some candidates who managed to answer this question correctly. In part (a), they were able to formulate correctly the required quadratic equation from the given information and solved it by using the appropriate methods to get the required ages and numbers in part (a); and the intended consecutive positive numbers. This shows that, those candidates had sufficient knowledge in solving real life word problems related to quadratic equations. Extract 10.2 shows a sample response from one of the candidates who answered this question correctly.

10	a) Let x be John's age
	y be Rachel's age
	$y = 3+x$ ---i)
	~~y+3~~ $((3+x)+3) \times (x+3) = 130$
	$(6+x) \times (x+3) = 130$
	$6(x+3) + x(x+3) = 130$
	$6x + 18 + x^2 + 3x = 130$
	$x^2 + 9x + 18 = 130$

10 a) $x^2 + 9x + 18 = 130$

$x^2 + 9x + 18 - 130 = 0$

$x^2 + 9x - 112 = 0$

So; The quadratic equation is $x^2 + 9x - 112 = 0$

$x^2 + 9x - 112 = 0$

$ax^2 + bx + c = 0$

$a = 1$, $b = 9$, $c = -112$

From the quadratic formula;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{\cancel{4a}\,2a}$$

$$= \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(-112)}}{\cancel{4}\,2(1)}$$

$$= \frac{-9 \pm \sqrt{81 + 448}}{2}$$

$$= \frac{-9 \pm \sqrt{529}}{2}$$

$$= \frac{-9 \pm 23}{2}$$

$= \frac{-9 + 23}{2}$ OR $= \frac{-9 - 23}{2}$

$= \frac{14}{2}$ OR $= \frac{-32}{2}$

$= 7$ OR $= -16$

Since there are no negative ages, $x \neq -16$

From; $y = 3 + x$

$= 3 + 7$

$= 10$

$\therefore$ Rachel's present age is 10 years and John's age is 7years.

10 b) Let 'n' and 'n + 1' be the ~~co~~ two consecutive numbers

$$n^2 + (n+1)^2 = 61$$
$$n^2 + n^2 + 2n + 1 = 61$$
$$2n^2 + 2n + 1 = 61$$
$$2n^2 + 2n + 1 - 61 = 0$$
$$\frac{2n^2}{2} + \frac{2n}{2} - \frac{60}{2} = \frac{0}{2}$$
$$n^2 + n - 30 = 0$$

Sum (1) = $^-5 + 6$

Product ($^-30$) = $^-5 \times 6$

$$(n-5)(n+6) = 0$$
$n - 5 = 0$ or $n + 6 = 0$

$n = 5$ or $n = -6$

But since there are positive numbers, $n \neq -6$

So;

$n = 5$

$n + 1 = 5 + 1$

$= 6$ ∴ The ~~co~~ numbers are 5 and 6

Extract 10.2: A sample of a candidate's correct response in question 10.

In Extract 10.2, the candidate formulated correctly the required equation from the given word problems and applied the quadratic formula in part (a) and the factorization method in part (b) to get the required results.

2.11 Question 11: Statistics

This question stated that: "*The following data represent the marks scored by 36 students of a certain school in Geography examination:*"

72	76	90	89	74	82	63	74	70	73	58	71
55	62	65	74	71	64	71	85	70	61	64	75
51	83	50	61	83	68	70	80	50	60	66	68

In part (a), the candidates were required to prepare a frequency distribution table representing the given data by using the class intervals: 50 – 54, 55 – 59, 60 – 64 and so on. In part (b), they were required to: (i) draw a histogram and (ii) calculate the median, correct to 2 decimal places by using the frequency distribution table obtained in part (a).

This question was attempted by 341,460 (78.1%) candidates. The analysis shows that 171,662 (50.3%) candidates scored from 3 to 10 marks, among them 22,932 (6.7%) scored full marks. This shows that the candidates' performance in this question was average. Contrarily, out of 169,798 (49.7%) candidates who scored below average, 85,059 (24.9%) scored 0 mark. The candidates' performance in this question is summarized in Figure 11.

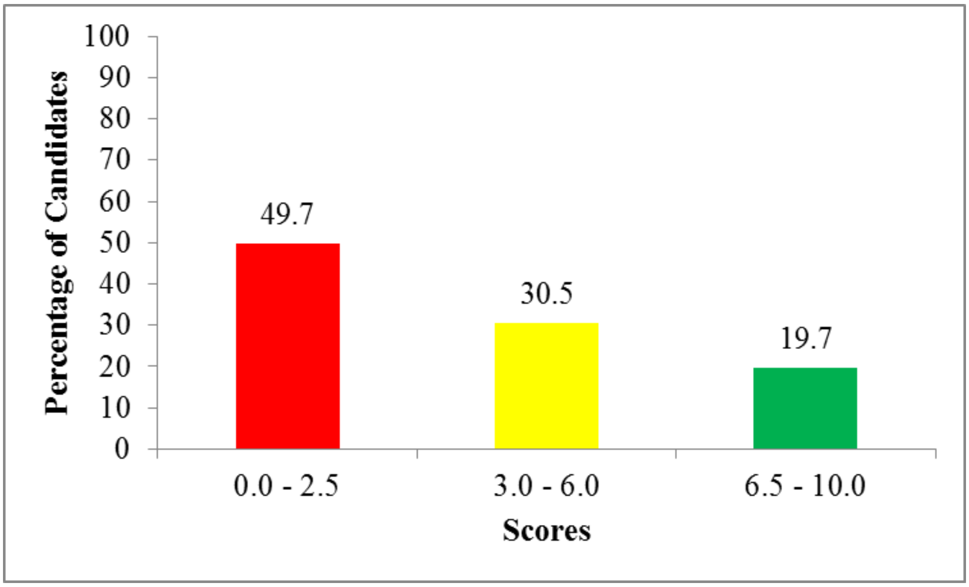
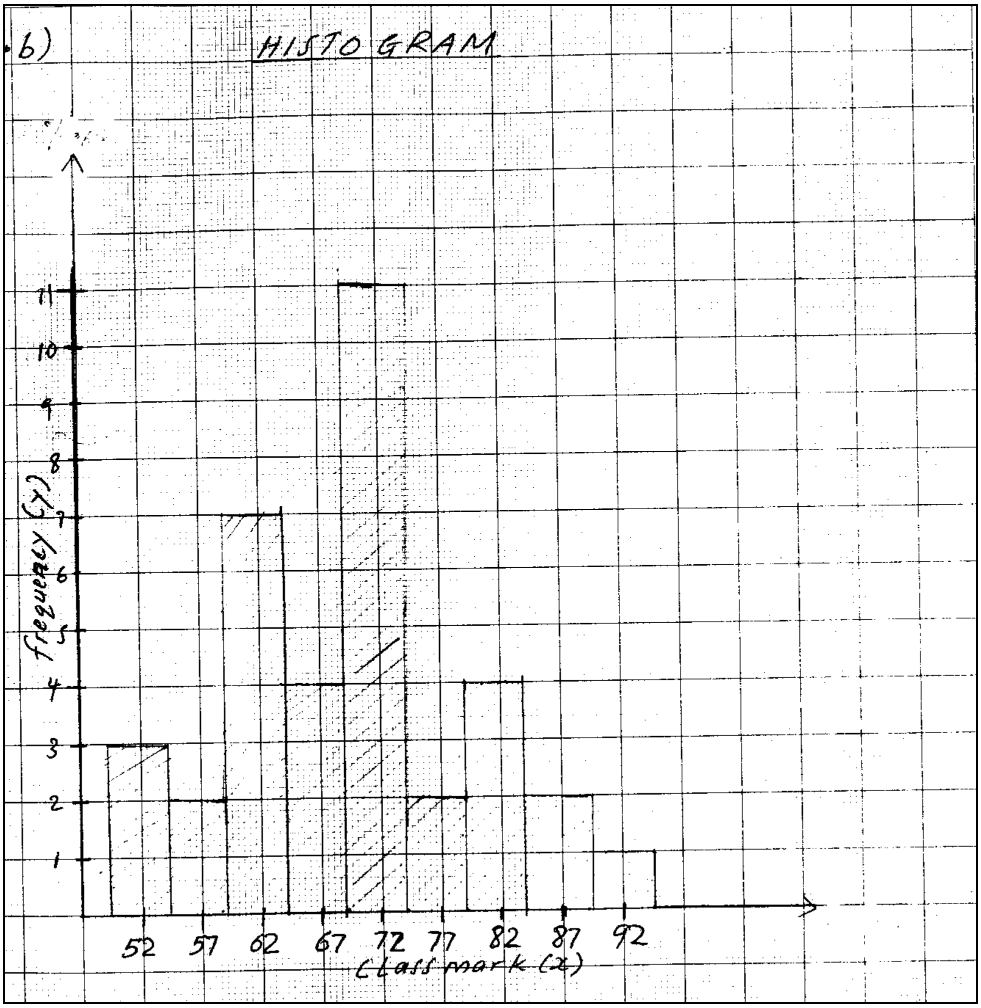


Figure 11: *Candidates' performance in question 11.*

The response analysis shows that, more than half of the candidates (50.3%) performed this question averagely. Among them, there were candidates who managed to score full marks. The candidates were able to present the 36 students' marks given in this question using the frequency distribution table and the correct class intervals. They were also able to identify the class intervals: 65 – 69, 70 – 74, 75 – 79, 80 – 84, 85 – 89 and 90 – 94 by using the given preceding intervals; and their corresponding frequencies correctly in part (a). As a result, they were able to draw an accurate histogram using the frequency on the vertical axis and class marks or class boundaries on the horizontal axis in part (b)(i). Similarly, they were able to use the results (grouped data) obtained in part (a) to calculate the median by using the correct formula in part (b)(ii). Extract 11.1 indicates the answer of the candidate who was able to answer this question correctly.

