THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2020

142 ADVANCED MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania has prepared this report on the analysis of the candidates' responses for Advanced Mathematics items for the Advanced Certificates Education Examination (ACSEE) 2020 in order to provide feedback to students, teachers and other education stakeholders on how the candidates responded to the questions.

The analysis of data and the candidates' responses is done to identify the strengths and weaknesses of the candidates on how they answered the questions. Principally, the report highlights the main areas where the candidates had good, average or poor performance.

The analysis shows that, the candidates performed well in questions that were set from topics of Functions, Logic, Algebra, Trigonometry, Numerical Methods, Linear Programming, Sets, Coordinate Geometry II, Statistics, Complex Numbers, Integration, Differentiation and Hyperbolic Functions. They did averagely in questions from topics of Vectors and Probability while the question that was set from the topic of Coordinate Geometry I had a weak performance.

The candidates' good performance was due to the following reasons: ability of the candidates to draw graphs of composite and rational functions, test the validity of an argument by using a truth table, use the principle of mathematical induction to prove mathematical statements and ability to solve three linear equations in three unknown simultaneously. Moreover, the ability to state, recall, apply and use correct formulae, techniques, laws, and mathematical identities enhanced their performance. In addition, the candidates had skills to formulate models and draw figures. However, the candidates' weak performance was caused by the lack of knowledge and skills on how to apply the formula for perpendicular distance from a point to a line to prove mathematical problems and failure to use ratio theorem in solving problems on locus.

Finally, the council would like to thank everyone who participated in the preparation of this report.

Dr. Charles E. Msonde

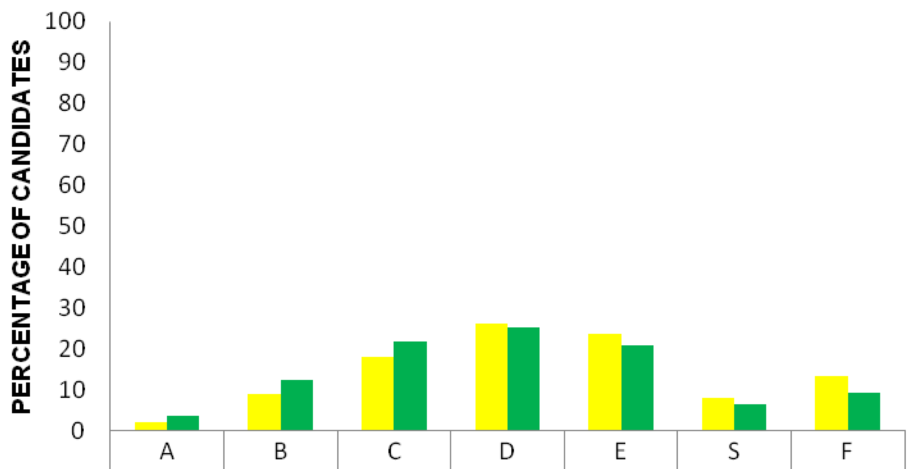
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Candidates' Items Response Analysis report (CIRA) for ACSEE 2020 was prepared based on the analysis of data and the candidates' responses. The report therefore provides the feedback to educational stakeholders on how the candidates performed well, average or poor in the examination questions.

Particularly, the report analyzed the candidates' performances in all examined topics for Advanced Mathematics. The Advanced Mathematics examination had two papers: The first paper had ten (10) compulsory questions where each question carried ten (10) marks. The second paper consisted of four (4) compulsory questions in section A where each question carried fifteen (15) marks and four (4) optional questions in section B from which the candidates were required to answer any two. In section B of the second paper, each question carried twenty (20) marks.

A total of 10,125 candidates sat for the Advanced Mathematics Examination, out of whom 9,176 (90.63%) candidates passed. This performance is better than that of 2019 whereby 86.74 percent of 10,649 candidates passed. This represents an increase of 3.89 percent in the candidates' performance. The candidates who passed Advanced Mathematics examinations obtained grades ranging from grade S to A as indicated in Figure 1.



	A	B	C	D	E	S	F
2019	2.1	8.9	18	26.2	23.6	7.9	13.3
2020	3.6	12.4	21.9	25.3	21.0	6.5	9.4

Figure 1: *Distribution of Grades for the 2019 and 2020 Advanced Mathematics Examinations*

The analysis of the individual questions is herein presented. It comprises a brief account of the requirements of the questions and the performance of candidates. The factors that accounted for good and poor performance in each question have been indicated and illustrated using samples of candidates' responses.

The analysis of the candidates' performance in each topic is shown in the appendices in which the green colour stands for good performance, the yellow colour for average performance and the red colour for poor performance. The percentage boundaries 0-34, 35-59 and 60-100 are used to represent poor, average and good performance. Finally, some recommendations are proposed at the end of the report. The recommendations may serve to help teachers and the government to improve the candidates' performance in future Advanced Mathematics examinations

2.0 ANALYSIS OF CANDIDATES’ PERFORMANCE IN EACH QUESTION

2.1 142/1 ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

This question had parts (a) and (b). In part (a), the candidates were required to use a non-programmable calculator to evaluate; (i) $\dfrac{\tan 25^{\circ}30' - \sqrt[5]{0.03e^{-.3}}}{\ln 3.2 + 0.006e^{0.3}}$ correct to six significant figures and (ii) $\sum_{n=4}^{7} \dfrac{2^{-n}(n!)}{\ln(0.3n)}$ correct to three decimal places. In part (b), the item instructed the candidates that the population in the city of Dar es Salaam is modelled by the equation $P(t) = P_0 e^{\lambda t}$ where $\lambda = 0.034657$ per year. They were required to use a non-programmable calculator to find the time t in years when the population in the city is three times the initial population P_0.

The analysis of data shows that 9898 (97.2%) candidates attempted this question. Amongst, 7502 (75.8%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was good. Figure 2 shows the percentage of candidates who obtained weak, average and good performance.

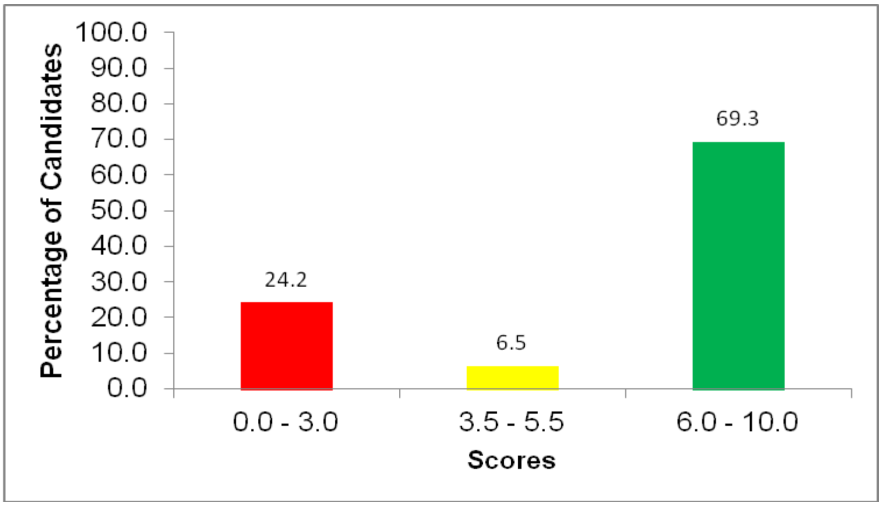


Figure 2: *Candidates' performance in question 1*

Figure 2 shows that 69.3 percent of candidates scored marks ranging from 6 to 10 whereby 3487 candidates equivalent to 35.2 percent scored all 10 marks. The candidates who scored all the marks in part (a) (i) performed calculations involving addition, subtraction, division and multiplication to evaluate $\frac{\tan 25^\circ 30' - \sqrt[5]{0.03e^{-.3}}}{\ln 3.2 + 0.006e^{0.3}}$ correct to six significant figures as 0.174857. The candidates who scored all marks in part (a) (ii) were able to formulate a series $\frac{2^{-4}(4!)}{\ln(0.3)(4)} + \frac{2^{-5}(5!)}{\ln(0.3)(5)} + \frac{2^{-6}(6!)}{\ln(0.3)(6)} + \frac{2^{-7}(7!)}{\ln(0.3)(7)}$ from $\sum_{n=4}^{7} \frac{2^{-n}(n!)}{\ln(0.3n)}$ by substituting $n=4$, $n=5$, $n=6$ and $n=7$. Also, they used a non-programmable calculator to find the sum of terms obtained in the formulated series and wrote the resulting answer correctly to 3 decimal places as 89.686. The candidates who did part (b) well replaced $P(t)$ by $3P_0$ in the equation $P(t) = P_0 e^{\lambda t}$; expressed t the subject of λ as $t = \frac{\ln 3}{\lambda}$; substituted λ with 0.034657 to get $t = \frac{\ln 3}{0.034657}$ and finally entered the expression $\frac{\ln 3}{0.034657}$ into a non-programmable calculator to obtain the time t as 32 years approximately. Extract 1.1 shows the sample of response by the candidates who answered the question correctly.

1 a)	i. 1.74857×10^{-1}	
	ii. 89.686	
1 b)	$P = P_0 e^{\lambda t}$	
	$3P_0 = P_0 e^{\lambda t}$	
	$3 = e^{\lambda t}$	
	Apply ln both sides	
	$\ln 3 = \ln e^{\lambda t}$	
	$\ln 3 = \lambda t \ln e$	
	$\ln e = 1$	
	$\ln 3 = \lambda t$	
	$t = \frac{\ln 3}{\lambda}$ given $\lambda = 0.034657$	
	$t = 31.7$ years	
	The time is 31.7 years.	

Extract 1.1: *A correct response from one of the candidates*

In Figure 2, it is also shown that 24.2 percent of the candidates scored marks ranging from 0 to 3 of which 911 (9.2%) candidates scored 0. The candidates who got wrong answers faced several challenges in answering this question: In part (a) (i), they did not set calculator in degree mode before using it in answering the question. Consequently, the candidates got wrong answers such as 0.184173. Also, a large number of candidates evaluated the numerator and denominator in the expression $\frac{\tan 25^{\circ}30' - \sqrt[5]{0.03e^{-.3}}}{\ln 3.2 + 0.006e^{0.3}}$ separately and then later divided the values to obtain the final answer. The most common challenge to majority of the candidates was failure to ensure that both values in the numerator and denominator are correct. It was also noted that a number of candidates could not present the final answer correctly to six significant figures as they wrote 0.174856741 contrary to the required one. The candidates who got part (a) (ii) wrong defined a series $\sum_{n=4}^{7} \frac{2^{-n}(n!)}{\ln(0.3n)}$ using one or two terms by substituting $n = 4$, $n = 7$ or both, ignoring $n = 5$, $n = 6$ and got an incorrect answer 8.2272222457. Others managed to compute the answer for the expression $\sum_{n=4}^{7} \frac{2^{-n}(n!)}{\ln(0.3n)}$ as 89.6859766 but did not present it into three decimal places as required. In part (b), some candidates failed to replace $P(t)$ with $3P_0$ into the equation $P(t) = P_0 e^{\lambda t}$ to solve for t. The candidates who did not show evidence of getting 31.69 did not recall the rounding procedure for approximating 31.69 to 32. These candidates faced the challenge of failure to increase the second digit in the number 31.69 by 1 because the next digit to 1 is greater than 5. Hence, they wrote 31 instead of 32 years. Extract 1.2 shows the response of a candidate who could not answer part (a) (i) correctly.

01. a/

$$\frac{\tan 25^{\circ} 30' - \sqrt[5]{0.03e^{-3}}}{\ln 3.2 + 0.006e^{0.3}}$$

Solution.

$\tan 25^{\circ}30' = 0.476975522$

$\sqrt[5]{0.030e^{-3}} = 0.193236387$

$\ln 3.2 = 1.16315081$

$0.006e^{0.3} = 8.099152845$

Then

In

$$\frac{0.476975532 - 0.193236387}{1.171349967}$$

$$= 0.242257279 \simeq 0.242257$$

$$\therefore \frac{\tan 25^\circ 30' - \sqrt[5]{0.07e^{-7}}}{\ln 7.2 + 0.006e^{0.7}} = 0.2422572\cancel{79}$$

Extract 1.2: *An incorrect response from one of the candidates*

In Extract 1.2, the candidate made computation slips in working out the terms of the numerator. He/she obtained the second term of the numerator as 0.27217458 instead of 0.193236387.

2.1.2 Question 2: Hyperbolic Functions

The question comprised parts (a) and (b). In part (a), the candidates were required to show that $(\cosh A - \cosh B)^2 - (\sinh A - \sinh B)^2 = -4\sinh^2\left(\frac{A-B}{2}\right)$. In part (b), the candidates were required to use the second derivative test to identify the nature of the stationary point of the function $f(t) = \cos 2t - 4\sinh t$.

The analysis shows that 38.5 percent of the candidates who attempted this question scored from 0 to 3 marks, 17.1 percent from 3.5 to 5.5 and 44.4 percent from 6 to 10 marks. Generally, the candidates' performance was good as 61.5 percent of the candidates got more than 3 marks. Figure 3 illustrates the candidates' performance in this question.

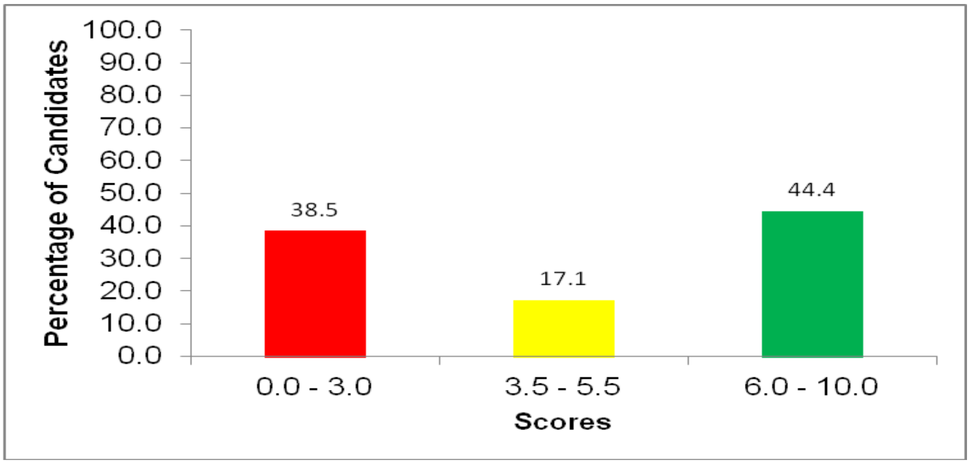


Figure 3: *Candidates' Performance in question 2*

The candidates, who did the question well, had adequate understanding of the concept tested. In part (a), they were capable of expanding the given expression to $\cosh^2 A - \sinh^2 A + \cosh^2 B - \sinh^2 B - 2\cosh A - B$. Then, with the aid of the identities $\cosh^2\theta - \sinh^2\theta = 1$ and $\cosh\theta = \cosh^2\frac{\theta}{2} + \sinh^2\frac{\theta}{2}$, the candidates managed to simplify the expression obtained in the previous step as $-4\sinh^2\left(\frac{A-B}{2}\right)$. The candidates who answered part (b) correctly, were able to determine the first derivative of the function $f(t) = \cos 2t - 4\sinh t$ as $f'(t) = -2\sin 2t - 4\cosh t$. Then, they were able to recognise that the first derivative consists of hyperbolic and trigonometric functions and thus the second derivative test cannot be applied to determine the nature of the stationary point of the function $f(t) = \cos 2t - 4\sinh t$. Extract 2.1 is a solution obtained by one of the candidates who answered correctly part (a) of this question.

2.(a) Considering L.H.S

$(\text{Cosh } A - \text{Cosh } B)^2 - (\text{Sinh } A - \text{Sinh } B)^2$

Using Factor Formula

$\Rightarrow \left[-2\,\text{Sinh}\,\frac{A+B}{2}\,\text{Sinh}\,\frac{A-B}{2}\right]^2 - \left[2\,\text{Sinh}\,\frac{A-B}{2}\,\text{Cosh}\,\frac{A+B}{2}\right]^2$

$= 4\left[\text{Sinh}^2\frac{A+B}{2}\,\text{Sinh}^2\frac{A-B}{2}\right] - 4\left[\text{Sinh}^2\frac{A-B}{2}\,\text{Cosh}^2\frac{A+B}{2}\right]$

$= 4\left[\text{Sinh}^2\frac{A+B}{2}\,\text{Sinh}^2\frac{A-B}{2} - \text{Sinh}^2\frac{A-B}{2}\,\text{Cosh}^2\frac{A+B}{2}\right]$

$= 4\left[\text{Sinh}^2\frac{A-B}{2}\left(\text{Sinh}^2\frac{A+B}{2} - \text{Cosh}^2\frac{A+B}{2}\right)\right]$

but ~~Sin~~ $\text{Cosh}^2 A - \text{Sinh}^2 A = 1$

$\text{Sinh}^2 A - \text{Cosh}^2 A = -1$

$= 4\left[\text{Sinh}^2\frac{A-B}{2}(-1)\right]$

$= -4\,\text{Sinh}^2\frac{A-B}{2}$

$\therefore (\text{Cosh } A - \text{Cosh } B)^2 - (\text{Sinh } A - \text{Sinh } A)^2 = -4\,\text{Sinh}^2\frac{A-B}{2}$

hence shown.

Extract 2.1: *A correct response from one of the candidates*

In Extract 2.1, the candidate proved the correctness of the given expression by using the relevant hyperbolic identities.

On the other hand, 908 (9.6%) candidates did not score any mark. Where as in part (a), most of them allowed the identities $\cosh A = \frac{e^A + e^{-A}}{2}$, $\sinh A = \frac{e^A - e^{-A}}{2}$, $\cosh B = \frac{e^B + e^{-B}}{2}$ and $\sinh B = \frac{e^B - e^{-B}}{2}$ to replace $\cosh A$, $\cosh B$, $\sinh A$ and $\sinh B$ in the given equation. The candidates could not proceed further to complete the proof as the resulting expression was difficult to deal with. The candidates who answered incorrectly part (b) were not able to differentiate $f(t) = \cos 2t - 4\sinh t$ properly. The analysis of the responses of candidates reveals that several candidates got incorrect function for the first derivative as $f'(t) = 2\sin 2t - 4\cosh t$. The correct one was $f'(t) = -2\sin 2t - 4\cosh t$. Further analysis shows that a number of candidates were not aware of the condition that at a turning point $f'(t) = 0$. Lastly, it was surprising to note how most candidates substituted $t = 0$ in the function for the second derivative. The candidates were supposed to set the function for the first derivative is equal to 0 that is $-2\sin 2t - 4\cosh t = 0$. Extract 2.2 is a sample solution from the candidate who did part (b) of this question badly.

5	$f(t) = \text{Cos}2t - 4\text{Sinh}t$
	$f(t)' = 2\text{Sin}2t - 4\text{Cosh}t$
	$f(t)'' = -4\text{Cos}2t + 4\text{Sinh}t$.
	at $f(0) = -4\text{Cos}(2(0)) + 4\text{Sinh}(0)$
	$= -4$.
	The Stationary point is -4.

Extract 2.2: *An incorrect response from one of the candidate*

In Extract 2.2, the candidate seems not to have comprehensive idea on how to identify the nature of the stationary point by using the second derivative test. The candidate replaced the value in $f(t) = \cos 2t - 4\sinh t$ with $t = 0$ and as the result failed to continue.

2.1.3 Question 3: Linear Programming

Part (a) of this question read "A farm stocks two types of local brews called Kibuku and Lubisi, both of which are produced in cans of the same size. He wishes to order fresh supplies and finds that he has room for up to 1500 cans. He knows that Lubisi is more popular and so proposes to order at least thrice as many cans of

Lubisi as Kibuku. He wishes, however, to have at least 120 cans of Kibuku and at most 950 cans of Lubisi. The profit on a can of Kibuku is sh. 3,000 and a can of Lubisi is sh. 4,000." Taking x to be the number of cans of Kibuku and y to be the number of cans of Lubisi which he orders, the candidates were required formulate a linear programming problem from this information. In part (b), the candidates were given that a cooperative society has two storage depots for storing beans. The storage capacity of depot 1 and 2 is 200 and 300 tons of beans respectively. The beans have to be sent to three marketing centres A, B and C. The demand of beans at A, B and C is 150, 150 and 200 tons respectively. The transport costs per ton from depot 1 to X, depot 1 to Y, depot 1 to Z, depot 2 to X, depot 2 to Y and depot 2 to Z are 10,000/=, 20,000/=, 14,000/=, 16,000/=, 30,000/= and 8,000/= respectively. The candidates were required to find the number of tons to be sent to each of the marketing centres.

The question was attempted by 9922 (97.5%) candidates out of whom 51.1 percent scored from 6 to 10 marks and 1.0 percent scored all 10 marks. Further analysis shows that 33.0 percent of the candidates had average performance; their scores ranged from 3.5 to 5.5 and 15.9 percent from 0 to 3 marks. Generally, the candidates' performance was good as shown in Figure 4.

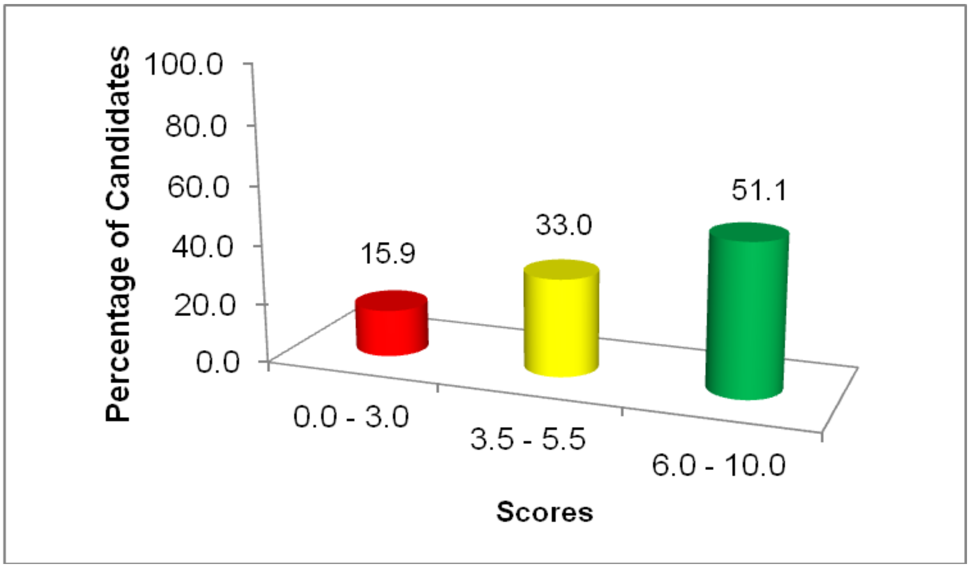
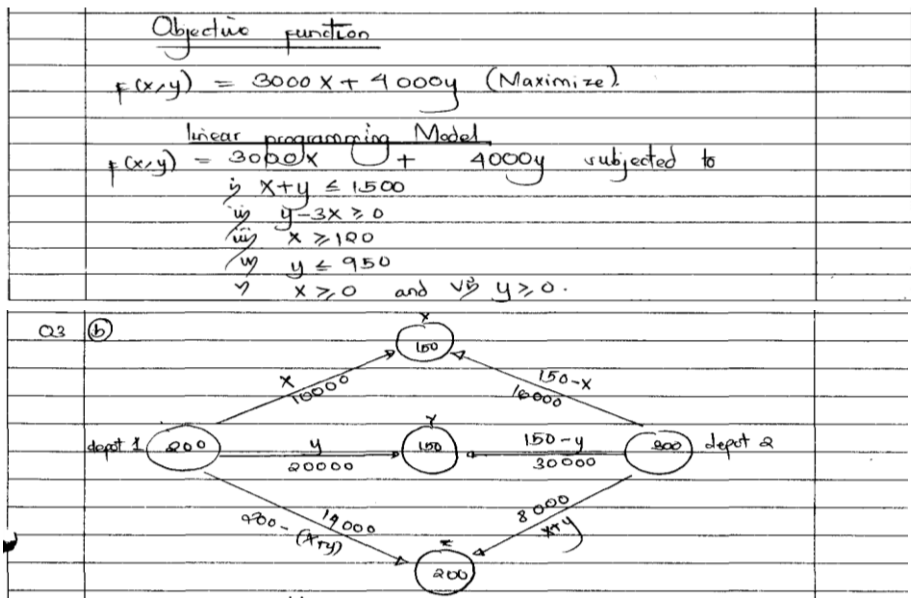


Figure 4: *Candidates' performance in question 3*

The candidates who did well were able to write down constraints representing the given linear programming problem in part (a). Such constraints were $x+y\le 1500$, $y\ge 3x$, $x\ge 120$, $y\le 950$ and $x, y\ge 0$. Further analysis shows that most candidates were able to write the correct objective function. Most of them wrote $f(x,y)=3000x+4000y$ and included the instruction to maximise the objective function. The candidates who answered part (b) correctly exhibited the following strengths: With the aid of a diagram, they were able to formulate the inequalities $x\le 150$, $y\le 150$, $x+y\le 200$, $x+y\ge 0$ and $x, y\ge 0$ and the objective function $f(x,y)=9{,}700{,}000-12{,}000x-16000y$. Then, they represented the inequalities graphically and indicated the feasible region as well as the corner points A (0, 0), B (15, 0), C (150, 50), D (50, 150) and E (0, 150). Then, they substituted the *x* and *y* value in the objective function with the x and y coordinate of the corner points A, B, C, D and E to get 9,700,000/=, 7,900,000/=, 7,100,000/=, 6,700,000/= and 7,300,000/= respectively. At last the candidates were able to conclude that the minimum cost occur when the point is $50, 150$ which gives 50, 100 and 200 tons of beans to be shifted from depot 1 to A, depot 2 to A and depot 2 to C respectively. Extract 3.1 is a solution obtained by one of the candidates who answered the question correctly.



