THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEMS RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2017

141 BASIC APPLIED MATHEMATICS

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141 BASIC APPLIED MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania has prepared this report on the analysis of the candidates' responses for Basic Applied Mathematics items for the Advanced Certificate of Secondary Education Examination (ACSEE) 2017 in order to provide feedback to students, teachers and other education stakeholders on how the candidates responded to the questions.

The analysis shows that, the candidates performed well in questions that were set from the topics of *Statistics* and *Linear Programming*; averagely in questions that were set from the topics of *Calculating Devices*, *Matrices*, *Exponential* and *Logarithmic* functions and had weak performance in the questions that were set from the topics of *Algebra, Functions, Trigonometry, Probability, Differentiation* and *Integration*.

The candidates' weak performance was due to the following reasons: inability to solve simultaneous equations; lack of knowledge on the sigma notation; lack of understanding of composite functions; inability to use the remainder theorem; inability to identify stationary points of a polynomial; inability to use the quotient and chain rule to differentiate functions; lack of knowledge of implicit differentiation; inability to apply knowledge of integration in answering questions; lack of knowledge on the basic trigonometric identities, ratios and basic rules of probability.

It is the expectation of the Council that this report will be useful in improving the candidates' performance in future Basic Applied Mathematics examinations.

The Council would like to thank the examiners, examination officers and all others who participated in preparing this report. The Council will also be grateful to receive constructive comments from the education stakeholders for improving future reports.

Dr. Charles Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report has been prepared from the analysis of the candidates' responses for the Basic Applied Mathematics examination in ACSEE 2017. The report has identified the areas in which many candidates faced problems as well as the areas they performed averagely and well. The Basic Applied Mathematics paper had a total of 10 compulsory questions, each carrying 10 marks.

In 2017, a total of 29,318 candidates sat for the Basic Applied Mathematics examination of which 14,427 (49.40%) candidates passed. The total number of candidates who sat for this examination in 2016 was 26,787, out of which 12,753 (47.95%) candidates passed, indicating that in 2017 the performance increased by 1.45 percent.

The analysis of the candidates' responses for each question is presented in section 2. In each question, the description of the requirements of the question and the performance of the candidates are provided. The performance of the candidates in each question was categorized based on the percentage of candidates who scored 35 percent or more of the marks in the questions, in the intervals 60 – 100 (good), 35 – 59 (average) and 0 – 34 (weak) respectively.

The third section presents the analysis of the candidates′ responses per topic examined. Furthermore, the factors which have contributed to weak performance in some of the topics examined in 2017 Basic Applied Mathematics examination are highlighted and the recommendations to improve the candidates' performance in this subject have been provided.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Calculating Devices

This question had parts (a) and (b). In part (a), the candidates were required to use a scientific calculator to evaluate:

(i) $\dfrac{458.4^3 \times 0.00274 - 7560 \div 3567^3}{458.4^3 \times 0.00274 + 9681 \div 1516^2}$ and (ii) $\dfrac{547}{250}\left[\sum_{\mathrm{i}=1}^{3} \mathrm{i}(\mathrm{i}+3)(\mathrm{i}+4)\right]^{\frac{1}{2}}$.

In part (b), they were required to find (i) $\log y$ correct to six decimal places given that $y = \dfrac{-\sqrt[3]{3.14}}{\sin 45^\circ - \log 7}$ and (ii) the value of q if $2.37q^3 + 0.625e^{\pi} = 314$.

This question was attempted by 96.7 percent of the candidates of which 39 percent scored from 3.5 to 10 marks and among them 3.1 percent scored all the 10 marks. The question was averagely performed.

The candidates who managed to answer this question correctly were able to accurately perform the computations in parts (a) and (b) by using a scientific calculator.

In part (a)(i), some of the candidates evaluated the numerator and the denominator separately to get $\dfrac{263927.345449}{263927.349661}$ and then converted this fraction to the decimal number 0.999999984. Other candidates entered the expression into their calculator exactly as it was given and got the correct answer 0.999999984. It was noted that several candidates also rounded this decimal number to the nearest integer.