	Frequency distribution table.		
11. a	Interval	Class mark	frequency
	50 − 54	52	3
	55 − 59	57	2
	60 − 64	62	7
	65 − 69	67	4
	70 − 74	72	11
	75 − 79	77	2
	80 − 84	82	4
	85 − 89	87	2
	90 − 94	92	1
			N = 36


b)
HISTOGRAM
Frequency (y)
1
2
3
4
5
6
7
8
9
10
11
52
57
62
67
72
77
82
87
92
Class mark (x)

b. ii) Median $= L + \left(\frac{\frac{N}{2} - Nb}{Nw}\right) \times i$

$L = 69.5$

$N = 36$

$Nb = 16$

$Nw = 11$

$i = 5$

Median $= 69.5 + \left(\frac{\frac{36}{2} - 16}{11}\right) \times 5$

$= 69.5 + \left(\frac{18 - 16}{11}\right) \times 5$

$= 69.5 + \left(\frac{2}{11} \times 5\right)$

$= 69.5 + \frac{10}{11}$

$= 69.5 + 0.91$

$\therefore$ Median $= 70.41$

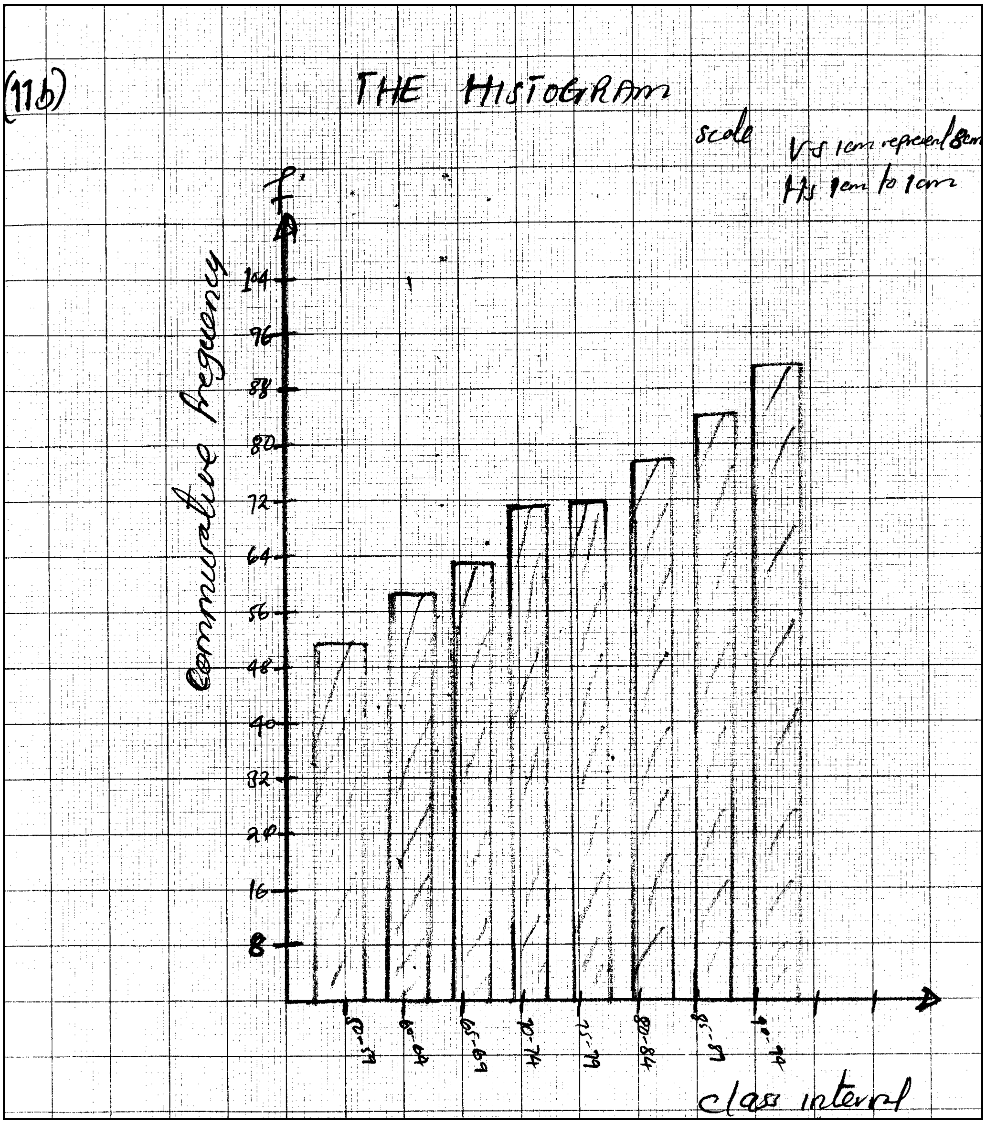
Extract 11.1: A sample of a candidate's correct response in question 11.

In Extract 11.1, the candidate managed to group the given data and write their frequencies correctly by using the given intervals in part (a). He/she used the obtained results to draw a histogram in part (b)(i). The candidate also managed to determine the lower class boundary of the median class, class size and all other necessary parameters required in the formula to calculate the median in part (b)(ii).

On the other hand, the analysis shows that, about quarter of the candidates (24.9%) failed to answer this question correctly. In part (a), they were not able to construct a frequency distribution table, because they could not tally the given values to get their frequencies for each interval. Others failed to get the correct succeeding class intervals because they lacked skills to identify the class size of the preceding class intervals. Also, some of them prepared a table of class intervals with their corresponding upper real limits instead of frequencies. In part (b)(i), the candidates were not able to draw a histogram correctly due to some reasons including the use of: incorrect frequencies against/or incorrect class intervals/boundaries or class marks. Others used the upper real limits on the horizontal axis. In part (b)(ii), the candidates were unable to calculate the required median because they committed the following errors: failure to identify the median class from the frequency distribution table they drew, use of incorrect formulae for calculating the median (m) like $m = L + \left(\frac{\sum f_w}{\sum f_w - \sum f_b} \right) i$, $m = L + \left(\frac{\frac{N}{2} + f_b}{f_w} \right) i$, $m = \left(\frac{t_1}{t_1 + t_2} \right) i$, and $m = L + \left(\frac{\frac{N}{2} - f_b}{f_w} \right)$ instead of using the correct formula, $m = L + \left(\frac{\frac{N}{2} - f_b}{f_w} \right) i$, where L is the lower class limit of the median class, N is the total number of frequencies, f_b is the sum of all frequencies just before the median class, f_w is the frequency in the median class and i is the class size. Extract 11.2 is a sample solution of a candidate who answered this question incorrectly.

11 solution

class interval	frequency	class mark	class boundary	class size
50 - 54	3	52	49·5 - 54·5	5
55 - 59	2	57	54·5 - 59·5	5
60 - 64	7	62	59·5 - 64·5	5
65 - 69	4	67	64·5 - 69·5	5
70 - 79	13	72	69·5 - 79·5	5
80 - 89	6	77	79·5 - 89·5	5
90 - 99	1	82	89·5 - 99·5	5
	$\Sigma F = 36$		cumulative	



(11b)
THE HISTOGRAM
scale
V-S 1cm represent 8cm
H-S 1cm to 1cm
f
Commulative frequency
104
96
88
80
72
64
56
48
40
32
24
16
8
50-59
60-64
65-69
70-74
75-79
80-84
85-89
90-94
class interval

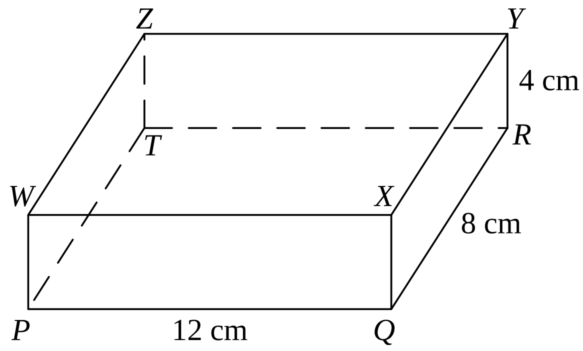
b	(ii) Median $= L + \left(\frac{t_1}{t_1+t_2}\right)C$
	$= 64.5 + \left(\frac{12}{12+20}\right)5$
	$= 64.5 + \left(\frac{12}{32} \times 5\right)$
	$= 64.5 + 1.875$
	$= 66.375$
	Correct to 2.dp ≈ 66.38
	$\therefore$ the median is 66.38

Extract 11.2: A sample of a candidate's incorrect response in question 11.

