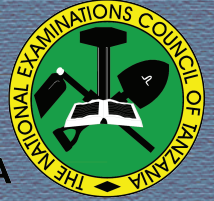




THE UNITED REPUBLIC OF TANZANIA  
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY  
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**CANDIDATES' ITEM RESPONSE ANALYSIS  
REPORT ON THE ADVANCED CERTIFICATE OF  
SECONDARY EDUCATION EXAMINATION  
(ACSEE) 2025**

**BASIC APPLIED MATHEMATICS**



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**(ACSEE) 2025**

**141 BASIC APPLIED MATHEMATICS**

*Published by:*

The National Examinations Council of Tanzania,

P.O. Box 2624,

Dar es Salaam, Tanzania.

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## FOREWORD

The National Examinations Council of Tanzania is pleased to present this report on the Candidates' Item Response Analysis (CIRA) for the Basic Applied Mathematics paper of the Advanced Certificate of Secondary Education Examination (ACSEE) 2025. This report provides a detailed evaluation of candidates' responses, offering valuable insights to guide improvements in mathematics education across Advanced secondary schools in Tanzania.

Analysis of the results reveals an overall satisfactory performance, with 33,936 (72.66%) candidates achieving passing grades. The examination covered eleven topics, with candidates attaining good performance in *Statistics, Functions* and *Differentiation*. In Statistics, they competently calculated range, standard deviation, and mean while accurately constructing cumulative frequency curves. Their skills in Functions were evident in determining slopes of straight lines, identifying asymptotes, and sketching and interpreting graphs of quadratic functions. In Differentiation, candidates successfully applied both the first principles and the product rule of differentiation, and effectively used derivatives to determine gradients of curves.

However, performance in *Algebra, Matrices and Linear Programming* and *Integration* was average. More concerning was the weak performance in *Trigonometry, Probability, Exponential and Logarithmic Functions* and *Calculating Devices*. The candidates faced various challenges including lack of understanding in using calculating devices (especially non-programmable calculators), failure to properly apply the laws of exponents and logarithms, inadequate grasp of probability concepts, and difficulty in applying trigonometric ratios and identities.

This report serves as an important resource for all education stakeholders in strengthening teaching and learning process in identified weak areas, while maintaining the positive momentum in topics where candidates performed well.

Finally, the Council extends its sincere appreciation to the examination officials, subject experts, and all professionals who contributed to the preparation of this report.

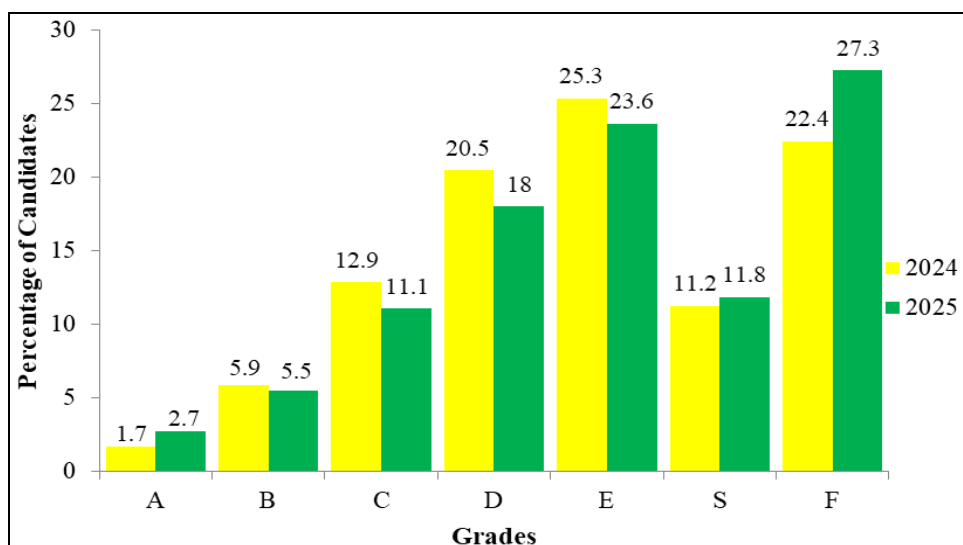


Prof. Said Ally Mohamed  
**EXECUTIVE SECRETARY**

## 1.0 INTRODUCTION

The 141 Basic Applied Mathematics examination paper for the Advanced Certificate of Secondary Education (ACSEE) 2025 was set in accordance with the Basic Applied Mathematics Syllabus for Advanced Secondary Education (2010) and the examination format of July 2019. The paper consisted of ten (10) compulsory questions, each carrying ten (10) marks, making a total of 100 marks.

In 2025, a total of 46,790 candidates sat for the examination, achieving a pass rate of 72.66 percent. This performance represents 4.89 percentage decline over the 2024 pass rate of 77.55 percent. A comparative analysis of grade distributions between 2024 and 2025 is illustrated in Figure 1.



**Figure 1:** *Distribution of Candidates by Grade in 2024 and 2025.*

The grade distribution analysis reveals a diverged performance trend in 2025 compared to 2024, with a 0.9 percent increase in Grade A. A decline pass rate in grades C and D by 1.8 percent and 2.5 percent respectively was observed as well as a sharp 4.9 percent rise in failures. These diverging performance patterns indicate widening achievement gaps, potentially highlighting the need for targeted instructional interventions.

This report systematically examines candidates' performance through sections. Section 2 presents a detailed question-wise analysis of candidate responses, identifying strengths and weaknesses. Section 3 provides a

comprehensive topic-based performance evaluation across the syllabus. The report concludes with evidence-based recommendations in Section 4, derived from the analytical findings.

Finally, the report includes Appendix I, which offers complete topic-level performance data for 2025, and Appendix II, showing a comparative performance analysis between the 2024 and 2025. These appendices provide additional details to support data-informed decision making by education stakeholders.

## 2.0 ANALYSIS OF CANDIDATES' RESPONSE IN EACH QUESTION

Candidates' overall performance on each question is classified according to the proportion of candidates who scored 3.5 marks or above. Performance is categorized as follows: good (60–100%), average (35–59%), and weak (0–34%). In the accompanying graphs and charts, these categories are visually represented by the colors green, yellow, and red, respectively. The subsequent analysis systematically evaluates the competences demonstrated and recurring challenges observed in candidates' responses, supported by extracts illustrating actual candidates' answers.

### 2.1 Question 1: Calculating Devices

This question examined candidates' ability to use non-programmable calculators to evaluate expressions involving trigonometric, logarithmic, and exponential terms as well as radicals. The question was stated as follows:

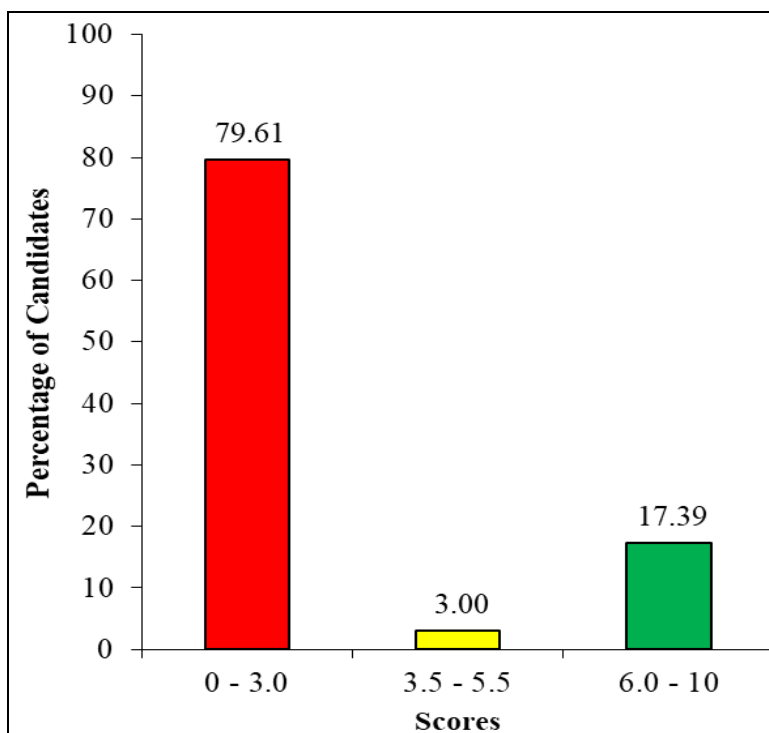
*Use a non-programmable calculator to evaluate the following expressions correct to 4 significant figures:*

$$(a) \quad \frac{\sin 49^\circ \times \ln \left( \frac{1 + \cos 50^\circ}{\sqrt{276}} \right)}{\sqrt{1234}}$$

$$(b) \quad \frac{e^{\log 5} \times {}^9C_6}{\sin \left( \frac{\pi}{6} \right) + \tan^{-1}(1.4)}$$

$$(c) \quad \sqrt[4]{\left( \frac{5e^{\left(\frac{5}{6}\right)} \div \sin^{-1}(0.5)}{\ln 3 \times \log_2 3} \right)^3}$$

This was the question with the weakest performance, with only 9,539 (20.39%) candidates passing. The analysis further reveals that a total of 21,862 (46.72%) candidates scored zero. Figure 2 shows the percentage of candidates who scored low, average, and high marks.



**Figure 2:** *Candidates performance on question 1*

In part (a), several candidates did not set their calculators to degree mode, which was necessary for correctly evaluating the values of  $\sin 49^\circ$  and  $\cos 50^\circ$ . As a result, their answers were incorrect due to the use of radian values instead of degrees. Another frequently observed error was the omission or incorrect use of brackets when entering the logarithmic expression. Some candidates failed to properly enclose the entire numerator  $\ln(1 + \cos 50^\circ)$  and the denominator  $\sqrt{276}$  within the brackets, leading to an incorrect value of the logarithmic part. These candidates entered the expression 
$$\frac{\sin 49^\circ \times \ln 1 + \cos 50^\circ \div \sqrt{276}}{\sqrt{1234}}$$
, which yielded an unintended value, 0.01488. It was also noted that some candidates confused the logarithmic functions, using the common logarithm (log) instead of the natural logarithm (ln). These candidates lacked the knowledge of some symbols on their calculators or had conceptual misunderstanding of the notation used in the question. Additionally, it was observed that some candidates misinterpreted the expression. They treated only the logarithmic part as being divided by the denominator  $\sqrt{1234}$ , instead of the entire

product in the numerator  $\sin 49^\circ \times \ln(1 + \cos 50^\circ \div \sqrt{276})$  was to be divided by the same denominator. This misinterpretation contributed to an incorrect answers. It was further noted that some candidates lacked an understanding of standard notation of numbers because they incorrectly wrote the number in the form  $-4.971 \times 10^{-2}$  which did not meet the condition  $1 \leq A < 10$  because  $A < 1$ . Moreover, other candidates failed to set their calculators to display answers in four significant figures. As a result, they ended up with incorrect answers, such as  $-0.0149$ .

In part (b), several candidates did not set their calculators to radian mode. Instead, they left them in degree mode, which led to an incorrect value of  $\sin\left(\frac{\pi}{6}\right)$ . Some candidates also lacked an understanding of inverse trigonometric functions, as they misinterpreted the term  $\tan^{-1}(1.4)$ ; instead, they computed the tangent of 1.4 rather than the inverse tangent of 1.4. Another common challenge was observed in the evaluation of the combination notation,  ${}^nC_r$ . Some candidates were not familiar with the calculator's combination function as they attempted to compute the value of  ${}^9C_6$  algebraically. Others confused combination with permutation notation or used an incorrect calculator key. Further, some candidates failed to use brackets properly, especially when entering the expression  $\sin\left(\frac{\pi}{6}\right) + \tan^{-1}(1.4)$ .

Similarly, in part (c), a number of candidates did not set their calculators to the correct mode (radian) when evaluating the expression

$$\sqrt[4]{\left(\frac{5e^{\left(\frac{5}{6}\right)} \div \sin^{-1}(0.5)}{\ln 3 \times \log_2 3}\right)^3}. \text{ As a result, they misinterpreted the output of the}$$

inverse sine operation, as well as the final answer. Another challenge was observed in the handling of the exponential term  $5e^{\left(\frac{5}{6}\right)}$ . Some candidates entered incorrect input of the exponent, causing the calculator to raise  $e$  to

5 instead of  $\frac{5}{6}$ . Others were not aware of the function key of the exponent ( $\wedge$ ), leading to incorrect entries. Further difficulties were observed in evaluating the inverse sine function, whereby some candidates incorrectly calculated  $\sin(0.5)$  instead of  $\sin^{-1}(0.5)$ , which resulted in an incorrect value and ultimately an entirely final answer. Another source of error arose from the incorrect interpretation of logarithmic expressions in the denominator. Candidates confused the natural logarithm ( $\ln$ ) with the common logarithm ( $\log$ ), and several were unfamiliar with steps of computing the term  $\log_2 3$ , which required the change of base, that is,  $\log_2 3 = \frac{\log 3}{\log 2}$  on calculators that do not directly support logarithms to base 2. In most cases, some candidates either used an incorrect base or entered the expression without proper brackets, resulting in an incorrect order of operations. Moreover, many candidates failed to apply the fourth root correctly. Some mistakenly took the square root instead of the fourth root, while others did not use the appropriate syntax or function keys on the calculator. A few candidates applied the root operation to only part of the fraction, rather than to the entire rational expression.

01.	
	(a) = -0.0497.
	(b) = 116.4947.
	(c) = 44.8260

**Extract 1.1:** A sample of incorrect responses to question 1

In Extract 1.1, the candidate failed to enter the expressions correctly, set the calculator to the appropriate mode, or configure it to display results with four significant figures.

Although the overall performance of the candidates was weak, 5.75 percent of them scored the full marks on this question. The analysis showed that these candidates had sufficient knowledge of the use of non-programmable calculators. In part (a), the candidates set the calculators to

degree mode and correctly entered the given expression. They then set their calculators to display results in four significant figures to get the correct answer  $-0.04971$ . In part (b), the candidates correctly evaluated the value of  $\frac{e^{\log 5} \times {}^9C_6}{\sin\left(\frac{\pi}{6}\right) + \tan^{-1}(1.4)}$  by setting the calculators to radian mode. In part

(c), these candidates set the calculators to radian mode and configured them to display four significant figures. They then correctly entered the

expression  $\sqrt[4]{\left(\frac{5e^{\frac{5}{6}} \div \sin^{-1}(0.5)}{\ln 3 \times \log_2 3}\right)^3}$  into their calculators and obtained 6.695.

Extract 1.2 highlights the correct responses on this question.