iii) $200 - (x+y) \geq 0$

$x + y \leq 200$

iv) $x + y \geq 0$

Non constraints

v) $x \geq 0$

vi) $y \geq 0$

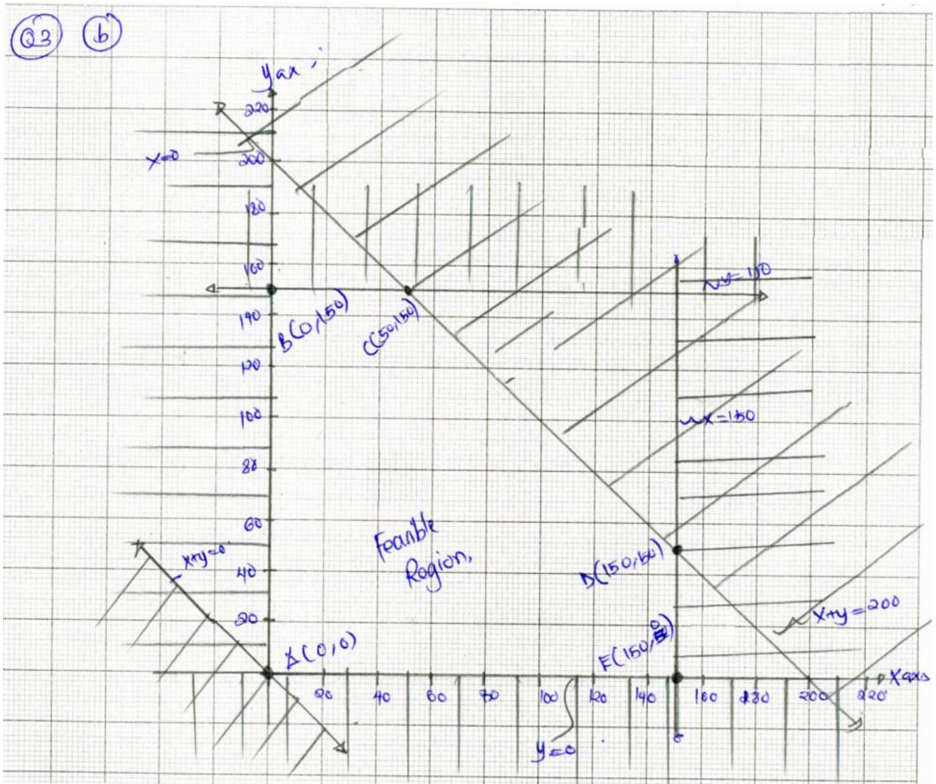
Q3 (b) Objective function

$f(x,y) = 10,000x + 20,000y + 2,800,000 - 14000x$

$- 14000y + 8000x + 8000y + 4,500,000 - 30000y$

$+ 2,400,000 - 16000x$

$f(x,y) = -12,000x - 16000y + 9700,000$



corner points	$f(x,y) = -12000x - 16000y + 9,700000$
A(0,0)	9700,000
B(0,150)	7,300,000
C(50,150)	6,700,000
D(150,50)	7,100,000
E(150,0)	7,900,000

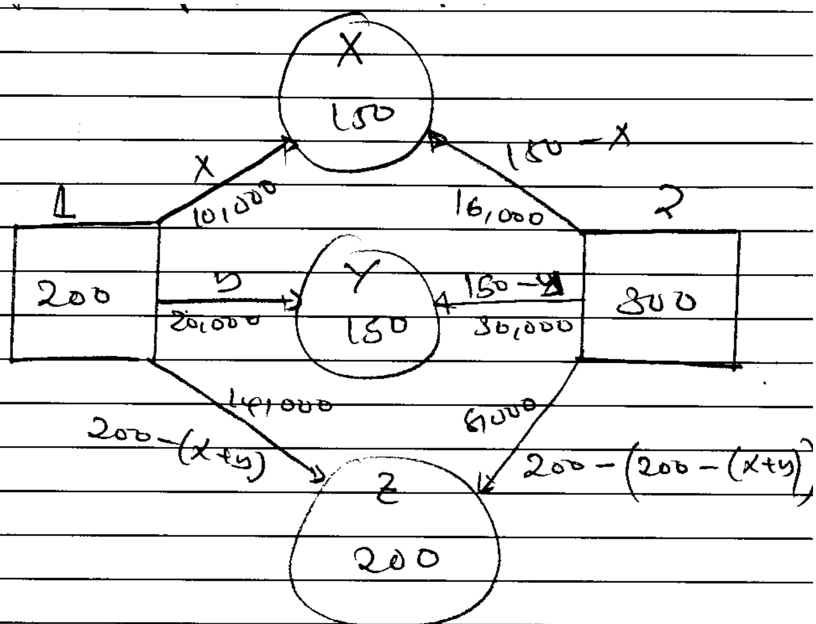
Q3	(b) The transportation of tons should be as follows.				
		X	Y	Z	
	Depot 1	50	150	0	
	Depot 2	100	0	200	

Extract 3.1: *A correct response from one of the candidates*

In Extract 3.1, the candidate was able to transform a word problem into a mathematical model in part (a). Also he/she was able to transform the transportation problem correctly into a mathematical model. Finally, the candidates solved the transportation problem graphically.

On the other hand, the candidates who got the question wrong faced the following challenges: In part (a), several candidates confused the inequality sign $\leq$ with $\geq$ indicating that they did not understand how to represent the phrases "at most" and "at least" with appropriate inequality. Most of them reversed the inequality. The frequently seen inequalities were $y \geq 3x, x \leq 120$, $y \leq 950$, $x + y \geq 1500$ and $x, y \leq 0$. Other candidates wrote a system of linear equations $y = 3x$, $x = 120$, $y = 950$ and $x + y = 1500$ instead of the inequalities. Another challenge was omission of the instruction to maximise the objective function. Most candidates did not write the word 'maximise" behind the function $f(x, y) = 3000x + 4000y$. The candidates who answered part (b) incorrectly failed to present the given transportation problem diagrammatically and therefore unable to formulate constraints and the objective function. Also, the candidates confused greater or equal to with less or equal sign. Such candidates wrote the incorrect inequalities such as $x + y \geq 200, x + y \geq 0, x \geq 150$ and $y \geq 150$. Other common errors were omitting the non negative constraints $x, y \geq 0$. However, it was pleasing to see how most candidates were able to draw the graphs for $x + y = 200$, $x + y = 0$, $x = 150$ and $y = 150$ but many did not label their graphs and others did not shade for feasible region and sometimes the feasible region was not indicated. Finally, some poor choices of scale were often seen so that the whole feasible region was not shown or was too small to be seen. Extract 3.2 is a response from a candidate who got part of question 3 (b) wrong.

3(b)	Solution
	Let "X" be amount of bean transfered from
	deposit 1 to X and "y" be amount of beans tran-
	fered from depot 1 to Y.



Inequalities / constraints:

$x \le 0$ ——— (i)

$y \le 0$ ——— (ii)

$200 - (x + y) \le 0$

$x + y \ge 200$ ——— (iii)

$x + y \le 0$ ——— (iv)

$150 - x \le 0$

$x \ge 150$ ——— (v)

$150 - y \ge 0$

$y \ge 150$ ——— (vi)

Extract 3.2: *An incorrect response from one of the candidates*

In Extract 3.2, the candidate reversed the inequality sign and as a result wrote the incorrect constraints.

2.1.4 Question 4: Statistics

The question comprised parts (a) and (b). In part (a), the candidates were required to show that $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$. In part (b), the candidates were given the following distribution table showing the masses in grams for a sample of potatoes:

Mass (g)	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	14	21	73	42	13	9	4	2

They were required to; (i) find the arithmetic mean by using coding method and assumed mean $A = 54.5$, (ii) use the mean obtained in (i) to find the variance and standard deviation correct to 2 decimal places and (iii) compute the 80th percentile correctly to three decimal places.

The analysis of data shows that the question was attempted by 10,019 (98.4%) candidates, out of which 77.3 percent of the candidates scored above 3 marks. Further analysis reveals that 22.7 percent of the candidates scored from 0 to 3 marks, 41.3 percent scored from 3.5 to 5.5 marks and 36.0 percent scored from 6 to 10 marks. The data also shows that 133 (1.3%) candidates scored all 10 marks while 108 (1.1%) candidates scored zero. Therefore, the candidates' performance was good. Some of these statistics are displayed in Figure 5.

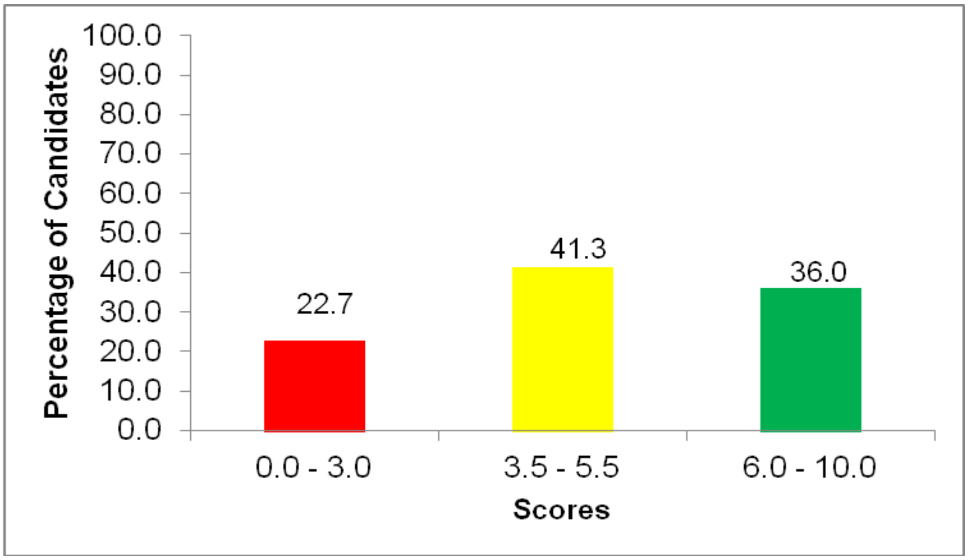


Figure 5: *Candidates' performance in question 4*

The candidates who responded to part (a) well were capable of showing that the left hand side of the equation $\sum_{i=1}^{n}(x_i - \bar{x})^2$ is equal to the right hand side $\sum x_i^{\,2} - n\bar{x}^2$ by expanding $(x_i - \bar{x})^2$ to $\sum_{i=1}^{n} x_i^{\,2} - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$. By using the substitution $\sum_{i=1}^{n} x_i = n\bar{x}$ and $\sum_{i=1}^{n} x_i^2 = n\bar{x}^2$, they were able to transform the equation $\sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$ into $\sum_{i=1}^{n} x_i^2 - n\bar{x}^2$. The candidates who answered part (b) correctly demonstrated the following strengths: In (i), they were able to; fill the frequency distribution table with the correct entries for the column x_i, $(x_i - A)$, $\mu_i = \frac{x_i - A}{c}$, fu, $(x_i - A)^2$, $f(x_i - A)^2$, cf, $f_i x_i$ and $f_i x_i^2$ and they were able to compute $\sum_{i=1}^{n} f_i u_i$ and $\sum_{i=1}^{n} f_i$ as -144 and 180 respectively. Also, the candidates used the formula $\bar{x} = A + C\frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i}$ together with the values for $\sum_{i=1}^{n} f_i u_i$ and $\sum_{i=1}^{n} f_i$ to calculate the arithmetic mean as 48.17 gram. In (ii), the candidates were able to calculate the variance and standard deviation by using the formulae $V = \frac{1}{n}\sum_{i=1}^{k} f_i X_i^2 - \bar{x}^2$ and $\sigma = \sqrt{V}$ to get 199.57 and 14.13 gram respectively. In (iii) the candidates managed to calculate the 80th percentile correctly to three decimal places using the formula $P_{80} = L + \left(\frac{\frac{8}{10}N - n_b}{n_w}\right)C$. A significant number of candidates managed to express the 80th percentile correct to three decimal places as 57.595 gram. A sample response of the candidate who performed well in this question is shown in Extract 4.1.

4. (a) Given:

$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x^2 - n\bar{x}^2$

From L.H.S:

$\sum_{i=1}^{n} (x_i - \bar{x})^2$

Expand $(x_i - \bar{x})^2$

$= ((x_i) - \bar{x})(x_i - \bar{x})$

$x_i^2 - 2\bar{x}x_i + \bar{x}^2$

$\sum_{i=1}^{n} (x_i^2 - 2\bar{x}x_i + \bar{x}^2)$

$\sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} \bar{x}x_i + \sum_{i=1}^{n} \bar{x}^2$

but Mean $= \sum_{i=1}^{n} x_i = \bar{x}$

$$\sum_{i=1}^{n} x_i^2 - 2\left(\sum_{i=1}^{n} \bar{x}\right)\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$$

$$\sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} \bar{x}\cdot\bar{x} + \sum_{i=1}^{n} \bar{x}^2$$

$$= \sum_{i=1}^{n} x_i^2 + \left(-2\sum_{i=1}^{n} \bar{x}^2 + \sum_{i=1}^{n} \bar{x}^2\right),$$

$$= \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} \bar{x}^2$$ let = since $\sum_{i=1}^{n} = n$:

$$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2.$$

Hence shown since L·H·S = R·H·S:

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2.$$

4. (b) Given: $A = 54.5$ $c = 10$:

Mass	X	f	CM.	d=X−A	u=d/c	u^2	fu	fu^2
10−19	14.5	2	2	−40	−4	16	−8	32.
20−29	24.5	14	16	−30	−3	9.	−42	126
30−39	34.5	21	37	−20	−2	4	−42	84
40−49	44.5	73	110	−10	−1	1.	−73	73
50−59	54.5	42	152	0	0	0	0	0
60−69	64.5	13	165	10	1	1	13	13
70−79	74.5	9	174	20	2	4	18	36
80−89	84.5	4	178	30	3	9.	12.	36
90−99.	94.5	2	180	40	4.	16.	8	32

$N = 180$ $\sum fu = -114.$ $\sum fu^2 = 432.$

(i) Arithmetic Mean:

$$\bar{X} = A + c\frac{\sum fu}{N}:$$

$$\bar{X} = 54.5 + 10\left(\frac{-114.}{180}\right)$$

$$\bar{X} = 48.1666667.$$

(ii) $$\delta^2 = c^2\left(\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2\right),$$

$$\text{Variance} = 10^2\left(\frac{432}{180} - \left(\frac{-114}{180}\right)^2\right).$$

Variance = . 199.88889 into 2 decimal place:

Variance = 199.89.

Standard deviation:

$$\sigma = c\sqrt{\left(\frac{\Sigma fu^2}{N}\right) - \left(\frac{\Sigma fu}{N}\right)^2}$$

$$\sigma = 10\sqrt{\left(\frac{432}{180}\right) - \left(\frac{-114}{180}\right)^2}$$

$$= 10\sqrt{1.9988889}$$

$$= 14.1382$$

Into 2 decimals.

$$\sigma = 14.14.$$

Standard deviation $= 14.14$.

(iii) 80 percentile

$$\frac{80}{100}\Sigma fx$$

$$= \frac{80}{100} \times 180$$

$$= 144$$

From: $= L + \left(\frac{80/100\, \Sigma fx - \Sigma fb}{fw}\right)c$

$$= 49.5 + \left(\frac{144 - 110}{42}\right)10$$

$$= 49.5 + 8.09524$$

$$= 57.595$$

$\therefore$ 80 percentile $= 57.595$.

Extract 4.1: *A correct response from one of the candidates*

In Extract 4.1, the candidate was able to prove the statistical expression in part (a). In part (b) he/she was able to calculate the measure of central tendency and measure of dispersion.

Despite the good performance, there were few candidates who performed poorly. The candidates who did not do part (a) well encountered the following challenges: failure of candidates to retain the sigma notation. The majority of candidates who ignored the sigma notation ended up writing the series $(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ...$ which did not help them to prove the correctness of the given expression. It was pleasing to see how several candidates expanded $\sum_{i=1}^{n}(x_i - \bar{x})^2$ to

$\sum_{i=1}^{n}\left(x_i^2 - 2\bar{x}x_i + \bar{x}^2\right)$ but could not express the obtained results into $\sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\bar{x}^2$ for easier replacement of $\sum_{i=1}^{n}x_i$ and $\sum_{i=1}^{n}\bar{x}^2$ with $n\bar{x}$ and $n\bar{x}^2$ respectively to get $\sum_{i=1}^{n}x_i^2 - n\bar{x}^2$. The candidates who did not respond to part (b) correctly used the incorrect and inappropriate formulae such as $\bar{x} = A - C\frac{\sum fu}{\sum f}$ to calculate the arithmetic mean in (i). In (ii) and (iii), most candidates failed to prepare the frequency distribution table with headings x_i, $(x_i - A)$, $\mu_i = \frac{x_i - A}{c}$, fu, $(x_i - A)^2$ and $f(x_i - A)^2$. The candidates were supposed to understand that the frequency distribution table is an important tool for calculating the variance, standard deviation and 80th percentile. Extract 4.2 is an example of an incorrect solution from a candidate who got part (a) of this question wrong.

4 a) $\sum_{i=1}^{n} [x_i - \bar{x})^2 = \sum_{i=1}^{n} (x_i^2 - n\bar{x}^2.$

Consider $\sum_{i=1}^{n} [x_i - \bar{x})^2.$

When i = 1.

$[x_1 - \bar{x})^2 + [x_2 - \bar{x})^2 + [x_3 - \bar{x}]^2 + (x_n - \bar{x})^2.$

$= x_i^2 - n\bar{x}^2.$

Hence shown.

Extract 4.2: *An incorrect response from one of the candidates*

In Extract 4.2, the candidate changed the left hand side of the given expression into the series. This approach was wrong as it could not give out the intended results.

2.1.5 Question 5: Sets

This question had parts (a) and (b). In part (a), the candidates were required to simplify $(A \cup B)' \cap (A \cap B)'$ by using the appropriate laws of sets. Part (b), of the question read “ The Malya social training college cultural group consists of 36 villagers, 25 of them participate in dancing, 28 participate in singing, while 26 among them participate in drama, 19 villagers dance and sing, 18 villagers dance and play drama and 15 participate in all three activities. If each villager participate in at least one of the activities, the candidates were required to use a Venn diagram to find the number of villagers; (i) who are either dancing or playing drama, (ii) who participate in at most two activities and (iii) who neither play drama nor sing.

The analysis of data shows that out of 10,117 (99.4%) candidates who attempted this question, 80 percent scored above 3 marks. The analysis has also shown that 20 percent of the candidates scored from 0 to 3 marks, 29.3 percent scored from 3.5 to 5.5 marks and 50.7 percent scored from 6 to 10 marks. It was also noted that 1094 candidates equivalent to 10.8 percent scored all 10 marks, while 178 candidates equivalent to 1.8 percent scored 0. Generally, this question was performed well as shown in figure 6.

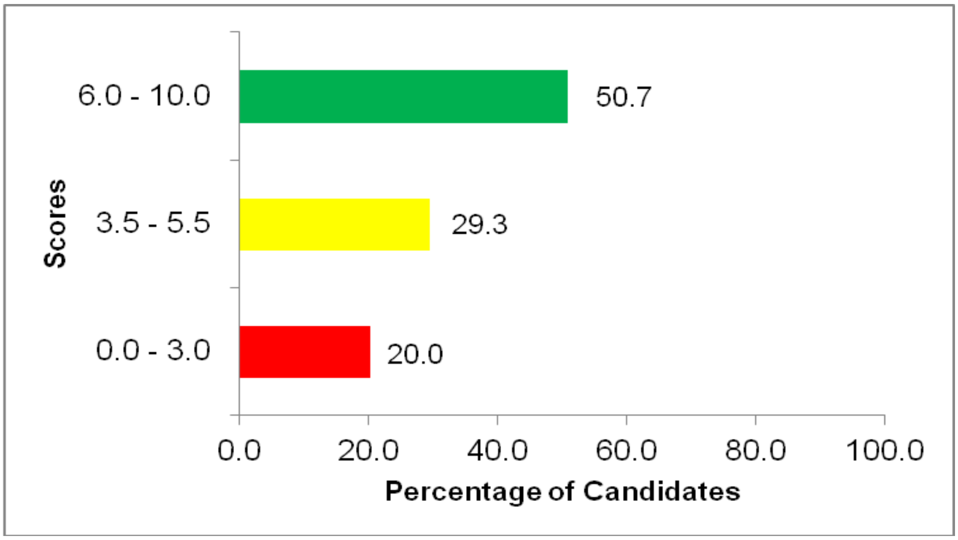


Figure 6: *Candidates' performance in question 5*

The candidates who did question 5 correctly demonstrated the following competences: In part (a), the candidates managed to use the laws of algebra of sets namely de Morgan’s law, distributive law, commutative law, associative law and the idempotent law to simplify the given set expression into $A' \cap B'$. In part (b), the candidates illustrated correctly the given word problem on the Venn diagram. By using Venn diagram, the candidates were able to answer sub-questions correctly as follows: (i) the villagers who are either dancing or playing drama are 6, (ii) the villagers who participate in at most two activities are 21 and (iii) the villagers who neither play drama nor sing are 3. Extract 5.1 is a sample response taken from the answer booklet of a candidate who answered the question correctly.

5 a) $(A \cup B)' \cap (A \cap B)'$ — given.

$= (A' \cap B') \cap (A' \cup B')$ — DeMorgans law

$= A' \cap (A' \cup B') \cap B' \cap (A' \cup B')$ — Distributive law

$= (A' \cup \phi) \cap (A' \cup B') \cap (B' \cup \phi) \cap (B' \cup A')$ — Identity law

$= A' \cup (\phi \cap B') \cap B' \cup (\phi \cap A')$ — Distributive law

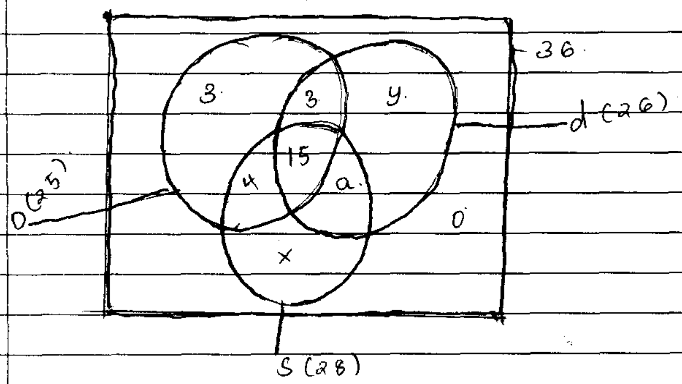
$= A' \cup \phi \cap B' \cup \phi$ — Identity law

$= A' \cap B'$ — Identity law.

b) Let D - represent dancing

S - represent singing

d - represent drama.



$3 + 3 + 15 + 4 + x + a + y = 36$

$a + y + x = 11$ — (1)

$4 + 15 + a + x = 28$

$a + x = 9$ — (2)

$3 + 15 + a + y = 26$

$a + y = 8$ — (3)

On solving

$a = \cancel{8}\ 6$

$x = 3$

$y = 2$.

5 b) i) Who are either dancing or playing drama

$= 3 + 3 + 15 + 4 + a + y$

$= \cancel{8}\ 25 + 6 + 2$

$= 33$ villagers

ii) who participate in atmost two activities

	$= 3 + 3 + 4 + a + x + y$	
	$= 3 + 3 + 4 + 6 + 3 + 2$	
	$= 21$ villagers.	
	iii) who neither play drama nor sing	
	$= 3$ villagers.	

Extract 5.1 *A correct response from one of the candidates*

In Extract 5.1, the candidate was able to apply the laws of algebra of sets to simplify the given expression and use Venn diagram to answer the sub-questions (i), (ii) and (iii).

Despite the good performance, there were 178 candidates equivalent to 1.8 percent who got the question wrong. In part (a), some candidates wrote the incorrect laws in front each step. Other candidates did not write the laws used though their working was correct. Also, several candidates committed errors right from the beginning. Hence, they failed to simplify the given expression into $A' \cap B'$. The candidates who answered part (b) incorrectly encountered the following challenges: One, failure of candidates to enclose the overlapping circle with a box defining the universal set. Two, some candidates were able to draw Venn diagram but could not position correctly the number of villagers in each region of the Venn diagram. Three, it was pleasing to see how a number of candidates managed to fill the information on the Venn diagram however answering the sub-question posed a big problem for many candidates. A sample answer from one of the candidates who answered part (a) incorrectly is shown in Extract 5.2.

5(a)	$(A \cup B)' \cap (A \cap B)'$ --- given	
	$(A' \cap B') \cap (A' \cup B')$ --- compliment rule.	
	$(A' \cap A') \cup (A' \cap B') \cup (B' \cap A') \cup (B' \cap B')$ --- distributive law	
	$\cancel{\emptyset} \cup (A' \cap B') \cup (B' \cap A') \cup \emptyset$ ---- de morgans	
	$(A' \cap B')$ ---- associative law	
	$(A \cup B)'$ --- idempotent law	
5(b)		

Extract 5.2: *An incorrect response from one of the candidates*

In Extract 5.2, the candidate wrote incorrect laws in front of each step.