In Extract 11.2, the candidate wrote incorrect intervals like 70 – 79, 80 – 89 and 90 – 99 and the incorrect corresponding frequency in part (a). He/she also drew a histogram using *cumulative frequencies* on the vertical axis instead of *frequencies* in part (b)(i). In addition, the candidate applied the formula for mode of grouped data in part (b)(ii) contrary to the requirement of the question.

2.12 Question 12: The Earth as a Sphere and Three dimensional Figures

This question had parts (a), (b) and (c). In part (a), the question stated that, "*Two towns, A and B, are located at* (10°S, 38°E) *and* (10°S, 43°E), *respectively*". The candidates were required to: (i) find the distance between the two towns in kilometers giving the answer to the nearest whole number, using radius of the Earth, R = 6,400km and $\pi = 3.14$, (ii) use the distance obtained in part (a)(i) to find the time taken by a ship sailing from town A to town B at a speed of 50 km/h. In part (b), the candidates were given a rectangular prism in which $\overline{PQ} = 12\,\text{cm}$, $\overline{QR} = 8\,\text{cm}$ and $\overline{RY} = 4\,\text{cm}$ as shown in the following diagram:



Then, they were instructed to find: (i) the total surface area and (ii) the angle between the planes *PTZW* and *QRZW*.

In part (c), they were required to find the volume of a cone whose base radius is 12 cm and slant height is 20 cm using $\pi = 3.14$

The analysis indicates that, this question was attempted by 245,236 (56.1%) candidates. The analysis also shows that, a total of 21,122 (8.6%) candidates scored from 3 to 10 marks and among them 398 (0.2%) candidates scored all 10 marks. Again, out of 224,114 (91.4%) candidates who scored below 3 marks, 166,954 (68.1%) candidates scored 0 mark. This shows that the candidates' performance in this question was weak. The candidates' performance in this question is summarized in Figure 12.

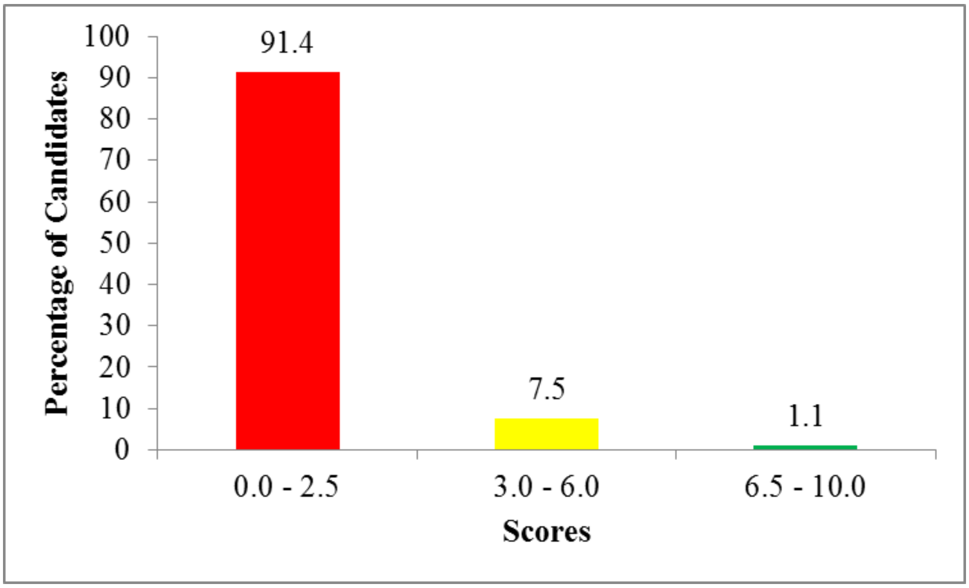


Figure 12: *Candidates' performance in question 12.*

The analysis shows that, there were reasons that caused the majority of candidates (68.1%) to score 0 mark. In part (a)(i), the candidates failed to use the correct formula when calculating the distance (d) between the points $(10^\circ\text{S}, 38^\circ\text{E})$ and $(10^\circ\text{S}, 43^\circ\text{E})$. They applied incorrect formulae such as $d = \frac{2\pi R\theta\cos\alpha}{180^\circ}$ and $d = \frac{\pi R}{180^\circ}$. Others wrote $d = \frac{\pi R\theta}{180^\circ}$, which is the formula for finding the distance along the same longitude. Instead, they were supposed to apply the formula for finding the distance along the same latitude, that is $d = \frac{\pi R\theta\cos\alpha}{180^\circ}$. Other candidates applied the correct formula but made wrong substitution of the angles. That is, they substituted $\theta = 10^\circ$ and $\alpha = 43^\circ - 38^\circ = 5^\circ$ instead of $\alpha = 10^\circ$ and $\theta = 43^\circ - 38^\circ = 5^\circ$ due to lack of adequate knowledge of the use of latitudes and longitudes in finding distance between two places. The candidates' failure to get the correct distance led to inability to get the required period of time in part (a)(ii). Furthermore, there were candidates who applied incorrect formulae for finding velocity, such as $\text{velocity} = \frac{\text{time}}{\text{distance}}$ instead of $\text{velocity} = \frac{\text{distance}}{\text{time}}$ when calculating the time. Those candidates lacked knowledge of solving problems related to navigation.

In part (b)(i), most of the candidates were unable to recognize that the total surface area of the given prism equals to the sum of areas of rectangles *TZYR*, *QXYR*, *WZYX*, *PQRT*, *PTZW* and *PQXW*. As a result, they applied incorrect formulae like $area = length \times width \times height$ and $area = length + width + height$. This indicates that, the candidates lacked adequate knowledge of the application of the surface areas of three dimensional figures. In part (b)(ii), the candidates were unable to identify the right angled triangle formed by the planes *PTZW* and *QRZW*, hence they failed to recognize the specific angle *PWQ*. In addition, the candidates failed to find the length of $\overline{WQ}$ which was an important step to get the intended angle. In general, these candidates lacked enough knowledge of angles between two intersecting planes. In part (c), most of the candidates used inappropriate formulae like $v = \pi r^2 h$, which is a formula for finding the volume of a cylinder, $v = 2\pi r^2$ and $v = \frac{4}{3}\pi r^2 h$ instead of $v = \frac{1}{3}\pi r^2 h$ in order to get the volume of a cone. Most of them failed to use the knowledge of Pythagoras theorem to get height h of the cone, as a result they regarded $20\,\text{cm}$ as the height instead of $h=\sqrt{20^2-12^2}$ cm. Extract 12.1 shows a sample response from one of the candidates who answered this question incorrectly.

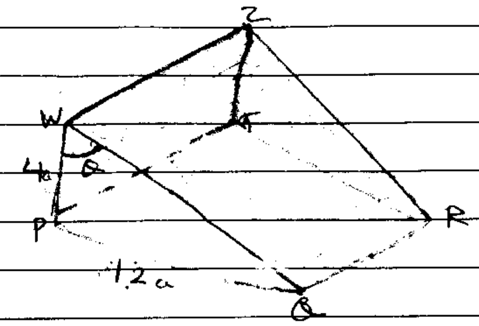
12a) town A C(10°S, 38°E)
B C(10°S, 43°E)

Distane = 2πr cos θ
= 2 × 3.14 × 6400 × cos 10 (α−β / 360)

12a) 6.28 × 6400 × cos 10
= 401,920 km
= 400,000 km distane (43−38 / 360)
= 400,000 × 5 / 360
= 5582.2
= 6000.0 km the distance between.

b) i Total surface area = (lw + lh + wh) cm²
= (12×8) + (12×4) + (8×4) cm²
= (96 + 48 + 32) cm²
= 176 cm²
∴ the total surface area = 176 cm²

12 b) (ii)



From $\sin\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

$\sin\theta = \frac{\cancel{12}^{3}}{\cancel{4}_{1}}$

$\sin\theta = 3$

$\theta = \sin^{-1} 3$

$\theta = \sin^{-1} 3$

12 c)

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \times 3.14 \times 12^2 \times 20$

$V = \frac{1}{3} \times 3.14 \times 144 \times 20$

$V = \frac{9043.2}{3}$

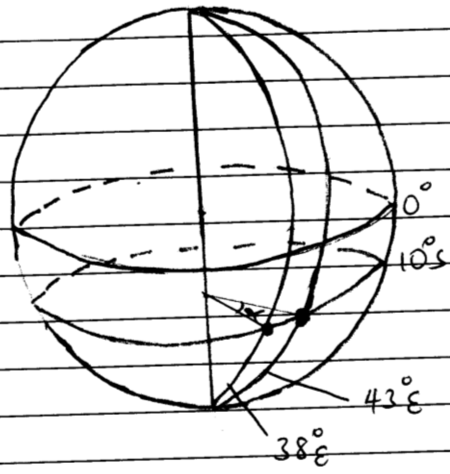
$V = 3014.4\,cm^3$

Extract 12.1: A sample of a candidate's incorrect response in question 12.