01:	
	(a) .
	$\frac{\sin 49^\circ \times \ln \left[ \frac{1 + \cos 50^\circ}{\sqrt{276}} \right]}{\sqrt{1234}}$
	$= -0.04971062254$ to 4 significant figures .
	Answer = <u><math>-0.04971</math></u>
	(b) .
	$\frac{e^{\log 5} \times {}^9C_6}{\sin\left(\frac{\pi}{6}\right) + \tan^{-1}(1.4)}$
	$=$ <u><math>116.5</math></u>
	(c) .
	$\sqrt[4]{\left(\frac{5e^{\frac{5}{6}} \div \sin^{-1}(0.5)}{\ln 3 \times \log_2 3}\right)^3}$
	$=$ <u><math>6.695</math></u>

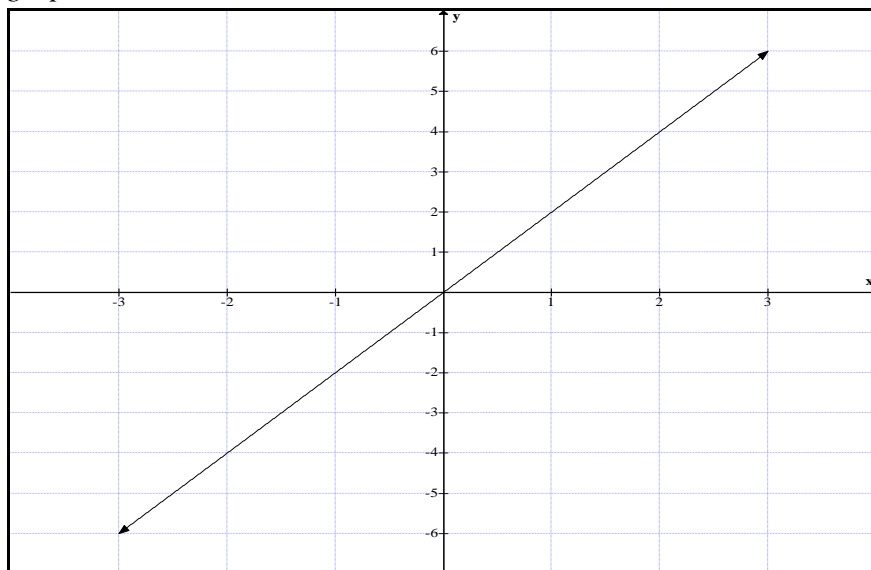
**Extract 1.2:** A sample of correct responses to question 1

In Extract 1.2, the candidate correctly used a non-programmable calculator to evaluate the given expressions correct to four significant figures.

## 2.2 Question 2: Functions

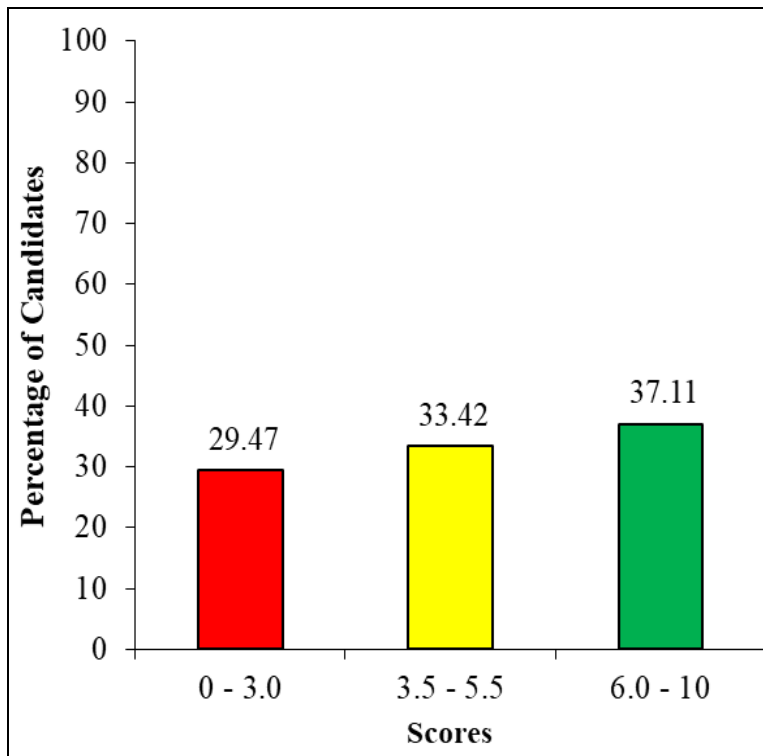
The question intended to examine the candidates' competence in finding the asymptotes of a rational function, as well as sketching and interpreting the graphs of a quadratic function. The question consisted of the following parts:

- (a) Calculate the slope of the function  $f(x)$  described by the following graph:



- (b) Given  $h(x) = \frac{1}{4-x}$ ,
- determine the vertical and horizontal asymptotes.
  - evaluate  $h(10)$ .
- (c) A particle moves along the path described by the function  $g(t) = -t^2 + 5t - 4$ .
- Sketch the graph of  $g(t)$ .
  - State the domain and range of  $g(t)$ .

The data show that 33,004 (70.53%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance on this question was generally good, as illustrated in Figure 3.



**Figure 3:** Candidates performance on question 2

A total of 161 (0.34%) candidates answered this question correctly. In part (a), the candidates correctly identified two points lying on the graph, such as  $(0, 0)$  and  $(1, 2)$ . They then applied the formula for calculating the slope ( $m$ ) of a straight line  $m = \frac{y - y_1}{x - x_1}$ , and substituted the identified points into the formula, resulting in  $m = 2$ .

In part (b)(i), the candidates correctly computed the vertical asymptote by equating the denominator to zero and correctly solved the resulting equation  $4 - x = 0$ , obtaining  $x = 4$ . For the case of horizontal asymptote, these candidates correctly evaluated the limit of  $h(x)$  as  $x$  becomes a very large number, that is,  $y = \lim_{x \rightarrow \infty} \frac{1}{4 - x}$ , and concluded that the horizontal asymptote is  $y = 0$  (see Extract 2.1). In part (b)(ii), the candidates correctly

computed  $h(10)$  by replacing  $x$  in  $h(x) = \frac{1}{4-x}$  with 10 and correctly performed basic operations, obtaining  $h(10) = -\frac{1}{6}$ .

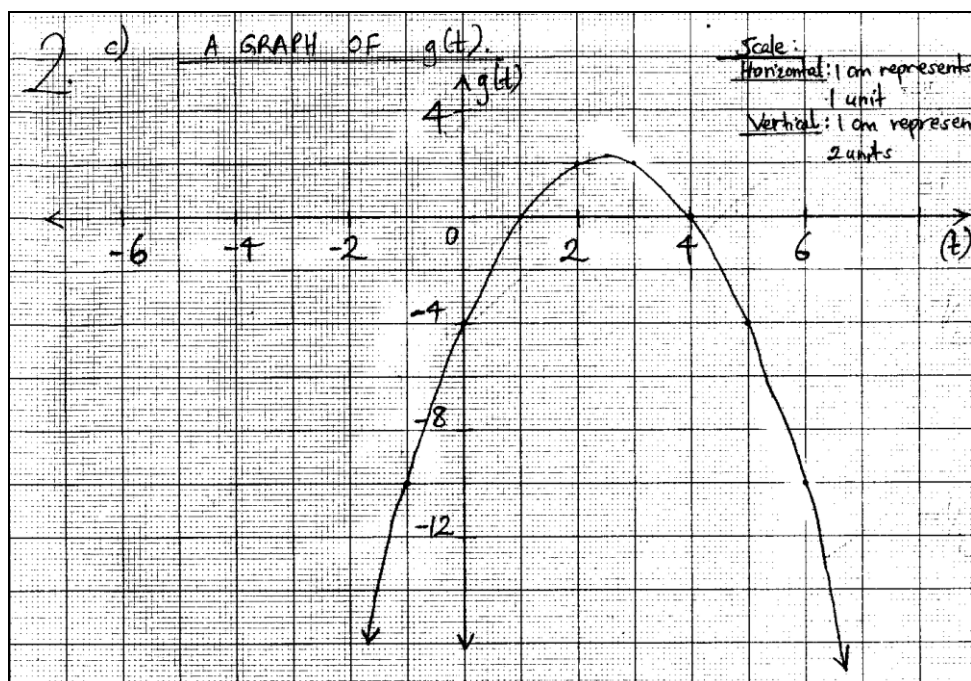
In part (c)(i), the candidates demonstrated a strong understanding of the shape and features of graph of a quadratic function, that is it forms a parabola. They correctly identified key characteristics of the graph, including the direction of the curve, opening downwards as coefficient of  $t^2$  is less than zero. They correctly calculated the  $x$ -intercepts (roots) by solving the equation  $-t^2 + 5t - 4 = 0$ , either by factoring, completing the square, or using the general quadratic formula, obtaining  $t = 1$  and  $t = 4$ . They also found the  $y$ -intercept by evaluating  $g(t)$  when  $t = 0$ , resulting in  $g(t) = -4$ . In addition, the candidates calculated the abscissa of the vertex of the parabola using the formula  $t = -\frac{b}{2a}$ , obtaining  $\frac{5}{2}$ . They then substituted this value into the function to determine the corresponding ordinate, obtaining  $\frac{9}{4}$ . Thus, they correctly identified the turning point of the graph,  $\left(\frac{5}{2}, \frac{9}{4}\right)$ . Finally, they used these information to sketch the graph

of the function, as illustrated in Extract 2.2. In part (c)(ii), the candidates correctly studied the graph and recognized that its domain is the set of all real numbers, since the function is defined for every value of  $t$ . Thus, they wrote  $\text{Domain} = \{t : t \in \mathbb{R}\}$ . For the range, they carefully examined the vertex of the parabola, which represents the maximum value of the function, as the parabola opens downwards. Therefore, they stated that  $\text{Range} = \left\{g(t) : g(t) \leq \frac{9}{4}\right\}$ .

2.	$b, y = 0$ Horizontal asymptote.
	from: $h(x) = \frac{1}{4-x}$
	divide each term by $x$ ;
	$\therefore h(x) = y = \frac{1/x}{4/x - x}$
	but $x \rightarrow 0$ .
	$\therefore y = \frac{0}{-1}$
	$y = 0$

**Extract 2.1:** A sample of correct responses to question 1

In Extract 2.1, the candidate correctly divided each term of both the numerator and the denominator by the highest power of  $x$ , resulting in the correct horizontal asymptote,  $y = 0$ .



**Extract 2.2:** A sample of correct responses to question 1

In Extract 2.2, the candidate sketched an accurate graph after correctly locating all the key points (intercepts and the vertex), and joining them using a smooth curve.

Despite the good performance, the analysis of data reveals that 13,787 (29.47%) candidates scored 3.0 marks or less. The candidates encountered several challenges. In part (a), some candidates selected points that did not lie exactly on the line. They either guessed or approximated the points, resulting in a slope that deviated from the actual value. This often stemmed from poor graph-reading skills or a lack of attention to instruction. Another frequently observed misconception was applying incorrect formulae. For instance, some candidates chose the appropriate points such as  $(0,0)$  and

$(2,1)$  but obtained an incorrect value of slope,  $m = \frac{1}{2}$ , instead of  $m = 2$ .

These candidates applied an incorrect formula for calculating slope, that is,

$m = \frac{x - x_1}{y - y_1}$ , indicating confusion between the change in  $y$  and the change in

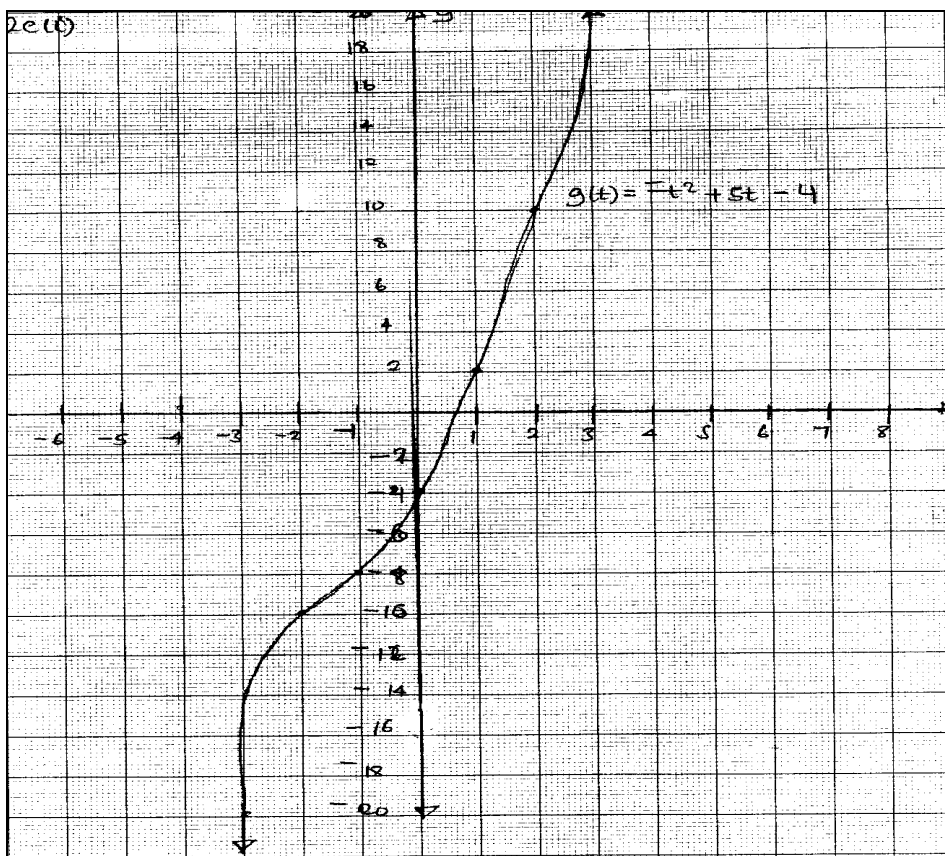
$x$ . Moreover, some candidates misinterpreted the slope as the length of the line, rather than the rate at which the vertical component changes with respect to the change in horizontal component. Thus, they performed irrelevant calculations, especially calculating the distance between two points.

In part (b)(i), confusion between horizontal and vertical asymptotes was frequently observed, as some candidates mistakenly identified the horizontal asymptote by setting the denominator equal to zero, which actually results in vertical asymptotes. Some candidates also incorrectly attempted to perform long division or plug in large values of  $x$ , leading to confusion and incorrect asymptotes. Furthermore, a few candidates mistakenly determined horizontal asymptote by plugging in some integers from the domain, such as  $x = 10$ , to observe the value to which the function approaches. While this approach may give an estimate, it is unreliable and lacks the precision and generality provided by rules based on degrees and coefficients. In part (b)(ii), many candidates committed errors in handling arithmetic with negative values in algebraic contexts. There were

candidates who correctly substituted  $x = 10$  into the function  $h(x) = \frac{1}{4 - x}$ ,

but failed to compute  $\frac{1}{4 - 10}$ , arriving at  $h(10) = -\frac{1}{6}$  instead of  $h(10) = \frac{1}{6}$ .

In part (c), one of the errors was failure to identify the general shape of the graph. Some candidates sketched a graph of a straight line, instead of a parabola. Others sketched inaccurate parabolas that open upwards, instead of parabola opening downwards. Another common mistake occurred during the calculation of the vertex. While the correct formula for the abscissa of the vertex is  $x = -\frac{b}{2a}$ , some candidates wrongly used incorrect formulae, such as  $x = -\frac{2a}{b}$ , or substituted values incorrectly, leading to a wrong location of the vertex. This misplacement affected the accuracy of the entire graph. Further analysis revealed that some candidates mishandled the process of finding  $x$ -intercept and  $y$ -intercept. For the  $x$ -intercept, it was observed that several candidates failed to correctly factorize the quadratic equation correctly, which led to incorrect solutions (roots) and consequently, inaccurate placement of the  $x$ -intercepts on the graph. Regarding the  $y$ -intercept, some candidates mistakenly set the expression equal to zero instead of substituting  $t = 0$  into the expression. This conceptual misunderstanding resulted in confusion when determining the graph's position relative to the coordinate axes. Another error observed among candidates' responses was the failure to recognize the symmetry of the parabola, as illustrated in Extract 2.3. Furthermore, some candidates located only the vertex or the intercepts, which resulted in poor-quality sketches lacking accuracy and some details.