2.1.6 Question 6: Functions

The question had parts (a) and (b). In part (a) (i), the candidates were informed that $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$. They were required to find $f o g$. In part (a) (ii), the candidates were required to copy and complete the following table of values:

x	-3	-2	-1	0	1	2
fog						

In part (a) (iii), the candidates were required to sketch the graph of $f o g$ by using the table of values in (ii). In part (b), the candidates were provided with the function $y = \dfrac{x^2 - 2x - 3}{x^2 - 4}$. They were required to find (i) the vertical and horizontal asymptotes and (ii) sketch the graph of y.

The analysis of data shows that the question was attempted by 10,147 (99.7%) candidates, out of which 97.5 percent of the candidates scored above 3 marks. Further analysis reveals that only 2.5 percent of the candidates scored from 0 to 3 marks, 11.8 percent scored from 3.5 to 5.5 marks and 85.7 percent scored from 6 to 10 marks. The data also shows that 192 candidates equivalent to 1.9 percent scored all 10 marks while 0.2 percent scored 0. Hence, the question was best performed in this examination for two consecutive years. Figure 7 shows the candidates' performance in this question.

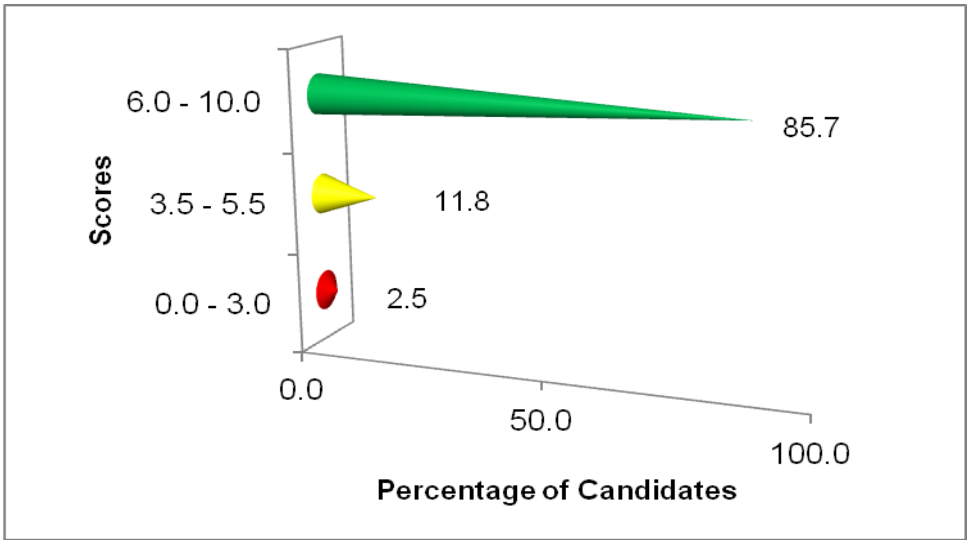


Figure 7: *Candidates' performance in question 6*

The analysis of the candidates' responses indicates that the candidates were able to find the composite function $fog(x) = x$ in part (a) (i). In part (a) (ii), they managed to fill in a table with the required entries by substituting $x = -3,\ -2,\ -1$, 0, 1 and 2 into $fog(x) = x$. In part (a) (iii), they correctly sketched the graph of fog by using the table of values in (ii). Further analysis shows that the candidates who answered part (b) correctly solved $x^2 - 4 = 0$ to get vertical asymptotes $x = 2$ and $x = -2$. Also, they evaluated $y = \lim_{x\to\infty}\left(\frac{x^2 - 2x - 3}{x^2 - 4}\right)$ to get horizontal asymptote $y = 1$. In part (b) (ii), the candidates solved $\frac{x^2 - 2x - 3}{x^2 - 4} = 0$ for the purpose of obtaining x - intercepts $-1, 0$ and $3, 0$. Also they were able to replace x with 0 in $y = \frac{x^2 - 2x - 3}{x^2 - 4}$ to come up with y – intercept $\left(0, \frac{3}{4}\right)$. Finally, the candidates used the information obtained in parts (i) and (ii) together with a table of values to trace the path of $y = \frac{x^2 - 2x - 3}{x^2 - 4}$ in three regions namely $x < -2$, $-2 < x < 2$, and $x > 3$. Extract 6.1 is a sample response taken from a candidate who did the question correctly.

6. (a)(i) $f(x) = x^2+1$

$g(x) = \sqrt{x-1}$

$fogx = f(g(x))$

$= (\sqrt{x-1})^2 + 1$

$= (x-1)+1$

$= x-1+1$

$\therefore fog = x$

(ii)

x	-3	-2	-1	0	1	2
fog	-3	-2	1	0	1	2

(iii) On the graph paper.

(b) $y = \dfrac{x^2 - 2x - 3}{x^2 - 4}$

(i) Horizontal Asymptote (H.A)

$y = \dfrac{\text{Leading Coefficient of Numerator polynomial}}{\text{Leading Coefficient of Denominator polynomial}}$

$y = \dfrac{1}{1}$

$\therefore$ Horizontal Asymptote is $y = 1$

Vertical Asymptote (V.A)

Let: $x^2 - 4 = 0$

$x^2 = 4$

$x = \sqrt{4}$

$x = \pm 2$

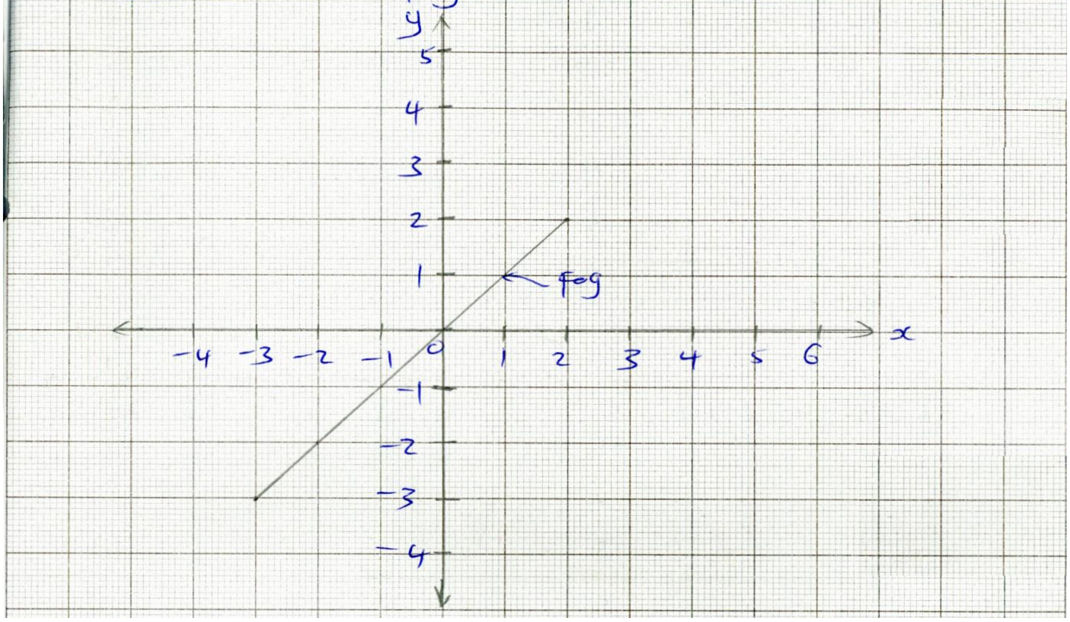
$\therefore$ Vertical Asymptotes are $x = -2$ and $x = 2$

Let: $y = f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 4}$

x	-3	-2	-1	0	1	2	3	4
y	2.4	∞	0	0.75	1.3	∞	0	0.4

6.(a)

GRAPH FOR $Fog = x$



Extract 6.1 *A correct response from one of the candidates*

In Extract 6.1, the candidate demonstrated competences in sketching the graph of composite functions.

Despite the best performance in this question, there were 254 candidates who scored low marks. The candidates found gof instead of fog in part (a) (i). They were not awarded the marks for the procedures of arriving at the final as $gof = x$. In part (a) (iii), several candidates did not use the table in (ii) to draw the graph for fog. As a result they drew the line $y = x$ as a continuous function with arrows on both sides. In part (b), the candidates confused the procedures of determining the horizontal asymptotes with vertical asymptote. Some candidates reversed the calculations of these asymptotes. Hence, they were unsuccessful in tracing the path for the graph of $y = \frac{x^2 - 2x - 3}{x^2 - 4}$. Extract 6.2 shows a sample response from a candidate who did part (b) of this question poorly.

06 (b) Soln (i)

$$Y = \frac{x^2 - 2x - 3}{x^2 - 4}$$

* Horizontal asymptote

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = 2$$

* Vertical asymptote

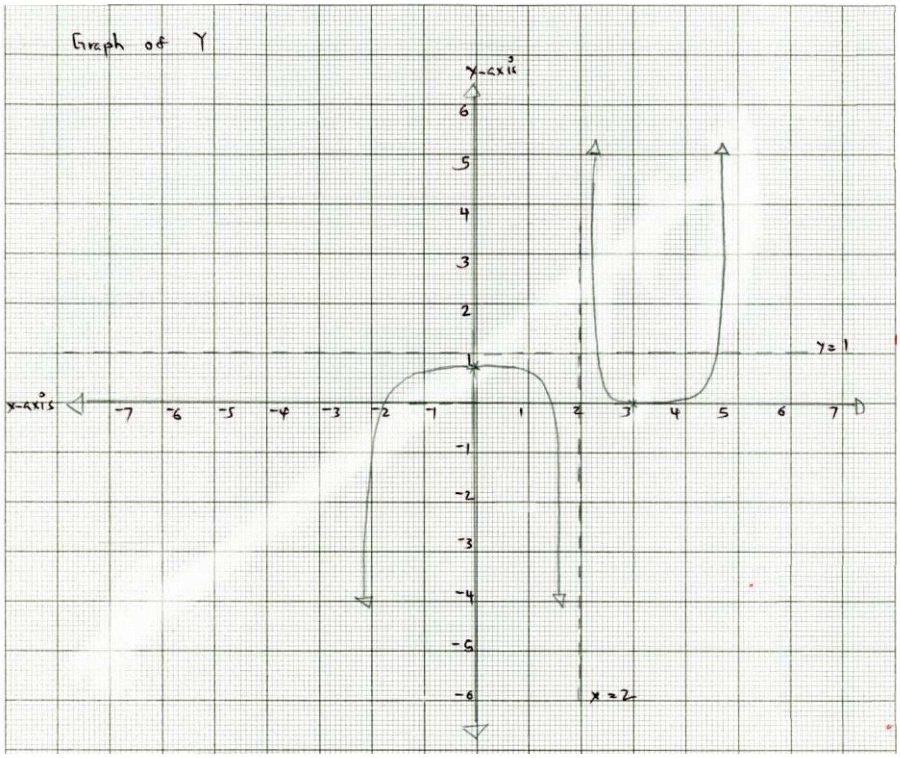
$$\frac{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \frac{1 - 2/x - 3/x^2}{1 - 4/x^2} : \cancel{\text{As } x\ \infty}$$

06(b) (i) $\frac{1 - 2/x - 3/x^2}{1 - 4/x^2}$ $\cancel{\text{As } 1}$

As X approach to ∞, so that $1/x = 0$

$$\frac{1 - 0 - 0}{1 - 0} = \frac{1}{1} = 1$$

$\therefore$ Vertical asymptote is 1 and horizontal asymptote is 2



Extract 6.2: *An incorrect response from one of the candidates*

In Extract 6.2, the candidate was not careful as he/she committed blunders in working out the asymptotes.

2.1.7 Question 7: Numerical Methods

The question had parts (a), (b), (c) and (d). In part (a), the candidates were required to use the Trapezium rule with 5 ordinates to find an approximate value for $\int_0^4 x\sqrt{9+x^2}\,dx$ correct to three decimal places. In part (b), the candidates were required to use the Simpson's rule with 5 ordinates to find an approximation for $\int_0^4 x\sqrt{9+x^2}\,dx$ correct to three decimal places. In part (c), the candidates were required to find the value of the integral $\int_0^4 x\sqrt{9+x^2}\,dx$. In part (d), the candidates were required to compare the results obtained in part (a) and (b) with the results in part (c) and then state the method which gives a better approximation of $\int_0^4 x\sqrt{9+x^2}\,dx$.

The question was attempted by 97.7 percent of the candidates, out of whom 15.4 percent scored from 0 to 3 marks, 16.5 percent from 3.5 to 5.5 marks and 68.1 percent from 6 to 10 marks. Generally, the candidates' performance was good, as 84.6 percent scored above 3 marks. Figure 8 is a summary of the candidates' performance in this question.

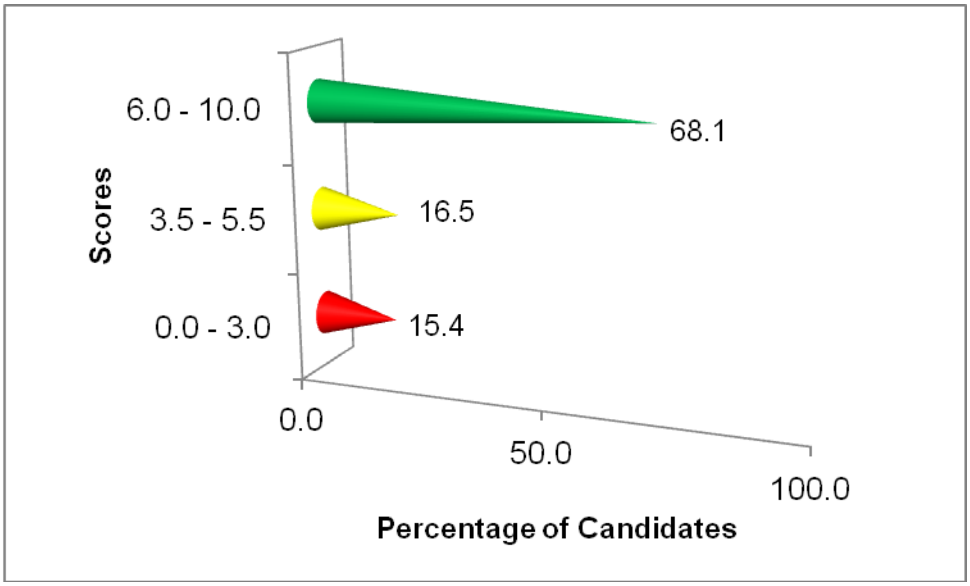


Figure 8: *Candidates' performance in question*

The candidates who answered this question correctly demonstrated the following strengths. In part (a), most of them substituted $a = 4$, $b = 0$ and $n = 5$ into the formula $d = \frac{b-a}{n-1}$ to obtain the value of $h = 1$, constructed a table of values with correct entries for x and $x\sqrt{9+x^2}$, and used a table of values and the Trapezoidal rule $A = \frac{d}{2}[y_1 + y_5 + 2\ \ y_2 + y_3 + y_4\ ]$ to approximate the value of $\int_0^4 x\sqrt{9+x^2}\,dx$ to three decimal places as 33.101. In part (b), the candidates were able to use the table of values in part (a) and the formula $A = \frac{d}{3}[y_1 + y_5 + 4(y_2 + y_4) + 2y_3]$ to approximate the value of $\int_0^4 x\sqrt{9+x^2}\,dx$ correct to three decimal places as 32.661.

In part (c) the candidates allowed $u = 9 + x^2$, $dx = \frac{du}{2x}$, $u = 9$ and $u = 25$ to take the place of $\sqrt{9 + x^2}$, dx and the limits of integration $x = 0$ and $x = 4$ in the integral $\int_0^4 x\sqrt{9 + x^2}\,dx$. Then, they evaluated $\frac{1}{2}\int_9^{25} u^{\frac{1}{2}}du$ correct to three decimal places as 32.667. In part (d), the candidates evaluated the absolute error in relation to Simpson's and Trapezium rule as 0.006 and 0.434 respectively. Finally, the candidates concluded by stating that Simpson's rule gives a better approximation for $\int_0^4 x\sqrt{9 + x^2}\,dx$ than trapezium rule. Extract 7.1 is a sample solution from one of the candidates who performed well.

7a

x	0	1	2	3	4
$y = x\sqrt{9+x^2}$	0	3.16228	7.21110	12.72792	20.
	y_1	y_2	y_3	y_4	y_5

$$A = \frac{d}{2}\left(y_1 + y_5 + 2(y_2 + y_3 + y_4)\right).$$

$$\int_0^4 x\sqrt{9+x^2}\,dx = \frac{1}{2}\left((0 + 20) + 2(3.16228 + 7.21110 + 12.72792)\right)$$

$$\int_0^4 x\sqrt{9+x^2}\,dx = \frac{1}{2}\left(20 + 46.2026\right).$$

$$\int_0^4 x\sqrt{9+x^2}\,dx = 33.1013$$

$$\int_0^4 x\sqrt{9+x^2}\,dx. = 33.101$$

7b Simpson's rule states that

$$A = \frac{d}{3}\left(y_1 + y_n + 4(\text{even ordinates}) + 2(\text{odd ordinates})\right).$$

$$A = \frac{d}{3}\left(y_1 + y_5 + 4(y_2 + y_4) + 2y_3\right)$$

$$\int_0^4 x\sqrt{9+x^2}\,dx = \frac{1}{3}\left((0+20) + 4(3.16228 + 12.72792) + 2(7.21110)\right)$$

$$\int_0^4 x\sqrt{9+x^2}\,dx = \frac{1}{3}\left(20 + 63.5608 + 14.4222\right)$$

$$\int_0^4 x\sqrt{9+x^2}\,dx = 32.661$$

7c	$\int_0^4 x\sqrt{9+x^2}\,dx$	
	~~let $u = 9+x^2$~~	
	~~$du = 2x\,dx$~~	
	$\frac{du}{2} = x\,dx$.	
	$= \int^{4} (\sqrt{u})\,\frac{du}{2}$	
	$= \frac{1}{2}\int (\sqrt{u})\,du = \frac{1}{2}\int u^{1/2}\,du$.	
7c	$= \frac{1}{2}\left(\frac{2u^{3/2}}{3}\right) + c$	
	$= \frac{u^{3/2}}{3} + c$	
	But $u = 9+x^2$	
	$= \frac{1}{3}(9+x^2)^{3/2}\Big]_0^4$	
	$= \frac{1}{3}\left[(25)^{3/2} - (9)^{3/2}\right]$.	
	$= \frac{1}{3}(125 - 27)$.	
	$\doteq 32.667$.	
	$\int_0^4 x\sqrt{9+x^2}\,dx = 32.667$.	
7d.	Absolute error $= \lvert$True value $-$ Measured value$\rvert$	
	Absolute error involved in using Trapezium rule.	
	Absolute error $= \lvert 32.667 - 33.101 \rvert = 0.434$.	
	In Simpson's rule	
	Absolute error $= \lvert 32.667 - 32.661 \rvert = 0.006$	
	Since $0.006 < 0.434$.	
	Hence Simpson's rule gives better approximation	

Extract 7.1 *A correct response from one of the candidates*

The candidates who did not answer the question correctly encountered the following challenges: In parts (a) and (b), the candidates used the formula $d = \frac{a-b}{n}$ instead of the formula $d = \frac{b-a}{n-1}$ to calculate the width for each strip.

These candidates got $d = 0.8$ instead of $h = 1$. The wrong width for each strip led to incorrect filling of a table with x values equal to 0, 0.8, 1.6, 2.4, 3.2, 4 and $x\sqrt{9+x^2}$ value equal to 0, 2.4849, 5.44, 9.2205, 14.0363, and 20. Such candidates obtained incorrect approximations for $\int_0^4 x\sqrt{9+x^2}\,dx$ in both Simpson's and Trapezoidal computations and hence, reversed the conclusions that Trapezoidal rule gives a better approximation for the definite integral $\int_0^4 x\sqrt{9+x^2}\,dx$, see Extract 7.2. However, it was surprising to see how several candidates filled the table of values with correct entries but reversed the calculations for the rules. In part (c), some of the candidates could not find the actual value of $\int_0^4 x\sqrt{9+x^2}\,dx$. This means they lacked the knowledge and skills for doing integration using substitution technique. In part (d), some candidates compared the value obtained by using the Trapezoidal rule with the value obtained by using the Simpson's rule, instead of comparing both values to the exact value for the definite integral.

7 (a) Trapezium rule

$$\frac{h}{2}\left[\text{Sum of Extreme} + 2\left(\text{Sum of Middle}\right)\right]$$

Given 5 ordinates.

$$\frac{4-0}{5} \Rightarrow 4/5 \Rightarrow 0.8$$

	Extremes	Middle
$x_1 = 0$	0	
$x_2 = 0.8$		$2.4839 \simeq 2.484$
$x_3 = 1.6$		5.440
$x_4 = 2.4$		9.220
$x_5 = 3.2$		14.036
$x_6 = 4$	20	
	20	31.18

$$\frac{0.8}{2}\left[20 + 2(31.18)\right]$$

$$\frac{0.8}{2}(82.36)$$

$$\int_0^4 x\sqrt{9+x^2}\,dx \Rightarrow \underline{32.944}$$

7 (b) Simpson's rule states.

$\frac{h}{3}\left[\begin{matrix}\text{Sum of}\\\text{Extreme}\end{matrix} + 2\left(\begin{matrix}\text{Sum of}\\\text{Odd}\end{matrix}\right) + 4\left(\begin{matrix}\text{Sum of}\\\text{Even}\end{matrix}\right)\right]$

$h = \frac{4-0}{5} = 0.8$

	Extreme,	Sum of Odd	Sum of Even
x_1	0		
x_2			2.484
x_3		5.440	
x_4			9.220
x_5		14.036	
x_6	20		
	20	19.476	11.704

$\frac{0.8}{3}\left[20 + 2(19.476) + 4(11.704)\right]$

$\frac{0.8}{3}\left[\overset{105.768}{\cancel{93.756}}\right]$

28.2048

A better approximation method is trapezium rule since its answer is near the true value from (c)

Extract 7.2: *An incorrect response from one of the candidates*

In Extract 7.2, the incorrect length of each strip led to wrong responses in every part of this question.

2.1.8 Question 8: Coordinate Geometry I

The question had parts (a), (b) and (c). In part (a), the candidates were instructed that "If *m* and *n* are the lengths of the perpendicular distance from the origin to the lines $x\cos\theta - y\sin\theta = p\cos 2\theta$ and $x\sec\theta + y\,\text{cosec}\,\theta = p$ respectively." They were required to prove that $m^2 + 4n^2 = p^2$. In part (b), the candidates were required to show that the bisector of the acute angle between $y = x + 1$ and the x-axis has the gradient of $-1+\sqrt{2}$. In part (c), the candidates were informed that a point P lies on the circle of radius 2 whose centre is at the origin. If A is the point $(4, 0)$, they were required to find the locus of a point which divides AP in the ratio $1:2$.

The analysis of data shows that 6624 (65.1%) candidates attempted this question. Therefore the question was skipped by many candidates (34.9%). Also, it was the worst performed question in this examination whereby 81.8 percent of the candidates scored from 0 to 3 marks. Figure 9 shows the percentage of candidates with low, average and good scores.

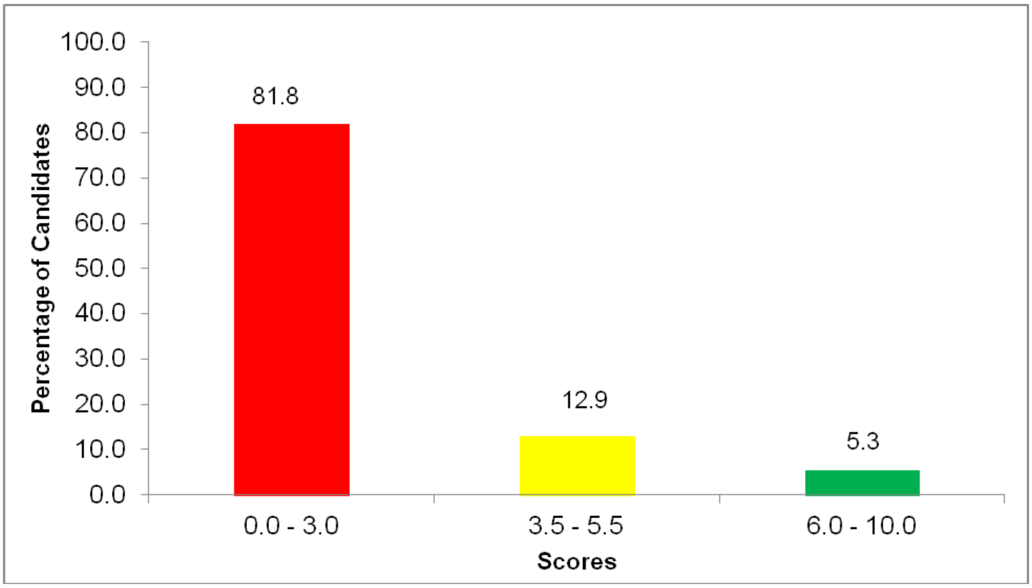


Figure 9: *Candidates' performance in question 8*

As shown in Figure 9 most candidates scored low marks of whom 34.6 percent of them scored 0. The candidates who scored 0 faced the following challenges: In part (a), the candidates failed to apply the formula for calculating the perpendicular distance from the point x_1, y_1 to the line $ax+by+c=0$ on $x\cos\theta - y\sin\theta = p\cos 2\theta$ and $x\sec\theta + y\operatorname{cosec}\theta = p$ in showing that $m^2+4n^2=p^2$. Similarly in part (b), several candidates were unable to apply the formula $d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$ in showing that the bisector of the acute angle between $y=x+1$ and the x-axis has the gradient $-1+\sqrt{2}$. Due to this challenge, most candidates could not identify that when the equation $y=0$ is rearranged to $0x+y+0=0$, the value of a_2 and b_2 in the right hand side of $\frac{|a_1x+b_1y+c|}{\sqrt{a_1^2+b_1^2}}=\frac{|a_2x+b_2y+c|}{\sqrt{a_2^2+b_2^2}}$ becomes 0 and 1 respectively. In part (c), the candidates failed to apply the ratio theorem $m(x, y)=\left(\frac{nx_1+mx_2}{n+m}, \frac{ny_1+my_2}{n+m}\right)$ to find the required equation of locus. The analysis of the candidates responses shows that many candidates used the formulae $x^2+y^2+2gx+2fy+c$ and $r=\sqrt{g^2+f^2-c}$, the radius 2 of a circle and point $(4, 0)$ to find the locus of a point which divides AP in the ratio 1:2. These candidates ended up getting $x^2+y^2+8x+12=0$ instead of $3x^2+3y^2-16x+20=0$. Extract 8.1 is a sample solution from a candidate who got the question wrong.