In Extract 12.1, the candidate applied incorrect formula when finding the distance along the same latitude in part (a). The candidate also did not take into account the total areas of all surfaces of the prism because he/she considered only three surfaces in part (b)(i). Again, in finding the angle between two planes, he/she applied $\sin\theta$ instead of $\tan\theta$ that relates to the required ratio $\frac{12}{4}$ in part (b)(ii). Lastly, the candidate substituted the slant height in the formula instead of height that could be obtained by using Pythagoras theorem, the given slant height and radius in part (c).

On the other hand, the analysis shows that, the candidates who scored full marks in this question were able to do the following: in part (a)(i), the candidates managed to use the correct formula when finding the distance between two towns located on the same and hence determine the time taken by the ship to sail from town A to B. In part (b)(ii), they were able to find the total surface area of the given prism and the angle between the stated planes. In part (c), the candidates managed to use the correct formula to calculate the volume of the given cone. Extract 12.2 is a sample of a candidates' correct response in this question.

12. a)



i) From $D = \frac{\alpha \pi R \cos\theta}{180}$

$\alpha = 43° - 38°$

$\alpha = 5°$

$\theta = 10°$, $R = 6400km$, $\pi = 3.14$

Then, $D = \frac{5 \times 3.14 \times 6400 km \cos 10°}{180°}$

$D = 549.742 km. \approx 550 km$

$\therefore$ The distance is 550 km

ii) Distance = 550 km

Speed = 50km/hr

Time = ?

From, speed = $\frac{\text{Distance}}{\text{Time}}$

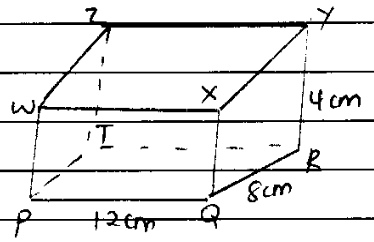
$50km/hr = \frac{550 km}{\text{Time}}$

12 ii) Time $= \frac{55\cancel{0}km}{5\cancel{0}km/hr}$

$= 11hrs$

$\therefore$ The ship will take 11 hours to reach town B

b)



i) Total surface area $= 2(lw + lh + wh)$

Where $L = 12cm$ (length)

$W = 8cm$ (width)

$h = 4cm$ (height)

Total surface area $= 2((12\times8) + (12\times4) + (8\times4))$

$= 2(96 + 48 + 32)$

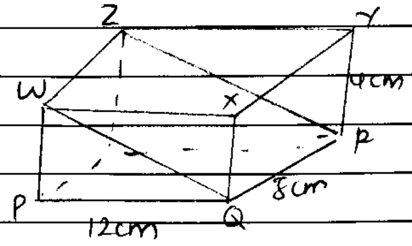
$= 2(176)$

$= 352cm^2$

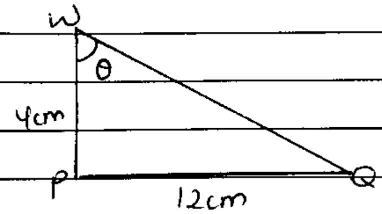
$\therefore$ The total surface area of the rectangular prism is $352\ cm^2$

ii) Angle between PTZW and QRZW

Consider angle $W\hat{P}Q$ or $R\hat{Z}T$



12. consider right angle $W\hat{P}Q$



Angle between PTZW and QRZW is θ

From, $\text{Tan}\,\theta = \dfrac{\text{opp.}}{\text{adjacent}}$

$\text{Tan}\,\theta = \dfrac{\cancel{12cm}\ 3}{\cancel{4cm}}$

$\text{Tan}\,\theta = 3\text{ cm}$

$\theta = \text{Tan}^{-1}(3)$

$\theta = 71.565^\circ$

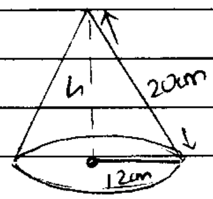
$\therefore$ The angle between is 71.565°

c) Given

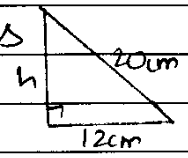
Radius is 12cm

slant height = 20cm

Cone formule = $V = \frac{1}{3}\pi r^2 h.$



1^{st} Determin h. from Δ



Apply pythaguras heorem

12.	c) $c^2=a^2+b^2$
	$20^2=12^2+h^2$
	$400=144+h^2$
	$\sqrt{256}=\sqrt{h^2}$
	$h=16cm$
	Then ~~Area~~
	Volume $=\frac{1}{\cancel{3}}\times 3.14\times\cancel{12}^{4}\times 12\times 16$
	volume $=2411.52cm^3$
	$\therefore$ The volume of the cone is $2411.52cm^3$

Extract 12.2: A sample of a candidate's correct response in question 12.

In Extract 12.2, the candidate applied the correct formulae, procedures and related concepts to get the required answers. This indicates that, he/she had sufficient knowledge in solving problems on distances along the same latitude, navigation as well as area and volumes of three dimensional figures.

2.13 Question 13: Matrices and Transformations

This question had parts (a), (b) and (c). In part (a), the candidates were required to find the matrix A whose inverse is $\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$. In part (b), the question stated that: "*Amani and Asha bought Coca-cola and Pepsi drinks for a farewell party. Amani spent Tshs. 9950 to buy 12 bottles of Coca-cola and 5 bottles of Pepsi drinks. Asha spent Tshs. 8150 to buy 9 bottles of Coca-cola and 5 bottles of Pepsi drinks*". The candidates were required to formulate a system of linear equations and apply the matrix method to find the price of one bottle of each type of the drinks. In part (c), the question stated that, "*Point* $\mathrm{A}(4, 2)$ *is reflected in the line* $y+x=0$ *followed by an anticlockwise rotation through* 90° *about the origin.*" The candidates were required to find the final image of point A.

The data analysis shows that, this question was attempted by 255,445 (58.4%) candidates. Further analysis shows that, 55,070 (21.6%) candidates scored from 3 to 10 marks, among them 1,452 (0.6%) scored full marks. Furthermore, 200,375 (78.4%) candidates scored from 0 to 2.5 marks, out of whom 148,453 (58.1%) candidates scored 0 mark. This reveals that the candidates' performance in this question was weak. The candidates' performance summary in this question is presented in Figure 13.

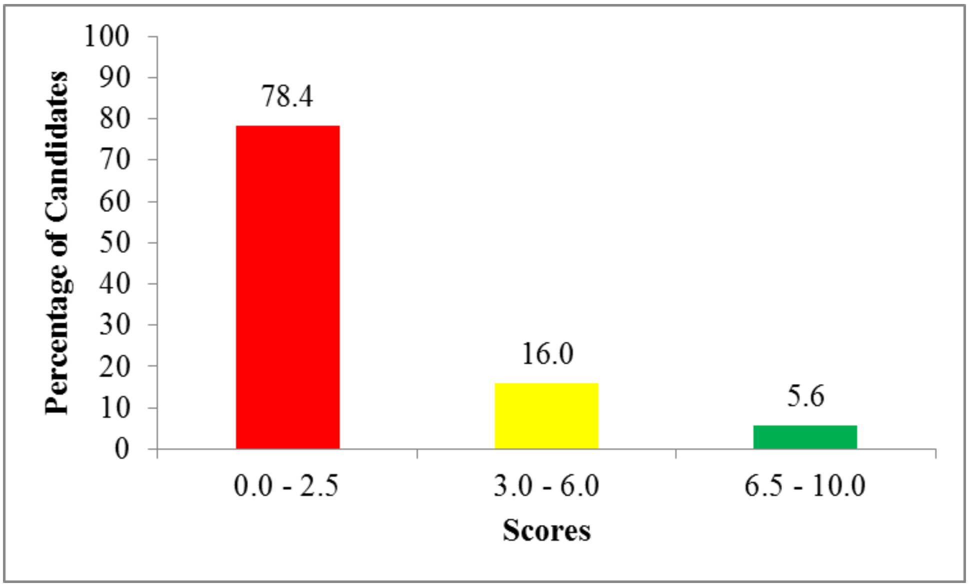


Figure 13: *Candidates' performance in question 13*.