**Extract 2.3:** A sample of incorrect responses to question 2

In Extract 2.3, the candidate drew an asymmetrical curve due to a failure to consider the property that a parabola has a line of symmetry that passes through the vertex,  $t = \frac{5}{2}$ .

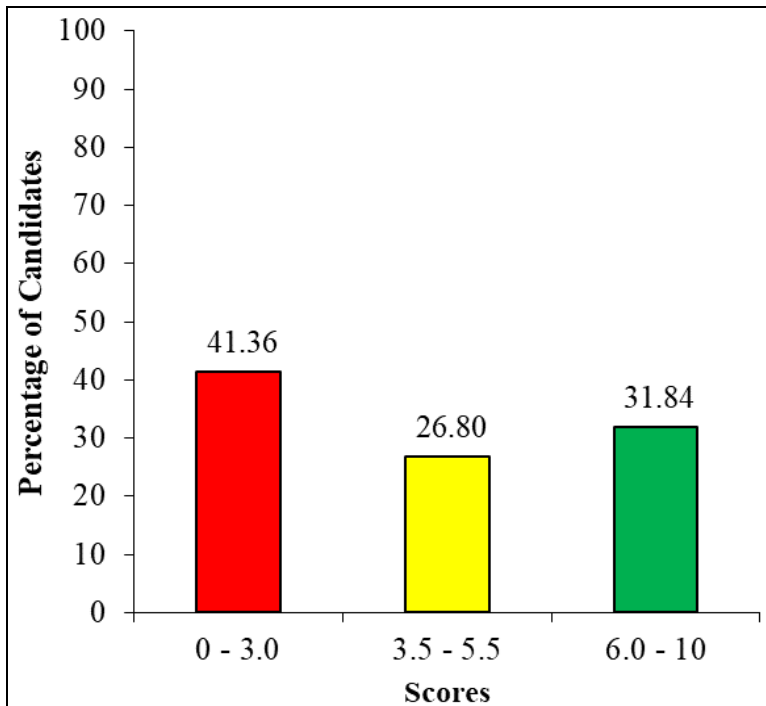
### 2.3 Question 3: Algebra

This question examined the candidates' ability to evaluate the values of expressions written in sigma notation, solve word problems involving sequences and series, and solve systems of simultaneous equations. The question comprised the following parts:

- (a) Find the value of  $\sum_1^4 (1+x)^2$ .

- (b) *The first and tenth terms of an arithmetic progression are  $-2$  and  $43$ , respectively. Find the sum of the first ten terms of the progression.*
- (c) *The difference between the areas of two squares is  $16$  and the sum of the length of a side of a large square and the length of a side of a small is  $8$ . Provided that  $x$  and  $y$  represent the lengths of the sides of small and large squares respectively;*
- (i) *formulate the simultaneous equations representing the given information.*
- (ii) *calculate the length of the side of the small square.*

The analysis of data reveals that 27,439 (58.64%) candidates scored between 3.5 and 10 marks inclusive, indicating that the overall candidates' performance was average. Figure 4 illustrates the performance on this question.



**Figure 4:** *Candidates performance on question 3*

In part (a), competent candidates demonstrated an understanding of how to interpret and expand expressions in sigma notation. They correctly

recognized that  $\sum_1^4 (1+x)^2$  represents the sum of expression  $(1+x)^2$  as  $x$  takes values of natural numbers from 1 to 4. Thus, they rewrote the series as  $(1+1)^2 + (1+2)^2 + (1+3)^2 + (1+4)^2$ , resulting in  $4+9+16+25$  and consequently 54.

In part (b), the candidates correctly recalled the basic formula for calculating the sum  $S_n$  of the first  $n$  terms of an Arithmetic Progression,

$S_n = \frac{n}{2}(A_1 + A_n)$ , where  $A_1$  is the first term and  $A_n$  is the  $n^{\text{th}}$  term of the progression. These candidates correctly interpreted the word problem and identified that  $n=10$ ,  $A_1 = -2$  and  $A_{10} = 43$ . They then substituted these values into the formula, obtaining  $S_{10} = \frac{10}{2}(-2+43)$  and finally  $S_{10} = 205$ .

Alternatively, other candidates approached this part by applying the general formula for calculating the sum  $S_n$  of the first  $n$  terms of an Arithmetic

Progression,  $S_n = \frac{n}{2}(2A_1 + (n-1)d)$ , where  $A_1$  is the first term and  $d$  is the common difference of the progression. Based on the formula for calculating the  $n^{\text{th}}$  term of an Arithmetic Progression,  $A_n = A_1 + (n-1)d$ , the candidates correctly formulated the equation  $43 = -2 + 9d$  after substituting  $n=10$ ,  $A_1 = -2$  and  $A_{10} = 43$  and solved it to get  $d = 5$ . Thereafter, they substituted the obtained values into the formula for calculating sum, that is,  $S_{10} = \frac{10}{2}(2(-2) + (10-1)5)$  and consequently obtained  $S_{10} = 205$ .

In part (c), the candidates correctly interpreted the word problem and applied the formula for calculating the area of a square. They realized that the area of the large square is  $y^2$  and the area of the small square is  $x^2$ . Therefore, these candidates correctly formulated the equation  $y^2 - x^2 = 16$  to represent the difference in areas, and  $x + y = 8$  to represent the given sum of the sides. They solved the equations and interpreted the solutions correctly, obtaining the length of the side of the small square as 3 units.

Extract 3.1 shows the correct responses from the candidate who answered this question correctly.

(c)	$x + y = 8$ .
	Simultaneous equations are
	$\begin{cases} x + y = 8 \\ y^2 - x^2 = 16 \end{cases}$
ii.	From: $y^2 - x^2 = 16$
	but: $y = 8 - x$ .
	$(8 - x)^2 - x^2 = 16$ .
	$64 - 16x + x^2 - x^2 = 16$ .
	$64 - 16x = 16$ .
	$64 - 16 = 16x$
	$48 = 16x$
	$x = \frac{48}{16}$
	$x = 3$ unit

**Extract 3.1:** A sample of correct responses to question 3

In Extract 3.1, the candidate correctly applied the substitution method to solve the simultaneous equations, resulting in the correct answer.

On the other hand, 42,477 (90.78%) candidates gave incorrect responses to some or all parts of this question. In part (a), one of the most frequent errors was misunderstanding the sigma notation. Some candidates wrongly assumed that the expression required to sum all values of  $x$  and then squaring the result, that is,  $(1+2+3+4)^2$ . Some candidates did not take other values of  $x$ , instead, they substituted  $x=1$  repeatedly, resulting in  $4+4+4+4$  and consequently 16. Moreover, some candidates only substituted the values of  $x$  from 1 to 4 into the expression  $(1+x)^2$  and obtained 4, 9, 16 and 25 without computing their sum. Additionally, some candidates incorrectly started from an incorrect lower limit, commonly  $x=0$  instead of  $x=1$ . Apart from misunderstanding the sigma notation, some candidates wrongly interpreted the expression  $(1+x)^2$  as  $(1+x^2)$ , failing to recognize that the exponent applies to the entire expression. This misunderstanding led to incorrect evaluation, such as  $(1+1^2)+(1+2^2)+(1+3^2)+(1+4^2)$ , which led to an incorrect answer.

In part (b), most candidates used the formula for the  $n^{\text{th}}$  term,  $A_n = A_1 + (n-1)d$ , instead of the formula for sum of the first  $n$  terms,  $S_n = \frac{n}{2}(A_1 + A_n)$  or  $S_n = \frac{n}{2}(2A_1 + (n-1)d)$ . Other candidates applied incorrect formulae for the sum of the first  $n$  term of an arithmetic progression,  $S_n = \frac{n}{2}(2A_1 - A_n)$  in particular. Additionally, a few candidates substituted incorrect values into the formula. For example, some of them incorrectly assigned the 10<sup>th</sup> term as the first term or confused the order of terms. Moreover, some of the candidates who used the formula  $S_n = \frac{n}{2}(2A_1 + (n-1)d)$  obtained an incorrect value of the common difference  $d$ , as Extract 3.2 shows.

In part (c), many candidates often confused the relationship between the length of a side and the area of a square. Some mistakenly assumed that the area of a square is directly proportional to the length of a side, rather than understanding that the area of a square is equal to the square of length of its side, leading to incorrect equations. Another frequent error was misinterpreting the phrases "difference between the areas" and "sum of the lengths". Several candidates formulated the equation  $x - y = 16$  instead of  $y^2 - x^2 = 16$ . Additionally, some candidates added the areas of the squares instead of the lengths of their sides, demonstrating a misinterpretation of the problem. Moreover, when setting up equations, some candidates incorrectly assigned the variables. For example, they let  $x$  represent the side of the larger square instead of the smaller one. leading to incorrect interpretation of solutions.

3. b)	
	$A_1 = -2$
	$A_{10} = 43$
	$A_1 + d = -2$
	$A_1 + 9d = 43$
	$0 + -8d = -45$
	$\frac{-8}{-8} \quad \frac{-8}{-8}$
	$d = 5.625$
	$\therefore$ <u>Common difference (d) = 5.625</u>
	$A_1 + 9d = 43$
	$A_1 + 9(5.625) = 43$
	$A_1 + 50.625 = 43$
	$A_1 = 43 - 50.625$
	$A_1 = -7.625$
3	$S_n = \frac{n}{2} (2A_1 + (n-1)d)$
	$S_{10} = \frac{10}{2} (2x(-7.625) + (10-1)5.625)$
	$= 5 (-15.25 + (9)5.625)$
	$= 5 (-15.25 + 50.625)$
	$= 5 (35.375)$
	$= 176.875$
	$\therefore$ <u>The sum of the first ten terms is 176.875</u>

**Extract 3.2:** A sample of incorrect responses to question 3

In Extract 3.2, the candidate incorrectly formulated the equation as  $A_1 + d = -2$ , resulting in incorrect values for the common difference and the sum of first ten terms.

## 2.4 Question 4: Differentiation

This question assessed the candidates' abilities to apply the first principles of differentiation, perform implicit differentiation, and apply differentiation to solve real-life problems. The question stated as follows:

- (a) Use the first principles of differentiation to find the derivative of the function  $f(x) = x^2 + 6x + 9$ .
- (b) Use the product rule to find the first derivative of the function  $f(x) = (x + 4)(3x^2 + 2x)$ .
- (c) A curve is described by the function  $3x^2 - 7y^2 + 4xy - 8x = 0$ . Calculate the gradient of the curve at the point  $(-1, 1)$ .

A total of 46,790 candidates attempted this question, of whom 31,724 (67.80%) candidates scored marks ranging from 3.5 to 10. This indicates that the candidates' performance on this question was good. Figure 5 provides a summary of the candidates' performance on this question.

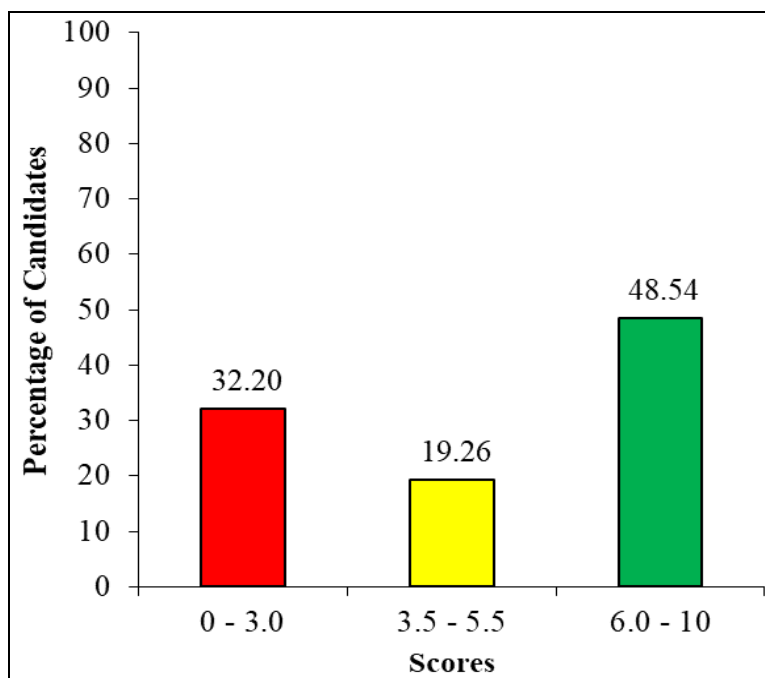


Figure 5: Candidates' performance on question 4

In this question, 6,995 (14.95%) candidates provided correct responses to all parts. In part (a), the candidates correctly recalled the first principles of differentiation, that is,  $f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$ . These candidates

developed  $f(x+h)$  by replacing  $x$  in  $f(x) = x^2 + 6x + 9$  with  $x+h$ , resulting in  $f(x+h) = (x+h)^2 + 6(x+h) + 9$ . They then substituted the expressions for  $f(x)$  and  $f(x+h)$  into the formula, obtaining

$$f'(x) = \lim_{h \rightarrow 0} \frac{\left( (x+h)^2 + 6(x+h) + 9 \right) - (x^2 + 6x + 9)}{h}. \quad \text{Thereafter, they}$$

evaluated the limit by firstly simplifying the expression

$$f'(x) = \frac{\left( (x+h)^2 + 6(x+h) + 9 \right) - (x^2 + 6x + 9)}{h} \quad \text{into}$$

$$f'(x) = \lim_{h \rightarrow 0} (2x + 6 + h) \quad \text{and then substituting } h = 0 \text{ into the expression,}$$

resulting in  $f'(x) = 2x + 6$ . In part (b), the candidates correctly recalled the

product rule of differentiation, that is  $\frac{d(u \cdot v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$ . These

candidates let  $u = x + 4$  and  $v = 3x^2 + 2x$ , that gave  $\frac{du}{dx} = 1$  and

$$\frac{dv}{dx} = 6x + 2. \quad \text{Hence, they substituted the expressions for } u, v, \frac{du}{dx} \text{ and}$$

$$\frac{dv}{dx} \text{ into the formula, obtaining } f'(x) = (3x^2 + 2x)(1) + (x + 4)(6x + 2).$$

They then correctly simplified the expression and concluded that the derivative of  $f(x) = (x+4)(3x^2 + 2x)$  is  $f'(x) = 9x^2 + 28x + 8$ .

In part (c), the candidates recognized that calculating the gradient of a curve at a specific point is equivalent to finding the value of the derivative of the curve at that point. Accordingly, they began by differentiating the given equation of the curve using implicit differentiation and obtained the

derivative  $\frac{dy}{dx} = \frac{8 - 6x - 4y}{4x - 14y}$ . They then substituted  $x = -1$  and  $y = 1$  into

the expression for  $\frac{dy}{dx}$ , simplified the result, and concluded that the gradient of  $3x^2 - 7y^2 + 4xy - 8x = 0$  at  $(-1, 1)$  is  $-\frac{5}{9}$ .