In part (a)(ii), the candidates showed good understanding on how to find the sum of the finite series, which was the essential step. They computed the required answer as follows:

$$\frac{547}{250}\left[\sum_{i=1}^{3} i(i+3)(i+4)\right]^{\frac{1}{2}}$$

$$= \frac{547}{250}\left[1(1+3)(1+4)+2(2+3)(2+4)+3(3+3)(3+4)\right]^{\frac{1}{2}}$$

$$= 31.40370781$$

In part (b)(i), the candidates were able to correctly enter the expression into their calculator from which they got $y = 10.61186312$ and thus $\log y = 1.025792$. In part (b)(ii), the candidates were able to make q subject of the equation and thereafter computed the required value of q. A sample solution of a candidate who answered this question correctly is shown in Extract 1.1.

Extract 1.1

1a) i) $\underline{0.999999984}$

ii) $\underline{31.40370781}$

b i) $\Rightarrow y = \frac{-\sqrt[3]{3.14}}{\sin 45^\circ - \log 7}$

$\Rightarrow \log(10.61186312)$

$\Rightarrow \therefore \log y = \underline{1.025792}$

ii) $2.37q^3 + 0.625e^{\pi} = 314$

$0.625e^{\pi} - 314 = -2.37q^3$

$2.37q^3 = 314 - 0.625e^{\pi}$

$q = \sqrt[3]{\frac{314 - 0.625e^{\pi}}{2.37}}$

$\therefore \underline{q = 5.018425}$

In Extract 1.1, the candidate was able to use a scientific calculator correctly in performing the computations.

However, 28.9 percent of the candidates who attempted this question scored zero. The analysis of their responses shows that these candidates lacked skills to use scientific calculators to perform computations. Some of the candidates wrote the answers in part (a)(i) as -15835.00092; 51.293 and 4.57754×10^{-5} instead of 0.999999984, indicating that they were unable to accurately enter the given expression in the calculators. It was noted that in part (a)(ii), most of the candidates were unable to identify the numbers to be summed, as a result ended up with incorrect answers. Wrong expansions of $\left[\sum_{i=1}^{3} i(i+3)(i+4)\right]^{\frac{1}{2}}$ such as $\sum_{i=1}^{3} 20^{\frac{1}{2}}+60^{\frac{1}{2}}+126^{\frac{1}{2}}$, $\left[\sum_{i=1}^{3} 1(1+3)(1+4)\right]^{\frac{1}{2}}$, $[3(3+3)(3+4)]^{\frac{1}{2}}$ e.t.c were usually seen in the candidates' scripts, an indicator of lack of understanding of the sigma notation.

In part (b)(i), some of the candidates obtained values of *y* such as $7.585021505, 17.426399$ and 2.379113 instead of $y=10.611863,$ a situation which shows that they entered wrong expressions in the calculators. For example some wrongly evaluated $\sqrt[3]{3.14}$ by dividing 3.14 by 3 while others were finding the square root of 3.14. It was noted that a number of candidates obtained values of *y* such as $\mathrm{y}=1.025961, 1.0259095$ and 1.025778 which lacked accuracy because they evaluated the numerator and the denominator separately by rounding each of the terms involved before reaching the final answer. For example they wrote, $\sqrt[3]{3.14}=1.46, \sin 45^{\circ}=0.7$ and $\log=0.8.$ It was also noted that some candidates obtained the correct value of *y* but did not proceed to find log *y* and mistakenly wrote that $\log y=10.611863.$

In part (b)(ii), several candidates were unable to obtain the final correct answer because they were unable to use the correct button on their calculator to find the cube root. It was noted that a number of candidates wrongly expressed $q=\sqrt{\frac{3.14-0.625e^{\pi}}{2.37}}$ instead of $q=\sqrt[3]{\frac{3.14-0.625e^{\pi}}{2.37}}$. Further analysis shows that, some candidates faced problems in finding the value of e^{π} and as a result obtained incorrect answer. Extract 1.2 is a sample answer showing some of the factors that made the candidates to perform poorly in this question.