The weak performance in this question was attributed to several factors. In part (a), some candidates failed to recall the property stating that, "*if* A^{-1} *is the inverse of matrix A, then the inverse of* A^{-1} *is A or shortly* $\left(A^{-1}\right)^{-1} = A$". Others were able to recall that $\left(A^{-1}\right)^{-1} = A$ but failed to apply the correct procedures to determine the inverse. For example, most of them failed to get the correct determinant of the given matrix A^{-1} because they got wrong determinant, that is $\left|A^{-1}\right| = 15 - 8 = 7$ instead of $\left|\mathrm{A}^{-1}\right| = 8 - 15 = -7$. Others failed to apply the correct procedure or formula for the inverse of a matrix correctly. There were some candidates who failed to interchange the elements in the main diagonal, for example they wrote $A^{-1} = \frac{1}{-7}\begin{pmatrix} 4 & -3 \\ -5 & 2 \end{pmatrix}$. However, some of them were unable to assign the elements in the leading diagonal with a negative sign. For example, they wrote $A^{-1} = \frac{1}{-7}\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ instead of $A^{-1} = \frac{1}{-7}\begin{pmatrix} 2 & -3 \\ -5 & 4 \end{pmatrix}$ that could give the required matrix A. In addition, there were some candidates who multiplied each element in the matrix $\begin{pmatrix} 2 & -3 \\ -5 & 4 \end{pmatrix}$ by -7 instead of $\frac{1}{-7}$. Those candidates lacked knowledge of the inverse of a matrix.

Others were able to state that $\mathrm{AA}^{-1} = \mathrm{I}$, where I is an identity matrix, implying that $\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ but failed to perform matrix multiplication of $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ by $\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$ which was an important step to get the values of a, b, c and d for the required matrix $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$. For example, some candidates wrote $\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$ and concluded that $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$ is the required matrix A.

In part (b), the candidates were not able to formulate the linear equations correctly from the given word problem. For example, some of them wrote incorrect equations like: $\begin{cases} 12x + 9y = 9950 \\ 5x + 5y = 8150 \end{cases}$ and $\begin{cases} 12x + 5y = 8150 \\ 9x + 5y = 9950 \end{cases}$ instead of $\begin{cases} 12x + 5y = 9950 \\ 9x + 5y = 8150 \end{cases}$, where x and y are unit prices of Coca cola and Pepsi drinks, respectively. Others were able to obtain the correct equations but failed to solve them by matrix method. However, few candidates managed to write $\begin{pmatrix} 12 & 5 \\ 9 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9950 \\ 8150 \end{pmatrix}$ but failed to find the inverse of the coefficient matrix $\mathrm{M} = \begin{pmatrix} 12 & 5 \\ 9 & 5 \end{pmatrix}$ so that $\begin{pmatrix} x \\ y \end{pmatrix} = \left(\mathrm{M}^{-1}\right)\begin{pmatrix} 9950 \\ 8150 \end{pmatrix}$. Likewise, some candidates got the correct inverse, that is $M^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{-3}{5} & \frac{4}{5} \end{pmatrix}$ but they re-arranged the final equation as $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 995 \\ 815 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{-1}{3} \\ \frac{-3}{5} & \frac{4}{5} \end{pmatrix}$ instead of $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{3} \\ \frac{-3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 995 \\ 815 \end{pmatrix}$. Those candidates failed to realize that, matrix multiplication is not commutative, as a result they ended by getting incorrect values of *x* and *y*.

In part (c), most of the candidates were unable to recall the correct matrix of reflection of a point. For example, some of them wrote $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin\alpha & \cos 2\alpha \end{pmatrix}$ instead of $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$. Others confused it with a rotation matrix, that is $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ in finding the image of point A(4, 2). Some candidates did not know that when point (*a*, *b*) is reflected in the line $y + x = 0$ implying that $y = -x$, the image will be (-*b*, -*a*). As a result, they wrote incorrect images like: (2, -4), (-2, 4) and (-4, -2) instead of (-2, -4). In that case, those candidates failed to get the correct final image after a rotation through 90° about the origin in anticlockwise direction. Few of them wrote the correct rotation matrix but failed to write the correct values for *sine* and *cosine* of angles 270° and 90°. In addition, some candidates were not aware that, when point (*p*, *q*) is rotated through 90°, the image will be (-*q*, *p*). As a result, they wrote incorrect final image of the point (-2, -4) as (-2, 4), (-4, 2) and (-4, -2) instead of (4, -2). Those candidates had insufficient knowledge of the transformation of matrices. Extract 13.1 illustrates a sample response from one of the candidates who was unable to answer this question correctly.

13 (a).

From,

$$A^{-1} = \frac{1}{|A|}(A).$$

let matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix} = \frac{1}{|A|}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} a/|A| & b/|A| \\ c/|A| & d/|A| \end{pmatrix}$$

Relate the values.

$$4 = a/|A|.$$

$$3 = b/|A|$$

$$5 = c/|A|$$

$$2 = d/|A|$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$$

$\therefore$ Matrix $A = \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$.

b) Let the bottles of cocacola be x
Let the bottles of pepsi be y

Amani: $12x + 5y \leq 9950$

Asha: $9x + 5y \leq 8150$

Non-negative objective function
$x \geq 0$ and $y \geq 0$

$f(x,y) := x + y$

$12x + 5y = 9950$

x-int $(y=0)$	y-int $(x=0)$
$12x = 9950$	$5y = 9950$
$/12 \quad /12$	$/5 \quad /5$
$x = 829.17$	$y = 1990$
$(829.17, 0)$	$(0, 1990)$

13. b) $9x + 5y = 8150$

x-int $(y=0)$	y-int $(x=0)$
$9x = 8150$	$5y = 8150$
$/9 \quad /9$	$/5 \quad /5$
$x = 905.5$	$y = 1630$
$(905.5, 0)$	$(0, 1630)$

Corner points $(0,0)$ $(0, 1630)$, $(829.17, 0)$ and $(701, 1254)$

$= (701, 1254)$

$\therefore$ The price of Cocacola is Tsh. 701 and of Pepsi is Tsh 1254.

B c) A (4, 2)

$$y + x = 0$$

$$y = -x$$

$$\theta = 45^\circ$$

MR (4, 2)

Rotation $\begin{pmatrix} \cos\theta & -\sin\theta^\circ \\ \sin\theta^\circ & \cos\theta^\circ \end{pmatrix}$

$$= \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & \cos 90 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Reflect m $= \begin{pmatrix} \cos 45^\circ & \cos 45^\circ \\ \sin 45^\circ & -\sin 45^\circ \end{pmatrix}\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$$= \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{2} + 2\sqrt{2} \\ \sqrt{2} - 2\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 4.2 \\ -1.4 \end{pmatrix}$$

Extract 13.1: A sample of a candidate’s incorrect response in question 13.

Part (a) of Extract 13.1 shows that, the candidate failed to apply the required formula and appropriate procedures to get a matrix whose inverse was given. In part (b), he/she confused the linear equation problems with linear inequality problems because he/she formulated inequalities $12x + 5y \le 9950$ and $9x + 5y \le 8150$ instead of equations $12x + 5y = 9950$ and $9x + 5y = 8150$.

In part (c), the candidate determined the image of point A(4, 2) after a rotation followed by reflection contrary to the given instructions. He/she was supposed to start with reflection followed by rotation to get the required final image.

Despite the weak performance in this question, there were a few candidates (0.6%) who managed to get the correct answers. In part (a), the candidates were able to find the inverse of the given matrix by using the correct procedures. In part (b), they were able to formulate and solve the linear equations by using the matrix method to get the price of one bottle of each type of the drinks. In part (c), they managed to find the image of point $A(4, 2)$ when reflected in the line $y + x = 0$ followed by an anticlockwise rotation through 90° about the origin. This indicates that, the candidates had sufficient knowledge of application of inverse of a matrix and transformation of a point in solving real life related problems. Extract 13.2 shows a sample response from one of the candidates who answered this question correctly.