04	ANSWERS:
	(a) Solution:
	From first principle of differentiation:
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 6(x+h) + 9 - (x^2 + 6x + 9)}{h}$
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{x^2 + h^2 + 2xh + 6x + 6h + 9 - x^2 - 6x - 9}{h}$
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{h^2 + 2xh + 6h}{h}$
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} (h + 2x + 6)$
	$\frac{dy}{dx} = 0 + 2x + 6$
	$\frac{dy}{dx} = 2x + 6$
	$\therefore$ The derivative of the function, $f'(x) = 2x + 6$ .

**Extract 4.1:** A sample of correct responses to question 4

As illustrated in Extract 4.1, the candidate correctly substituted the expressions for  $f(x)$  and  $f(x+h)$  into the formula and performed the necessary algebraic simplification, obtaining the required derivative.

In spite of the good performance, 15,067 (32.20%) candidates scored low marks. In part (a), many candidates exhibited difficulties due to conceptual and algebraic misconceptions. One of the most common errors was the use of a wrong definition of the derivative from first principles. They used incorrect formulae, such as  $f'(x) = \lim_{x \rightarrow \infty} \left( \frac{f(x) - f(x+h)}{h} \right)$  instead of

$f'(x) = \lim_{x \rightarrow \infty} \left( \frac{f(x+h) - f(x)}{h} \right)$ , as a result could not produce the correct

answers. Another mistake arose during the formulation of  $f(x+h)$ . Many

candidates correctly got  $f'(x) = \lim_{h \rightarrow 0} \frac{\left( (x+h)^2 + 6(x+h) + 9 \right) - (x^2 + 6x + 9)}{h}$ ,

but could not expand the term  $(x+h)^2$  correctly, leading to incorrect answers such as  $x^2 + h^2$  or  $x^2 + h$ . Such algebraic errors distorted the entire expression and led to incorrect simplification. Some candidates also misused brackets. As a consequence, they did not apply the negative sign to all terms in  $f(x)$ , leading to incorrect cancellation and a defective numerator. Therefore, the expression that is supposed to be divisible by  $h$  remains jumbled with terms that cannot be appropriately simplified. Since dividing by  $h$  and taking the limit as  $h \rightarrow 0$  is the core of the method, those mistakes led to failure in factoring out  $h$  from the numerator after subtraction, and hence, invalid results. Other candidates substituted  $h = 0$  before simplifying the expression, leading to division by zero or loss of meaningful terms in the derivative. Moreover, a few candidates simply ignored the step of evaluating the limit or failed to interpret it correctly. They left the derivative as a simplified quotient in terms of  $h$  without proceeding with the evaluation of the limit, and thus provided incomplete answers.

In part (b), many candidates faced difficulties due to incorrect application of the product rule or errors in algebraic manipulation. One of the common misconceptions was the misunderstanding of the product rule. Some candidates mistakenly differentiated both functions and multiplied the derivatives directly, writing  $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dv}{dx}$ , leading to completely incorrect

results. Another mistake occurred in identifying and differentiating the expressions involved in the product. For example, with a function  $f(x) = (x+4)(3x^2+2x)$ , some candidates incorrectly differentiated  $x+4$  to get  $x$  instead of 1 or differentiated  $3x^2+2x$  to get  $6x$  after differentiating  $3x^2$  only. It was also noted that some candidates failed to expand the resulting expression correctly, especially when multiplying polynomials, leading to missing or incorrectly combined terms. Others forgot to collect like terms or failed to arrange the final expression in

standard form, which made their work appear incomplete or confusing. In some cases, candidates wrongly applied inappropriate differentiation rules, commonly the quotient rule or the chain rule, instead of the product rule. This indicates that the candidates had conceptual confusion between a function formed with the product of two expressions and composite functions.

In part (c), most candidates confused the concept of the gradient of a curve with that of a straight line. They used the formula for the slope of a straight

line,  $m = \frac{y - y_1}{x - x_1}$  Instead of applying differentiation. This demonstrates a

lack of understanding of the gradient of a curve at a given point. In some cases, candidates substituted the given point directly into the given expression of the curve  $3x^2 - 7y^2 + 4xy - 8x = 0$ , instead of first finding

$\frac{dy}{dx}$ . Furthermore, some candidates confused the gradient of the curve with

the gradient of the normal. Instead of finding the derivative at the given point, they calculated the negative reciprocal of the derivative, which gave the gradient of the normal, not that of the curve. Moreover, some candidates performed differentiation incorrectly. Most of these candidates did not realise the need for implicit differentiation. Extract 4.2 provides a highlight of incorrect responses to this question.

4 (c)	$6x + 14y \frac{dy}{dx} + 4xy - 4 = 4$
	$6x - 4 = 14y \frac{dy}{dx} - 4y \frac{dy}{dx}$
	$\frac{6x-4}{14y-4} = \frac{d(14y-4)}{14y-4} \frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{6x-4}{14y-4}$
	$\frac{dy}{dx} = \frac{6(-1)-4}{14(1)-4}$
	$\frac{dy}{dx} = \frac{-6-4}{14-4}$
	$\frac{dy}{dx} = \frac{-10}{10}$
	$\frac{dy}{dx} = -1$
	$\therefore$ The gradient of the curve at the point $(-1, 1)$ is $-1$ .

**Extract 4.2:** A sample of incorrect responses of question 4

In Extract 4.2, the candidate did not apply the knowledge of implicit differentiation when differentiating the term  $4xy$  in the equation  $3x^2 - 7y^2 + 4xy - 8x = 0$ .

## 2.5 Question 5: Integration

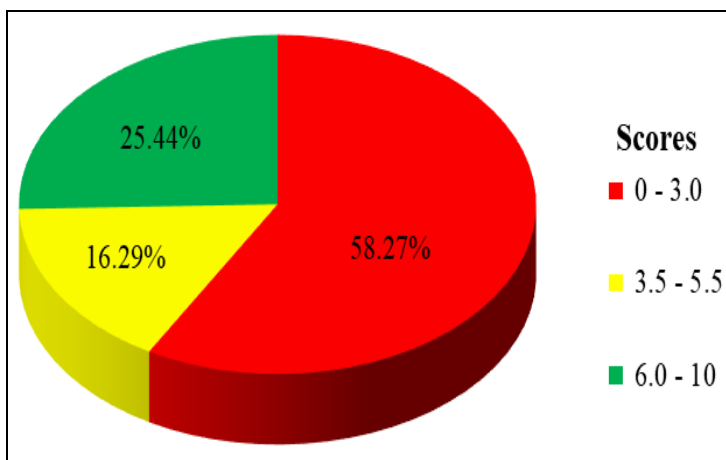
This question intends to examine the candidates' ability to determine indefinite and definite integrals of polynomial functions, as well as to apply integration techniques to find the volume of a solid generated by rotating a region about an axis. It comprised the following parts:

- (a) (i) Solve  $\int (4x+5)^{10} dx$ .

(ii) Find the value of  $\int_2^5 (x^2 - 2x + 1) dx$ .

- (b) Find the volume of the solid generated by rotating the region bounded by the curve given by the function  $y = x^2$  and the straight line represented by the equation  $y - 2 = 0$  about the  $y$ -axis.

The data revealed that 19,528 (41.73%) candidates scored more than 3.0 marks, of whom 7,622 (16.29%) candidates scored between 3.5 and 5.5 marks and 11,906 (25.44%) scored between 6.0 and 10 marks. Therefore, the overall performance of the candidates on this question was average. Figure 6 illustrates the performance of the candidates on this question.

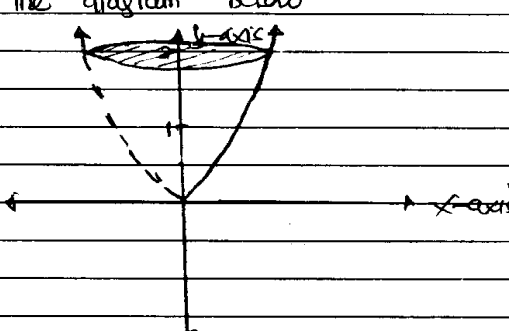


**Figure 6:** Candidates' performance on question 5

In part (a)(i), competent candidates correctly identified that the most appropriate method for evaluating the given integral was the substitution method, due to the composite structure of the function. These candidates let  $u = 4x + 5$  and then differentiated it to obtain  $\frac{du}{dx} = 4$  or  $dx = \frac{du}{4}$ . They then replaced  $4x + 5$  and  $dx$  in  $\int (4x + 5)^{10} dx$  with  $u$  and  $dx = \frac{du}{4}$  respectively, resulting in  $\int (u^{10}) \left( \frac{du}{4} \right)$ . They then applied the power rule for integration, that gave  $\frac{1}{4} \left( \frac{u^{11}}{11} \right) + C$ . Finally, they substituted  $u = 4x + 5$  and expressed the final answer as  $\frac{1}{44} (4x + 5)^{11} + C$ . Similarly, in part

(a)(ii), the candidates correctly applied the power rule for integration to each term of the quadratic expression  $x^2 - 2x + 1$ , obtaining  $\int_2^5 (x^2 - 2x + 1) dx = \left[ \frac{1}{3}x^3 - x^2 + x \right]_2^5$ . Thereafter, they evaluated the definite integral over the interval  $[2, 5]$  by calculating the value of the antiderivative at the upper limit  $x=5$  and the value at the lower limit  $x=2$ , and finally taking their difference, resulting in the correct value of the definite integral as 21.

In part (b), the candidates correctly identified that the most suitable method for finding the volume of the solid formed by rotation was the disc method, since the functions could be expressed in terms of  $y$ . They rewrote the given equation  $y = x^2$  as  $x = \sqrt{y}$ , to reflect the radius of rotation in terms of  $y$ . They also correctly determined that the region of interest lies between  $y=0$  and  $y=2$ , which became the lower and upper limits of integration respectively. Using the disc method formula for rotation about the  $y$ -axis,  $V = \int_a^b \pi x^2 dy$ , these candidates substituted  $x = \sqrt{y}$  into the formula, resulting in  $V = \int_0^2 \pi y dy$ , which was integrated correctly, giving  $2\pi$  cubic units.

5B	Soln
	Given; $y = x^2$
	$y - 2 = 0$
	$y = 2$
	Consider the diagram below
	
	Volume = $\int_a^b \pi x^2 dy$ . But $x^2 = y$
	Volume = $\int_0^2 \pi y dy$
	Volume = $\pi \left[ \frac{y^2}{2} \right]_0^2$
	Volume = $\frac{\pi}{2} [y^2]_0^2$
	Volume = $\frac{\pi}{2} [2^2 - 0^2]$
	Volume = $\frac{\pi}{2} [4 - 0]$
	Volume = $\frac{\pi}{2} \times 4$
	Volume = $2\pi$ cubic unit
	$\therefore$ The volume of the solid cylinder is $2\pi$ cubic unit

**Extract 5.1:** A sample of correct responses to question 5

In Extract 5.1, the candidate correctly interpreted the problem using a diagram and applied the disc method appropriately to solve it.

On the other hand, 44,961 (96.09%) candidates lost marks of this question, and among them, 12,680 (27.10%) candidates scored zero. The analysis of responses revealed that the candidates who performed poorly on this question had a number of misconceptions that affected the accuracy of their

solutions. In part (a)(i), a common mistake was the wrong application of the power rule for integration. For example, some candidates wrote

$$\int (4x+5)^{10} dx = \frac{(4x+5)^{10}}{40} + C, \text{ showing that they forgot to increase the}$$

power by one. Some candidates treated the term  $(4x+5)^{10}$  wrongly after making a change of variable, as Extract 5.2 illustrates. Although

$$\int (4x+5)^{10} dx \text{ is an indefinite integral, some candidates did not add the}$$

constant of integration  $C$ . In part (a)(ii), many candidates demonstrated a lack of understanding of integration rules. Although most recognized that each term of the quadratic expression had to be integrated separately, some incorrectly applied the power rule. Specifically, they either failed to increase the exponent by one or made errors when calculating the new

coefficients. For instance, instead of integrating  $x^2$  to get  $\frac{x^3}{3}$ , some wrote

$3x^2$ , indicating confusion between integration and differentiation rules.

Other candidates incorrectly substituted the limits of integration. Most of them swapped the upper and lower limits, resulting in a negative value of the correct answer. Some candidates also substituted the limits directly into the original function rather than the antiderivative. Furthermore, some candidates omitted or incorrectly used brackets when evaluating expressions, such as  $F(b) - F(a)$ , which resulted in sign errors and incorrect final answers. These procedural mistakes greatly affected the accuracy of their solutions. In addition, a few candidates mistakenly included a constant of integration ( $C$ ) in their final answer despite working with a definite integral, indicating confusion between definite and indefinite integration.

In part (b), a common error was the failure to express the integrand correctly in terms of the variable of rotation. Since the rotation was about the  $y$ -axis, candidates were expected to express  $x$  as a function of  $y$ , that is  $x = \sqrt{y}$ . However, several candidates proceeded with integration in terms of  $x$  by substituting  $y = x^2$ , which is appropriate only when the rotation is about the  $x$ -axis. Similarly, some candidates employed inappropriate or

incorrect formulae, commonly  $V = \int_a^b \frac{1}{3} \pi x^2 dx$  or  $V = \pi r^3$ , instead of the correct formula  $V = \int_a^b \pi x^2 dy$  for rotation about the  $y$ -axis. Another common error was the wrong application of the formula for finding volumes. Several candidates failed to correctly interpret the geometric boundaries of the region, which led to the use of incorrect limits or integrating beyond the region bounded by  $y = 2$ . For instance, some candidates incorrectly used limits corresponding to the  $x$ -axis ( $-2$  and  $2$ ) instead of the limits along the  $y$ -axis ( $0$  and  $2$ ). Additionally, some made arithmetic errors while evaluating definite integrals or neglected to include the factor  $\pi$  in their final answers. These errors highlight the candidates' lack of knowledge of the concepts about the volume of a solid of revolution. Extract 5.2 describes the sample of incorrect responses to this question.

5	$\int (4x+5)^{10} dx$
	let $4x+5$ be $u$
	$\int u^{10} dx$
	$= \frac{u^{11}}{11}$
	$u = 4x+5$
	$\frac{d}{dx} = \frac{(4x+5)^{11}}{11}$
	$\frac{d}{dx} = \frac{(4x+5)^{11}}{11}$

**Extract 5.2:** A sample of incorrect responses to question 5

In Extract 5.2, the candidate incorrectly applied the rule directly to the expression  $(4x+5)^{10}$  without considering the inner linear expression, leading to a mathematical error.