Q Given: $r=2$.

$(-g,-f)=(0,0)$

from $x^2+y^2+2gx+2fy+c=0$

$r=\sqrt{g^2+f^2-c}$

$(2)^2=(\sqrt{g^2+f^2-c})^2$

$4=g^2+f^2-c$

$c=g^2+f^2-4$

$c=$

$g=-4,\ f=0$

$c=(-4)^2+(0)^2-4$

$c=16-4$

$c=12$

Then

$x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+8x+0y+12=0$

$\therefore$ The equation of the point is $x^2+y^2+8x+12=0$.

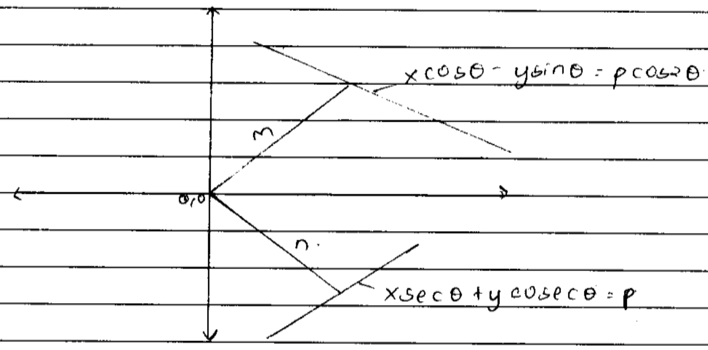
Extract 8.1: *An incorrect response from one of the candidates*

In Extract 8.1, the candidates lacked competence in solving a locus problem.

Only 19, out of the 6624 candidates who attempted the question, scored all 10 marks. In part (a), the candidates were able to rearrange the given lines into

$x\cos\theta - y\sin\theta - p\cos 2\theta = 0$ and $x\sec\theta + y\,\mathrm{cosec}\,\theta - p = 0$ respectively, find the perpendicular distances $m = p\cos 2\theta$ and $n = p\sin\theta\cos\theta$ of origin $(0, 0)$ from the given lines and they were able to add m^2 and n^2 to arrive at $m^2 + 4n^2 = p^2$. In part (b), the candidates were able to equate the distances of a point on the bisector of the acute angle from the line $y = x+1$ and the x-axis and manipulate the relation $\frac{x-y+1}{\sqrt{2}} = \frac{y}{\sqrt{1}}$ to show that the bisector has the gradient of $-1+\sqrt{2}$. In part (c), they were able to apply the formula for the point which divides the line segment AP internally whereby the point $P(a, b)$ is on the circle, express the coordinates of P in terms of x and y as $a = 3x-8$ and $b = 3y$ and substitute a and b into $a^2 + b^2 = 4$ to get the locus $3x^2 + 3y^2 - 16x + 20 = 0$. Extract 16.2 shows a solution of the candidates who performed well in this question

8



$m = \left| \frac{x\cos\theta - y\sin\theta - p\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} \right|$

but $(x, y) = (0, 0)$; $\cos^2\theta + \sin^2\theta = 1$

so, $m = \left| \frac{-p\cos 2\theta}{1} \right|$

$n = \left| \frac{x\sec\theta + y\,\mathrm{cosec}\,\theta - p}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}} \right|$

but $(x, y) = (0, 0)$

$\sec^2\theta + \mathrm{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta\sin^2\theta}$

$\sec^2\theta + \mathrm{cosec}^2\theta = \frac{1}{\cos^2\theta\sin^2\theta}$

$$n = \left| \frac{-p}{\sqrt{\frac{1}{\cos^2\theta \sin^2\theta}}} \right|$$

$n = -(\cos\theta \sin\theta)\, p$

$m^2 = (p\cos 2\theta)^2$

$4n^2 = 4(\cos^2\theta \sin^2\theta)\, p^2$

8. $m^2 + 4n^2 = p^2(\cos 2\theta)^2 + p^2(4\cos^2\theta \sin^2\theta)$

$= p^2 [(\cos 2\theta)^2 + (\sin 2\theta)^2]$

$= p^2$

$\therefore m^2 + 4n^2 = p^2$ – Hence prooved

b). $y = x + 1$

$y = 0$ – x-axis

from

$$\left| \frac{Ax + By + C}{\sqrt{A^2 + B^2}} \right| = \left| \frac{ax + by + c}{\sqrt{a^2 + b^2}} \right|$$

$$\left| \frac{x - y + 1}{\sqrt{1^2 + (-1)^2}} \right| = \left| \frac{0x + y + 0}{\sqrt{0^2 + 1^2}} \right|$$

$$\frac{x - y + 1}{\sqrt{2}} = \pm (y)$$

for acute angle we take the positive value

so,

$x - y + 1 = y\sqrt{2}$

$y(\sqrt{2} + 1) = x + 1$

$$y = \frac{x}{\sqrt{2} + 1} + 1$$

$y = mx + c$

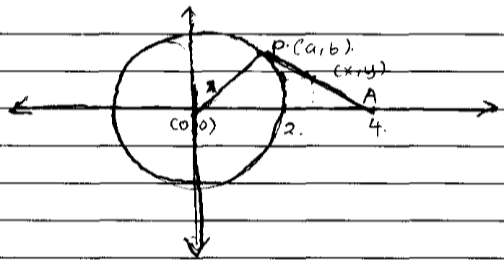
$$m = \frac{1}{\sqrt{2} + 1}$$

On rationalizing

$$m = \frac{1(\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

Slope, $m = -1 + \sqrt{2}$ Hence shown

8 (c)



from $(x,y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$

$m : n = 1 : 2$

$(x_1, y_1) = (4, 0)$

$(x_2, y_2) = (a, b)$

$x^2 + y^2 = 4$ — circle.

$a^2 + b^2 = 4$ — ①

$(x,y) = \left(\frac{1(a) + 2(4)}{3}, \frac{1(b) + 2(0)}{3}\right)$

$(x,y) = \left(\frac{8 + a}{3}, \frac{b}{3}\right)$

$x = \frac{8+a}{3}$ $\quad y = \frac{b}{3}$

$a = 3x - 8$ $\quad b = 3y$

Into eqn ①

$(3x-8)^2 + (3y)^2 = 4$

$(3x)^2 - 48x + 64 + 9y^2 = 4$

$x^2 + y^2 - \frac{16}{3}x + \frac{20}{3} = 0$

$\therefore$ The locus is $x^2 + y^2 - \frac{16}{3}x + \frac{20}{3} = 0$

Extract: 8.2 *A correct response from one of the candidates*

2.1.9 Question 9: Integration

The question consisted of three parts (a), (b) and (c). In part (a), the candidates were required to find the indefinite integral $\int \frac{\sin x}{1+\cos x} dx$. In part (b), the candidates were required to evaluate the definite integral $\int_1^{e^2} \ln x \, dx$. In part (c), the candidates were required to find the length of an arc of the curve given by the parametric equations $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ from $\theta = 0$ to $\theta = 2\pi$.

The analysis of data shows that 8520 (83.7%) candidates attempted this question. Amongst, 6258 (73.5%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was good. Figure 10 gives the percentage of candidates who obtained weak, average and good performance.

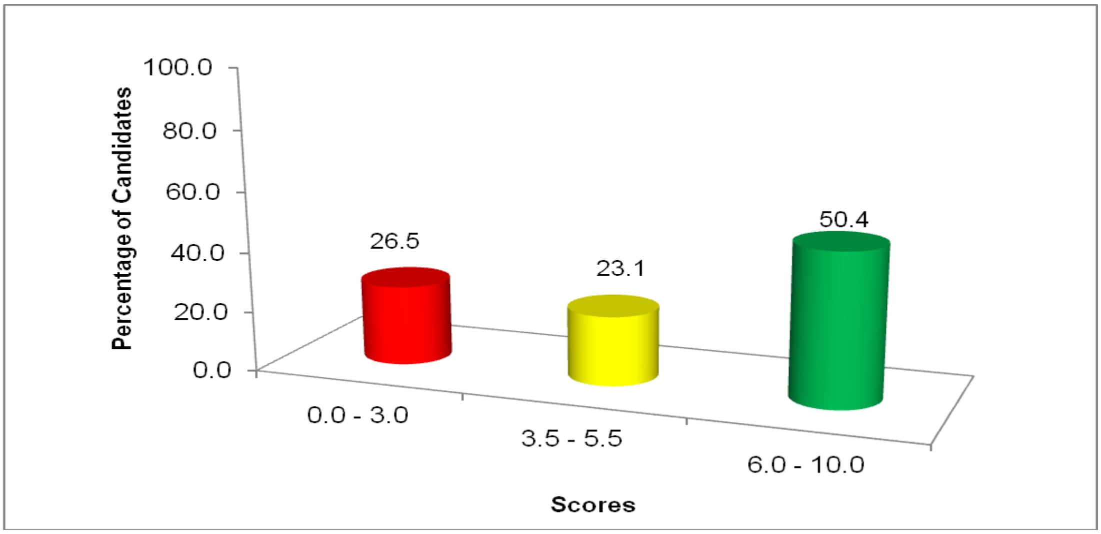


Figure 10: *Candidates' performance in question 9*

Figure 10 shows that 50.4 percent of candidates scored marks ranging from 6.0 to 10 of which 1327 equivalent to 15.6 percent scored all 10 marks. The candidates who scored all marks demonstrated the following competences. In part (a), they used the t-formula while others used the substitution $u = 1 + \cos x$ to find the indefinite integral $\int \frac{\sin x}{1+\cos x}dx$ as $\ln\left|\sec^2 \frac{x}{2}\right| + c$. In part (b), the candidates used the integration by parts technique $\int udv = uv - \int vdu$ to evaluate the given definite integral. In this case they identified suitable functions representing u ($u = \ln x$) and $dv = \int dx$ then differentiated u to get $du = \frac{dx}{x}$ integrated dv to get $v = x$. Finally they evaluated $\int_1^{e^2} \ln x dx$ by putting $u = \ln x$, $v = x$ and

$du = \frac{dx}{x}$ into $\int udv = uv - \int vdu$ to obtain 8.38906. In part (c), the candidates were able to find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ of $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ as $a\ \theta\cos\theta$ and $a\ \theta\sin\theta$ respectively, evaluate the length an arc by putting the values of $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ into the formula $l = \int_{\theta_1}^{\theta_2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$ ending up with $2\pi^2 a$ units. Extract 9.1 represents a sample solution from one of the candidates who had adequate skills on the concepts of integration.

9a) $\int \frac{\sin x\ dx}{1+\cos x}$ Soln,

$\int \frac{\sin x\ dx}{1+\cos x}$

let $1+\cos x$ be u

$du/dx = -\sin x$

$dx = -du/\sin x$

$= \int \frac{\sin x}{u} \cdot \frac{-du}{\sin x}$

$-\int \frac{du}{u}$

$= -\ln u + c$, but $u = 1+\cos x$

$= -\ln(1+\cos x) + C.$

b) $\int_1^{e^2} \ln x\, dx$ Soln,

from integration by parts, ILATE

$\int_1^{e^x} x^0 \ln x dx$

let $\ln x$ be u,

$du/dx = 1/x$, $du = dx/x$

$\int dv = \int x^0 dx$

$V = x$

$\int udv = uv - \int vdu$

$\int x^0 \ln x dx = x\ln x - \int x \frac{dx}{x}$

$\int \ln x dx = x\ln x - \int dx$

$\int_1^{e^2} \ln x\ dx = x\ln x - x$

$$\int_1^{e^2} \ln x\,dx = (e^2\ln e^2 - e^2) - ((1)\ln(1) - (1))$$

$$\int_1^{e^2} \ln x\,dx = 2e^2 - e^2 + 1$$

$$\int_1^{e^2} \ln x\,dx = e^2(2-1) + 1$$

$$\int_1^{e^2} \ln x\,dx = \underline{\underline{e^2 + 1}}$$

9c) Soln;

$$S = \int \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta.$$

$$x = a(\cos\theta + \theta\sin\theta)$$

$$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$$

$$\frac{dx}{d\theta} = a\theta\cos\theta$$

$$y = a(\sin\theta - \theta\cos\theta)$$

$$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$$

$$\frac{dy}{d\theta} = a\theta\sin\theta$$

$$S = \int \sqrt{(a\theta\cos\theta)^2 + (a\theta\sin\theta)^2}\,.\,d\theta$$

$$S = \int \sqrt{(a\theta)^2[\cos^2\theta + \sin^2\theta]}\,.\,d\theta$$

$$S = \int \sqrt{(a\theta)^2}\,d\theta$$

$$S = \int_0^{2\pi} a\theta\, d\theta$$

$$S = a\int_0^{2\pi} \theta\, d\theta$$

$$a\left[\frac{\theta^2}{2}\right]_0^{2\pi}$$

$$a\frac{(2\pi)^2}{2} = \frac{a4\pi^2}{2} = a2\pi^2$$

$$\therefore \quad S = a2\pi^2$$

Extract 9.1: *A correct response from one of the candidates*

On the other hand, 26.5 percent of the candidates scored from 0 to 3 marks of which 613 (7.2%) candidates scored 0. The candidates who scored no marks faced the following challenges: In part (a), the candidates were unable to apply the substitution $u = 1 + \cos x$ to the given integral. For instance, one of the candidates selected $u = 1 + \cos x$ but during the process of finding the first derivative expressed dx in terms of du and $\sin x$ as $dx = \frac{du}{\sin x}$ instead of $dx = -\frac{du}{\sin x}$. The candidate got $\int \frac{\sin x}{1 + \cos x} dx = \ln|1 + \cos x| + c$ instead of $\int \frac{\sin x}{1 + \cos x} dx = \ln\left(\frac{1}{1 + \cos x}\right) + c$. Some candidates who applied the t-formula committed errors in quoting the formula of $\sin x$, $\cos x$ and dx therefore they were unable to reach the final answer. In part (b), some candidates used the substitution $u = \ln x$ which did not help them to write the original integral in terms of u as they found themselves required to find $\int uxdu$. Others integrated $\ln x$ to get $\frac{1}{x}$ which was wrong. In part (c), the candidates used wrong formulae to calculate the required length of the curve. The incorrect formula which were frequently noted by the markers include $l_{AB} = \int_{\theta_1}^{\theta_2} \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$ and $l_{AB} = \int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dy}{d\theta}\right)^2}$. Such candidates ended up with the incorrect length such as $4a^2\pi$ units instead of $2\pi^2 a$ units. Another common mistake was failure of candidates to correctly differentiate $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ with respect to θ to get $a\ \theta\cos\theta$ and $a\ \theta\sin\theta$. Extract 9.1 is a sample answer from one of the candidates who answer part (c) incorrectly.

		use only
	~~[illegible]~~	
9(c)	$x = a(\cos\theta + \theta\sin\theta)$	
	$x = a\cos\theta + a\theta\sin\theta$.	

$\frac{dx}{d\theta} = -a\sin\theta + a\cos\theta \cdot$

$y = a\sin\theta - a\theta\cos\theta,$

$\frac{dy}{d\theta} = a\cos\theta + a\sin\theta \cdot$

$$L_A = \int_{\theta_1}^{\theta_2} \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 \cdot d\theta \cdot$$

$$= \int_{0}^{2\pi} (-a\sin\theta + a\cos\theta)^2 + (a\cos\theta + a\sin\theta)^2 \cdot d\theta \cdot$$

$$\int_{0}^{2\pi} (a^2\sin^2\theta + a^2\cos^2\theta) + (a^2\cos^2\theta + a^2\sin^2\theta)\, d\theta \cdot$$

$$\int_{0}^{2\pi} a^2(\sin^2\theta + \cos^2\theta) + a^2(\cos^2\theta + \sin^2\theta)\, d\theta$$

$$\int_{0}^{2\pi} a^2 + a^2 \; d\theta$$

$$2a^2 \int_{0}^{2\pi} d\theta$$

$$2a^2\theta \Big]_{0}^{2\pi}$$

$$= 2a^2(2\pi)$$

$$= 4a^2\pi \text{ units.}$$

Extract 9.2: *An incorrect answer from one of the candidates*

In Extract 9.2, the candidates lacked an understanding on how to differentiate the parametric equations and also he/she was unable to recall the correct formula to find the length of an arc.

2.1.10 Question 10: Differentiation

In this question, the candidates were required to (a) find the derivative of x^n from first principles, (b) use Taylor's theorem to expand $\cos\left(\frac{\pi}{6} + h\right)$ in ascending powers of h up to the term containing h^3 and (c) find the least possible value of x^2+y^2 given that $x+y=10$.

The analysis of data shows that out of 8699 (85.5%) candidates who responded to the question, 5359 (61.6%) candidates scored the marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was good. Figure 11 shows the percentage of candidates who had weak, average and good scores.

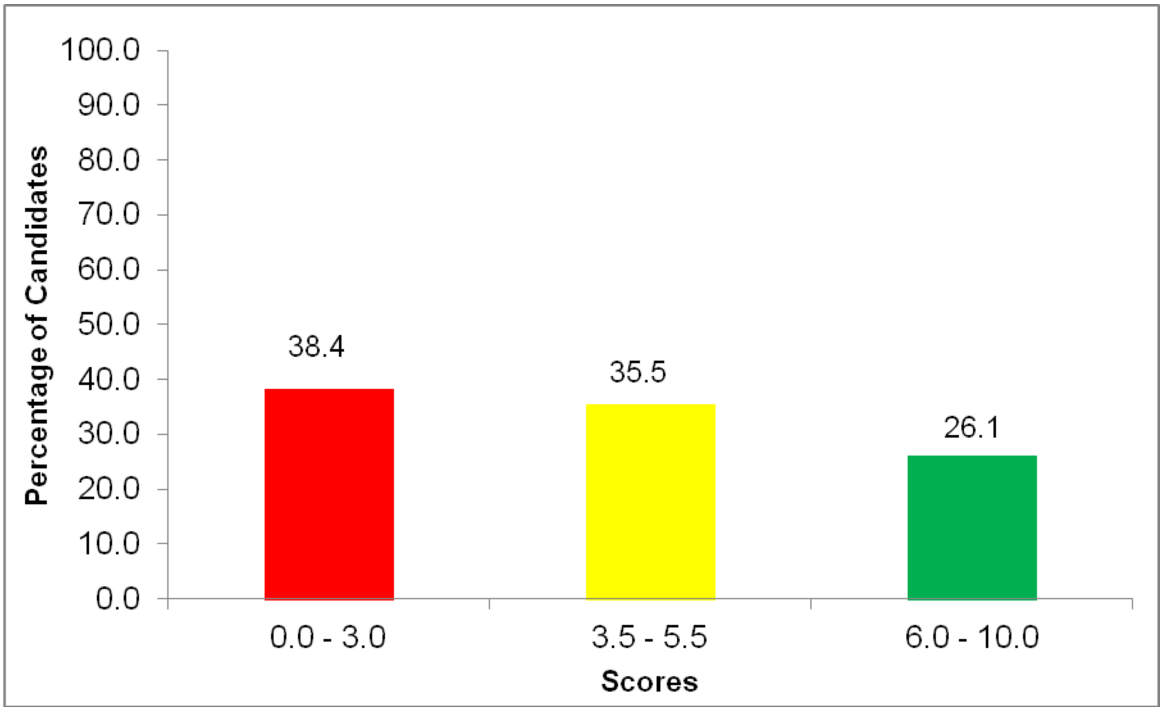


Figure 11: *Candidates' performance in question 10*

Figure 11 shows that 2271 (26.1%) candidates scored from 6 to 10 marks with 160 (1.8%) candidates scoring all 10 marks. The candidates who answered part (a) correctly were able to expand $(x+h)^n$ to $x^n+nx^{n-1}+\dfrac{n(n-1)x^{n-2}h^2}{2!}+\dots$, substitute x^n and the expansion of $(x+h)^n$ into the formula $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ to get $f'(x^n)=nx^{n-1}$. The candidates who answered part (b) correctly were able to replace the terms of the Taylor expression $f\left(\dfrac{\pi}{6}+h\right)=f\left(\dfrac{\pi}{6}\right)+f'\left(\dfrac{\pi}{6}\right)h+\dfrac{f''\left(\dfrac{\pi}{6}\right)h^2}{2!}+\dfrac{f'''\left(\dfrac{\pi}{6}\right)h^3}{3!}+\dots$ with $f\left(\dfrac{\pi}{6}\right)=\dfrac{\sqrt{3}}{2}$, $f'\left(\dfrac{\pi}{6}\right)=\dfrac{-1}{2}$, $f''\left(\dfrac{\pi}{6}\right)=\dfrac{-\sqrt{3}}{2}$ and $f'''\left(\dfrac{\pi}{6}\right)=\dfrac{1}{2}$ to get

$\cos\left(\frac{\pi}{6}+h\right)=\frac{\sqrt{3}}{2}-\frac{1}{2}h-\frac{\sqrt{3}h^2}{4}+\frac{h^3}{12}$. The candidates who answered part (c) correctly were able to: express p in terms of x as $p=x^2+(10-x)^2$; find the first derivative of p ($\frac{dp}{dx}=4x-20$), set the first derivative of p is equal to 0 to get $x=5$, identify that p has a least value since $\frac{d^2p}{dx^2}=4>0$ and were able to replace x with 5 in the equation $p=x^2+(10-x)^2$ to get the least value of x^2+y^2 as 50. Extract 10.1 is a sample solution from a candidate who did this question well.

Qn 10 (a) solution.

Derivative of x^n from the first principles.

from the formula.

$$\frac{dy}{dx}=f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

where by

$$f(x)=x^n.$$

$$f(x+h)=(x+h)^n.$$

$$f'(x)=\lim_{h\to 0}\left(\frac{(x+h)^n-x^n}{h}\right) \quad \text{——— (1)}$$

from Binomial expansion.

$$(x+h)^n=x^n+nx^{n-1}h+\frac{n(n-1)}{2!}x^{n-2}h^2+\frac{n(n-1)(n-2)}{3!}x^{n-3}h^3$$

Substitute expansion into equation (1).

$$f'(x)=\lim_{h\to 0}\left(\frac{x^n+nx^{n-1}h+\frac{n(n-1)}{2!}x^{n-2}h^2+\frac{n(n-1)(n-2)}{3!}x^{n-3}h^3-x^n}{h}\right)$$

$$f'(x)=\lim_{h\to 0}\left(\frac{nx^{n-1}h+\frac{n(n-1)}{2!}x^{n-2}h^2+\frac{n(n-1)(n-2)}{3!}x^{n-3}h^3}{h}\right)$$

$$f'(x)=\lim_{h\to 0}\left(nx^{n-1}+\frac{n(n-1)}{2!}x^{n-2}h+\frac{n(n-1)(n-2)}{3!}x^{n-3}h^2\right)$$

$$f'(x)=\lim_{h\to 0}\left(nx^{n-1}+\frac{n(n-1)}{2!}x^{n-2}(0)+\frac{n(n-1)(n-2)}{3!}x^{n-3}(0^2)\right)$$

$$\therefore f'(x)=nx^{n-1}$$

$\therefore$ Derivatives of $x^n=nx^{n-1}$.

Qn 10 (b) solution.

use Taylor theorem to expand $\cos\left(\frac{\pi}{6}+h\right)$ to h^3.

from Taylors theorem below.

$$f(x+h) = f(h) + f'(h)x + \frac{f''(h)x^2}{2!} + \frac{f'''(h)x^3}{3!} + \frac{f^{iv}(h)x^4}{4!}$$

$f(h) = \cosh$

let $\cos\left(\frac{\pi}{6}+h\right) = \cos(x+h)$

$f(h) = \cosh$, $f(x) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

$f'(h) = -\sinh$, $f'(x) = -\sin\frac{\pi}{6} = -\frac{1}{2}$.