13 a) Given $A^{-1} = \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$

From;

$$A \cdot A^{-1} = I$$

$$A \cdot \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4a+5b & 3a+2b \\ 4c+5d & 3c+2d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$4a+5b = 1$ ---i)

$3a+2b = 0$ ---ii)

$$\begin{matrix} 3 \\ 4 \end{matrix}\begin{cases} 4a + 5b = 1 \\ 3a + 2b \quad 0 \end{cases}$$

$$-\begin{cases} 12a + 15b = 3 \\ 12a + 8b = 0 \end{cases}$$

$$\frac{7b}{7} = \frac{3}{7}$$

$$b = 3/7$$

From eqn ii)

$$3a + 2b = 0$$

$$3a + 2\left(\frac{3}{7}\right) = 0$$

$$3a + \frac{6}{7} = 0$$

$$\frac{1}{3} \times 3a = \frac{-6}{7} \times \frac{1}{3}$$

$$a = \frac{-2}{7}$$

13 a) $4c + 5d = 0$ --- i)

$3c + 2d = 1$ --- ii)

$$3\begin{cases} 4c + 5d = 0 \end{cases}$$
$$4\begin{cases} 3c + 2d = 1 \end{cases}$$

$$-\begin{cases} 12c + 15d = 0 \\ 12c + 8d = 4 \end{cases}$$

$$\frac{7d}{7} = \frac{-4}{7}$$

$$d = -4/7$$

From eqn i)

$$4c + 5d = 0$$

$$4c + 5\left(\frac{-4}{7}\right) = 0$$

$$4c + \frac{-20}{7} = 0$$

$$\frac{1}{4} \times 4c = \frac{20}{7} \times \frac{1}{4}$$

$$c = \frac{5}{7}$$

$$\therefore \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -2/7 & 3/7 \\ 5/7 & -4/7 \end{pmatrix}$$

13 b) Let x be the price of bottle of Coca-cola and y be the price of a bottle of Pepsi.

$$12x + 5y = 9950 \text{ ---i)}$$
$$9x + 5y = 8150 \text{ ---ii)}$$

In matrix form;

$$\begin{pmatrix} 12 & 5 \\ 9 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9950 \\ 8150 \end{pmatrix}$$

Where; $|A| = (12\times5) - (5\times9)$
$$= 60 - 45$$
$$= 15$$

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} 5 & -5 \\ -9 & 12 \end{pmatrix}$$
$$= \frac{1}{15}\begin{pmatrix} 5 & -5 \\ -9 & 12 \end{pmatrix}$$
$$= \begin{pmatrix} 5/15 & -5/15 \\ -9/15 & 12/15 \end{pmatrix}$$

So;

$$\begin{pmatrix} 5/15 & -5/15 \\ -9/15 & 12/15 \end{pmatrix}\begin{pmatrix} 12 & 5 \\ 9 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5/15 & -5/15 \\ -9/15 & 12/15 \end{pmatrix}\begin{pmatrix} 9950 \\ 8150 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 49750/15 + -40,750/15 \\ -89,550/15 + 97,800/15 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9000/15 \\ 8,250/15 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 600 \\ 550 \end{pmatrix}$$

$x = 600$ and $y = 550$

$\therefore$ The price of one bottle of Coca-cola is 600/= and of Pepsi is ~~5~~ 550/=

13	c) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ Reflected in line $y = -x$ $\theta = 135^{\circ}$	

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2(135^{\circ}) & \sin 2(135^{\circ}) \\ \sin 2(135^{\circ}) & -\cos 2(135^{\circ}) \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \cos 270^{\circ} & \sin 270^{\circ} \\ \sin 270^{\circ} & -\cos 270^{\circ} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

Followed by rotation, $\theta = 90^{\circ}$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$= \begin{pmatrix} \cos 90^{\circ} & -\sin 90^{\circ} \\ \sin 90^{\circ} & \cos 90^{\circ} \end{pmatrix} \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

$\therefore$ The final image of point A is at $(4, -2)$

Extract 13.2: A sample of a candidate's correct response in question 13.

In Extract 13.2, the candidate applied the formula $AA^{-1} = I$ to get the required inverse in part (a). In part (b), he/she formulated and solved the simultaneous equations $12x + 5y = 9{,}950$ and $9x + 5y = 8{,}150$ by matrix method to get the price of each type of drink per bottle. The candidate also managed to find the reflection of point $A(4,2)$ through 135° followed by an anticlockwise rotation through 90° about the origin to get (4, -2) as the final image.

2.14 Question 14: Functions and Linear Programming

This question had parts (a) and (b). In part (a), the candidates were required to find the domain and range of the inverse of the function $f(x) = (x+2)^2$. In part (b), the question stated that, "*A businessman plans to buy at most 210 sacks of Irish and sweet potatoes. Irish potatoes cost shs. 30,000 per sack and sweet potatoes cost shs. 5,000 per sack. He can spend up to shs. 2,500,000 for his business. The profit on a single sack of Irish potatoes is shs. 12,000 and for sweet potatoes is shs. 10,000*". The candidates were required to find the number of sacks of each type of potatoes to be bought in order to realize the maximum profit.

The data analysis indicates that, this question was attempted by 226,873 (51.9%) candidates. Further analysis shows that 37,425 (16.5%) candidates scored from 3 to 10 marks, among them 343 (0.2%) scored 10 marks. Moreover, out of 189,448 (83.5%) candidates who scored below 3 marks, 166,954 (68.1%) candidates scored 0 mark, indicating that the performance in this question was weak. The summary of candidates' performance in this question is shown in Figure 14.

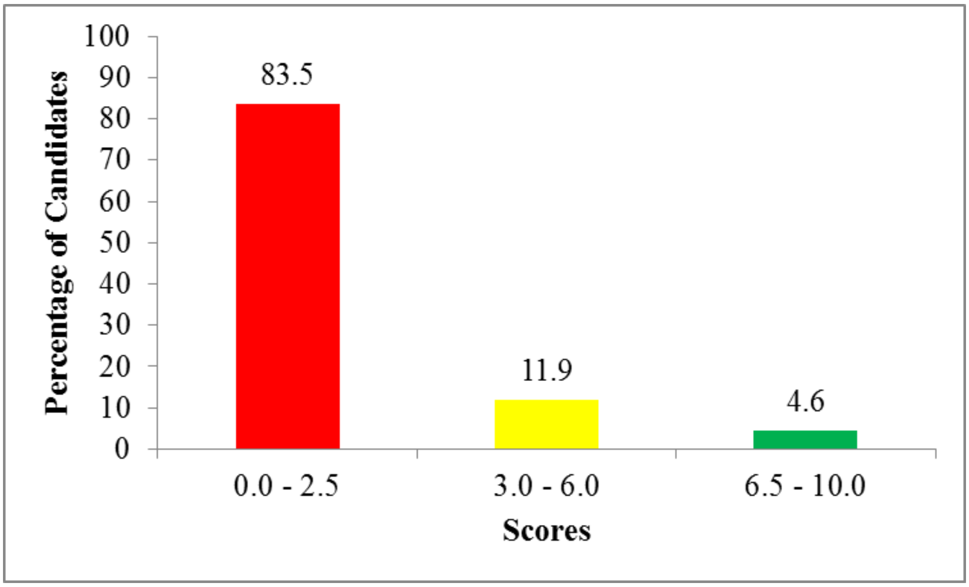


Figure 14: *Candidates' performance in question 14.*

The candidates' response analysis indicates that, the majority of candidates (47.1%) failed to get the correct answer in this question because of the following reasons: in part (a), some candidates determined the domain and range of $f(x)=(x+2)^2$ instead of finding the domain and range of its inverse, $f^{-1}(x)$. Also, there were candidates who failed to apply the correct procedures to get the inverse of the function $f(x)=(x+2)^2$. For example, some of them interchanged the variable *x* with $f(x)$ to get $x=(f(x)+2)^2$ but failed to make $f(x)$ the subject. They wrote incorrect inverse like $f(x)^{-1}=\pm\sqrt{y}-2$ and $f(x)^{-1}=\pm x^2-2$ instead of $f^{-1}(x)=\pm\sqrt{x}-2$ which was a necessary step to get the domain and range of $f^{-1}(x)$. This shows that, the candidates lacked adequate knowledge of transposition of formulae with terms involving exponents.

In part (b), the candidates formulated incorrect linear inequalities like: $x+y\geq 210$ and $30{,}000x+5{,}000y\geq 2500{,}000$ instead of $x+y\leq 210$ and $30{,}000x+5{,}000y\leq 2500{,}000$. In addition, there were some candidates who used the given profits to formulate the linear inequality $12{,}000x+10{,}000y\leq 2500{,}000$. Those candidates were unable to formulate the correct objective function from the given word problem. Others wrote incorrect objective function like: $f(x,y)=30{,}000x+5{,}000y$ by using the given costs instead of using the given profits to get $f(x,y)=12{,}000x+10{,}000y$. As a result, they ended by drawing incorrect graphs from which they could not get the required feasible region and corner points which were the sufficient conditions to calculate the number of sacks of Irish and sweet potatoes and maximum profit. Extract 14.1 represents a sample of incorrect response from one of those candidates.

14. a)

solution

$f(x) = (x+2)^2$

$f(x) = (x+2)(x+2)$

$= x^2+2x+2x+4$

$= x^2+4x+4$

$f(x) = x^2+4x+4$

But inverse of function f,

$f(x) = x^2+4x+4$

interchange variables

$x = y^2+4y+4$

make y the subject

y^2+4y+4

sum(4) = 2+2

product (4) = 2×2

$y^2+2y+2y+4$

$y(y+2)+2(y+2)$

$(y+2)(y+2)$

$y=-2$

Hence, domain = $\{x : x \in R\}$

Range = $\{y : y = -2\}$

14. b). Solution

let. Irish potatoes be x.

Sweet potatoes be y.