## 2.6 Question 6: Statistics

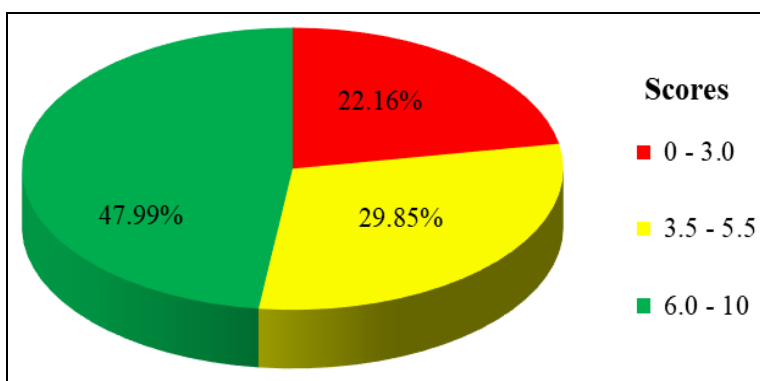
This question examined the candidates' ability to compute the range and standard deviation for an ungrouped data, calculate the mean using the assumed mean method, and represent statistical data using a cumulative frequency curve. The question had the following parts:

- (a) *The weights (in kg) of ten people who attended a meeting were as follows: 60, 72, 82, 66, 102, 123, 79, 88, 93 and 81.*
- (i) *Find the range of their weights.*
- (ii) *Compute the standard deviation of their weights.*
- (b) *The following table shows the weights (in kg) of patients admitted at Godegode Hospital in March 2024.*

<b>Weight (in kg)</b>	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
<b>Number of patients</b>	10	16	19	15	18	2

- (i) *Use the assumed mean,  $A = 55$  to calculate the mean weight of patients.*
- (ii) *Draw a cumulative frequency curve representing the information.*

A total of 36,420 (77.84%) candidates scored between 3.5 and 10 marks inclusive. Therefore, the overall performance in this question was good. Figure 7 presents the distribution of candidates who scored high, average, and low marks on this question.



**Figure 7:** Candidates' performance on question 6

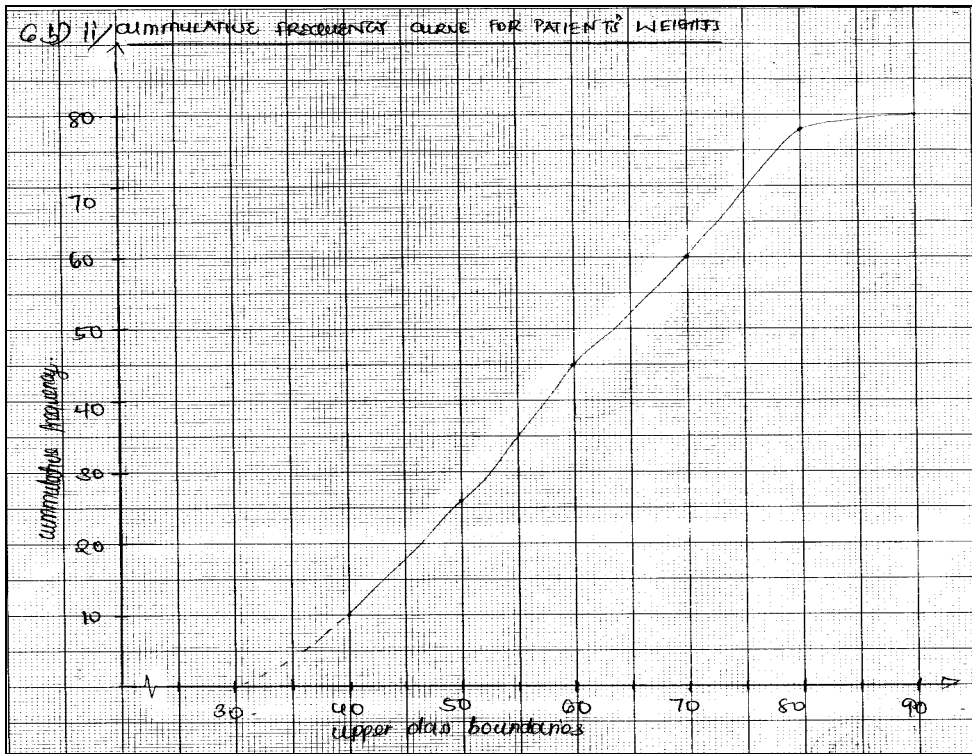
The analysis further revealed that 1,802 (3.85%) candidates scored full marks (10). In part (a)(i), the candidates correctly recalled that range is the difference between the maximum and minimum values in the data set. Accordingly, they identified 123 kg as the maximum weight and 60 kg as the minimum, and accurately computed the range to get 63 kg.

In part (a)(ii), the candidates followed the correct procedure for calculating the standard deviation of ungrouped data. They correctly determined the mean of the data set, then calculated the square of deviation of each weight from the mean, and summed the obtained squared deviations, getting 3020.36. To obtain the variance, they divided 3020.36 by the number of observations (10), resulting in 302.036. Finally, they computed the square root of the variance and correctly obtained the standard deviation of 17.379 kg.

In part (b)(i), the candidates correctly applied the formula for calculating the mean ( $\bar{x}$ ) using assumed mean method,  $\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$  where  $A$  is the assumed mean,  $d_i$  is the deviation ( $x_i - A$ ) of each data value from the assumed mean,  $f_i$  is the frequency of each class. They constructed a frequency distribution table with the necessary columns for  $x_i$ ,  $f_i$ ,  $d_i$  and  $f_i d_i$ , which enabled them to determine  $\sum f_i = 80$  and  $\sum f_i d_i = 210$ . They then substituted these values into the formula and correctly obtained the mean of  $\bar{x} = 57.625$  (see Extract 6.1).

In part (b)(ii), the candidates began by computing the cumulative frequencies corresponding to the upper class boundaries of each class interval. They then located the cumulative frequency points on the graph, placing the upper class boundaries on the  $x$ -axis and the corresponding cumulative frequencies on the  $y$ -axis. Using these points, the candidates correctly drew a cumulative frequency curve (ogive) by smoothly joining the points with a free hand curve. Extract 6.1 provides an overview of correct responses on this question.

6) Cumulative Frequency Distribution Table for the Data.							
weight class intervals	class mark $x$	frequency $f$	$d = x - \bar{x}$	$f d$	cumulative frequency	upper class boundaries	
30-40	35	10	-20	-200	10	40	
40-50	45	16	-10	-160	26	50	
50-60	55	19	0	0	45	60	
60-70	65	15	10	150	60	70	
70-80	75	18	20	360	78	80	
80-90	85	2	30	60	80	90	
	$\Sigma f = 80$			$\Sigma fd = 210$			
✓ Mean weight							
$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$							
$= 55 + \frac{210}{80}$							
$= 57.625 \text{ kg.}$							



**Extract 6.1:** A sample of correct responses to question 6

In Extract 6.1, the candidate presented a correct frequency distribution table with all the necessary columns, which facilitated the accurate calculation of the mean and the correct construction of the cumulative frequency curve.

Conversely, 10,371 (22.16%) candidates scored low marks due to a number of misconceptions. In part (a)(i), some candidates incorrectly assumed that the range is the average of the data values or the difference between the first and the last values in the list. Additionally, some candidates mistakenly added the maximum and minimum values, instead of subtracting them. This fundamental misunderstanding led to the application of incorrect operations and ultimately resulted in wrong answers. In other cases, candidates confused the range with the interquartile range, as they calculated the upper and lower quartiles and subtracted them. Another frequent error was the failure to correctly identify the maximum and minimum values in the data set. For example, in the given data set, the smallest number is 60 and the largest is 123. However, some candidates incorrectly identified 66 as the smallest and 93 as the largest number, thus could not get the correct answer, particularly 27. This mistake was largely

attributed to poor data organization or failing to arrange the data in ascending order before identifying the smallest and largest numbers.

In part (a)(ii), a frequent weakness was the confusion between standard deviation, variance, and mean deviation. Some candidates incorrectly calculated the mean of the data taking the absolute differences (applicable only to mean deviation). Other candidates computed the variance by finding the mean of the squared deviations but did not take the square root to obtain the intended standard deviation. Additionally, some candidates

applied incorrect formulas, such as  $\delta = \sqrt{\frac{\sum (x - \bar{x})}{N}}$ , resulting in wrong answers. Arithmetic errors were also common during the squaring and summation process. Some candidates incorrectly squared the deviations or added the squared values inaccurately, which significantly affected the final answer.

In part (b)(i), many candidates confused class boundaries, class marks, and class intervals. Some candidates used class limits instead of class marks when calculating the deviations, which led to inaccurate deviations, and consequently an incorrect mean value. Other weaknesses stemmed from poor organization of the frequency distribution table, resulting in confusion and omission of critical steps. Several candidates failed to present their work in a proper tabular form, omitting essential column(s) like frequency  $f_i$ , class marks  $x_i$ , deviations  $d_i$ , or the product of  $f_i$  and  $d_i$ . As a result, they skipped some values, which negatively affected their final answers. Furthermore, some candidates applied incorrect formulae, such as

$\bar{x} = A + \frac{\sum fx}{\sum f}$ , which led to a wrong answer of  $\bar{x} = 112.625$ .

In part (b)(ii), a common misconception arose from the incorrect use of class boundaries. Several candidates plotted cumulative frequencies against class marks or lower class boundaries instead of the upper boundaries. This error distorted the shape of the curve and led to inaccurate representation of the data distribution. Additionally, some candidates failed to compute cumulative frequencies correctly. They either added frequencies incorrectly, skipped some classes, or started from an incorrect baseline (such as a non-zero cumulative value), compromising the accuracy of the

ogive. Plotting errors were also frequent. Some candidates inaccurately positioned cumulative frequency points on the graph by placing them at incorrect coordinates or using unevenly scaled axes. This often occurred because candidates did not clearly label the axes or failed to use appropriate and consistent scales, resulting to misleading or unreadable curves. Moreover, some candidates joined the points with straight lines instead of drawing a smooth curve, contrary to the methods of constructing standard ogive. Furthermore, several candidates omitted the essential elements such as axis labels, titles, or keys, hence reducing the clarity and interpretability of their cumulative frequency curves. Extract 6.2 shows a sample of incorrect responses on this question.

6. b) <u>Soln</u>						
center interval	c. mark	f	$d = x - A$	$fd$	Cum. Frequency	
30 - 40	35	10	-20	-200	10	
40 - 50	45	16	-10	-160	26	
50 - 60	55	19	0	0	45	
60 - 70	65	15	10	<del>150</del> 150	60	
70 - 80	75	18	20	360	78	
80 - 90	85	2	30	60	80	
		$\Sigma f = 80$		$\Sigma fd = 210$		
D) Mean by assumed mean:						
$A = 55$						
$C = 10$						
$\Sigma fd = 210$						
$N = 80$						
From						
$\text{Mean } (\bar{x}) = A + \left( \frac{\Sigma fd}{N} \right) C$						
$= 55 + \left( \frac{210}{80} \right) 10$						
$= 55 + 26.25$						
$= 81.25$						
$\therefore$ Mean weight of the patient = 81.25						

**Extract 6.2:** A sample of incorrect responses to question 6

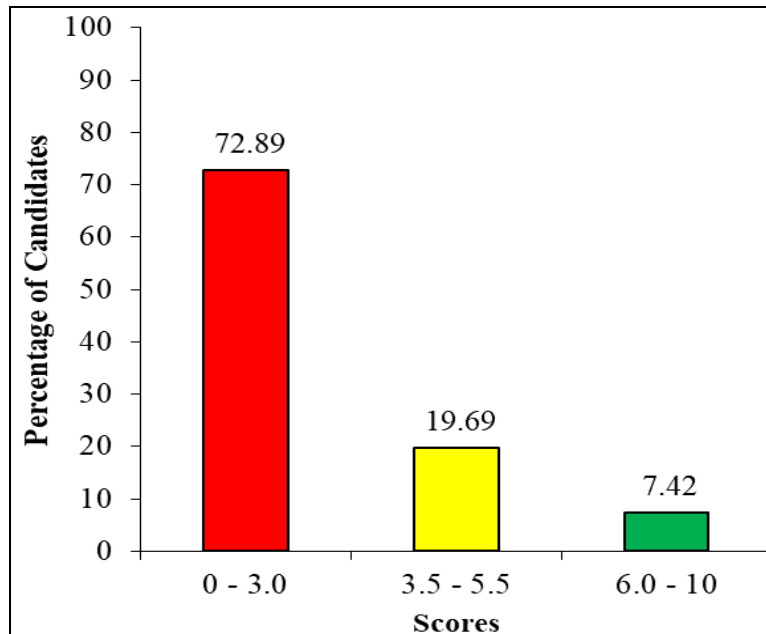
In Extract 6.2, the candidate applied an incorrect formula for computing the mean by assumed mean method, using  $\bar{x} = A + \left( \frac{\Sigma fd}{N} \right) C$ , instead of the applying correct formula  $\bar{x} = A + \frac{\Sigma fd}{N}$ .

## 2.7 Question 7: Probability

This question assessed the candidates' ability to determine the probability of independent events and apply the concept of permutations in real-life situations. The question stated as follows:

- (a) (i) Evaluate  ${}^{12}C_9 + {}^7P_3$  using factorial notation.
- (ii) A reception office has one bench with 5 seats. In how many ways should 11 people be seated on the bench?
- (b) A biased coin is such that the probability of obtaining a head when it is tossed is  $\frac{3}{7}$ . If the coin is tossed twice, determine the probability of obtaining:
- (i) 2 tails.
- (ii) at least one tail.

As illustrated in Figure 8, the majority of candidates (72.89%) scored low marks, indicating that the overall candidates' performance on this question was weak.



**Figure 8:** Candidates' performance on question 7

The analysis of data revealed that 46,048 (98.41%) candidates lost some marks, of whom 12,998 (27.78%) scored zero, due to various reasons. In part (a)(i), a frequently observed misconception was the use of inappropriate or incorrect formulae. For instance, while the correct formula for combinations is  ${}^nC_r = \frac{n!}{r!(n-r)!}$ , some candidates incorrectly used

${}^nC_r = \frac{n!}{(n-r)!}$ , which is the formula for permutations,  ${}^nP_r$ . These

candidates lacked the conceptual understanding of the difference between permutations and combinations. Another common error was the omission or incorrect use of brackets during simplification, which led to errors in the order of operations, particularly when using calculators. Additionally, some candidates mishandled factorial expressions, especially when dealing with larger numbers such as 12!, 9!, or 7!. They either simplified the factorials incorrectly or attempted to compute them manually without cancelling common factors, leading to unnecessary arithmetic complexity and further mistakes.

In part (a)(ii), a common misconception was interpreting the problem based on combinations instead of permutations. Some candidates incorrectly assumed that since only 5 people were to be seated from a group of 11, the task involved merely choosing 5 individuals, disregarding the significance of order. As a result, they used the formula for combinations, which counts selections without considering the order, instead of applying the appropriate formula for calculating permutations. Another frequent mistake involved ignoring the constraint of 5 seats and arranging all 11 individuals. Some candidates erroneously calculated 11!, assuming that all people could be seated, which led to overcounting, as only 5 out of the 11 were to be arranged. Furthermore, some candidates failed to distinguish the terms selection and arrangement. They first attempted to choose 5 people using combinations and then arrange all 11, failing to recognize that permutations fundamentally account for both selection and arrangement in a single step. Additionally, computation errors were commonly observed in calculations. For example, some candidates failed to properly simplify the denominator

or made mistakes in evaluating expressions like  $\frac{11!}{6!}$ , leading to incorrect final answers.