$f''(h) = -\cosh$, $f''(x) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

$f'''(h) = \sinh$, $f'''(x) = \sin\frac{\pi}{6}$ $\frac{1}{2}$.

from..

$$f(x+h) = f(h) + f'(h)x + \frac{f''(h)x^2}{2!} + \frac{f'''(h)x^3}{3!}$$

$$\cos\left(\frac{\pi}{6}+h\right) = \frac{\sqrt{3}}{2} + \left(\frac{-1}{2}\right)x + \left(-\frac{\sqrt{3}}{2}\right)\frac{x^2}{2!} + \left(\frac{1}{2}\right)\frac{x^3}{3!}$$

$$\cos\left(\frac{\pi}{6}+h\right) = \frac{\sqrt{3}}{2} - \frac{x}{2} - \frac{\sqrt{3}\cdot x^2}{4} + \frac{x^3}{12}.$$

$$\cos\left(\frac{\pi}{6}+h\right) = \frac{6\sqrt{3} - 6x - 3\sqrt{3}x^2 + x^3}{12}$$

Qn10. (c) Solution.

$x+y = 10$. ——— equation (i).

Least possible value of x^2+y^2.

from.

$(x+y)^2 = x^2+y^2+2xy$.

$x^2+y^2 = (x+y)^2 - 2xy$

$x^2+y^2 = (x+y)^2 - 2xy$

from equation (i).

$x+y = 10$.

$x^2+y^2 = (10)^2 - 2xy$.

$x^2+y^2 = 100 - 2xy$.

$y = (10-x)$

$x^2+y^2 = 100 - 2x(10-x)$

$x^2+y^2 = 100 - 20x + 2x^2$

$x^2+y^2 = 2x^2 - 20x + 100$.

$\frac{dy}{dx}(x^2+y^2) = 4x - 20$

from $\frac{dy}{dx}(x^2+y^2) = 0$.

$4x - 20 = 0$.

$\frac{4x}{4} = \frac{20}{4}$

	$x = 5.$	
	$\frac{d^2}{dx^2}(x^2+y^2) = 4.$	
	From $(x^2+y^2) = 2x^2 - 20x + 100.$	
	$(x^2+y^2) = 2(5)^2 - 20(5) + 100$	
	$(x^2+y^2) = 50$	
	$\therefore$ The values possible is $(5, 50)$	

Extract 10.1: *A correct response from one of the candidates*

On the other hand, 38.4 percent of the candidates scored from 0 to 3 marks out of which 3.7 percent scored 0. The candidates who answered this question incorrectly faced difficulties as follows: In part (a), some candidates did not show the procedures on how they arrived at $f'(x^n) = nx^{n-1}$. Other candidates failed to expand $(x+h)^n$ correctly. Most of them expanded it as $x^n + nx^{n-1}h + n(n-1)x^{n-2}h^2 + ...$ or $x^n + nx^{n-1}h + n(n-1)x^{n-2}h^2 + ...$ and several formulated it $f(x+h)$ as x^{n+h}. In part (b), the majority of candidates recalled correctly Taylor's series expression for $f(a+h)$ but they were unable to evaluate the derivatives of $f\ x$ at $x = \frac{\pi}{3}$. In part (c), some candidates differentiated $x + y = 10$ instead of $p = x^2 + y^2$. Other candidates differentiated $p = x^2 + y^2$ implicitly as $2y\frac{dy}{dx} + 2x = 0$ instead of $\frac{dp}{dx} = 2x + 2y\frac{dy}{dx}$. Such candidates were unable to find the required least value. Extract 10.2 is a sample solution of a candidate who did part (c) of this question badly.

10c	$x + y = 10$	
	Given x^2+y^2	
	$2y\,dy/dx + 2x = 0$	
	$2y\,dy/dx = -2x$	
	$dy/dx = x/y.$	
	at $dy/dx = 0$	
	$x = 0.$	
	then, $0 + y = 10$	
	$y = 10$	

$y\,dy/dx - x = 0$

$(dy/dx)^2 + y\,d^2y/dx^2 - 1 = 0$

at $y = 10$

$10\,dy/dx = 1$

$dy/dx = 1/10$ at $x = 0$

$\therefore$ the least possible value is 0.

Extract 10.2: *An incorrect response from one of the candidates*

In Extract 10.2, the candidate failed to solve a problem involving minimum values.

2.2 142/2 ADVANCED MATHEMATICS 2

2.2.1 Question 1: Probability

The question was

(a) Eggs are packed in boxes of 500. On average 0.7% of the eggs are found to be broken. Find the probability that in a box of 500 eggs;

(i) exactly 3 eggs are broken

(ii) at least 2 eggs are broken.

(Write your answers in four significant figures)

(b) As an experiment, a temporary roundabout is constructed at the crossroads. The time, X in minutes, which vehicles have to wait before entering the roundabout is a random variable having the following probability density function $f(x) = \begin{cases} 0.8 - 0.32x, & 0 \le x \le 2.5 \\ 0, & \text{otherwise} \end{cases}$. Find the mean waiting time for vehicles and standard deviation for the distribution.

(c) The mean weight of 600 male villagers in a certain village is 79.7 kg and the standard deviation is 6 kg. Assuming that the weights are normally distributed, find how many villagers weigh more than 90 kg.

(d) How many possible combinations of six questions are there in an examination paper consisting of a total of eight questions?

The analysis shows that the question was attempted by 10015 (98.4%) candidates of whom 15.7 percent scored from 9 to 15 marks, 28.7 percent from 5.5 to 8.5 marks and 55.7 percent from 0 to 5 marks. Further analysis indicates that 31 (0.3%) candidates did the question well and scored all 15 marks while 1256 (12.5%) candidates scored 0. The candidates' performance in this question was average as shown in Figure 12.

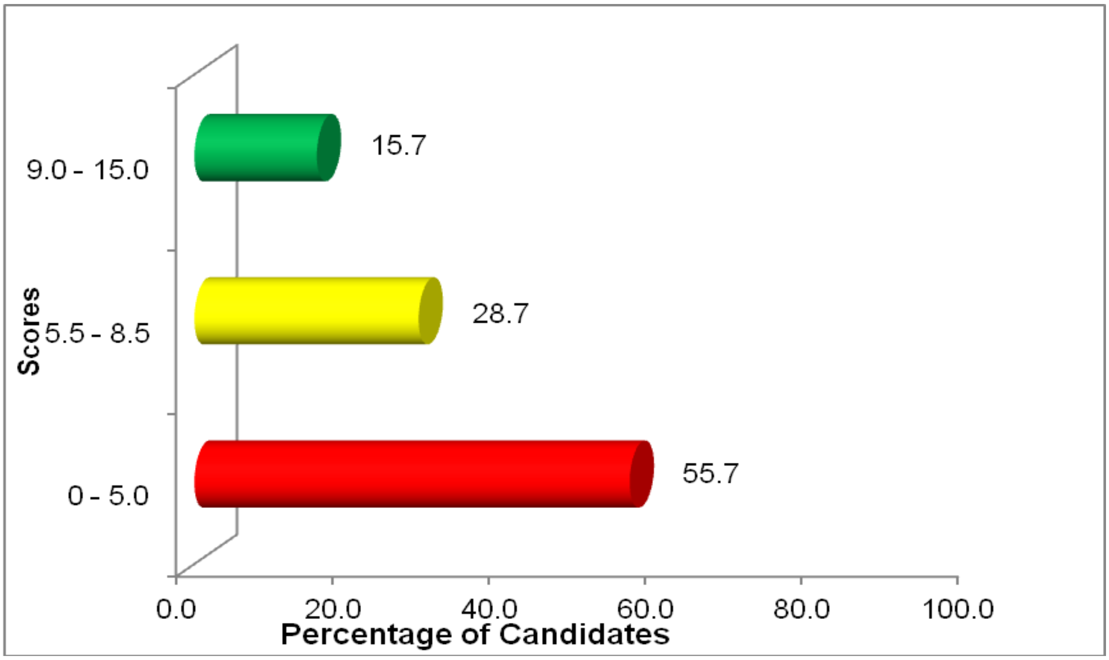


Figure 12 *Candidates' performance in question 1*

The analysis of the candidates' solutions reveals that the candidates who answered this question correctly demonstrated the following strengths. In part (a), the candidates realised that $n > 50$ and $p < 0.1$ and therefore the problem can be solved by using the Poisson's distribution formula $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where $\lambda = 500 \times 0.07 = 3.5$. Hence, they correctly computed the required probabilities to 4 significant figures as 02158 and 08641 respectively. In part (b), the candidates were able to find the mean waiting time by using the formula $E(x) = \int_a^b xf(x)dx$ to get 0.8333 and standard deviation using the formula $\sigma = \sqrt{E(x)^2 - E(x)^{2}}$ and $E(x^2) = \int_a^b x^2 f(x)dx$ to obtain 0.5893. In part (c), the candidates were able to identify that the villagers weighing more than 90 kg must weigh at least 90.5 kg of which when expressed in standard units is equal to $z = \frac{90.5 - 79.7}{6} = 1.8$. Then, they used calculators or statistical tables to evaluate the proportion of villagers as the area to right of $z = 1.8$ to get 0.03593. Finally, they multiplied 0.03593 with 600 to get 22 as the number of villagers weighing more than 90 kg. The candidates who answered part (d) used the formula ${}^nC_r = \frac{n!}{r!(n-r)!}$ to find the number of possible combinations of six questions out of eight questions as 28. A sample answer from one of such candidates who answered part (c) and (d) correctly is shown in Extract 11.1

1 (c) Soln.

Given, mean (μ) = ~~600~~ 79.7kg

Standard Deviation (σ) = 6kg

Required, Number of weigh more than 90kg

1st $P(X > 90) = ?$

From

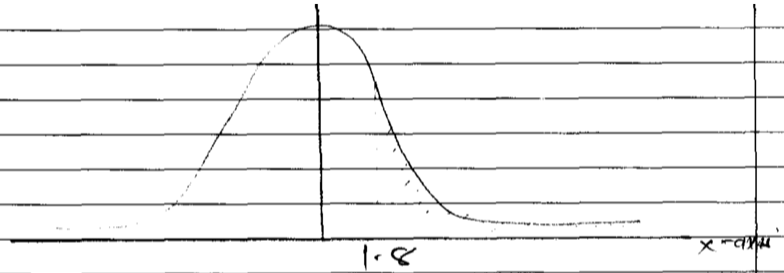
$$z = \frac{x - \mu}{\sigma}$$

For continuity

$$P(X > 90.5) = P\left(z > \frac{90.5 - 79.7}{6}\right)$$

$$P(X > 90.5) = P(z > 1.8)$$

Consider, x-axis.



$P(Z > 1.8) = P(1.8)$
$= 0.03593$

1 (c) Number of villages:
$= 0.03593 \times 600$

$= 22$ villagers

∴ Number of villagers weigh more than 90kg = 22 villagers

1 (d) soln

Given,
$n = 8$
$r = 6$

from: ${}^{n}C_r = \frac{n!}{(n-r)!\,r!}$

${}^{8}C_6 = \frac{8!}{(8-6)!\,6!}$

${}^{8}C_6 = \frac{8!}{2!\cdot 6!}$

$= \frac{8 \times 7 \times 6!}{2 \times 6!}$

$= 56/2 = 28$

∴ There are 28 combination of six questions.

Extract 11.1: *A correct response from one of the candidates*

On the other hand, 1256 (12.5%) candidates who got the question wrong encountered the following challenges. In part (a), most of them used the binomial distribution formula $P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$ instead of the poison distribution formula $P(X = x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$. These candidates did not know the assumptions underlying the application of the two probability distributions. In part (b), many candidates evaluated the definite integrals $E\ x = \int_0^{2.5} 0.8 - 0.32x\ dx$ and $E\ x^2 = \int_0^{2.5} 0.8 - 0.32x\ ^2 dx$ to get the mean waiting time and expectation of x^2. The candidates were supposed to understand that the mean waiting time is the expectation of x and is given by $E(x) = \int_0^{2.5} (0.8x - 0.32x^2)dx$ while $E\ x^2$ is used to find the standard deviation and it is given by the expression $E(x^2) = \int_a^b x^2 f(x)dx = \int_0^{2.5} (0.8x^2 - 0.32x^3)dx$. In part (c), the candidates did not add 0.5 kg to 90 kg to get x is equal to 90.5 kg. Such candidates ended up calculating z score directly using $x = 90$ kg instead of $x = 90.5$ kg and thus they incorrectly calculated the number of villagers weighing more than 90 kg as 26 instead of 22. In part (d), many candidates confused the formula for calculating combination with permutation. The frequently noted error was using the formula ${}^{8}P_{6}$ to get 20,160 instead of ${}^{8}C_{6}$ to get 28 as the expected answer. Extract 11.2 is a sample response of a candidate who did part (a) badly.

1a) $n = 500$

$p = 0.7\% = 0.007$

$p + q = 1$

$0.007 + q = 1$

$q = 1 - 0.007$

$q = 0.993$

1a)(i) $P(x = 3) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{500}C_{3} (0.007)^{3} \cdot (0.993)^{500-3}$

$= 0.2164$

$\therefore$ Probability $= 0.2164$

1a(ii) $P(X \geq 2)$

$= 1 - [P(X=0) + P(X=1)]$

$P(X=0) = {}^{500}C_0 (0.007)^0 (0.993)^{500-0}$

$P(X=0) = 0.02983$

$P(X=1) = {}^{500}C_1 (0.007)^1 (0.993)^{500-1}$

$P(X=1) = 0.1051$

$P(X \geq 2) = 1 - (0.02983 + 0.1051)$

$\therefore P(X \geq 2) = 0.8651$

$\therefore$ Probability $= 0.8651$

Extract 11.2: *An incorrect answer from one of the candidates*

In Extract 11.2**,** the candidate could not solve the given question in part (a) using Poisson distribution.

2.2.2 Question 2: Logic

This question had parts (a), (b) and (c). In part (a), the candidates were required to use the laws of propositions of algebra to simplify the compound statement $(p \wedge q) \vee [\sim r \wedge (q \wedge p)]$. In part (b), the candidates were given the argument "If he begs pardon then he will remain in school. Either he is punished or he does not remain in school. He will not be punished. Therefore, he did not beg pardon". They were required to describe this argument in symbolic form and test its validity by using a truth table. In part (c), the candidates were required to construct a network diagram for the proposition $(p \wedge q) \wedge [(r \vee s) \wedge w]$.

The analysis of data shows that the question was attempted by 10169 (99.9%) candidates. Also, the analysis shows that 7 (0.1%) candidates did not attempt this question. Further analysis shows that 92.9 percent of the candidates scored above 5 marks of which 18.9 percent scored all 15 marks. Thus, the question was among four questions which were best performed in this examination rating the second position in rank order of the candidates' performance. Figure 13 shows the percentages of candidates with weak, average and good performance.

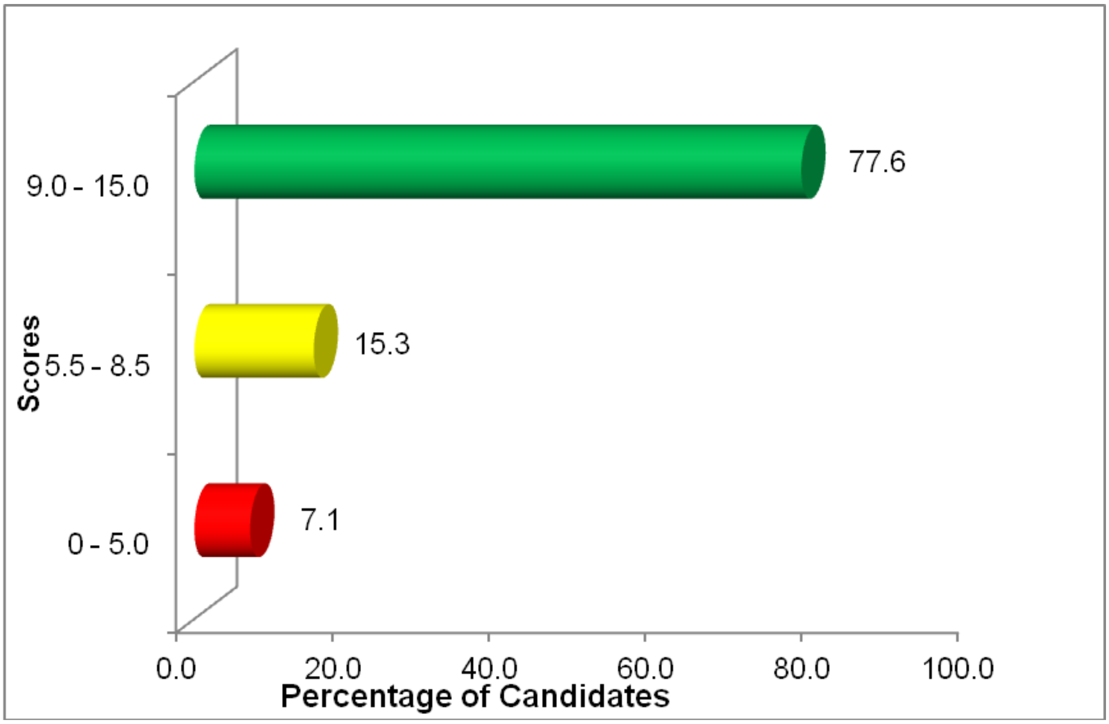


Figure 13: *Candidates' performance in question 2*

As shown in Figure 13, many candidates (77.6%) scored from 9 to 15 marks. These candidates demonstrated the following strengths: In part (a), the candidates managed to simplify the given statement into $p \wedge q$ by using commutative, identity, idempotent and distributive laws of propositions of algebra. In part (b), the candidates considered the letters p, q and r to represent the components he begs pardon, he remains in school and he is punished respectively. Then, they connected letters with logical connectives to represent the given argument symbolically as $[(p \rightarrow q) \wedge (r \vee \sim q) \wedge \sim r] \rightarrow \sim p$. Also, the candidates constructed a truth table for $[(p \rightarrow q) \wedge (r \vee \sim q) \wedge \sim r] \rightarrow \sim p$ and were able to conclude that the argument is valid since the last column at the right contains only T. In part (c), the candidates were able to draw the diagram of network that correspond to the compound statement $(p \wedge q) \wedge [(r \vee s) \wedge w]$. Extract 12.1 shows the solution of a candidate who answered this question correctly.

1a Solution

$(p \wedge q) \vee (\sim r \wedge (q \wedge p))$ ... Given

$(p \wedge q) \vee ((p \wedge q) \wedge \sim r)$ ... Commutative law

$[(p \wedge q) \wedge (T)] \vee [(p \wedge q) \wedge \sim r]$ ... Identity law

$(p \wedge q) \wedge [T \vee \sim r]$ ... Distributive law

$(p \wedge q) \wedge T$ ... Complement law

$p \wedge q$. Identity law

1b Solution

let Statement

p - He begs pardon

q - He remain in School

r - He is punished

Compound Statement

$[(p \to q) \wedge (r \vee \sim q) \wedge \sim r] \to \sim p$

Truth table

						a	b	c	d	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \to q$	$r \vee \sim q$	$a \wedge b$	$c \wedge \sim r$	$d \to \sim p$
T	T	T	F	F	F	T	T	T	F	T
T	T	F	F	F	T	T	F	F	F	T
T	F	T	F	T	F	F	T	F	F	T
T	F	F	F	T	T	F	T	F	F	T
F	T	T	T	F	F	T	T	T	F	T
F	T	F	T	F	T	T	F	F	F	T
F	F	T	T	T	F	T	T	T	F	T
F	F	F	T	T	T	T	T	T	T	T

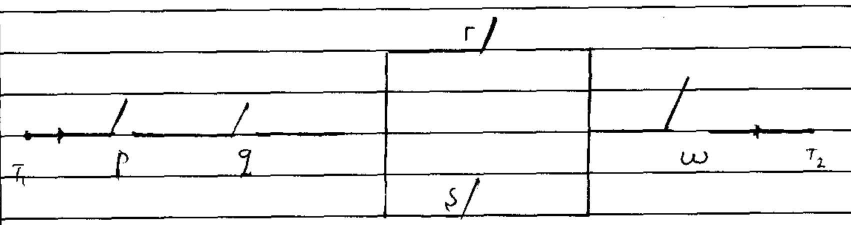
$a \equiv p \to q \quad b \equiv r \vee \sim q \quad c \equiv [(p \to q) \wedge (r \vee \sim q)]$

$d \equiv [(p \to q) \wedge (r \vee \sim q)] \wedge \sim r$

Since there are only T values in the last column of the truth table

Then the Argument is Valid

(c) Electrical network for $(p \wedge q) \wedge [(r \vee s) \wedge w]$



Extract 12.1: *A correct response from one of the candidates*

Despite the good performance there were 73 (0.7%) candidates who did not score any mark. In part (a), the majority of the candidates used laws of algebra of proposition incorrectly to simplify $(p \wedge q) \vee [\sim r \wedge (q \wedge p)]$. As a result, they ended up with incorrect simplified statements such as $p \wedge q \wedge \sim r$. Also, there were candidates who applied correct laws but did not state the appropriate names of laws or did not state the names at all. In part (b), most candidates failed to connect the premises $p \to q$, $r \vee \sim q$, $\sim r$ and the conclusion ~p with logical connectives to get $[(p \to q) \wedge (r \vee \sim q) \wedge \sim r] \to \sim p$. Thus the candidates were unable to fill the truth table with correct T and F values. The candidates who answered part (c) incorrectly failed to connect the switches *p*, *q* and *w* in series with the parallel connection of the switches *r* and *s* in order to construct a network diagram. Extract 12.2 is a sample responses from one of the candidates who did the part (b) and (c) badly.

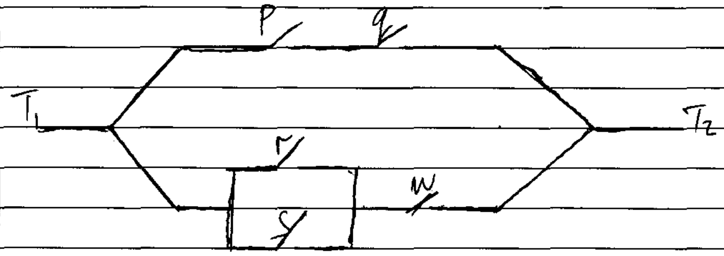
2b Let P be He beg pardon
Q be He will remain in school
R be He punished
Then:
$[(P \to Q) \wedge (R \vee Q) \wedge \sim R] \to \sim P$

Let $(P \to Q) \wedge (R \vee Q)$ be a Also $a \wedge \sim R$ be b

Truth table

P	Q	R	P→Q	R∨Q	a	a∧~R	~R	~P	b→~P
T	T	T	T	T	T	F	F	F	T
T	T	F	T	T	T	T	T	F	F
T	F	T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T	F	T
F	T	T	T	T	T	F	F	T	T
F	T	F	T	T	T	T	T	T	T
F	F	T	T	T	T	F	F	T	T
F	F	F	T	F	F	F	T	T	T

2c $(p \wedge q) \wedge [(r \vee s) \wedge w)]$



Extract 12.2: *An incorrect answer from one of the candidates*

In Extract 12.2 the candidate failed to test the validity of an argument in part (b). In part (c), the candidate failed to represent a compound statement with a network diagram.

2.2.3 Question 3: Vectors

This question comprised parts (a), (b) and (c). In part (a), the candidates were required to find a unit vector perpendicular to both vectors $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$, where $\underline{a}=3\underline{i}+2\underline{j}+2\underline{k}$ and $\underline{b}=\underline{i}+2\underline{j}-2\underline{k}$. In part (b), the candidates were given the area of a parallelogram as $5\sqrt{6}$ square units. If the adjacent sides of the parallelogram are $\underline{i}-2\underline{j}+\lambda\underline{k}$ and $2\underline{i}+\underline{j}-4\underline{k}$ respectively. They were required to find the positive value of λ. In part (c), the candidates were informed that "a particle is moving so that at any instant its velocity $\underline{v}$ is given by $v = 3t\underline{i} - 4\underline{j} + t^2\underline{k}$. If the particle is at point $P(1,0,1)$ when $t = 0$. They were required to find (i) the displacement vector when $t = 2$ and (ii) the magnitude of the acceleration when $t = 2$.

The analysis of data shows that 10112 (99.4%) candidates attempted the question, of which 5854 candidates (57.9%) scored marks ranging from 5.5 to 15. Therefore, the candidates' performance in this question was average. Figure 14 summarizes the analysis of data in the candidates' performance.