$x + y \geq 210$

$30{,}000x + 5000y \geq 2500{,}000$

$x \geq 0$

$y \geq 0$

from the intercept

$x + y = 210$

x	210	0
y	0	210

$30{,}000x + 5000y = 2500{,}000$

x	~~500~~ 83.3	0
y	0	500

a = 0, 0

b = 83.3, 0

d = 0, 210

c =

from the equation

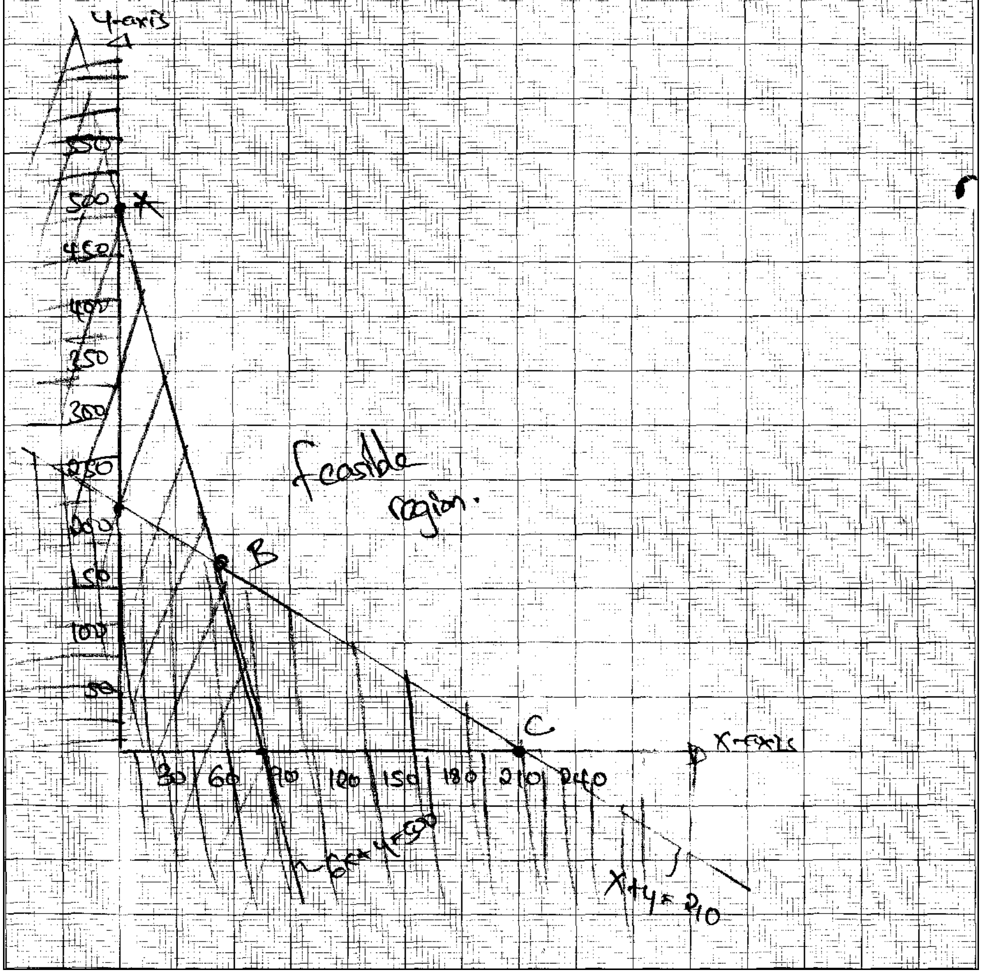
$x + y = 210$

$30{,}000x + 5000y = 2500{,}000$

$\cdot 3 \mid x + y = 210$

$1 \mid 30x + 5y = 250$

b)


y-axis
550
500
450
400
350
300
250
200
150
100
50
A
B
C
feasible region.
x-axis
30
60
90
120
150
180
210
240
6x+y=500
x+y=210

14 b) $3X - 3X + 3y - 5y = 210 - 250$

$3y - 5y = -40$

$-2y = -40$

$y = 20$

$x + y = 210$

$X = 190$

$X, y = 190, 20$

Corner point	objective fx. $12000x + 10,000y$	max & min
0,0	0+0	0
83.3,0	83.3×12000	999600
0, 210	2100,000	2100000
190,20	2280000+200,000	2480000

~~The~~ in 190,20 The product

Extract 14.1: A sample of a candidate’s incorrect response in question 14.

In Extract 14.1, the candidate failed to make y the subject of $x = y^2 + 4y + 4$ to get the inverse of the given function, as a result he/she got the incorrect domain and range in part (a). In part (b), he/she formulated the linear inequalities with the signs “$\geq$” instead of “$\leq$”. As a result, he/she got incorrect graph and wrong values of the objective function.

Although the performance was weak, there were some candidates (0.2%) who answered this question correctly. In part (a), the candidates managed to find the inverse of the given function and were able to determine the domain and range. In part (b), they were able to identify the decision variables for the number of sacks of Irish and sweet potatoes and use them to formulate the objective function $f(x, y) = 12000x + 10000y$ and linear inequalities $x + y \leq 210$ and $30,000x + 5,000y \leq 2500,000$ including $x \geq 0$ and $y \geq 0$. Then, they drew the graph of these inequalities and identified the corner points of the feasible region from the graph. Lastly, they substituted the corner points to the objective function to determine a point that would give the required number of sacks of Irish and sweet potatoes and the maximum profit as shown in Extract 14.2.

14. a) Soln

$f(x) = (x+2)^2$

Find it's inverse

~~y = (x + y~~

$y = (x+2)^2$

$x = (y+2)^2$

make y the subject

$\sqrt{x} = \sqrt{(y+2)^2}$

$y + 2 = \sqrt{x}$

$y = \sqrt{x} - 2$

Domain is $\{x : x \geq 0\}$.

Range is $\{y : y$ all real numbers$\}$.

b) let the number of Irish potatoes be x

let the number of sweet potatoes be y.

Table of summary.

	Irish	Sweet	Total
Number	x	y	210.
Price.	$30,000x$	$5,000y$	2,500,000

14 Constrains :

$x + y \leq 210$ . . . . (i)

$30{,}000x + 5{,}000y \leq 2{,}500{,}000$ . . . . (ii)

Non - negative constrains

$x \geq 0, \quad y \geq 0$.

Objective function

$f(x) = 12{,}000(x) + 10{,}000(y)$.

Table values :

$x + y = 210$.