In part (b), many candidates incorrectly assumed that the coin is fair, even though the question explicitly stated that it was biased. As a result, they used  $P(H) = \frac{1}{2}$  and  $P(T) = \frac{1}{2}$  instead of the given biased probability

$P(H) = \frac{3}{7}$ , which implies  $P(T) = \frac{4}{7}$ . This incorrect assumption led to flawed computations and ultimately inaccurate results. Another common error involved confusing the probabilities of heads and tails. Some candidates mistakenly used  $P(T) = \frac{3}{7}$ , the given probability for heads, instead of subtracting it from 1 to obtain the correct probability of obtaining a tail. This fundamental misunderstanding produced incorrect values when determining the probability of outcomes such as obtaining two tails.

In part (b)(i), several candidates failed to recognize that the events resulted from tossing a coin twice are independent. As a result, they incorrectly added the individual probabilities instead of multiplying them. For example, some candidates wrote  $P(TT) = \frac{4}{7} + \frac{4}{7}$  giving  $P(TT) = \frac{8}{7}$ , which is incorrect, as probabilities cannot exceed 1. In part (b)(ii), many candidates misinterpreted the event “at least one tail” as “exactly one tail.” Consequently, they calculated only the probability of obtaining exactly one tail, instead of accounting for both one tail and two tails, leading to incorrect answers. Another common mistake was failing to consider all possible outcomes that yield exactly one tail. For instance, some candidates considered only the outcome “head then tail” and neglected “tail then head,” thereby undercounting the total probability. Additionally, some candidates attempted to use the complementary approach but did so incorrectly by multiplying the probabilities of a head and a tail, then subtracting that product from 1.

7. (a) (i)	Soln.
	${}^{12}C_9 + {}^7P_3.$
	For
	${}^nC_r = \frac{n!}{(n-r)!r!}, \quad {}^nP_r = \frac{n!}{(n-r)!r!}.$
	$\frac{12!}{(12-9)!} + \frac{7!}{(7-3)!3!}$

7. (a) (i)	$\frac{12!}{(12-9)!} + \frac{7!}{(7-3)!3!}$
	$\frac{12!}{3!} + \frac{7!}{4!3!}$
	$\frac{479001600}{6} + \frac{5040}{24 \times 6}$
	$\frac{479001600}{6} + \frac{5040}{144}$
	$= 79833635.$
	$\therefore$ The answer will be 79833635.

**Extract 7.2:** A sample of incorrect responses to question 7

In Extract 7.2, the candidate demonstrated confusion between the combination and permutation formulae by incorrectly applying them interchangeably.

Despite the overall weak performance, a total of 743 candidates, equivalent to 1.59 percent, correctly answered the question and scored full marks. In part (a)(i), the candidates correctly recalled the definitions of both combination  ${}^nC_r$  and permutation  ${}^nP_r$  using factorial notation, that is,

$${}^nC_r = \frac{n!}{r!(n-r)!} \quad \text{and} \quad {}^nP_r = \frac{n!}{(n-r)!}.$$

They also understood that

$n! = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$  and the fact that  $0! = 1$ . Thus, they correctly evaluated the value of  ${}^{12}C_9 + {}^7P_3$ , obtaining the correct answer of 430.

In part (a)(ii), the candidates recognized that the question required the use of permutations, as it involved selecting and arranging 5 people from a group of 11 where the order of seating matters. They evaluated  ${}^{11}P_5$ , either algebraically or using a calculator, and obtained the correct answer of 55,440. Alternatively, some candidates approached the problem by considering the number of ways to assign individuals to each of the five seats successively as  $11 \times 10 \times 9 \times 8 \times 7$ , also arriving at 55,440 ways.

In part (b), the candidates recognised that the given experiment involved complementary and independent events. They correctly used the given probability of obtaining a head,  $P(H) = \frac{3}{7}$ , to determine the probability of

obtaining a tail as  $P(T) = 1 - \frac{3}{7} = \frac{4}{7}$ . Using a tree diagram, the candidates systematically analysed all possible outcomes and their corresponding probabilities. For the event of obtaining two tails, they correctly applied the formula for independent events,  $P(E) = \frac{4}{7} \times \frac{4}{7}$ , resulting in  $P(E) = \frac{16}{49}$ .

Most of them also identified the event "at least one tail" as the complement of obtaining two heads, and accurately computed its probability as

$P(E) = 1 - \left(\frac{3}{7} \times \frac{3}{7}\right)$ , giving  $P(E) = \frac{40}{49}$ . There were few candidates who

used a different approach to get the correct answer, as illustrated in Extract 7.2.

7b)

i) 2 tails (TT)

$$= \frac{4}{7} \times \frac{4}{7}$$

$$= \frac{16}{49}$$

$\therefore$  The probability of getting two tails is  $\frac{16}{49}$ .

ii) at least one tail.

$$= P(HT) + P(TH) + P(TT)$$

$$= \left(\frac{3}{7} \times \frac{4}{7}\right) + \left(\frac{3}{7} \times \frac{4}{7}\right) + \left(\frac{4}{7} \times \frac{4}{7}\right)$$

$$= \frac{12}{49} + \frac{12}{49} + \frac{16}{49}$$

$$= \frac{40}{49}$$

$\therefore$  The probability of getting at least one tail is  $\frac{40}{49}$ .

**Extract 7.1:** A sample of correct responses to question 7

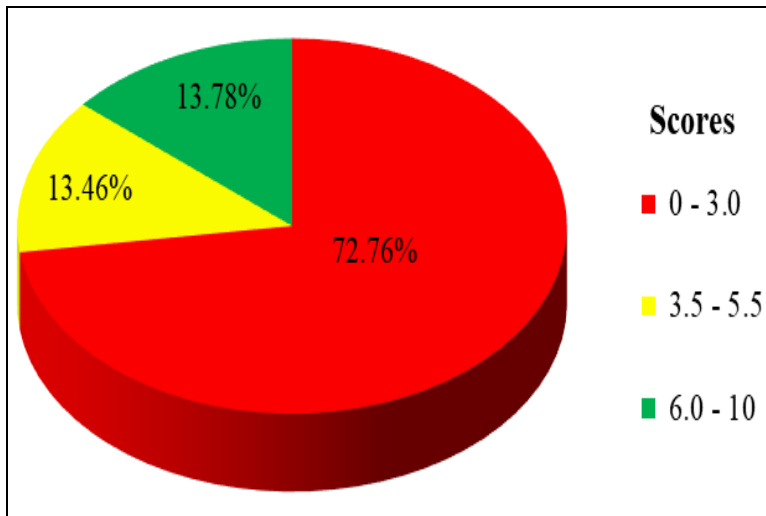
In Extract 7.2, the candidate correctly interpreted the probabilities using a tree diagram, applied the appropriate formulae to accurately evaluate the probability of obtaining at least one tail.

## 2.8 Question 8: Trigonometry

This question intended to examine the candidates' competence in applying special angles, using double angle identities for sine and cosine, and solving problems involving substitution and integration of trigonometric functions. The question consisted of the following parts:

- (a) (i) Find the value of  $\sin 45^\circ \cos 60^\circ + \tan 45^\circ \cos 45^\circ$  and express the answer in surd form.
- (ii) Show that  $\frac{2\cos^2 A - 1}{\sin A \cos A} = 2 \cot 2A$ .
- (b) (i) Solve  $\int \sec^2(2x+10)dx$ .
- (ii) Given  $y = \sin(x^2 - 4)$ , find  $\frac{dy}{dx}$ .

The candidates' performance on this question was generally weak, with only 12,747 (27.24%) candidates scoring between 3.5 and 10 marks inclusive. Figure 9 presents the distribution of candidates according to low, average and high score categories.



**Figure 9:** Candidates' performance on question 8

The analysis further revealed that 45,985 (98.28%) candidates lost some marks on this question. Among them, 20,285 (43.35%) candidates scored zero.

In part (a)(i), most candidates did not recall correct trigonometric ratios of special angles. Many candidates confused the standard values of sine, cosine, and tangent at  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ . For example, some incorrectly wrote  $\sin 45^\circ = \frac{1}{2}$ ,  $\cos 45^\circ = \frac{\sqrt{3}}{3}$  or  $\tan 45^\circ = \frac{\sqrt{3}}{2}$ . This mistakes led

them to make substitutions of wrong values, consequently getting wrong answers. Another frequent error was the wrong application of trigonometric formulae. Instead of directly substituting the known values, some candidates attempted to manipulate the expression using double-angle formula unnecessarily. Additionally, improper use of brackets during multiplication, especially with radicals or decimals, resulted in mistakes due to incorrect order of operations. Some candidates also did not adhere to the instruction to express the answer in surd form. They converted trigonometric values into decimals, which led to approximates rather than exact answers, contrary to the requirements of the question. Furthermore, arithmetic errors were common when simplifying expressions involving fractions or radicals. These included inaccurate multiplication or simplification, which affected the numerical accuracy of the answers.

In part(a)(ii), most candidates applied incorrect trigonometric identities. Many candidates failed to recognize that is equivalent to  $\cos 2A$ , which is a standard double-angle identity. Instead of applying this identity directly, several candidates attempted to simplify each term of the expression separately, resulting in unnecessary complexity and ultimately incorrect answers. Additionally, many candidates did not realize that  $\sin A \cos A$  is equal to  $\frac{1}{2} \sin 2A$ , hence missing a key simplification that could have made

the problem more manageable. Some candidates also confused the term  $\cot 2A$  with other terms such as  $\cot A$ , leading to incorrect substitutions and simplifications. Another common error was treating the identity as an algebraic equation to be solved rather than a proof. Instead of transforming the left-hand side to match the right-hand side, some candidates manipulated both sides simultaneously, sometimes even "cross-multiplying", which is inappropriate in identity verification tasks. Additionally, some candidates prematurely or improperly cancelled the terms. For instance, they cancelled  $\cos A$  in the numerator and denominator of the expressions  $\frac{2\cos^2 A - 1}{\sin A \cos A}$  without acknowledging the structural role of these terms within the identity.

In part (b)(i), most candidates demonstrated a partial understanding of the integral of  $\sec^2(2x+10)$ , but struggled to correctly account for the linear

inner function. For example, some candidates evaluated  $\int \sec^2(2x+10)dx$  directly as  $\tan(2x+10)+C$ . These candidates failed to make the substitution  $u = 2x+10$  and  $du = 2dx$ , which could introduce a coefficient of  $\frac{1}{2}$  in  $\tan(2x+10)+C$ . Some candidates confused the integral of  $\int \sec^2(2x+10)dx$  with unrelated standard integrals such as  $\int \sec(2x+10)dx$  or  $\int \tan(2x+10)dx$ , resulting in incorrect expressions involving logarithms or combination of trigonometric functions. Others unnecessarily applied integration by parts, which indicated a lack of knowledge on integration techniques. Additionally, several candidates omitted the constant of integration  $C$  in their final answers. This oversight suggests a gap in understanding the basic structure of indefinite integrals.

In part (b)(ii), many candidates experienced difficulties in correctly applying the chain rule when differentiating composite functions. A common misconception was differentiating the outer sine function without accounting for derivative of the inner quadratic function. For instance, some candidates gave the derivative of  $\cos(x^2-4)$ , omitting the multiplication by the derivative of the inner quadratic function,  $2x$ . There were also cases where candidates recognized the need for the chain rule but incorrectly differentiated the inner function. For example, some wrote the derivative of  $x^2-4$  as  $x$  instead of  $2x$ . Others applied the chain rule in the wrong order or confused it with other differentiation techniques, resulting in structurally incorrect expressions. Additionally, some candidates mistakenly differentiated  $\sin(x^2-4)$  as  $2x \cos x$ , erroneously treating it as a power function rather than a composition of functions. There were candidates who applied the product rule unnecessarily, interpreting the expression as a product of  $\sin x$  and  $x^2-4$ . Extract 8.1 provides a highlight of incorrect responses on this question.

8. b/.	i.	<u>soln.</u>
		$= \int \sec^2(2x+10) dx$
		$= \tan \int (2x+10) dx$
		$= \tan \left( \frac{2x^2}{2} + 10x \right)$
		$= \tan(x^2 + 10x)$
		$\therefore \int \sec^2(2x+10) dx = \tan(x^2 + 10x)$
	ii/	<u>soln.</u>
		$y = \sin(x^2 - 4)$
		$\frac{dy}{dx} = \cos(2x)$
		$\therefore \frac{dy}{dx} = \cos(2x)$

**Extract 8.1:** A sample of incorrect responses to question 8

In Extract 8.1, the candidate attempted to solve the question by separately performing the integration and differentiation of the inner and outer functions. However, instead of applying the appropriate substitution, the candidate inserted the obtained results into the original expression, leading to a conceptually incorrect approach.

In contrast, 806 (1.72%) candidates correctly answered this question, scoring full marks. For instance, in part (a)(i), the candidates demonstrated

an understanding of the special angles. They correctly recalled the exact values of sine, cosine, and tangent for special angles, that is  $\sin 45^\circ = \frac{\sqrt{2}}{2}$ ,  $\cos 45^\circ = \frac{\sqrt{2}}{2}$ ,  $\cos 60^\circ = \frac{1}{2}$  and  $\tan 45^\circ = 1$ . They substituted these values accurately into the given expression and simplified it to obtain the required answer,  $\frac{3}{4}\sqrt{2}$ .

In part (a)(ii), the candidates correctly recalled the double-angle trigonometric identities, such as  $\cos 2A = 2\cos^2 A - 1$ ,  $\sin 2A = 2\sin A \cos A$  and  $\cot 2A = \frac{\cos 2A}{\sin 2A}$ . They substituted these identities appropriately on either the left-hand side or right-hand side of the expression and simplified correctly to verify the identity  $\frac{2\cos^2 A - 1}{\sin A \cos A} = 2\cot 2A$ .

In part (b)(i), the candidates recognized that the integral involved a composite function and appropriately applied the substitution method. They changed the variable by letting  $u = 2x + 10$ , which implied that  $\frac{du}{dx} = 2$  or  $dx = \frac{du}{2}$ . Substituting into the integral, they obtained  $\frac{1}{2} \int \sec^2 u du$ , and consequently  $\frac{1}{2} \tan u + C$ . Finally, they substituted back  $u = 2x + 10$ , obtaining  $\frac{1}{2} \tan(2x + 10) + C$ .

Also, in part (b) (ii), the candidates correctly identified that the function required the application of the chain rule, as it involved composite functions. Extract 8.2 illustrates a sample of correct responses on this question.

	Q10	solo
		$y = \sin(x^2 - 4)$
		let
		$u = x^2 - 4$
		$\frac{du}{dx} = 2x$
		Then
		$y = \sin u$
		$\frac{dy}{du} = \cos u$
		Now
		$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
		$\frac{dy}{dx} = (\cos u) \times (2x)$
		$\frac{dy}{dx} = 2x \cdot \cos u$
		$\frac{dy}{dx} = 2x \cdot \cos(x^2 - 4)$
		$\therefore \frac{dy}{dx} = 2x \cdot \cos(x^2 - 4)$

**Extract 8.2:** A sample of correct response to question 8

In Extract 8.2, the candidate correctly identified the function as a composition of an outer function  $\sin u$  and an inner function  $u = x^2 - 4$  and correctly applying the chain rule.