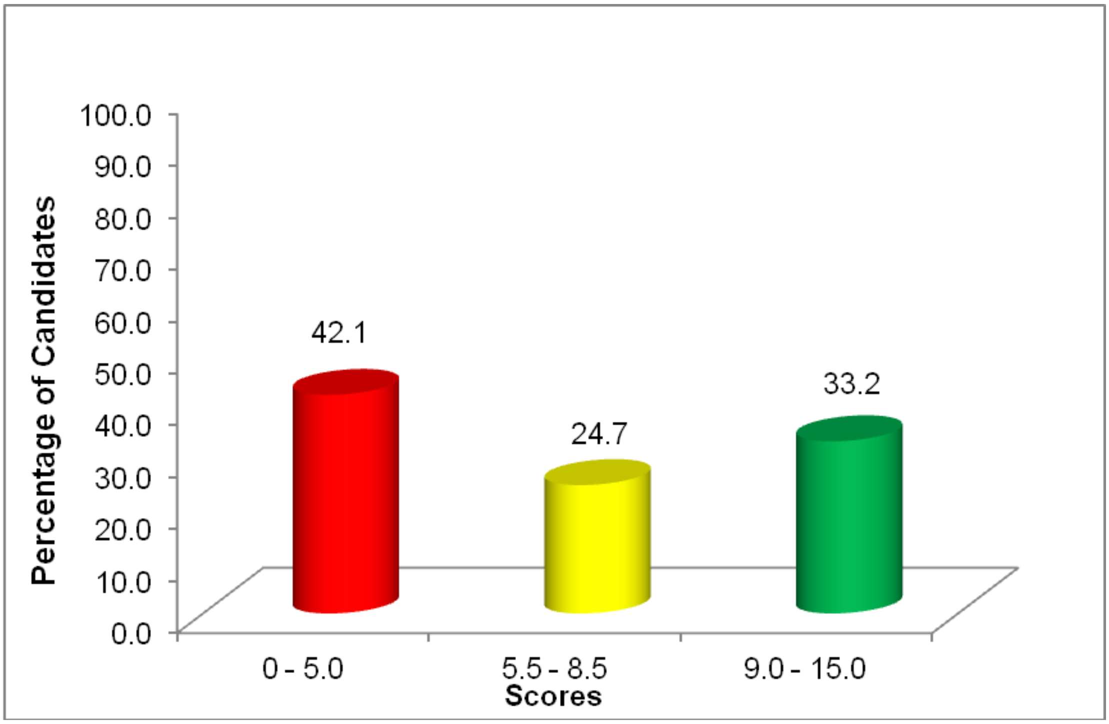
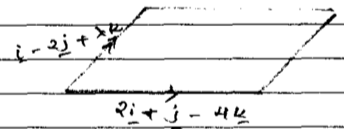


Figure 14: *Candidates' performance in question 3*

The candidates who correctly answered part (a) were able to cross multiply vector $\underline{a} + \underline{b}$ and $\underline{a} - \underline{b}$ to get $16\underline{i} - 16\underline{j} - 8\underline{k}$. Also, they were able to compute the unit vector perpendicular to $\underline{a} + \underline{b}$ and $\underline{a} - \underline{b}$ using the formula $\frac{(\underline{a}+\underline{b})\times(\underline{a}-\underline{b})}{|(\underline{a}+\underline{b})\times(\underline{a}-\underline{b})|}$ to get $\frac{1}{3}(2\underline{i} - 2\underline{j} - \underline{k})$. In part (b), the candidates were able to cross multiply the adjacent sides of a parallelogram to get the vector $\underline{i}(8-\lambda)+\underline{j}(4+2\lambda)+5\underline{k}$. Then, they equated the magnitude of $\underline{i}(8-\lambda)+\underline{j}(4+2\lambda)+5\underline{k}$ with the area of the parallelogram ($5\sqrt{6}$) to obtain $\lambda=3$. The candidates who correctly answered part (c) (i) integrated the velocity $\underline{v}=3t\underline{i}-4\underline{j}+t^2\underline{k}$ to get the expression for the displacement as $\underline{s}=\frac{3}{2}t^2\underline{i}-4t\underline{j}+\frac{1}{3}t^3\underline{k}+c$ where the constant of integration at t = 0 is $\underline{c}=\underline{i}+\underline{k}$. Also, the candidates managed to obtain the displacement vector as $7\underline{i}-8\underline{j}+\frac{11}{3}\underline{k}$ when $t=2$. In part (c) (ii), the candidates differentiated $\underline{v}=3t\underline{i}-4\underline{j}+t^2\underline{k}$ with respect to *t* to get the acceleration at $t=2$ equal to 5 units. Extract 13.1 is a sample solution from of one the candidates who correctly answered part (b) and (c) of this question.

b)

Area $= 5\sqrt{6}$



A = let $\underline{a} = 2i + \underline{j} - 4\underline{k}$

$\underline{b} = \underline{i} - 2\underline{j} + \lambda\underline{k}$

$A = |\underline{a} \times \underline{b}|$

$\underline{a} \times \underline{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -4 \\ 1 & -2 & \lambda \end{vmatrix}$

$\underline{a} \times \underline{b}' = i(\lambda - 8) - j(2\lambda + 4) + k(-4 - 1)$

$= (\lambda - 8)\underline{i} - (2\lambda + 4)\underline{j} - 5\underline{k}$

3. b) $|\underline{a} \times \underline{b}| = \sqrt{(\lambda - 8)^2 + (2\lambda + 4)^2 + 25}$

$5\sqrt{6} = \sqrt{(\lambda - 8)^2 + (2\lambda + 4)^2 + 25}$

~~25t~~

$150 = (\lambda - 8)^2 + (2\lambda + 4)^2 + 25$

$150 = \lambda^2 - \cancel{16}\lambda + 64 + 4\lambda^2 + \cancel{16\lambda} + 16 + 25$

$150 = 5\lambda^2 + 105$

$5\lambda^2 = 150 - 105$

$5\lambda^2 = 45$

$\lambda^2 = 9$

$\lambda = \pm\sqrt{9}$

$\lambda = \pm 3 \quad \therefore \lambda = 3.$

$\therefore$ The value of λ is 3

c) $\underline{v} = 3t\underline{i} - 4\underline{j} + t^2\underline{k}$

$P(1, 0, 1) \quad t = 0$

i) $\underline{v} = dr/dt$

$\frac{dr}{dt} = 3t\underline{i} - 4\underline{j} + t^2\underline{k}$

$\int dr = \int(3t\underline{i} - 4\underline{j} + t^2\underline{k})\,dt$

$r = \frac{3t^2}{2}\underline{i} - 4t\underline{j} + \frac{t^3}{3}\underline{k} + c$

$t = 0 \quad \underline{r} = \underline{i} + \underline{k}$

$\underline{i} + \underline{k} = c$

$\therefore r(t) = \frac{3t^2}{2}\underline{i} - 4t\underline{j} + \frac{t^3}{3}\underline{k} + \underline{i} + \underline{k}$

$\underline{r} = \left(\frac{3}{2}t^2 + 1\right)\underline{i} - 4t\underline{j} + \left(\frac{t^3}{3} + 1\right)\underline{k}$

$t = 2$

$\underline{r}(2) = \left(\frac{3}{2}(2)^2 + 1\right)\underline{i} - 4(2)\underline{j} + \left(\frac{2^3}{3} + 1\right)\underline{k}$

3. c)

i) $\underline{r}(2) = 7\underline{i} - 8\underline{j} + \frac{11}{3}\underline{k}$

$\therefore$ Displacement vector when $t = 2$ $\underline{r} = 7\underline{i} - 8\underline{j} + \frac{11}{3}\underline{k}$

ii)

$\underline{v} = 3t\underline{i} - 4\underline{j} + t^2\underline{k}$

$a = \frac{dv}{dt}$

Differentiate $\underline{v}$ wrt t

$dv/dt = 3\underline{i} + 2t\underline{k}$

$\underline{a} = 3\underline{i} + 2t\underline{k}$

when $t = 2$

$\underline{a}(2) = 3\underline{i} + 2(2)\underline{k}$

$\underline{a}(2) = 3\underline{i} + 4\underline{k}$

$|\underline{a}| = \sqrt{3^2 + 4^2}$

$|\underline{a}| = \sqrt{25}$

$|\underline{a}| = 5$ units$/s^2$

$\therefore$ Magnitude of acceleration when $t = 2$ is 5 unit$/s^2$.

Extract 13.1: *A correct response from one of the candidates*

Further analysis shows that 42.1 percent of candidates scored low marks. These candidates encountered the following challenges. In part (a), majority of the candidates obtained the correct vectors for $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$ but committed errors in finding $\underline{a}+\underline{b} \times \underline{a}-\underline{b}$. Other candidates found the unit vector for $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$ separately. This indicates that they did not understand the requirements of the question. In part (b), most candidates applied the formula for calculating the area of a parallelogram as *length x width* instead of cross multiplying vectors $\underline{i}-2\underline{j}+\lambda\underline{k}$ and $2\underline{i}+\underline{j}-4\underline{k}$. It was surprising to note how several candidates were able to equate the given area of the parallelogram with the magnitude of $\underline{i}(8-\lambda)+\underline{j}(4+2\lambda)+5\underline{k}$ but could not solve the equation $5\lambda^2+105=150$ to get $\lambda=3$. In part (c) (i), it was noted that many candidates differentiated $\underset{\sim}{v}=3t\underset{\sim}{i}-4\underset{\sim}{j}+t^2\underset{\sim}{k}$ to find the displacement vector. The candidates were supposed to understand that the displacement can be obtained using the relation $\underline{s}=\int \underline{v}dt$. It was also noted that a number of candidates substituted $t=2$ in $\underline{s}=\frac{3}{2}t^2\underline{i}-4t\underline{j}+\frac{1}{3}t^3\underline{k}+c$ to get $\underline{s}=6\underline{i}-8\underline{j}+\frac{8}{3}\underline{k}$ ignoring the constant of integration. The candidates were supposed to be aware that at $t=0$ $\underline{s}=\underline{i}+\underline{k}$ and therefore the constant of integration is $\underline{c}=\underline{i}+\underline{k}$. In part (c) (ii), some candidates failed to differentiate $\underline{v}=3t\underline{i}-4\underline{j}+t^2\underline{k}$ with respect to t to get $\underline{a}=3\underline{i}+2t\underline{k}$. For instance, a considerable number of candidates differentiated the given vector to get the incorrect vector $\underline{a}=3\underline{i}-4\underline{j}+2t\underline{k}$. Others were able to obtain $\underline{a}=3\underline{i}+4\underline{k}$ at $t=2$ but could not find its magnitude to get acceleration equal to 5 units. Also, several candidates did not substitute $t=2$ into $\underline{a}=3\underline{i}+2t\underline{k}$ to get $\underline{a}=3\underline{i}+4\underline{k}$. Such candidates evaluated the magnitude as $|\underline{a}|=\left|\sqrt{3^2+2^2}\right|$ instead of $|\underline{a}|=\left|\sqrt{3^2+4^2}\right|$. Extract 13.2 is a sample responses showing some mistakes done by the candidates.

3(c) solution

$V = 3ti - 4j + t^2k$

$P = (1, 0, 1)$

i) Displacement vector

$\frac{dP}{dt} = V$

$P = \int V$

$= \int 3ti - 4j + t^2k \; dt$

$= \frac{3t^2}{2}i - 4tj + \frac{t^3}{3}k + c$

$t = 2$

$= \frac{3(2)^2}{2}i - 4(2)j + \frac{2^3}{3}.k$

$= 2i - 8j + \frac{8}{3}k$

Total displacement $= (2+1)i - (8+0)j + (\frac{8}{3}+1)k$

$\therefore$ Displacement vector $= 3i - 8j + \frac{17}{8}k = 3i - 8j + \frac{17}{8}k$

ii) Acceleration $= \frac{dV}{dt}$

$\frac{dv}{dt} = 3i - 0 + 2tk$ at $t = 2$

$= 3i + 2(2)k$

$= 3i + 4k$

$\therefore$ Acceleration $= 3i + 4k$ units.

Extract 13.2 *An incorrect response from one of the candidates*

In Extract 13.2, the candidate could not integrate the given vector for velocity to determine the required displacement in part (c) (i). In part (c) (ii), the candidate did not compute the magnitude of vector $3\underline{i} + 4\underline{k}$ to get the acceleration as a scalar quantity.

2.2.4 Question 4: Complex Numbers

The question was

(a) If $z = a + ib$, prove that $z\bar{z}$ is a real number for all complex numbers z.

(b) Given that $z = \cos\theta + i\sin\theta$, express $\cos^4\theta$ as the sum of cosines of multiple of θ.

(c) If $z = \cos\alpha + i\sin\alpha$, show that $\frac{1}{1+z} = \frac{1}{2}\left(1 - i\tan\left(\frac{\alpha}{2}\right)\right)$.

The analysis of data indicates that 10041 (98.7%) candidates attempted the question, out of which the scores of 7683 (76.5%) candidates ranged from 5.5 to 15 marks. Therefore, the general performance of the candidates in this question was good. Figure 15 shows the percentage distribution of candidates who got low, average and good scores.

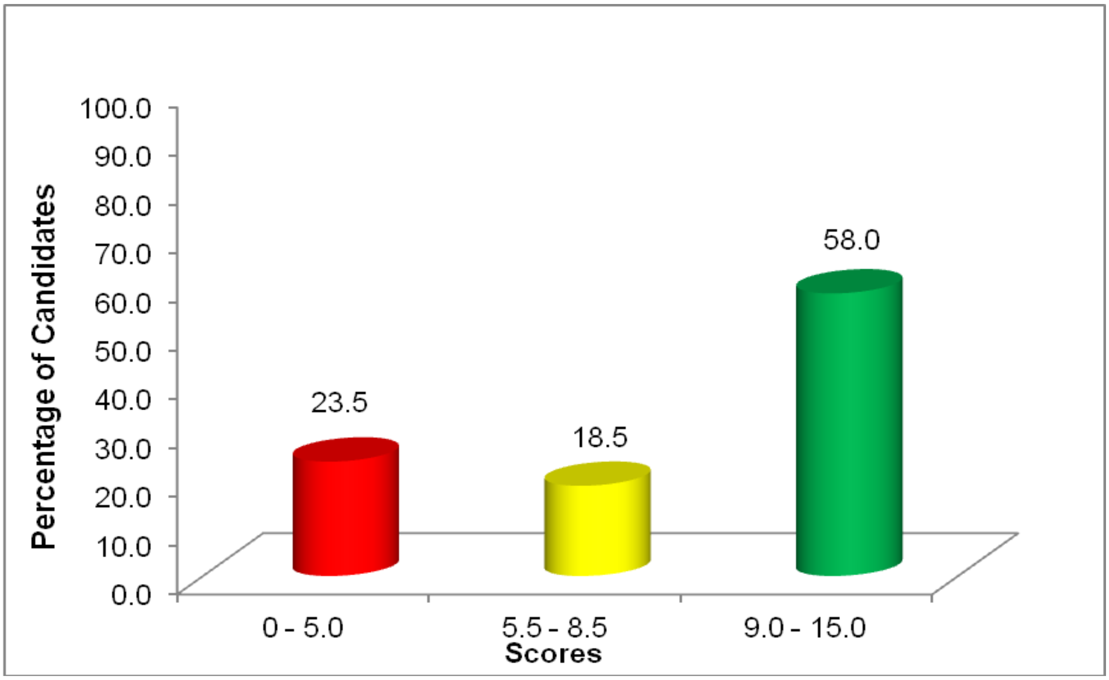


Figure 15: *Candidates' Performance in question 4*

The analysis of data shows that 2437 candidates (23.9%) scored all 15 marks. The candidates who scored all marks demonstrated the following strengths: In part (a), the candidates were able to show that when the complex number $z = a + ib$ is multiplied by its corresponding conjugate $\overline{z} = a - ib$, the result $z\overline{z}$ is a real number $a^2 + b^2$. In part (b), the candidates expanded $(z + \overline{z})^4$ to $z^4 + 4z^3\overline{z} + 6z^2\overline{z}^2 + 4z\overline{z}^3 + \overline{z}^4$. Then, they substituted the complex number

$z=\cos\theta+i\sin\theta$ and its conjugate $\bar{z}=\cos\theta-i\sin\theta$ in the expansion $(z+\bar{z})^4$ in order to express $\cos^4\theta$ as the sum of cosines of multiple of θ is equal to $\frac{1}{8}\cos 4\theta+\frac{1}{2}\cos 2\theta+\frac{3}{8}$. In part (c), the candidates were able to substitute $z=\cos\alpha+i\sin\alpha$ into $\frac{1}{1+z}$ to get $\frac{1}{1+\cos\alpha+i\sin\alpha}$. Then, the candidates used the conjugate of $1+\cos\alpha+i\sin\alpha$ together with the double angle formula for $\cos\alpha$ and $\sin\alpha$ to simplify $\frac{1}{1+\cos\alpha+i\sin\alpha}$ into $\frac{1}{2}\left(1-i\tan\left(\frac{\alpha}{2}\right)\right)$. Extract 14.1 shows an example from of one the candidates who correctly answered this question.

b) $z=\cos\theta+i\sin\theta$ required $\cos^4\theta$ in term of multiple of θ.

Recall that $z=\cos\theta+i\sin\theta$

$1/z=\cos\theta-i\sin\theta$.

So, $z+1/z=2\cos\theta$.

Also for $z^n+1/z^n=2\cos n\theta$.

From $z+1/z=2\cos\theta$ by

$=(z+1/z)^4=(2\cos\theta)^4$

$=z^4+4z^2+6+4/z^2+1/z^4=16\cos^2\theta$.

$=\left(z^4+1/z^4\right)+4\left(z^2+1/z^2\right)+6=16\cos^2\theta$.

but $z^4+1/z^4=2\cos 4\theta$

$z^2+1/z^2=2\cos 4\theta$

$=2\cos 4\theta+4(2\cos 2\theta)+6=16\cos^4\theta$

$=2\cos 4\theta+8\cos 2\theta+6=16\cos^2\theta$

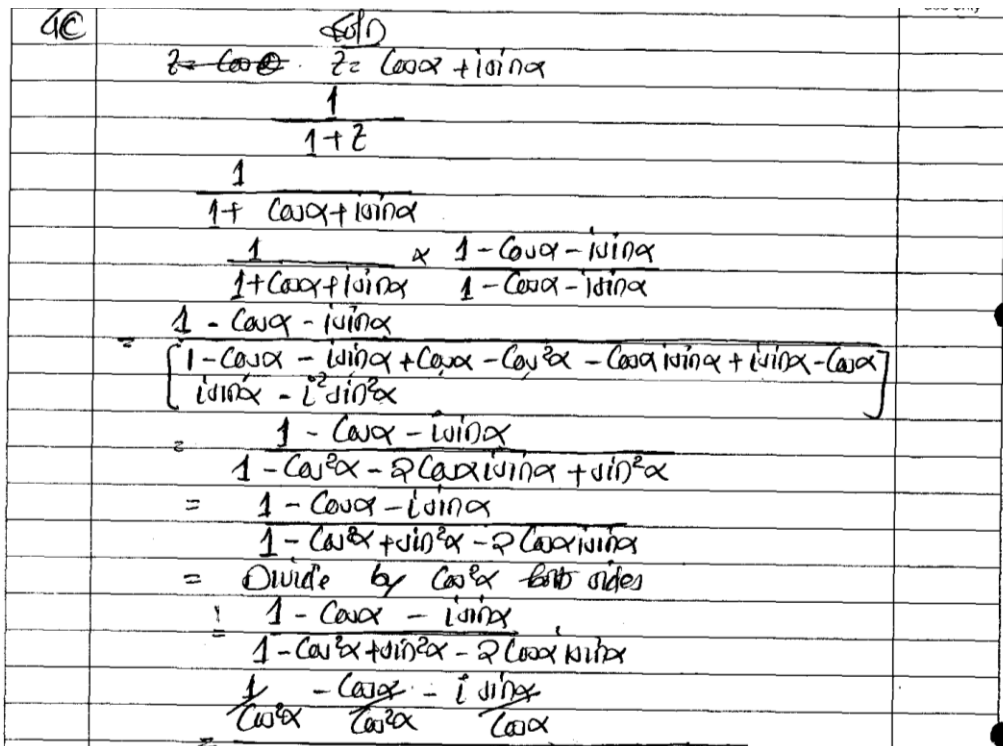
4 b) $=2\cos 4\theta+8\cos 2\theta+6=16\cos^4\theta$.

$=\frac{2(\cos 4\theta+4\cos 2\theta+3)}{16}=\frac{16\cos^4\theta}{16}$

$\therefore \cos^4\theta=\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$

Extract 14.1: *A correct response from one of the candidates*

Further analysis shows that 248 (2.5%) candidates did not score any mark. These candidates committed several errors. In part (a), the candidates failed to show that when a complex $z = a + bi$ is multiplied by its complex conjugate $\overline{z} = a - bi$ the result $a^2 + b^2$ is a real number. In part (b), the candidates who applied Domoivre's theorem to expand $(\cos\theta + i\sin\theta)^4$ ended up with the equation $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. This equation was too difficult to simplify as it demanded the candidates to recall the double angle formula $\cos 2\theta = 2\cos^2\theta - 1$ in order to change it into multiples of θ. Other candidates expressed $(z + \overline{z})^4$ in terms θ as $\cos 4\theta$ instead of $16\cos^4\theta$. Such candidates ended up getting $\cos 4\theta = \cos 4\theta + 4\cos 2\theta + 6$ which was wrong. In part (c), the candidates failed to identify the appropriate conjugate. The common mistake was using $1 - \cos\alpha - i\sin\alpha$ as the conjugate for the denominator of $\dfrac{1}{1 + \cos\alpha + i\sin\alpha}$ instead of $1 + \cos\alpha - i\sin\alpha$. Extract 4.2 is sample solutions from one of the candidates who got part (c) wrong.


4c

Soln

~~z = cos~~ $z = \cos\alpha + i\sin\alpha$

$\dfrac{1}{1+z}$

$\dfrac{1}{1+\cos\alpha + i\sin\alpha}$

$\dfrac{1}{1+\cos\alpha + i\sin\alpha} \times \dfrac{1-\cos\alpha - i\sin\alpha}{1-\cos\alpha - i\sin\alpha}$

$= \dfrac{1-\cos\alpha - i\sin\alpha}{\left[1-\cos\alpha - i\sin\alpha + \cos\alpha - \cos^2\alpha - \cos\alpha\, i\sin\alpha + i\sin\alpha - \cos\alpha\, i\sin\alpha - i^2\sin^2\alpha\right]}$

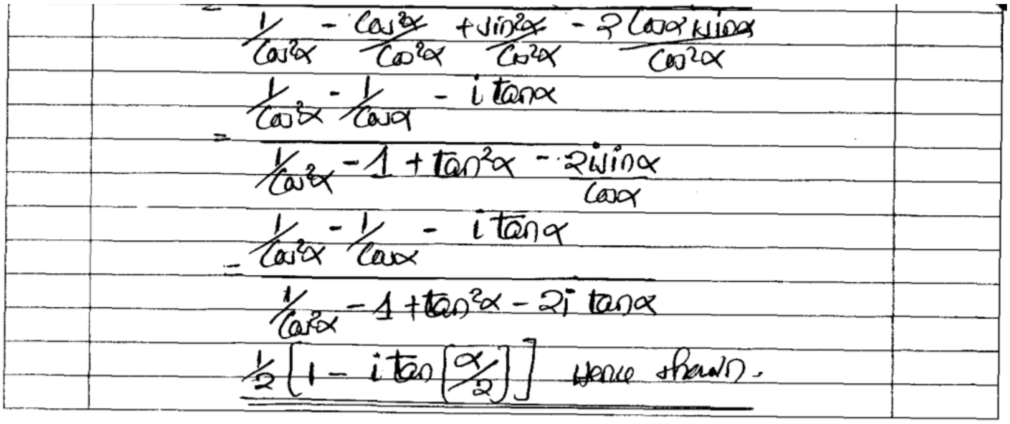
$= \dfrac{1-\cos\alpha - i\sin\alpha}{1-\cos^2\alpha - 2\cos\alpha\, i\sin\alpha + \sin^2\alpha}$

$= \dfrac{1-\cos\alpha - i\sin\alpha}{1-\cos^2\alpha + \sin^2\alpha - 2\cos\alpha\, i\sin\alpha}$

= Divide by $\cos^2\alpha$ ~~both~~ sides

$= \dfrac{1-\cos\alpha - i\sin\alpha}{1-\cos^2\alpha + \sin^2\alpha - 2\cos\alpha\, i\sin\alpha}$

$\dfrac{1}{\cos^2\alpha} - \dfrac{\cos\alpha}{\cos^2\alpha} - \dfrac{i\sin\alpha}{\cos\alpha}$



$$= \frac{1}{\cos^2\alpha} - \frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha} - \frac{2\cos\alpha\, i\sin\alpha}{\cos^2\alpha}$$

$$\frac{1}{\cos^2\alpha} - \frac{1}{\cos\alpha} - i\tan\alpha$$

$$= \frac{1}{\cos^2\alpha} - 1 + \tan^2\alpha - \frac{2i\sin\alpha}{\cos\alpha}$$

$$\frac{1}{\cos^2\alpha} - \frac{1}{\cos\alpha} - i\tan\alpha$$

$$= \frac{1}{\cos^2\alpha} - 1 + \tan^2\alpha - 2i\tan\alpha$$

$$\frac{1}{2}\left[1 - i\tan\left[\frac{\alpha}{2}\right]\right]$$ hence shown.

Extract 14.2: *An incorrect response from one of the candidates*

In Extract 4.2, the candidate was unable to use the complex conjugate of $z = 1 + \cos\theta + i\sin\theta$ to simplify the given expression.

2.2.5 Question 5: Trigonometry

The question had parts (a), (b) and (c). In part (a), the candidates were required to (i) simplify the expression $\frac{1}{\sqrt{x^2 - a^2}}$ where $x = a\operatorname{cosec}\theta$ and (ii) prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$. In part (b), the candidates were required to (i) express $2\cos\theta + 5\sin\theta$ in the form of $R\sin(\theta - \alpha)$ and (ii) express $\cos(\alpha - \beta)$ in terms of *m* and *n* where $\cos\alpha - \cos\beta = m$ and $\sin\alpha - \sin\beta = n$. In part (c), the candidates were required to use t-substitution to find the general solution of the equation $3\cos\theta - 4\sin\theta + 1 = 0$.

The analysis of data shows that 68.5 percent of the candidates opted for this question. It was further observed that 88.6 percent of the candidates scored from 7 to 20 marks. Moreover, the data shows that 11.4 percent of the candidates scored below 7 marks, 29.8 percent scored from 7 to 11.5 marks and 58.8 percent scored from 12 to 20 marks. In general this question was among four questions which were best performed as illustrated in figure 16.