Extract 1.2

1. a(i) $\dfrac{458.4^3 \times 0.00274 - 7560 \div 3567^3}{458.4^3 \times 0.00274 + 9681 \div 1516^2}$

$= \dfrac{263927.3454 - 7560 \div 4.538 \times 10^{10}}{263927.3454 + 9681 \div 2298256}$

$= \dfrac{-1.66593 \times 10^{13}}{263927.3454 + 4.212 \times 10^{-3}}$

$= \dfrac{1.66593 \times 10^{13}}{263927.3496}$

$= 63,120,779.35$

ii) $\dfrac{547}{250}\left[\sum_{i=1}^{3} i\,(i+3)(i+4)\right]^{1/2}$

~~Soln.~~

$= 2.188\left[5\right]^{1/2}$

$= 5.47$

$\therefore = 5.47$

b) i) $y = \dfrac{-\sqrt[3]{3.14}}{\sin 45^\circ - \log 7} = \dfrac{1.464344351}{0.7 - 0.845}$

$= \dfrac{1.464344351}{0.6155}$

$\therefore = 2.379113$

ii) $2.37q^3 + 0.625e^{\pi} = 314$

$2.37q^3 + 0.625e^{180} = 314$

$\dfrac{3\pi}{3} = \dfrac{314}{3} = 104 \quad = 2.37q^3 + 0.625 \times 1.48$

$= 104$

$q = 104$

Extract 1.2 shows that, the candidate lacked skills to use scientific calculator to perform computations and the basic mathematics knowledge to handle algebra.

2.2 Question 2: Functions

This question had parts (a) and (b). In part (a), the candidates were required to find (i) $f \circ g(25)$ and (ii) $g \circ f(14)$ given that $f(x) = 3x - 1$ and $g(x) = \sqrt{2x-1}$. In part (b), they were required to (i) verify that $x + 4$ is not a factor of the polynomial function $f(x) = x^3 - 9x^2 + 10x - 24$ and (ii) describe the nature of the stationary points of the function $f(x) = 2x^3 - 15x^2 + 24x$ and show them on the graph.

Many candidates (79.7%) answered this question, where 80.7 percent of them scored 0 to 3 out of 10 marks. Notably, 34.5 percent of the candidates scored 0 and only 19.3 percent scored from 3.5 to 10 marks. This question was therefore poorly performed.

In part (a), many candidates showed no understanding of composite functions. They did not understand that the composition of functions $f \circ g(x)$ or $g \circ f(x)$ is a process through which one function is substituted into another function. For example to create the new function $f \circ g(x)$, they were supposed to replace everywhere there is an x in $f(x)$ by the function $g(x)$. Evaluation of $f \circ g(x)$ and $g \circ f(x)$ at the points $x = 25$ and $x = 14$ was simply to plug in these values into the new functions. Contrary to this, the candidates applied incorrect concepts, for instance some multiplied the functions and then substituted the given values. Other candidates evaluated $g(25)$ instead of $f \circ g(25)$ and $f(14)$ instead of $g \circ f(14)$ while several candidates solved the equations $3x - 1 = 25$ and $\sqrt{2x-1} = 14$.

In part (b)(i), many candidates failed to verify that $x + 4$ is not a factor of the given polynomial function. They could neither divide the polynomial by the linear factor nor substitute $x = -4$ in the polynomial in order to carry out the test. The candidates were applying incorrect concepts such as equating $x + 4$ and $x^3 - 9x^2 + 10x - 24$ and thereafter solving the resulting equation for x, drawing the graphs of the linear factor and the polynomial or finding the first and the second derivatives of the polynomial function.

In (b)(ii), several candidates were unable to correctly determine the stationary points because they were unable either to differentiate the given function correctly to obtain $f'(x) = 6x^2 - 30x + 24$ or solve the quadratic equation

$f'(x) = 0$ to obtain the stationary points (1, 11) and (4, -16) . The candidates did not understand that a stationary point is a point on a curve for which the value of the first derivative is equal to zero. The analysis of responses shows that, most of the candidates used incorrect formulae, concepts and table of values in answering the question, see Extract 2.1.

Extract 2.1

2. (a) given

$f(x) = 3(x) - 1$ and $g(x) = \sqrt{2x-1}$.

(i) $f \cdot g(25)$.

Therefore,

$f \cdot g(25)$.

$g(25) = \sqrt{2 \times 25 - 1}$.

$= \sqrt{50 - 1}$.

$= \sqrt{49}$

$= 7$.