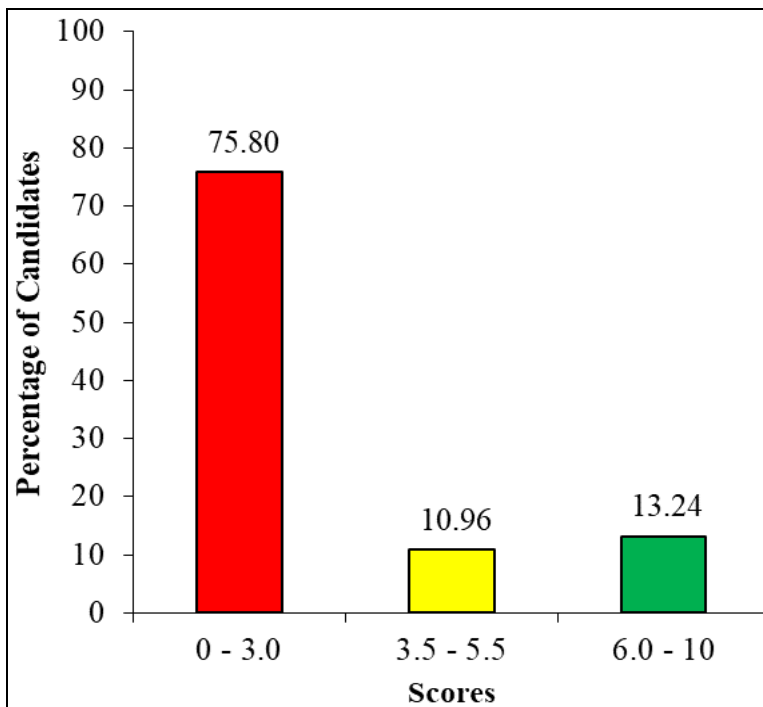
## 2.9 Question 9: Exponential and Logarithmic Functions

This question assessed candidates' ability to apply the knowledge of logarithmic and exponential functions in solving real-life problems. It consisted of the following parts:

- (a) (i) Find the first derivative of the function  $f(x) = \ln(2x + 1)$ .
- (ii) Find the solution of  $\int e^{9x+2} dx$ .

- (b) Sketch the graphs of both  $f(x) = 2^x$  and  $g(x) = \log(3+x)$  on the same  $xy$ -plane.
- (c) The analysis of the 2022 demographic census in Tanzania revealed that the population doubles after every 50 years. In how many years does the population be triple?

In this question, 11,322 (24.20%) candidates scored 3.0 marks or more, indicating that the overall candidates' performance was weak. The distribution of candidates who scored low, average, or high marks is illustrated in Figure 9.



**Figure 10:** Candidates' performance on question 9

The data further reveals that 45,874 (98.04%) candidates did not score full marks on this question, with 22,495 (48.08%) candidates scoring zero. In part (a)(i), most candidates exhibited misconceptions due to partial understanding of differentiation rules. A common error involved treating the derivative of the logarithmic function  $\ln(2x+1)$  as simply

$\frac{d}{dx}(2x+1)\times\ln(2x+1)$ , instead of the correct procedure,  $\frac{\frac{d}{dx}(2x+1)}{2x+1}$ .

Others correctly identified that the derivative of  $\ln x$  is  $\frac{1}{x}$ , but failed to apply the chain rule by neglecting to multiply by the derivative of the linear function. As a result, these candidates produced wrong answers such as  $f'(x) = \frac{1}{2x+1}$  instead of the correct one,  $f'(x) = \frac{2}{2x+1}$ . Some candidates mistakenly applied the product or quotient rule, treating the logarithmic function  $\ln(2x+1)$  as a product of two separate functions, namely  $u = \ln x$  and  $v = 2x+1$ .

In part (a)(ii), many candidates showed a lack of understanding of integration techniques and the relationship between differentiation and integration. A common mistake was writing the integral of  $e^{ax+b}$  as simply  $e^{ax+b}$ , omitting division by the coefficient of  $x$ , giving the incorrect answer  $e^{9x+2} + C$ . Some candidates incorrectly obtained  $\int e^{9x+2} dx = \frac{1}{9(e^{9x+2})} + c$ , as illustrated in Extract 9.1. Others made errors in substitution, either integrating term by term or treating the linear expression  $9x+2$  in the exponent as a single unit, resulting in invalid manipulation and incorrect answers.

In part (b), most candidates demonstrated limited understanding of the general shape and properties of the exponential and logarithmic graphs. For the exponential function, while the curve should increase rapidly as  $x$  increases and approach the  $x$ -axis asymptotically as  $x$  decreases, some candidates incorrectly sketched it as a straight line or parabola, indicating confusion with linear and quadratic functions. Additionally, several candidates misunderstood the concept of a horizontal asymptote. Instead of recognizing that exponential curves approach but never touch the  $x$ -axis, some candidates drew graphs that crossed the axis or wrongly identified the asymptote as a vertical line. Furthermore, other candidates failed to locate key points, such as  $(0, 1)$ , in which all exponential functions of the form  $f(x) = a^x$  pass through. This mistake highlights a lack of knowledge on

exponential functions. Similar misconceptions were observed in the sketching logarithmic graphs. Some candidates confused the location and nature of the vertical asymptote, incorrectly placing it along the y-axis, despite the function being undefined for  $x \leq -3$ . Others omitted the asymptote and extended the graph into the region where  $x < -3$ , which is mathematically invalid. Some candidates incorrectly reflected or shifted the graph, placing it in the region  $x > 3$  rather than  $x > -3$ . Some candidates could not distinguish between logarithmic and exponential functions.

In part (c), majority of candidates demonstrated a misconception by interpreting population growth as linear instead of exponential model. They assumed that the population increases by a fixed amount every 50 years. Consequently, they incorrectly recognized that the time it takes for a population to triple is simply 1.5 times the doubling time, thus obtained 75 years. This approach is incorrect, as population growth behaves exponentially. Although some candidates recognized that logarithms were needed to solve for the time, many were unable to apply logarithmic laws correctly. Instead of isolating the exponent using valid algebraic steps, they used flawed methods such as dividing by the base or taking square roots, which are not appropriate in exponential equations. These errors reflected limited understanding of both exponential growth and the use of logarithms in solving such real-life problems. Extract 9.1 shows a sample of incorrect responses on this question.

	ii)
	$\int e^{9x+2} dx.$
	Let $9x+2 = u$
	$\frac{du}{dx} = 9$
	$dx = \frac{du}{9}$

9	a) us
	$\int e^u \cdot \frac{du}{9}$
	$\frac{1}{9} \int e^u du$
	$\frac{1}{9} (e^u)^{-1}$
	$\therefore \int e^{9x+2} dx = \frac{1}{9(e^{9x+2})}$

**Extract 9.1:** A sample of incorrect responses to question 9

In Extract 9.1, the candidate used the incorrect formula

$$\int e^{ax+b} dx = \frac{1}{a(e^{ax+b})} + C \text{ instead of } \int e^{ax+b} dx = \frac{1}{a}(e^{ax+b}) + C.$$

Conversely, 917 (1.96%) candidates answered this question correctly. In part (a)(i), the candidates demonstrated a good understanding of the differentiation of a natural logarithmic functions. They correctly applied the

rule that the derivative of  $\ln(g(x))$  is  $\frac{1}{g(x)} \times g'(x)$ . Given

$f(x) = \ln(2x+1)$ , they correctly identified that  $g(x) = 2x+1$  and

$g'(x) = 2$ . Substituting into the rule, they obtained  $f'(x) = \frac{2}{2x+1}$ .

Alternatively, some candidates successfully applied the chain rule to reach the same result. In part (a) (ii), most candidates applied substitution method

by letting  $u = 9x+2$ , which gave  $\frac{du}{dx} = 9$  such that  $dx = \frac{du}{9}$ . Substituting

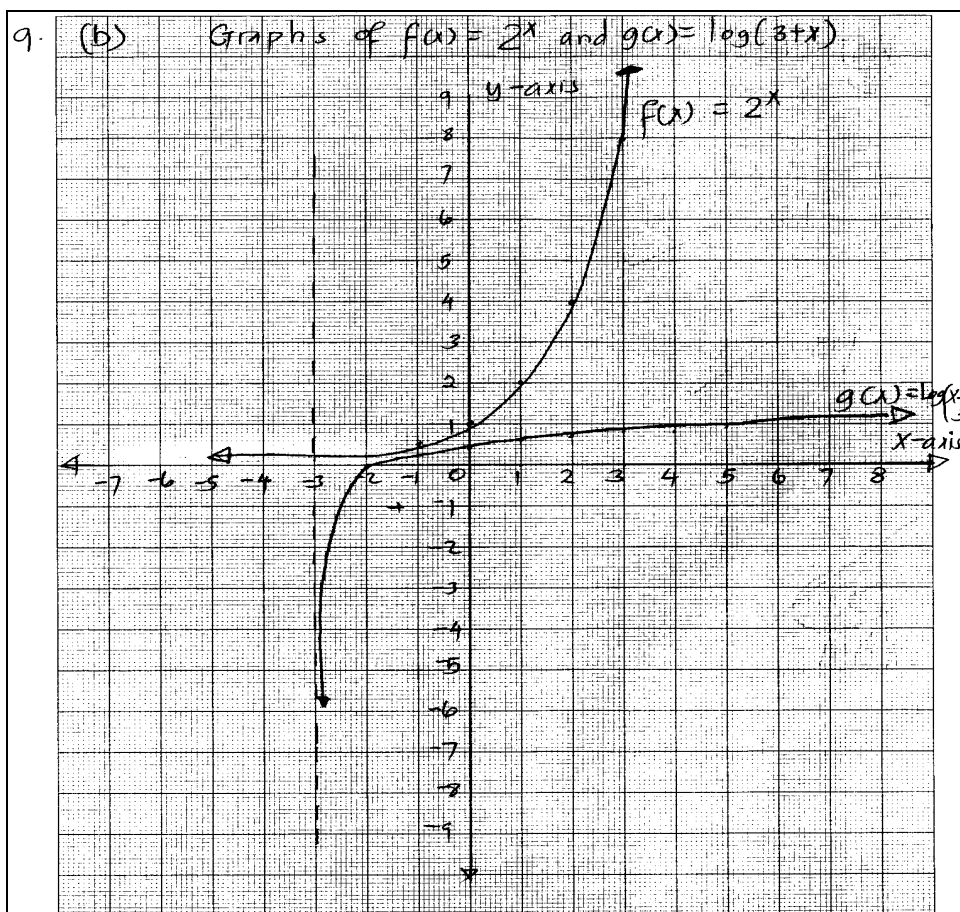
into the integral, they obtained  $\int (e^u) \left( \frac{du}{9} \right)$ , equivalent to  $\frac{1}{9} e^u + C$ . Upon

substituting  $u = 9x+2$  in  $\frac{1}{9} e^u + c$ , they arrived at the correct answer,

$\frac{1}{9}e^{9x+2} + c$ . Other candidates used the standard exponential integral formula,  $\int (e^{g(x)}) dx = \frac{1}{g'(x)} \times e^{g(x)} + C$  to get the correct answer.

In part (b), the candidates demonstrated a good understanding of the general shapes and key properties of logarithmic and exponential functions by accurately sketching both graphs. They correctly identified that the graph of  $y = 2^x$  passes through the point  $(0, 1)$ , increases rapidly as  $x$  increases, and approaches the  $x$ -axis without touching it, indicating that  $y = 0$  is a horizontal asymptote. Similarly, they recognized that the graph of  $y = \log(3+x)$  passes through the point  $(-2, 0)$ , is only defined for  $x > -3$ , and has a vertical asymptote at  $x = -3$ , and produced correct graphs.

In part (c), the candidates correctly interpreted the word problem as an exponential growth and applied the general formula,  $P(t) = P_0 e^{kt}$ , where  $P_0$  is the initial population,  $k$  is the growth constant, and  $t$  is the time in years. Given that the population doubles in 50 years, they understood that when time,  $t$  is 50 years, the population,  $P(50)$ , becomes  $2P_0$ . Therefore, they substituted  $t = 50$  into the formula and obtained the equation  $2P_0 = P_0 e^{50k}$  and solved it to get  $k = \frac{\ln 2}{50}$ . Substituting this value into the original formula, it gave the general equation  $P(t) = P_0 e^{\frac{\ln 2}{50}t}$ . They also understood that tripling the population means  $P(t)$  becomes  $3P_0$ , and therefore, the equation  $3P_0 = P_0 e^{\frac{\ln 2}{50}t}$  was obtained and correctly solved to get  $t = 79.248$ . This indicated that the population triples after approximately 79.2 years. Extract 9.2 provides a sample of correct responses on this question.



**Extract 9.2:** A sample of correct responses to question 9

In Extract 9.2, the candidate correctly drew the graphs of both exponential and logarithmic functions.

## 2.10 Question 10: Matrices and Linear Programming

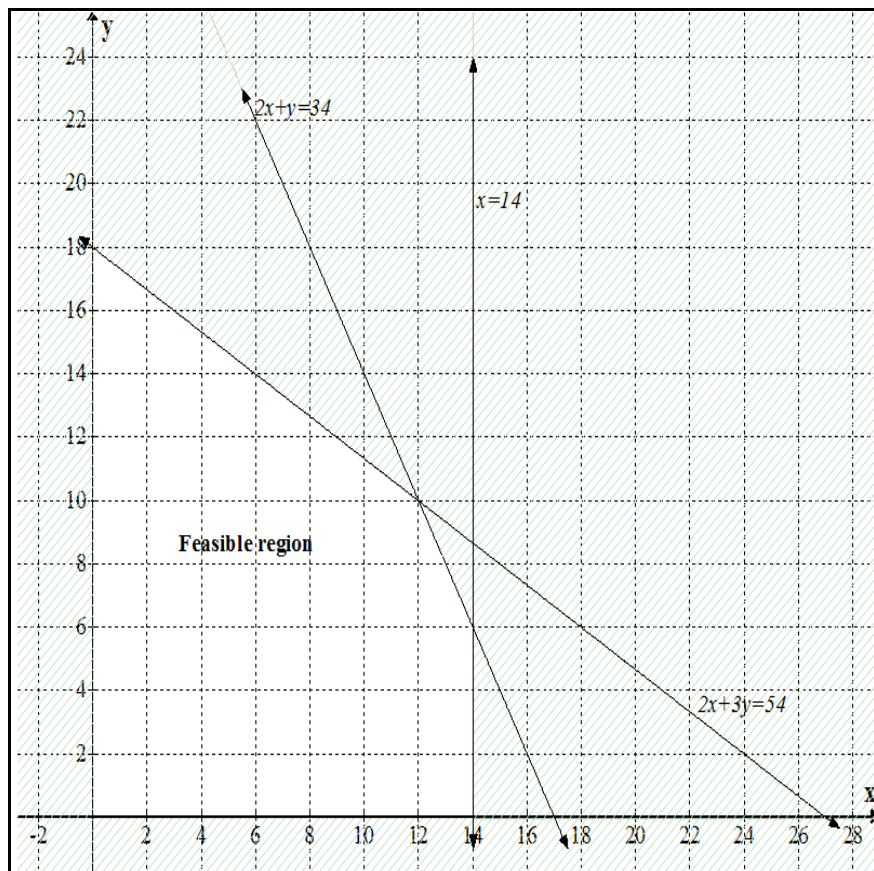
The question examined the candidates' ability to perform basic operations on matrices and to formulate constraints from a graph representing a linear programming problem. The question stated as follows:

- (a) *A company produces three types of products, namely P, Q and R. Each product is inspected by three different experts before being sold. The following table shows the time (in minutes) spent by each expert to inspect a product.*

	<i>P</i>	<i>Q</i>	<i>R</i>	<i>Total time available per day</i>
<i>1<sup>st</sup> expert</i>	<i>10</i>	<i>8</i>	<i>12</i>	<i>184</i>
<i>2<sup>nd</sup> expert</i>	<i>7</i>	<i>9</i>	<i>14</i>	<i>193</i>
<i>3<sup>rd</sup> expert</i>	<i>12</i>	<i>14</i>	<i>16</i>	<i>270</i>

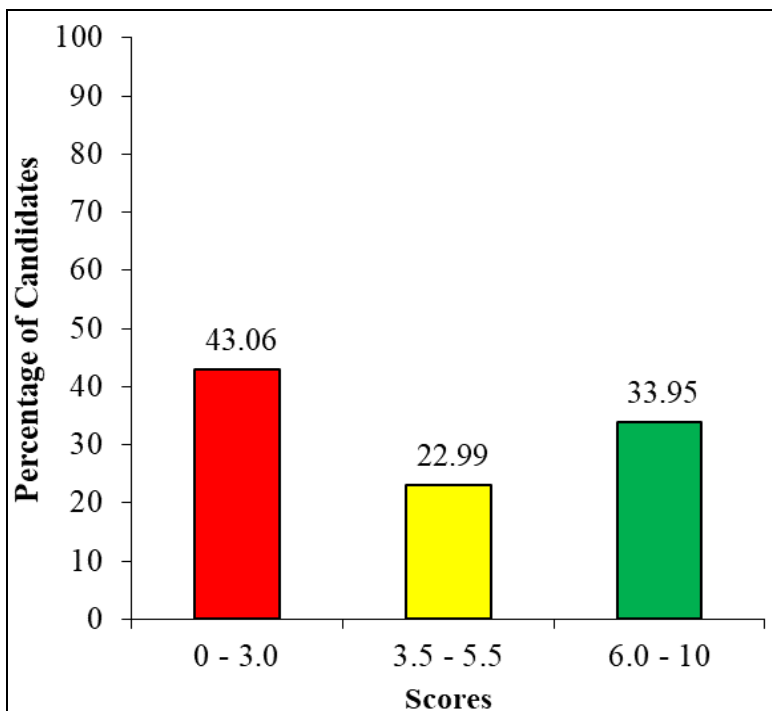
Use Cramer's rule to find the number of product *P* inspected per day.

- (b) The following graph illustrates a certain problem related to linear programming.



Using the information given in the graph, formulate the constraints of the problem.

The overall performance of candidates on this question was average. A total of 10,757 (22.99%) candidates scored between 3.5 and 5.5 marks inclusive, while 15,886 (33.95%) candidates scored between 6.0 and 10 marks. The distribution of candidates who attained low, average, and high scores is presented in Figure 11.



**Figure 11:** Candidates' performance on question 10.

The data also reveals that 1,329 candidates, equivalent to 2.84 percent, correctly answered all parts of this question. In part (a), the candidates began by translating the information from the table into a system of three linear equations, where  $x$ ,  $y$ , and  $z$  represent the number of products P, Q, and R inspected per day, respectively. Based on inspection times, they formulated the equations  $10x + 8y + 12z = 184$ ,  $7x + 9y + 14z = 193$  and

$$12x + 14y + 16z = 270, \quad \text{and} \quad \text{consequently} \quad \begin{pmatrix} 10 & 8 & 12 \\ 7 & 9 & 14 \\ 12 & 14 & 16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 184 \\ 193 \\ 270 \end{pmatrix}.$$

Taking  $A = \begin{pmatrix} 10 & 8 & 12 \\ 7 & 9 & 14 \\ 12 & 14 & 16 \end{pmatrix}$  and the matrix  $A_x$  being formed by replacing the

first column of matrix  $A$  with the constants from the right-hand side of the

equations, that is,  $A_x = \begin{pmatrix} 184 & 8 & 12 \\ 193 & 9 & 14 \\ 270 & 14 & 16 \end{pmatrix}$ . To find the value of  $x$  (product P)

using Cramer's rule, the candidates correctly evaluated and divided the determinant of matrix  $A_x$  by the determinant matrix  $A$ , obtaining  $x = 4$ .

In part (b), the candidates correctly interpreted the graph and identified the feasible region. They correctly studied the decision variables  $x$  and  $y$  and carefully observed that the boundary lines defining the feasible region were solid lines. This indicated that the corresponding inequalities were inclusive (i.e., involved  $\leq$  or  $\geq$ ). For each line, they determined the correct inequality sign of each equation by testing a point within the feasible region, such as  $(2, 2)$ , where if the point satisfied the equation, they retained the inequality direction; otherwise, they reversed it. Through this method, they correctly formulated the constraints;  $2x + y \leq 34$ ,  $2x + 3y \leq 54$  and  $x \leq 14$  based on the boundary lines  $2x + y = 34$ ,  $2x + 3y = 54$  and  $x = 14$ , respectively. In addition, they accurately included the non-negativity constraints,  $x \geq 0$  and  $y \geq 0$ , recognizing that the feasible region was confined to the first quadrant. Extract 10.1 shows a sample of correct responses on this question.

10	a) Given.																				
	<table border="1"> <thead> <tr> <th></th> <th>P</th> <th>Q</th> <th>R</th> <th>Total time</th> </tr> </thead> <tbody> <tr> <td>1<sup>st</sup> expert</td> <td>10</td> <td>8</td> <td>12</td> <td>184</td> </tr> <tr> <td>2<sup>nd</sup> expert</td> <td>7</td> <td>9</td> <td>14</td> <td>193</td> </tr> <tr> <td>3<sup>rd</sup> expert</td> <td>12</td> <td>14</td> <td>16</td> <td>270</td> </tr> </tbody> </table>		P	Q	R	Total time	1 <sup>st</sup> expert	10	8	12	184	2 <sup>nd</sup> expert	7	9	14	193	3 <sup>rd</sup> expert	12	14	16	270
	P	Q	R	Total time																	
1 <sup>st</sup> expert	10	8	12	184																	
2 <sup>nd</sup> expert	7	9	14	193																	
3 <sup>rd</sup> expert	12	14	16	270																	
	Required number of P using cramer's rule																				
	$10p + 8q + 12r = 184$ $7p + 9q + 14r = 193$ $12p + 14q + 16r = 270$																				
	In Matrix form																				
	$\text{Let } A = \begin{pmatrix} 10 & 8 & 12 \\ 7 & 9 & 14 \\ 12 & 14 & 16 \end{pmatrix} \begin{pmatrix} P \\ Q \\ R \end{pmatrix} = \begin{pmatrix} 184 \\ 193 \\ 270 \end{pmatrix}$																				
	Determinant of matrix A-																				
	$ A  = \begin{vmatrix} 10 & 8 & 12 \\ 7 & 9 & 14 \\ 12 & 14 & 16 \end{vmatrix}$																				
	$ A  = 10 \begin{vmatrix} 9 & 14 \\ 14 & 16 \end{vmatrix} - 8 \begin{vmatrix} 7 & 14 \\ 12 & 16 \end{vmatrix} + 12 \begin{vmatrix} 7 & 9 \\ 12 & 14 \end{vmatrix}$																				
	$= 10(9 \times 16 - 14 \times 14) - 8(7 \times 16 - 14 \times 12) + 12(7 \times 14 - 9 \times 12)$																				
	$ A  = (10 \times -52) + (-8 \times -56) + (12 \times -10)$																				
	$= -520 + 448 - 120$																				
	$= -192$																				
	$ A  = -192$																				
10	a) $= (-272 \times 10) + 3408 + (-1840)$																				
	$= -1152$																				
	For value of P = $\frac{ P }{ A }$																				
	$= \frac{-768}{-192}$																				
	$= 4$																				
	Number of product P per day = 4																				

Extract 10.1: A sample of correct responses to question 10

In Extract 10.1, the candidate correctly applied cofactor expansion to evaluate the values of determinants, and accurately determined the number of product P inspected per day,  $x = 4$ .

However, the data showed that 20,148 (43.06%) candidates lost marks on this question, with 10,451 (22.34%) scoring zero. In part (a), a common challenge was inability to translate the word problem into a matrix equation. Many candidates failed to formulate the required linear equations, hence they could not form the matrix equation. For instance, some candidates mismatched matrix dimensions, leading them to compute the time required to inspect one unit of product P rather than the number of units inspected per day. Another frequent error was inability to perform matrix multiplication. Some candidates attempted to multiply matrices with incompatible dimensions, such as multiplying a  $3 \times 1$  matrix by another  $3 \times 3$  matrix, indicating a limited understanding of matrix operations. When the dimensions were correctly set, several candidates made arithmetic errors during the multiplication process, such as summing products incorrectly within the rows and columns. Further analysis showed that a significant number of candidates failed to compute the determinant of a  $3 \times 3$  matrix. Others attempted to find the inverse of matrices, despite this not being required by the question. Additionally, some candidates focused excessively on numerical calculations without interpreting their final answers in context.

In part (b), most candidates failed to determine the correct inequalities for the constraints. Although the boundary equations were provided, some candidates wrote  $2x + y \geq 34$  instead of  $2x + y \leq 34$ , or  $2x + 3y \geq 54$  instead of  $2x + 3y \leq 54$ . They could not check whether these inequalities satisfy the feasible region. Another common error was writing only the boundary equations, such as  $2x + y = 34$ ,  $2x + 3y = 54$  and  $x = 14$ . These candidates failed to recognize that in linear programming, constraints describe regions, not just lines. It was also noted that most candidates omitted the non-negativity constraints,  $x \geq 0$  and  $y \geq 0$ , which are essential because the feasible region lies in the first quadrant. Furthermore, some candidates failed to identify the corresponding equation for the particular boundary line. Without verifying the intercepts or testing some points along the lines, they misidentified equations, leading to mismatched or incorrect

constraints. Extract 10.2 represents a sample of incorrect responses on this question.

10	b	By using the graph above the following are the formulation of the constraints to the problem.
		constraints. is
		$2x + y \geq 34$
		$x \geq 14$
		$2x + 3y \geq 54$ .
		$x \geq 0$
		$y \geq 0$ .

**Extract 10.2:** A sample of incorrect responses to question 10

In Extract 10.2, the candidate formulated the incorrect constraints due to failure to identify the appropriate inequality signs.

### 3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Basic Applied Mathematics paper covered 11 topics, with candidates' performance showing distinct variations. Topics demonstrating good performance included *Statistics* (77.84%), *Functions* (70.53%), and *Differentiation* (67.80%). These results indicate candidates generally mastered core concepts in data interpretation and statistical measures, sketching and interpreting graphs of functions, and applying both first principles and the product rule of differentiation, and knowledge of derivatives to determine gradients of curves.

An average performance was observed in *Algebra* (58.64%), *Matrices and Linear Programming* (56.94%), and *Integration* (41.73%). While showing reasonable competence in algebraic manipulation, candidates displayed moderate understanding of matrix operations and interpretation of graphs related to linear programming problem, as well as integration techniques.

The examination identified four (04) topics with pass rates below 35 percent: *Trigonometry* (27.24%), *Probability* (27.11%), *Exponential and Logarithmic Functions* (24.20%), and *Calculating Devices* (20.39%). These results highlight particular difficulties with trigonometric concepts, insufficient grasp of concepts related to probability, and the application of exponential and logarithmic functions. The notably low performance in

*Calculating Devices* suggests many candidates struggled with using computational tools. Appendix I details the candidates' performance in each topic.

## **4.0 CONCLUSION AND RECOMMENDATION**

### **4.1 Conclusion**

In the 2025, a total of 46,790 candidates sat for the Basic Applied Mathematics examination, with 72.66 percent achieving a passing marks. This represents a 4.89 percent decline from the 77.55 percent pass rate recorded in 2024. Appendix II illustrates the trend of candidates' performance in each topic for the years 2024 and 2025.

### **4.2 Recommendation**

To improve student's performance in Basic Applied Mathematics examinations, teachers are recommended to take the following initiatives:

- (a) Adopt teaching methods that promote deep conceptual understanding rather than rote memorization. These approaches should focus on developing students' logical reasoning abilities while strengthening their proficiency in using proper mathematical notation and procedures.
- (b) Explain mathematical concepts through multiple modalities: verbal explanations, visual diagrams, real-world examples, and technological tools that enable students to verify concepts through visualization rather than abstract reasoning alone. Additionally, educators should incorporate error-analysis exercises where students identify and explain mistakes in sample solutions.
- (c) Provide students with regular, targeted practice designed to build long-lasting understanding and retention. This includes: structured problem-solving sessions, application-based learning activities, opportunities for conceptual reinforcement and progressive skill development exercises.

To improve student's performance in the identified weak topics, the following strategies should be implemented:

- (a) Effective use of calculators; teachers should establish a structured approach to ensure students correctly set their calculators to the appropriate mode (radian or degree) before calculations. Additionally, in-depth training on calculator syntax should be provided, reinforced with daily practice exercises to enhance proficiency.
- (b) Mastering exponents and logarithms; teaching students the laws of exponents and logarithms clearly, emphasizing their application in simplifying expressions. Using digital tools (Desmos, GeoGebra) to visualize exponential and logarithmic functions, including their asymptotic behavior. Ensuring students can identify key points for logarithmic exponential functions. Explaining graph transformations (shifts, stretches, reflections) and reinforce understanding of domain and range. Applying real-world examples (e.g., population growth, compound interest) to illustrate exponential growth, comparing linear and exponential trends using spreadsheets or graphs.
- (c) Strengthening probability concepts; providing detailed instruction on probability rules, techniques for determining sample space (e.g., probability tree diagrams), and concepts like independent and conditional probability; clearly differentiating between combinations and permutations, using visual aids (seating arrangements, card selections, outcome tables); and emphasizing correct mathematical notation, especially for factorials.
- (d) Improving trigonometry understanding; ensure students memorize and apply exact trigonometric ratios for standard angles ( $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$ ); teaching trigonometric identities conceptually, focusing on proof techniques rather than rote memorization; differentiating between proving identities and solving trigonometric equations; introducing integration techniques involving trigonometric functions (substitution, integration by parts); strengthening comprehension of composite functions and the chain rule for differentiation.

**Appendix I****Summary of Candidates' Performance in Each Topic in 141 Basic Applied Mathematics ACSEE 2025**

<b>S/N</b>	<b>Topic</b>	<b>Questions Number</b>	<b>Percentage of Candidates who Passed</b>	<b>Remarks</b>
1	Statistics	6	77.84	Good
2	Functions	2	70.53	Good
3	Differentiation	4	67.80	Good
4	Algebra	3	58.64	Average
5	Matrices and Linear Programming	10	56.94	Average
6	Integration	5	41.73	Average
7	Trigonometry	8	27.24	Weak
8	Probability	7	27.11	Weak
9	Exponential and Logarithmic Functions	9	24.20	Weak
10	Calculating Devices	1	20.39	Weak

## Appendix II

### Candidates' Performance in Each Topic for the years 2024 and 2025