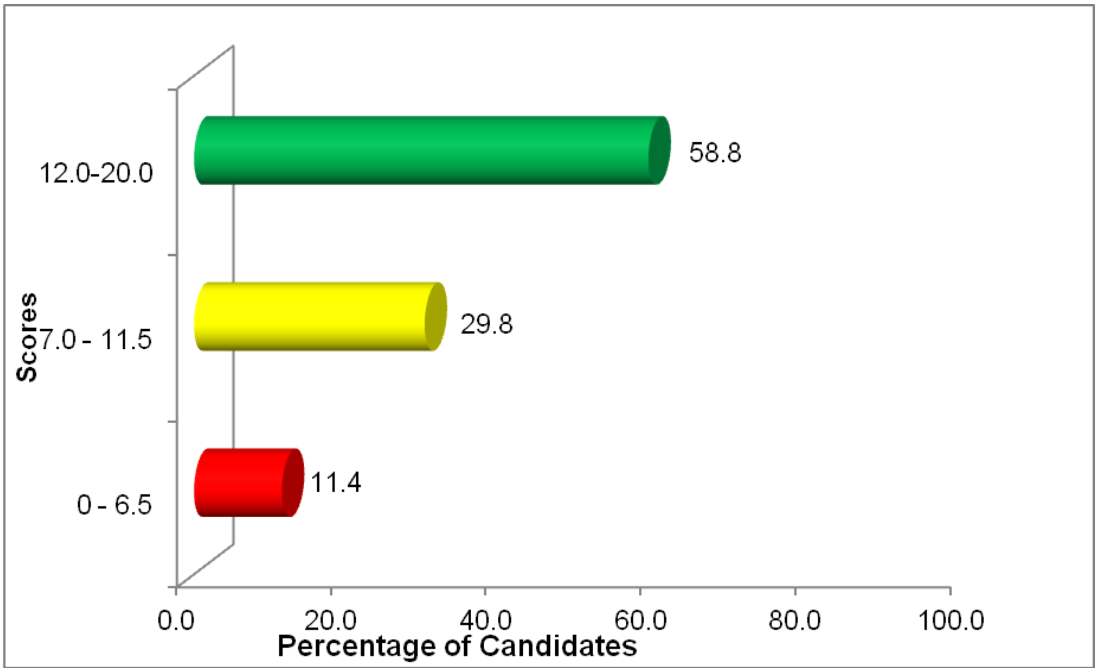


Figure 16: *The candidates' performance in question 5*

The candidates who correctly answered this question demonstrated the following skills. In part (a), the candidates substituted $x = a\,\mathrm{cosec}\,\theta$ into $\dfrac{1}{\sqrt{x^2 - a^2}}$ to get $\dfrac{1}{\sqrt{a^2\left(\cos ec^2\theta - 1\right)}}$. Then, by using the identity $\cos ec^2\theta - 1 = \cot^2\theta$ the candidates were able to simplify $\dfrac{1}{\sqrt{a^2\left(\cos ec^2\theta - 1\right)}}$ into $\dfrac{1}{a}\tan\theta$. In part (a) (ii), the candidates applied the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\,\mathrm{cosec}\,\theta$. In part (b) (i), the candidates changed $R\sin(\theta - \alpha)$ into $-R\cos\theta\sin\alpha + R\sin\theta\cos\alpha$; divided $-R\sin\alpha = 2$ to $R\cos\alpha = 5$ to get $\alpha = -21.80^\circ$; added $R^2\sin^2\alpha$ to $R^2\cos^2\alpha$ to get $R = \sqrt{29}$; finally they correctly expressed $2\cos\theta + 5\sin\theta$ in the form $R\sin(\theta - \alpha)$ as $\sqrt{29}\sin\ \theta + 21.80^\circ$. In part (b) (ii), majority of the candidates squared *m* and *n* to get the expressions $\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta$ and $\sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta$ respectively;

added m^2 and n^2 to get $2-2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=m^2+n^2$. Finally, the candidates allowed $\cos(\alpha-\beta)$ to take the place of $\cos\alpha\cos\beta+\sin\alpha\sin\beta$ so as to express $\cos(\alpha-\beta)$ in term of *m* and *n* as $\cos(\alpha-\beta)=1-\frac{1}{2}\ m^2+n^2$. In part (c), the candidates were able to substitute the t-formula $\cos\theta=\frac{1-t^2}{1+t^2}$ and $\sin\theta=\frac{2t}{1+t^2}$ into the equation $3\cos\theta-4\sin\theta+1=0$ to get $t^2+4t-2=0$. Then, they solved the equation $t^2+4t-2=0$ to get t_1 and t_2 equal to 0.4495 and -4.4495 respectively. Finally, by using $t=\tan\left(\frac{\theta}{2}\right)$ and $\frac{\theta}{2}=180n+\alpha$, the candidates ended up with the required general solution of $\theta=360n+48.4°$ and $\theta=360n-154.6$. Extract 15.1 is a sample solution from one of the candidates who answered this question correctly.

(b) (i) $R\sin(a-\alpha) = 2\cos a + 5\sin a$

$\Rightarrow R\sin a\cos\alpha - R\sin\alpha\cos a = 2\cos a + 5\sin a$

Comparing; $R\cos\alpha = 5$ and

$R\sin\alpha = -2$

squaring and adding;

$R^2(\sin^2\alpha + \cos^2\alpha) = 5^2 + (-2)^2$

5. (b) (i) $\Rightarrow R^2 = 29$ $[\because \sin^2 a + \cos^2 a = 1]$

$\Rightarrow R = \pm\sqrt{29}$

Dividing; $\Rightarrow \tan\alpha = -2/5$

$\Rightarrow \alpha = \tan^{-1}(-2/5)$

$= -21.8°$

$\therefore 2\cos a + 5\sin a = \pm\sqrt{29}\sin(a + 21.8°)$

(ii) Given; $m = \cos\alpha - \cos\beta$

$n = \sin\alpha - \sin\beta$

squaring and adding;

$\Rightarrow m^2 + n^2 = (\sin^2\alpha + \cos^2\alpha) + \cancel{\sin^2\alpha}$

$(\sin^2\beta + \cos^2\beta) - 2\cos\alpha\cos\beta$

$- 2\sin\alpha\sin\beta$

$= 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$[\because \sin^2 a + \cos^2 a = 1]$

$= 2 - 2\cos(\alpha - \beta)$

$\Rightarrow 2\cos(\alpha-\beta) = 2 - (m^2 + n^2)$

	$\therefore \cos(\alpha-\beta) = 1 - \left(\frac{m^2+n^2}{2}\right).$	

Extract 15.1: *correct response from one of the candidates*

In Extract 15.1, the candidate demonstrated competences in applying the appropriate principles and identities to solve the given trigonometric questions.

On the other hand there were several candidates (11.4%) who could not answer the question properly. The candidates poor performance in part (a) (i) was due to failure of candidates to apply the identity $\operatorname{cosec}^2\theta - 1 = \cot\theta$. Most candidates used $\operatorname{cosec}^2\theta - 1$ instead $\cos^2\theta - 1$ ending with $\frac{1}{a}\operatorname{cosec}\theta$. Others expressed $\frac{1}{\sqrt{a^2\cos ec^2\theta - a^2}}$ as $\frac{1}{a\operatorname{cosec}\theta - a}$. These candidates were supposed to understand that the next step after putting $x = a\operatorname{cosec}\theta$ into $\frac{1}{\sqrt{x^2 - a^2}}$ is $\frac{1}{a\sqrt{(\cos ec^2\theta - 1)}}$. In part (a) (ii), the candidates experienced difficulties in using the identity $\cos^2\theta + \sin^2\theta = 1$ to prove the given trigonometric equation. In part (b) (i), the candidates encountered the following challenges: One, most candidates compared the terms of $2\cos\theta + 5\sin\theta$ and $-R\cos\theta\sin\alpha + R\sin\theta\cos\alpha$ to get $R\cos\alpha = 2$ and $R\sin\alpha = -5$ instead of $R\sin\alpha = -2$ and $R\cos\alpha = 5$ respectively. The candidates obtained the expression for $2\cos\theta + 5\sin\theta$ as $\sqrt{29}\sin(\theta - 291.8°)$ instead of $\sqrt{29}\sin(\theta + 21.80°)$. In part (b) (ii), most candidates expanded $\cos(\alpha - \beta)$ to get $\cos\alpha\cos\beta - \sin\alpha\sin\beta$. Then, they substituted $\cos\alpha = m + \cos\beta$ and $\sin\alpha = m + \sin\beta$ to get $\cos\ \alpha - \beta\ = m\cos\beta + n\sin\beta + 1$ which was difficult to express in term of *m* and *n*. These candidates were supposed to use $\cos(\alpha - \beta)$ as a substitution for $\cos\alpha\cos\beta - \sin\alpha\sin\beta$ in the equation $2 - 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = m^2 + n^2$. In part (c), some candidates used inappropriate t-formula for $\cos\theta$ and $\sin\theta$. The analysis of the candidates' responses reveals that candidates quoted incorrectly the formula for $\cos\theta$ and $\sin\theta$. Such formula include $\frac{1+t^2}{1-t^2}$ and $\frac{2t}{1-t^2}$ instead of $\frac{1-t^2}{1+t^2}$ and $\frac{2t}{1+t^2}$ respectively. Other candidates failed to recall that the general solution of $\tan\frac{\theta}{2}=t$ is $\frac{\theta}{2}=180n+\tan^{-1}t$ degrees or $\frac{\theta}{2}=\pi n+\tan^{-1}t$ radians. Several candidates equated $\frac{\theta}{2}$ to $\tan^{-1} 0.4995$ and $\tan^{-1} -4.4995$ to get $\theta=48.4^{\circ}$, $\theta=205.34^{\circ}$ and $\theta=408.4^{\circ}$ instead of $\theta=360n+48.4^{\circ}$ and $\theta=360n-154.6$. Also, a number of candidates solved correctly $t^2+4t-2=0$ to get the values of t but did not multiply by 2 to the first term in the expression $180n+\tan^{-1}t$. The candidates ended up getting $\theta=180n+48.4^{\circ}$ and $\theta=180n-154.6$. Extract 15.2 illustrates a wrong answer in part (b) (ii) from one candidate who lacked knowledge of trigonometric concepts.

5· b) ii) Soln.

$$\text{Cos}\,\alpha - \text{Cos}\,\beta = m$$
$$\text{Sin}\,\alpha - \text{Sin}\,\beta = n.$$
$$\text{Cos}\,(\alpha-\beta) = \text{Cos}\,\alpha\,\text{Cos}\,\beta + \text{Sin}\,\alpha\,\text{Sin}\,\beta$$
$$\text{But}\ \text{Cos}\,\alpha = m + \text{Cos}\,\beta$$
$$\text{Sin}\,\alpha = n + \text{Sin}\,\beta$$
$$\text{Cos}\,(\alpha-\beta) = (m + \text{Cos}\,\beta)\,\text{Cos}\,\beta + (n + \text{Sin}\,\beta)\,\text{Sin}\,\beta$$
$$= m\,\text{Cos}\,\beta + \text{Cos}^2\beta + n\,\text{Sin}\,\beta + \text{Sin}^2\beta$$

5· b) ii)

$$\text{Cos}\,(\alpha-\beta) = m\,\text{Cos}\,\beta + n\,\text{Sin}\,\beta + \text{Cos}^2\beta + \text{Sin}^2\beta$$
$$\text{But}\ \text{Cos}^2\beta + \text{Sin}^2\beta = 1.$$
$$\text{Cos}\,(\alpha-\beta) = m\,\text{Cos}\,\beta + n\,\text{Sin}\,\beta + 1$$
$$\text{Cos}\,(\alpha-\beta) = (m\,\text{Cos}\,\beta)^2 + (n\,\text{Sin}\,\beta)^2 + (1.)^2$$
$$= m^2\,\text{Cos}^2\beta + n^2\,\text{Sin}^2\beta + 1.$$
$$= m^2n^2\left(\frac{\text{Cos}^2\beta}{n^2} + \frac{\text{Sin}^2\beta}{m^2}\right) + 1.$$

Extract 15.2: *An incorrect response from one of the candidates*

In Extract 15.2, the candidate was unable to express $\cos\ \alpha-\beta$ in term of *m* and *n*.

2.2.6 Question 6: Algebra

The question had parts (a), (b) and (c). In part (a), the candidates were required to use the principle of mathematical induction to prove that $3^{2(n+1)}-8n-9$ is divisible by 8. In part (b), the candidates were required to find the inverse of the matrix $A=\begin{pmatrix} 3 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{pmatrix}$. In part (c), the candidates were required to use the inverse matrix obtained in (b) to find the values of x, y and z in the simultaneous equations $\begin{cases} 3x-y+2z=11 \\ 2x+3y+z=-1 \\ x+2y-z=-6 \end{cases}$.

The analysis of data shows that the question was opted by 8122 (79.8%) candidates and 2054 (20.2%) candidates did not attempt it. Further analysis shows that 89.0 percent of the candidates scored above 6.5 marks of which 18.9 percent scored all 20 marks. Thus, the question was among four questions which were best performed in this examination occupying the third position in rank order of the candidates' performance. Figure 17 shows the percentages of candidates with weak, average and good performance.

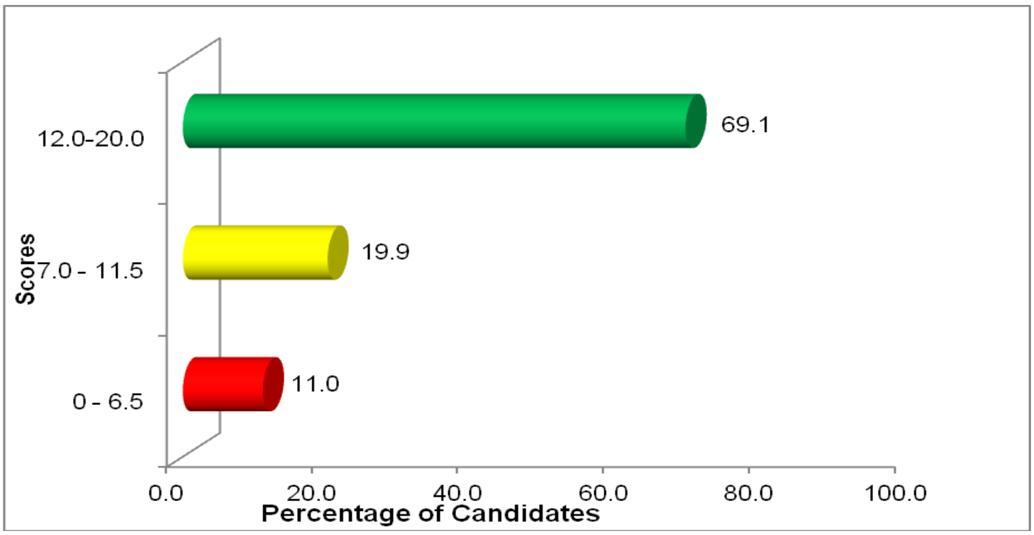


Figure 17: *Candidates' performance in question 6*

As shown in Figure 17 many candidates (69.1%) scored high marks. In part (a), the candidates were able to equate the given statement with $P(n)$. Then they were able to substitute $n=1$, $n=k$, $n=k+1$ into the given mathematical statement to get 64, $8m+8k+9$ and $8(9m+8k+8)$ which are divisible by 8. In part (b), most candidates were able to obtain the determinant, cofactors and adjoint of the matrix $A=\begin{pmatrix} 3 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{pmatrix}$ as $\det(A)=-16$, $cof(A)=\begin{pmatrix} -5 & 3 & 1 \\ 3 & -5 & -7 \\ -7 & 1 & 11 \end{pmatrix}$ and $Adj(A)=\begin{pmatrix} -5 & 3 & -7 \\ 3 & -5 & 1 \\ 1 & -7 & 11 \end{pmatrix}$. Also, majority of the candidates used the formula $A^{-1}=\dfrac{1}{\det(A)}\times Adj(A)$ to find the required inverse as $A^{-1}=-\dfrac{1}{16}\begin{pmatrix} -5 & 3 & -7 \\ 3 & -5 & 1 \\ 1 & -7 & 11 \end{pmatrix}$.

In part (c), many candidates were able to change the given system of simultaneous equation in matrix form as $\begin{pmatrix} 3 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 11 \\ -1 \\ -6 \end{pmatrix}$. Then, they multiplied the inverse obtained in part (b) on both sides of the matrix equation to get $x=1$, $y=-2$ and $z=3$. Extract 16.1 is a sample solution from a candidate who answered part (a) correctly.

Qn 6: (a) Verification when $n = 1$

$= 3^{2(n+1)} - 8n - 9$

$= 3^{2(1+1)} - 8(1) - 9$

$= 81 - 8 - 9$

$= 64$

$= 8(8)$

It is true for $n = 1$

Assume it is true for $n = k$.

$\Rightarrow 3^{2(k+1)} - 8k - 9 = 8M$

$3^{2k}.3^2 - 8k - 9 = 8M$

$3^{2k}.3^2 = 8M + 8k + 9$ --- (i)

Required to prove is when $n = k+1$

$= 3^{2(k+1+1)} - 8(k+1) - 9$

$= 3^{2(k+1+1)} - 8(k+1) - 9$

$= 3^{2(k+1)} \cdot 3^2 - 8(k+1) - 9$

but

$3^{2(k+1)} = 8M + 8k + 9$

$= (8M + 8k + 9).3^2 - 8(k+1) - 9$

$= 9(8M + 8k + 9) - 8k - 8 - 9$

$= 72M + 72k + 81 - 8k - 17$

$= 72M + 64k - 64$

$= 8(9M + 8k - 8)$ it is true for $n = k+1$

$\therefore$ Hence it is divisible by 8.

Extract 16.1: *A correct response from one of the candidates*.

Despite the good performance, there were few candidates (11.0%) who scored low marks. The candidates weak performance in part (a) was due to the following reasons: One, failure of candidates to express $3^{2k+2} - 8k - 9$ as a multiple of 8 when $n = k$ i.e. $3^{2k+2} - 8k - 9 = 8m$; two, failure of candidates to replace 3^{2k+2} with $8m + 8k + 9$ in the equation $3^2(3^{2k+2}) - 8k - 8 - 9$ to get $8(9m + 8k + 8)$ which is also divisible by 8 when $n = k+1$. In part (b), several candidates used the cofactors of the matrix A and the incorrect formula $A^{-1} = \frac{1}{\det(A)} \times Cof(A)$ to find inverse matrix as $-\frac{1}{16}\begin{pmatrix} -5 & 3 & 1 \\ 3 & -5 & -7 \\ -7 & 1 & 11 \end{pmatrix}$. The candidates were supposed to change rows of the cofactor matrix into columns to get the adjoint matrix then use the formula $A^{-1} = \frac{1}{\det(A)} \times Adj(A)$ to find the inverse matrix. Other candidates were unable to use the given matrix to find the elements of the cofactor matrix and hence unable to find the inverse as well as solving the given linear equations simultaneously in part (c). It was surprising to note how a significant number of candidates used Crammer's rule to solve for *x*, *y* and z in a given system of simultaneous equations contrary to the requirements of the question which instructed them to use the inverse obtained in part (b). Such candidates were not awarded marks for procedures instead they were awarded marks for each correct value of x, y and z. Extract 16.2 is a response showing some mistakes done by the candidates in part (b).

6. b) $\begin{bmatrix} (-3-2) & -(-2-1) & (4-3) \\ -(1-4) & (-3-2) & -(6+1) \\ (-1-6) & -(3-4) & (9+2) \end{bmatrix} = \begin{bmatrix} +5 & -3 & -1 \\ -3 & +5 & +7 \\ +7 & -1 & -11 \end{bmatrix}$.

Then transfer the matrix $= (A^C)^T$

$(A^C)^T = \begin{bmatrix} +5 & -3 & +7 \\ -3 & +5 & -1. \\ -1 & 7 & -11 \end{bmatrix}$

Find the determinant of $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{bmatrix}$

$\frac{1}{|A|} = \frac{1}{[(-3-2)-(-2-1)+(4-3)]}$

$= \frac{1}{-16}$

$\therefore \begin{bmatrix} 3 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{bmatrix} = \frac{1}{-16} \begin{bmatrix} +5 & -3 & +7 \\ -3 & +5 & -1 \\ -1 & +7 & -11 \end{bmatrix}$

Extract 16.2: *An incorrect response from one of the candidates*

In Extract 16.2, the candidate did not have skills on how to find the inverse of a matrix by using adjoint of the matrix.

2.2.7 Question 7: Differential Equations

The question was

(a) Solve the differential equation $y\frac{d^2y}{dx^2} + 25 = \left(\frac{dy}{dx}\right)^2$ given that $\frac{dy}{dx} = 4$ when $y = 1$ and $y = \frac{5}{3}$ when $x = 0$.

(b) Solve the differential equation $xy^2 + x^2y\frac{dy}{dx} = \sec^2 2x$.

(c) The rate at which atoms in a mass of a radioactive material are disintegrating is proportional to the number of atoms (N) present at any time t. If N_0 is the number of atoms present at time $t=0$. Solve the differential equation that represents this information.

(d) If half of the original mass disintegrates in 152 days, find the constant of proportionality for the solution obtained in (c). (Give your answer in three significant figures)

This question was opted by few candidates (14%), out of which, 69.4 percent of the candidates passed. Further analysis indicated that 30.6 percent of the candidates scored from 0 to 6.5 marks, 44.0 percent scored from 7 to 11.5 marks and 25.4 percent scored from 12 to 20 marks. Moreover, the analysis shows that 21 (1.5%) candidates scored all 20 marks and 5.0 percent of the candidates scored 0. Generally, this question was well performed as summarized in Figure18.

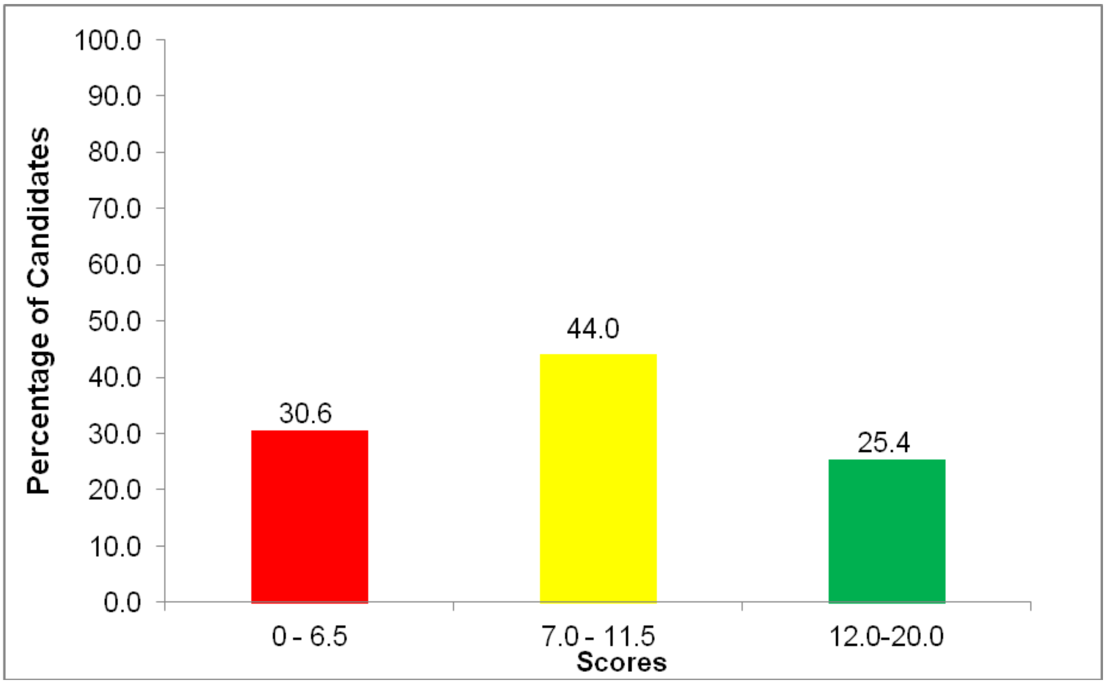


Figure 18: *The candidates' performance in question 7*

As shown in Figure 18, 25.4 percent of the candidates scored high marks. The good performance in this question was due to the fact that the candidates had adequate knowledge and skills in the topic of differential equations. In part (a), the candidates managed to replace $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in the equation $y\frac{d^2y}{dx^2}+25=\left(\frac{dy}{dx}\right)^2$ with $\frac{dP}{dy}.P$ and P respectively to form the first differential equation $\frac{p}{p^2-25}dp=\frac{1}{y}dy$; integrate the equation $\frac{p}{p^2-25}dp=\frac{1}{y}dy$ with respect to p and y to get $\left(\frac{dy}{dx}\right)^2-25=ky^2$ where $P=\frac{dy}{dx}$; substitute $\frac{dy}{dx}=4$ and $y=1$ in $\left(\frac{dy}{dx}\right)^2-25=ky^2$ to get the constant of integration $k=-9$. Also, the candidates were able to substitute $3y=5\cos\theta$, $dy=-\frac{5}{3}\sin\theta d\theta$ as well as the initial conditions $y=\frac{5}{3}$ when $x=0$ into $\int\frac{dy}{\sqrt{25-9y^2}}=\int dx$ to get the particular solution $y=\frac{5}{3}\cos(3x)$. In part (b), the candidates multiplied by 2 on both sides of the given differential equation to get $2xy^2+2x^2y\frac{dy}{dx}=2\sec^2 2x$. Then, the candidates were able to identify that $2xy^2+2x^2y\frac{dy}{dx}$ is the derivative of x^2y^2. Thus, they integrated the relation $\frac{d}{dx}(x^2y^2)=2\sec^2 2x$ with respect to x to obtain $y=\frac{\sqrt{\tan 2x+c}}{x}$. In part (c), the candidates were able to formulate the differential equation $\frac{dN}{dt}=-kN$ from the given word problem. Then, they solved it to get $N=N_0e^{-kt}$. In part (d), the candidates were able to identify that if half of original mass disintegrates in 152 days then $N=\frac{1}{2}N_0$. Also, they were able to replace N and t in the equation $N=N_0e^{-kt}$ with $\frac{1}{2}N_0$ and 152 days respectively to get the value of k correct to three significant figures as $k = 0.00456$. Extract 17.1, is a sample work from one of the candidates who correctly answered part (a).