x	0	210
y	210	0

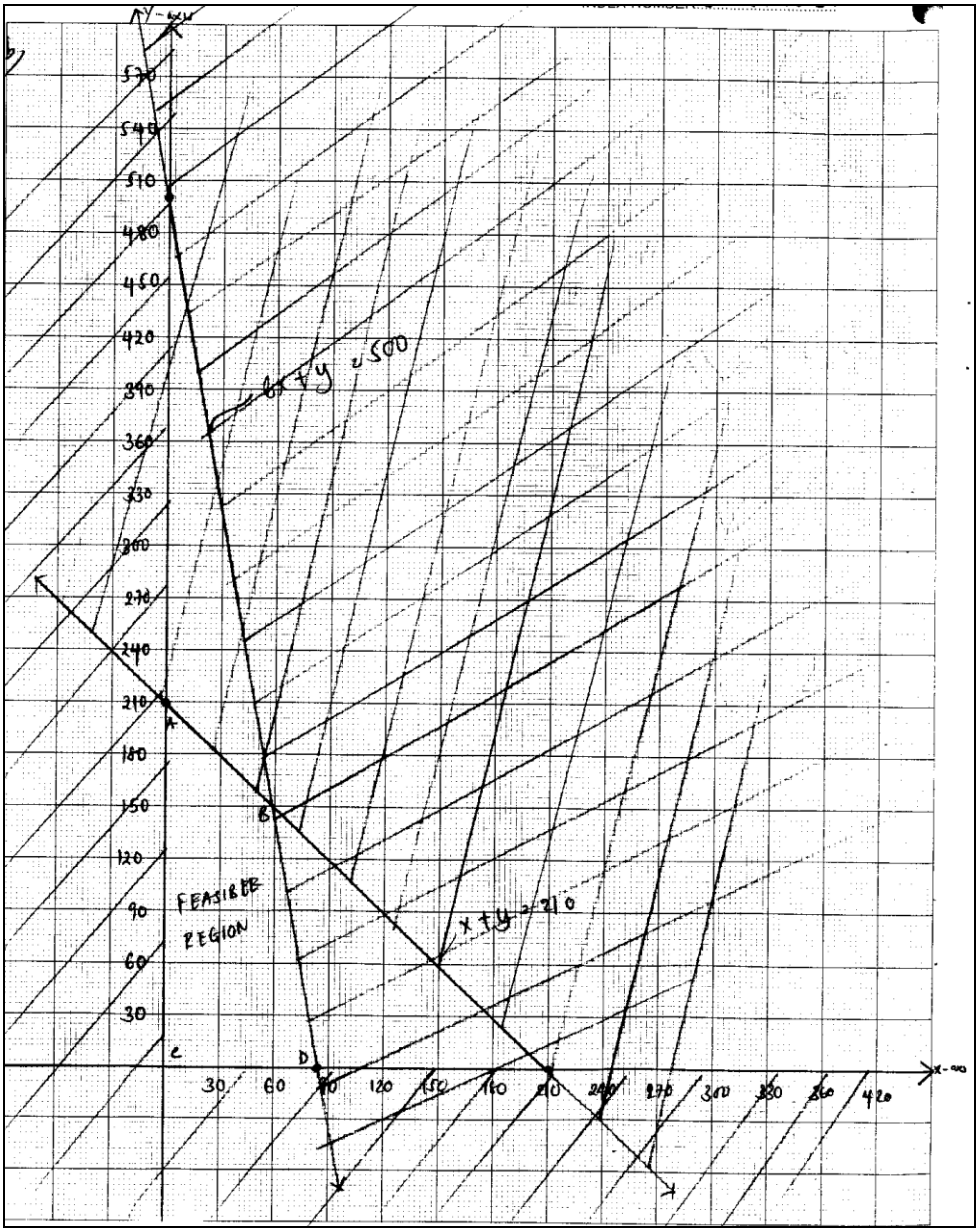
$6x + y = 500$

x	0	83.3
y	500	0

On graph .

Corner point	Objective function	profit
A (0, 210)	12,000 (0) + 10,000 (210)	2,100,000
B (58, 152)	12,000 (58) + 10,000 (152)	2,216,000
C (0, 0)	12,000 (0) + 10,000 (0)	0
D (83.3, 0)	12,000 (83.3) + 10,000 (0)	999,600

∴ For maximum profit business man have to buy 58 sacks of irish potatoes and 152 sacks of sweet potatoes



Extract 14.2: A sample of a candidate's correct response in question 14.

In Extract 14.2, the candidate was able to get the required domain and range of the inverse of the given function by using the appropriate procedures in part (a). In part (b), he/she formulated the correct mathematical model representing the given problem and solved it graphically to get the number of sacks of Irish and sweet potatoes required to maximize the profit.

3.0 PERFORMANCE OF THE CANDIDATES IN EACH TOPIC

The 041 Basic Mathematics examination paper had fourteen (14) questions that were set from 23 topics. Those topics include: *Statistics, Sequences and Series*, *Accounts*, *Ratio, Profit and Loss*, *Rates and Variations, Pythagoras Theorem, Trigonometry, Matrices and Transformations, Perimeters and Areas*, *Coordinate Geometry, Vectors, Functions, Linear Programming, Exponents and Radicals, Logarithms, Numbers*, *Decimals and Percentages, The Earth as a Sphere, Three Dimensional Figures*, *Sets, Probability*, *Quadratic Equations* and *Algebra*.

The statistical analysis shows that, there was no topic that had a good performance in CSEE 2020 in Basic Mathematics examination. Further analysis shows that, five topics namely: *Statistics* (50.3%), examined in question 11, *Sequences and Series* (39.9%) examined in question 8, *Accounts* and *Ratio, Profit and Loss* (39.6%) examined in question 7 and *Rates and Variations* (31.5%) examined in question 6 had an average performance. The remaining topics had a weak performance of 25.6 to 6.3 percent. The summary of the performance in each topic is shown in the Appendix.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis of the candidates' performance show that in CSEE 2020, four topics, namely: *Statistics* (50.3%), *Sequence and Series* (39.9%), *Accounts*, *Ratio*, *Profit and Loss* (39.6%) and *Rates and Variations* (31.5%) had average performance while in CSEE 2019, only three topics, namely: *Statistics* and *Circles* (53.4%) and *Rates and Variations* (33.9%) had average performance. None of the topics had good performance in both years.

The main reasons that contributed to weak performance in these topics include the candidates' failure to: apply correct formulae, rules, theorems, properties and procedures, formulate expressions, inequalities and equations from word problems, perform correct mathematical operations, draw diagrams and graphs as well as interpreting figures correctly.

4.2 Recommendations

In order to improve the candidates' performance in Basic Mathematics examinations in the future, teachers are strongly recommended to:

(a) guide students through pair-share thinking to formulate expressions by using letters, simplify expressions by using the BODMAS rule, formulate and solve equations with one to two unknowns and linear inequalities with one unknown, especially from real life related word problems in the topic of Algebra.

(b) demonstrate to students in small groups and employ the proper methods and formulae in solving quadratic equations step by step as well as discussing the properties of quadratic equations and techniques used in solving related word problems in the topic of Quadratic Equations.

(c) use teaching and learning tools such as identical objects in managing students' classroom discussions on applying the rules, formulae and diagrams in solving the real life set and probability related problems and evaluating their competence in each concept in the topics of Sets and Probability.

(d) use flat shapes and hall shapes in teaching 3 dimensional figures and simple techniques for finding angles, side lengths and diagonals, angles formed between two planes, angles between line segment and plane and procedures for finding areas and volume of hall shapes including prisms, cylinders, pyramids, cone and building models in the topic of Three Dimensional Figures.

(e) use a variety of tools, such as globe, orange, water melon, atlas and graphs to guide students to discuss how to use latitude and longitude to find distances (in kilometers and nautical miles) between two places on the Earth. Teachers should also supervise students to discuss how to find the distance between two towns located on the same latitude and two different latitudes; and the distance between two cities located on the same longitude and two different longitudes and how to apply the knowledge acquired from all concepts in the topic of The Earth as a Sphere to solve problems related to navigation.

(f) guide students in their groups to discuss how to use the concepts of Numbers, Decimals and Percentages in simplifying mathematical expressions, forming and solving real life related problems involving different operations.

(g) Demonstrate to students how to apply the rules of exponents to verify the rules of logarithm, simplify terms and calculate equations using rationalize the denominators of the expressions involving radicals and assess students' proficiency in all concepts of Logarithms, Exponents and Radicals.

(h) use participatory methods to guide students’ discussion on the characteristics of relations and functions by using the teaching and learning resources available in their localities, how to draw graphs step by step and solve simultaneous equations by graphical method. This step is important before teaching the topic of Linear Programming.

(i) use the teaching and learning resources available in their environment to explain all the necessary procedures that should be followed when formulating the inequalities/equations and objective functions, drawing graphs and calculating maximum values and minimum values using real examples pertaining to Linear Programing.

(j) lead students' discussions in small groups on how to formulate linear equations by using gradient and intercepts as well as two points; and steps to consider in drawing graphs of linear equations using a table of values and intercepts in the *xy*-plane in the topic of Coordinate Geometry.

(k) use various techniques when teaching all concepts in Vectors, including the basic operations of vectors, they should assess students’ competence through real life related problems pertaining to velocities, displacements and forces of various objects.

(l) use real flat-shaped objects in demonstrating how to calculate the perimeters and areas and solve real life related problems associated with these concepts in the topic of Perimeters and Areas.

(m) use creative and participatory techniques in teaching students how to multiply matrices, demonstrate all necessary steps to determine the inverse of matrices, solve simultaneous equations by using the matrix methods. By the same token, teachers should impart to students complete knowledge in Geometrical Transformations (Form Two topic) using proper teaching and learning resources, such as rulers, colored chalks, plane mirrors and graphs before they teach transformations in the topic of Matrices and Transformations (Form Four topic).

(n) use teaching and learning resources, such as square cuttings, flat objects that are in a shape of a right angled triangle, square root tables and square tables in guiding students on how to apply the Pythagoras theorem in solving related miscellaneous problems.

(o) demonstrate how to use sine, cosine and tangent in unraveling mysteries about angle of elevation and angle of depression; and calculate the heights and angles of real structures by employing the knowledge from Trigonometry and Pythagoras Theorem.

APPENDIX

ANALYSIS OF CANDIDATES' PERFORMANCE PER TOPIC IN BASIC MATHEMATICS – CSEE 2020

S/N	Topic(s)	Question Number	Percentage of Candidates who scored 30 marks or more	Remarks
1	Statistics	11	50.3	Average
2	Sequence and Series	8	39.9	Average
3	Accounts, Ratio, Profit and Loss	7	39.6	Average
4	Rates and Variations	6	31.5	Average
5	Pythagoras Theorem and Trigonometry	9	25.6	Weak
6	Matrices and Transformations	13	21.6	Weak
7	Perimeters and Areas	5	17.3	Weak
8	Coordinate Geometry and Vectors	4	17.1	Weak
9	Functions and Linear Programming	14	16.5	Weak
10	Logarithms, Exponents and Radicals	2	14.4	Weak
11	Numbers, Decimals and Percentages	1	13.3	Weak
12	The Earth as a Sphere and Three Dimensional Figures	12	8.6	Weak
13	Sets and Probability	3	8.2	Weak
14	Quadratic Equations and Algebra	10	6.3	Weak