07. a) $y\frac{d^2y}{dt^2} + 25 = \left(\frac{dy}{dx}\right)^2$

let $p = \frac{dy}{dx}$

$\frac{dp}{dx} = \frac{d^2y}{dt^2}$

$y\frac{dp}{dx} = p^2 - 25$

but $\frac{dp}{dx} = \frac{dp}{dy}\cdot\frac{dy}{dx}$

$\frac{dp}{dt} = p\frac{dp}{dy}$

$yp\frac{dp}{dy} = p^2 - 25$

$\frac{p\,dp}{p^2-25} = \frac{dy}{y}$

Intergrating

$\frac{1}{2}\ln(p^2-25) = \ln y + \ln A$

$\ln\sqrt{p^2-25} = \ln Ay$

$\sqrt{p^2-25} = Ay$

$p^2 - 25 = (Ay)^2$

$p^2 = (Ay)^2 + 25$

07. a) $p^2 - 25 = A^2y^2$

$\left(\frac{dy}{dt}\right)^2 - 25 = A^2y^2$

$(4)^2 - 25 = By^2$ $\therefore A^2 = B$

$-9 = B(1)$

$$\left(\frac{dy}{dt}\right)^2 = 25 - 9y^2$$

$$\frac{dy}{dt} = \sqrt{25-9y^2}$$

$$\frac{dy}{\sqrt{25-9y^2}} = dx$$

let $9y^2 = 25\sin^2\theta$

$3y = 5\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{3y}{5}\right)$

$3\frac{dy}{d\theta} = 5\cos\theta$, $dy = \frac{5}{3}\cos\theta\, d\theta$

$$\frac{5/3\cos\theta\, d\theta}{5\sqrt{1-\sin^2\theta}} = dx$$

$$\frac{1}{3}\int d\theta = \int dx$$

$$\frac{1}{3}\theta = x + A$$

07. a)

$$\frac{1}{3}\theta = x + A$$

$$\frac{1}{3}\left|\sin^{-1}\frac{3y}{5}\right| = x + A$$

putting $y = 5/3$ and $x = 0$

$$\frac{1}{3}\left|\sin^{-1}(1)\right| = A$$

$$\frac{\pi}{6} = A$$

$$\sin^{-1}\frac{3y}{5} = 3x + \frac{\pi}{6} \times 3$$

$$\frac{3y}{5} = \sin\left(3x + \frac{\pi}{2}\right)$$

$$\frac{3y}{5} = \cos 3x$$

$$\therefore \quad y = \frac{5}{3}\cos 3x$$

Extract 17.1: *A correct response from one of the candidates*

Despite the good performance, there were several candidates (30.6%) who scored low marks. The analysis of the candidates' responses in this category shows that they lacked knowledge and skills in differentiation and integration techniques. In part (a), the candidates could not express $\frac{d^2y}{dx^2}$ as $\frac{dP}{dy}.P$. Most of them presented it as $\frac{dP}{dy}$ and therefore were unable to proceed toward the particular solution of $y = \frac{5}{3}\cos 3x$. In part (b), the candidates failed to realise that the given differential equation has to be multiplied by two so as to transform it into an exact differential equation $2xy^2 + 2x^2y\frac{dy}{dx} = 2\sec^2 2x$. The candidates who did not answer (c) correctly transformed the information that "The rate at which atoms in a mass of a radioactive material are disintegrating is proportional to the number of atoms (N) present at any time t" as $\frac{dN}{dt} \propto N$ instead of $-\frac{dN}{dt} \propto N$. The candidates solved the equation $\frac{dN}{N} = kdt$ to get $N = N_0e^{kt}$ instead of $N = N_0e^{-kt}$. The candidates were supposed to be aware that the numbers of atoms were decreasing. In part (d), the candidates substituted N and t in the equation $N = N_0e^{kt}$ with $\frac{1}{2}N_0$ and 152 days respectively to get $k = -0.00456$ instead of $k = 0.00456$. Extract 17.2 shows a sample solution from one of the candidates who incorrectly answered part (c) and (d).

7 c)

Rate in mass of radioactive material is $\frac{dN}{dt}$

$\frac{dN}{dt} \propto N$

$\frac{dN}{dt} = KN$

by separating variables

$\frac{dN}{N} = k\,dt$

Intergrating to both sides of equation:

$\int \frac{dN}{N} = \int K\, dt$

$\int dN/_N = \ln N$

$\ln N = kt + c$

Initially $N = N_0$, $t = 0$

$\ln N_0 = K(0) + C$

$C = \ln N_0$

$\ln N = Kt + \ln N_0$

7 d)

Time = 152 days

$1/2$ of origrnal sample

<u>solution</u>

$\ln N = Kt + \ln N_0$

$N = \frac{1}{2} N_0$

$t = 152$ days

$\ln \frac{1}{2} N_0 = 152K + \ln N_0$

$\ln \frac{1}{2} N_0 - \ln N_0 = 152K$

$\ln \left(\frac{\frac{1}{2} N_0}{N_0}\right) = 152K$

$\ln (\frac{1}{2}) = 152K$

$k = \frac{1}{152} \ln(\frac{1}{2})$

$K = \frac{1}{152}(-0.693)$

$K = -0.00456$

$\therefore$ Constant of proportionality $K = -0.00456$

Extract 17.2: *An in correct response from one of the candidates*

In Extract 17.2, the candidate failed to apply the knowledge of first order differential equations in solving a real life problem.

2.2.8 Question 8: Coordinate Geometry II

The question was

(a) Find the equation of a tangent to the ellipse $4x^2 + y^2 = 6$ at $\left(\frac{1}{2}, \sqrt{5}\right)$ in the form $ax + by + c = 0$.

(b) The points $P(at_1{}^2, 2at_1)$ and $Q(at_2{}^2, 2at_2)$ lie on the parabola $y^2 = 4ax$ and that the tangents at the points P and Q intersect at R. Find the coordinates of R.

(c) Convert the following polar equations into Cartesian equations:

(i) $r^2 = 4Sin2\theta$.

(ii) $r = 3(1 + Cos\theta)$.

(d) A curve is defined by the parametric equations $x = t^2$ and $y = \frac{2}{t}$ where $t \neq 0$. Show that the equation of the normal at the point $Q\left(p^2, \frac{2}{p^2}\right)$ is $p^4x - py + 2 = p^6$.

The analysis of data shows that 3685 (36.2%) candidates opted this question out of which, 78.8 percent of the candidates scored above 6.5 marks. Further analysis shows that 21.2 percent scored from 0 to 6.5 marks, 27.6 percent scored from 7 to 11.5 marks and 51.2 percent scored from 12 to 20 marks. The analysis has also indicated that 245 (6.6%) candidates scored all 20 marks, while 130 (3.5%) candidates score 0. Therefore, the general performance in the question was good. Figure 19 displays the candidates' performance in this question.

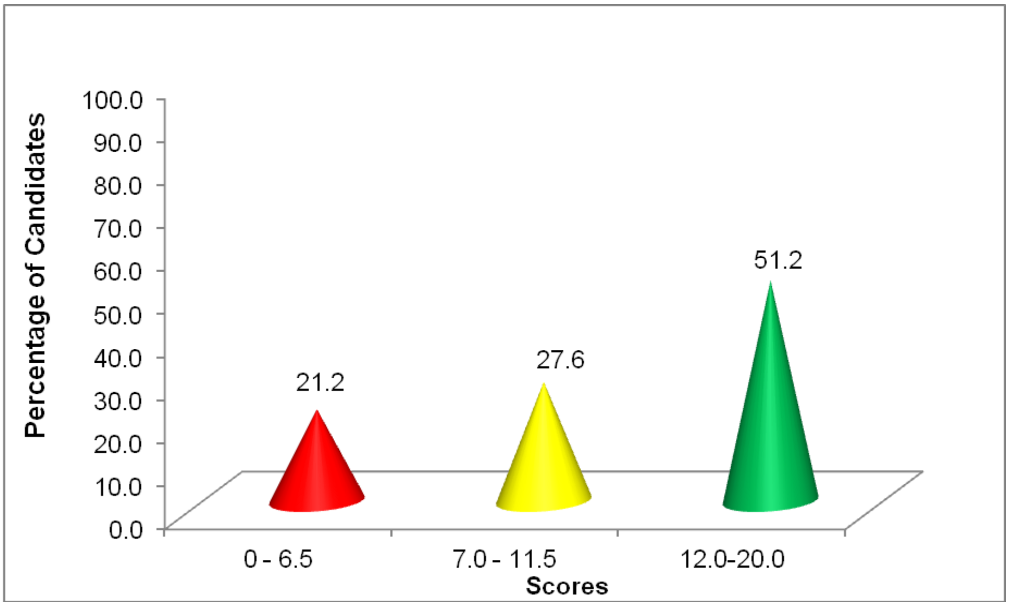


Figure 19: *Candidates' performance in question 8*

The analysis of the candidates' responses shows that a considerable number of candidates (51.2%) had adequate knowledge on the topic. The candidates who correctly attempted part (a) were able to: differentiate the equation $4x^2 + y^2 = 6$ with respect to x to get $\frac{dy}{dx} = \frac{-4x}{y}$, substitute the x and y coordinates of the point $\left(\frac{1}{2}, \sqrt{5}\right)$ into $\frac{-4x}{y}$ to get the slope of the tangent $m = \frac{-2}{\sqrt{5}}$ and by using the formula $y = m(x - x_1) + y_1$, they were able to evaluate the equation of the tangent in the form $ax + by + c = 0$ as $2x + \sqrt{5}y - 6 = 0$. In part (b), the candidates were able to solve the equations of tangents to the parabola $y^2 = 4ax$ at the points P and Q to get x and y coordinates of the point R as $x = at_1t_2$ and $y = a(t_2 + t_1)$. In part (c), most candidates used the relations $r^2 = (x^2 + y^2)$, $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$ to convert the polar equations $r^2 = 4\sin 2\theta$ and $r = 3(1 + \cos\theta)$ into Cartesian equations $(x^2 + y^2)^2 = 8xy$ and $x^2 + y^2 = 3\left(\sqrt{x^2 + y^2} + x\right)$ respectively. In part (d), majority of the candidates were able to show that $p^4x - py + 2 = p^6$ is not an equation of the normal to the curve at the point $Q\left(p^2, \frac{2}{p^2}\right)$. Extract 18.1 shows a solution of a candidate who performed well in parts (a) and (b) of this question.

8a $4x^2 + y^2 = 6$.

Differentiating to obtain slope.

$8x + 2y\frac{dy}{dx} = 0$.

$2y\frac{dy}{dx} = -8x$.

$y\frac{dy}{dx} = -4x$.

$\frac{dy}{dx} = \frac{-4x}{y}$.

Slope $= \frac{-4 \times 1/2}{\sqrt{5}}$

$= \frac{-2}{\sqrt{5}}$.

$M = \frac{y - y_0}{x - x_0}$.

$\frac{-2}{\sqrt{5}} = \frac{y - \sqrt{5}}{x - 1/2}$

$\sqrt{5}(y - \sqrt{5}) = -2(x - 1/2)$

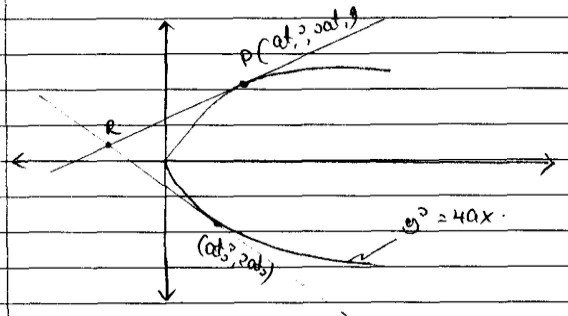
$\sqrt{5}y - 5 = -2x + 1$

$2x + \sqrt{5}y - 5 - 1 = 0$.

$2x + \sqrt{5}y - 6 = 0$.

Equation of tangent is $2x + \sqrt{5}y - 6 = 0$.

10b



Equation of tangent

$y^2 = 4ax.$

$2y\frac{dy}{dx} = 4a.$

$y\frac{dy}{dx} = 2a$

$\frac{dy}{dx} = \frac{2a}{y}$

$\frac{dy}{dx} = \frac{2a}{2at_1}$

$M = \frac{1}{t_1}$

$\frac{1}{t_1} \times \frac{y - 2at_1}{x - at_1^2}$

$x - at_1^2 = t_1y - 2at_1^2.$

$x - at_1^2 + 2at_1^2 - t_1y = 0.$

$x + at_1^2 - t_1y = 0$. – Equation of tangent

8b 2nd tangent

$M = \frac{1}{t_2}.$

$\frac{1}{t_2} \times \frac{y - 2at_2}{x - at_2^2}$

$x - at_2^2 = yt_2 - 2at_2^2.$

$x - yt_2 - at_2^2 + 2at_2^2 = 0$

$x - yt_2 + at_2^2 = 0$. – – ii

Since the two tangents intersect.

then insert equation ii to i

$x = yt_2 - at_2^2.$

$yt_2 - at_2^2 + at_1^2 - t_1y = 0$

$yt_2 - at_2^2 + at_1^2 - t_1y = 0$

$y(t_2 - t_1) = at_2^2 - at_1^2$

$y(t_2 - t_1) = a(t_2^2 - t_1^2)$

$y(t_2 - t_1) = a(t_2 - t_1)(t_2 + t_1)$

$y = a(t_1 + t_2)$

but $x = yt_2 - at_2^2.$

$x = (at_1 + at_2)t_2 - at_2^2.$

$x = at_1t_2 + at_2^2 - at_2^2.$

$x = at_1t_2.$

thus point $R = (at_1t_2, a(t_1 + t_2))$

Extract 18.1: *A correct response from one of the candidates*

Despite the good performance in this question, several candidates (21.2%) scored low marks. The reason for such performance was associated with mistakes they made. In part (a), some candidates found the equation of the normal to the ellipse which is $y = 2\sqrt{5}x$ instead of the equation of a tangent to the ellipse. Others incorrectly substituted the wrong x and y coordinate in the equation $y = m(x - x_1) + y_1$.The candidates who incorrectly answered part (b) computed the coordinates of a point where normal lines perpendicular to tangents at P and Q intersect contrary to the requirements of the question. Other candidates computed the incorrect slopes of the tangents and therefore they were unable to find the coordinates of point R. In some cases, the candidates drew the chord $\overline{PQ}$ passing through the point $R(a,0)$. In part (c), the candidates failed to recall the relationships $r^2 = x^2 + y^2$, $x = r\cos\theta$ and $y = r\sin\theta$. Hence, they could not get the correct Cartesian equations for polar equations $r^2 = 4\sin 2\theta$ and $r = 3(1 + \cos\theta)$. Other candidates were unable to recall the double angle formula for $\sin 2\theta$ i.e. $\sin 2\theta = 2\sin\theta\cos\theta$ to be used in part (c) (i). In part (d), the candidates failed to show that the equation of the normal at the point $Q\left(p^2, \frac{2}{p^2}\right)$ is not $p^4x - py + 2 = p^6$. Extract 18.2 shows a sample solution of a candidate who attempted part (b) incorrectly.

8(b) as $y^2 = 4ax$

$$2y\frac{dy}{dx} = 4a.$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

at point $(at_1^2, 2at_1)$ at point $(at_2^2, 2at_2)$

$$\frac{dy}{dx} = \frac{4a}{2at_1} \qquad \frac{dy}{dx} = \frac{4a}{2at_2}$$

$$\frac{dy_1}{dx} = \frac{2}{t_1} \qquad \frac{dy_2}{dx_2} = \frac{2}{t_2}$$

$$\frac{y - 2at_1}{x - at_1^2} = \frac{2}{t_1}$$

$$yt_1 - 2at_1^2 = 2x - 2at_1^2$$

$$yt_1 = 2x.$$

$$y_1 = \frac{2x}{t_1}$$

at point $(at_2^2, 2at_2)$ and $\frac{dy}{dx} = \frac{2}{t_2}$

$$\frac{y - 2at_2}{x - at_2^2} = \frac{2}{t_2}$$

$$yt_2 - 2at_2^2 = 2x - 2at_2^2$$

$$y_2 = \frac{2x}{t_2}$$

$$y_2 \cdot t_2 = 2x$$

Extract 18.2: *An incorrect response from one of the candidates*

In Extract 18.2, the candidate replaced y in the formula $\frac{dy}{dx} = \frac{4a}{2y}$ with at_1 and at_2 instead of $2at_1$ and $2at_2$ respectively.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Advanced Mathematics Examination had two papers namely Advanced Mathematics 1 and Advanced Mathematics 2. The examination tested 18 topics out of which 10 topics were in paper one and eight topics in paper two. The topics which were tested in paper one are Calculating Devices, Hyperbolic Functions, Linear Programming, Statistics, Sets, Functions, Numerical Methods, Coordinate Geometry I, Integration and Differentiation. The topics which were tested in paper two are Probability, Logic, Vectors, Complex Numbers, Trigonometry, Algebra, Differential Equations and Coordinate Geometry II.

The analysis of data shows that the candidates' performance was good in 15 topics. The topics which had good performance in this examination are Functions, Logic, Algebra, Trigonometry, Numerical Methods, Linear Programming, Sets, Coordinate Geometry II, Statistics, Complex Numbers, Calculating Devices, Integration, Differential Equations, Differentiation and Hyperbolic Functions.

The good performance in those topics was attributed by the candidates' ability to:

(a) draw graphs of composite and rational functions.

(b) test the validity of an argument by using a truth table.

(c) represent the compound statements with a network diagram.

(d) use the principle of mathematical induction to prove mathematical statements.

(e) apply knowledge of the inverse of a matrix to solve three linear equations in three unknowns simultaneously.

(f) interpret and formulate linear programming models.

(g) recognise suitable and relevant integration techniques.

(h) withstand high degree of carefulness and avoiding silly mistakes, blunders and errors.

On the other hand the candidates' performance was average in 02 topics and weak in 01 topic. The topics which were averagely performed are Vectors (57.9%) and Probability (44.3%) while the poorly performed topic is Coordinate Geometry I (18.2%). In 2019, the topics which had average performance are Numerical Methods (51.1%), Differential Equations (36.6%), Differentiation (43%) and Coordinate Geometry I (40.6%) while the two topics which were poorly performed are Probability (23.4%) and Integration (29.3%), see appendix I and II.

The candidates' poor performance in Coordinate Geometry I was due to

(a) The candidate's inability to apply the perpendicular distance from a point to a line to prove mathematical problems.

(b) The candidate's inability to apply the ratio theorem in solving locus problems.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The CIRA report has been specifically written to provide awareness to stakeholders about the candidates' responses in ACSEE 2020. The focus of the analysis was therefore to identify the strengths and weaknesses of the candidates' responses in various items. The Candidates' Items Response Analysis (CIRA) in Advanced Mathematics 2020 shows that 90.63 percent of the candidates passed the examination, in comparison to 86.74 percent who passed the examination in 2019.

The analysis for 2020 shows that 15, topics were well performed, 02 topics were averagely done and 01 topics was poorly performed. The analysis of data for 2019 reveals that, 12 topics had good performance, 4 topics had average performance and 2 topics were poorly performed. The comparison of the candidates' performance in each topic for three years consecutive is shown in appendix III.

Primarily, the report has revealed the main areas where the candidates had good, average or poor performance. It is expected that the stakeholders will use the recommendation of this report to improve the performance in future Advanced Mathematics Examinations.

4.2 Recommendations

For the purpose of improving future candidates' performance in this subject especially the topics with poor performance, it is suggested that:

(i) The students should be encouraged to study in groups so as to consolidate and apply the formula for calculating the perpendicular distance of a point from a line in solving the related questions in different settings and situations.

(ii) The teachers should identify slow learners and conduct remedial teachings on how to apply the ratio theorems in solving the related questions.

(iii) The students should be provided with sufficient exercises on how to apply special distributions such as normal distribution in answering examination questions.

(iv) The Ministry of Regional Administration and Local Government should conduct in-service training to teachers on the topic of coordinate geometry I.

Appendix I

Analysis of Candidates' Performance in each Topic in the 2020 Advanced Mathematics Examination

S/N	Topic	Number of Questions	The % of Candidates who Scored 35 %or more
1	Functions	1	97.5
2	Logic	1	92.9
3	Algebra	1	89.0
4	Trigonometry	1	88.6
5	Numerical Methods	1	84.6
6	Linear Programming	1	84.1
7	Sets	1	80.0
8	Coordinate Geometry II	1	78.8
9	Statistics	1	77.3
10	Complex Numbers	1	76.5
11	Calculating Devices	1	75.8
12	Integration	1	73.5
13	Differential Equations	1	69.4
14	Differentiation	1	61.6
15	Hyperbolic Functions	1	61.5
16	Vectors	1	57.9
17	Probability	1	44.3
18	Coordinate Geometry I	1	18.2

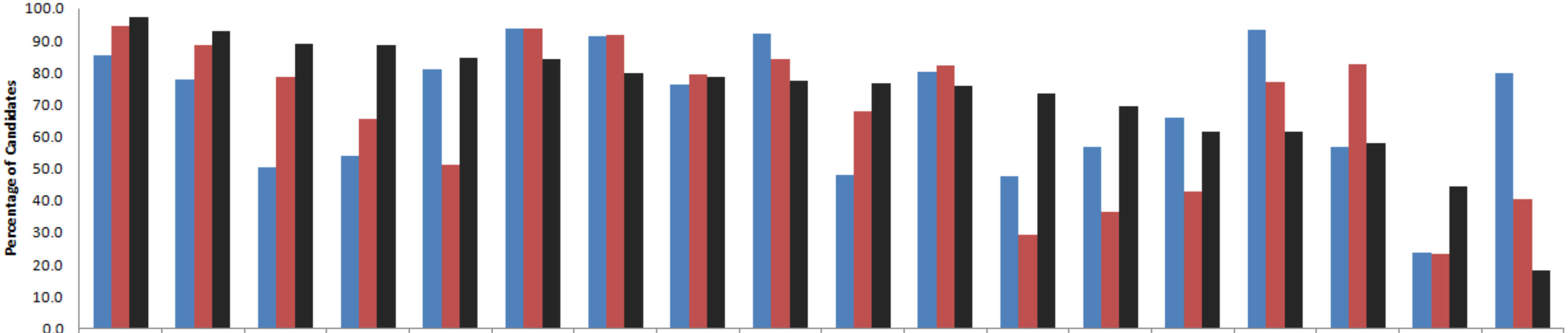
Appendix II

Analysis of Candidates' Performance in each Topic in the 2019 & 2020 Advanced Mathematics Examination

S/N	Topic	2019			2020		
		Number of Question	The % of Candidates who scored at 35 or above	Remarks	Number of Question	The % of Candidates who scored at 35 or above	Remarks
1	Functions	1	94.5	Good	1	97.5	Average
2	Logic	1	88.7	Good	1	92.9	Good
3	Algebra	1	65.3	Good	1	89.0	Weak
4	Trigonometry	1	78.6	Good	1	88.6	Weak
5	Numerical Methods	1	51.1	Average	1	84.6	Good
6	Linear Programming	1	93.7	Good	1	84.1	Good
7	Sets	1	79.6	Good	1	80.0	Weak
8	Coordinate Geometry II	1	79.6	Good	1	78.8	Good
9	Statistics	1	84.3	Good	1	77.3	Average
10	Complex Numbers	1	67.8	Good	1	76.5	Good
11	Calculating Devices	1	82.3	Good	1	75.8	Average
12	Integration	1	29.3	Weak	1	73.5	Weak
13	Differential Equations	1	36.6	Average	1	69.4	Good
14	Differentiation	1	43	Average	1	61.6	Good
15	Hyperbolic Functions	1	76.9	Good	1	61.5	Average
16	Vectors	1	82.7	Good	1	57.9	Good
17	Probability	1	23.4	Weak	1	44.3	Weak
18	Coordinate Geometry I	1	40.6	Average	1	18.2	Weak

Appendix III

The candidates' performance topic-wise in 2018, 2019 and 2020



	Functions	Logic	Algebra	Trigonometry	Numerical Methods	Linear Programming	Sets	Coordinate Geometry II	Statistics	Complex Numbers	Calculating Devices	Integration	Differential Equations	Differentiation	Hyperbolic Functions	Vectors	Probability	Coordinate Geometry I
2018	85.5	78	50.4	53.8	81.2	93.6	91.5	76.4	92.1	47.8	80.1	47.4	56.8	65.8	93.5	56.6	23.9	79.7
2019	94.5	88.7	78.6	65.3	51.1	93.7	91.9	79.6	84.3	67.8	82.3	29.3	36.6	43.0	76.9	82.7	23.4	40.6
2020	97.5	92.9	89.0	88.6	84.6	84.1	80.0	78.8	77.3	76.5	75.8	73.5	69.4	61.6	61.5	57.9	44.3	18.2