$\underline{f \cdot g(25) = 7}$.

(ii) $g \cdot f(14)$.

Therefore,

$g \cdot f(14)$.

$f(14) = 3 \times 14 - 1$

$= 42 - 1$

$= 41$.

$\underline{g \cdot f(14) = 41}$.

(b) (i)

$(x+4)(x+4)(x+4)$

$(x^2 + 8x + 16)(x+4)$

$x^3 + 8x^2 + 16x + 4x^2 + 32x + 64$

$x^3 + 12x^2 + 48x + 64$

Since the values $x^3 + 12x^2 + 48x + 64$ is Not equal to the function then $(x+4)$ is Not a factor of polynomial function.

2b	(i) $x = \frac{-b}{2a}$
	$x = \frac{-(-15)}{2 \times 2}$
	$x = \frac{15}{4}$
	$x = 3.75$
	$y = \frac{4ac - b^2}{4a}$
2b	(ii) $y = \frac{(4 \times 2 \times 24) - (-15)^2}{4 \times 2}$
	$y = -4.125$
	$\therefore$ Points are; $(3.75, -4.125)$

Extract 2.1 shows that, the candidate lacked knowledge on composite functions, factors and stationary points of a polynomial.

Only 0.001 percent of the candidates scored all the 10 marks. These candidates demonstrated good understanding of composite functions, factorizing polynomials and determining the maximum and minimum points of a polynomial. Extract 2.2 shows a sample answer from one of the candidates.

Extract 2.2

2(a)	$f \circ g(x) = f(g(x)) = 3(\sqrt{2x-1}) - 1$
	Thus
	$f \circ g(25) = 3(\sqrt{50-1}) - 1$
	$= 3\sqrt{49} - 1$
	$= 20$
	$\therefore f \circ g(25) = 20$
	(ii) $g \circ f(4)$
	From
	$g \circ f(x) = g(f(x))$
	$g \circ f(x) = g(f(x)) = \sqrt{2(3x-1) - 1}$

Then

$$gof(14) = \sqrt{2(3\times14-1) - 1}$$
$$= \sqrt{81}$$
$$= 9$$

$\therefore$ The value of $gof(14) = 9$

2(b) (i) Given
$f(x) = x^3 - 9x^2 + 10x - 24$

Required: To verify that $x+4$ is not a factor of $f(x)$

Then
for $x+4$ to be a factor of $f(x)$, the reminder
2b) should be equal to zero

let $x+4 = 0$
$x = -4$

Then the reminder, $R(x)$ is calculated by substituting the value of $x = -4$ in $f(x)$

$R(x)$ = Reminder of $f(x)$

$\Rightarrow$ Reminder of $f(x) = (-4)^3 - 9(-4)^2 + 10(-4) - 24$

Reminder $= -64 - 144 - 40 - 24$
Reminder $= -272$

Since the reminder is not equal to zero

$\therefore$ $x+4$ is not a factor of $f(x)$

hence verified

2(b) (ii) Given;
$f(x) = 2x^3 - 15x^2 + 24x$.

for stationary points
Firstly, find $f'(x)$

$f'(x) = 6x^2 - 30x + 24$

let $f'(x) = 0$ -- for stationary points

$0 = 6x^2 - 30x + 24$

on simplifying the equation above.

$x^2 - 5x + 4 = 0$

2b)(i)

$x^2 - x - 4x + 4 = 0$

$x(x-1) - 4(x-1) = 0$

$(x-4)(x-1) = 0$

$x - 4 = 0$ or $x - 1 = 0$

$x = 4$ or $x = 1$

when $x = 4$, $y = ?$

$y = f(4) = 2(4)^3 - 15(4)^2 + 24(4)$

$y = -16$

when $x = 1$

$y = f(1) = 2(1)^3 - 15(1)^2 + 24(1)$

$= 11$

$y = 11$

$\therefore (x, y) = (4, -16)$ or $(1, 11)$

For maximum and minimum points

$f''(x) = 12x - 30$

For maximum point

$f''(x) < 0$

$f''(1) = 12(1) - 30 = -18$

$f''(1) < 0$

$\therefore$ The ~~point~~ maximum point is $(1, 11)$

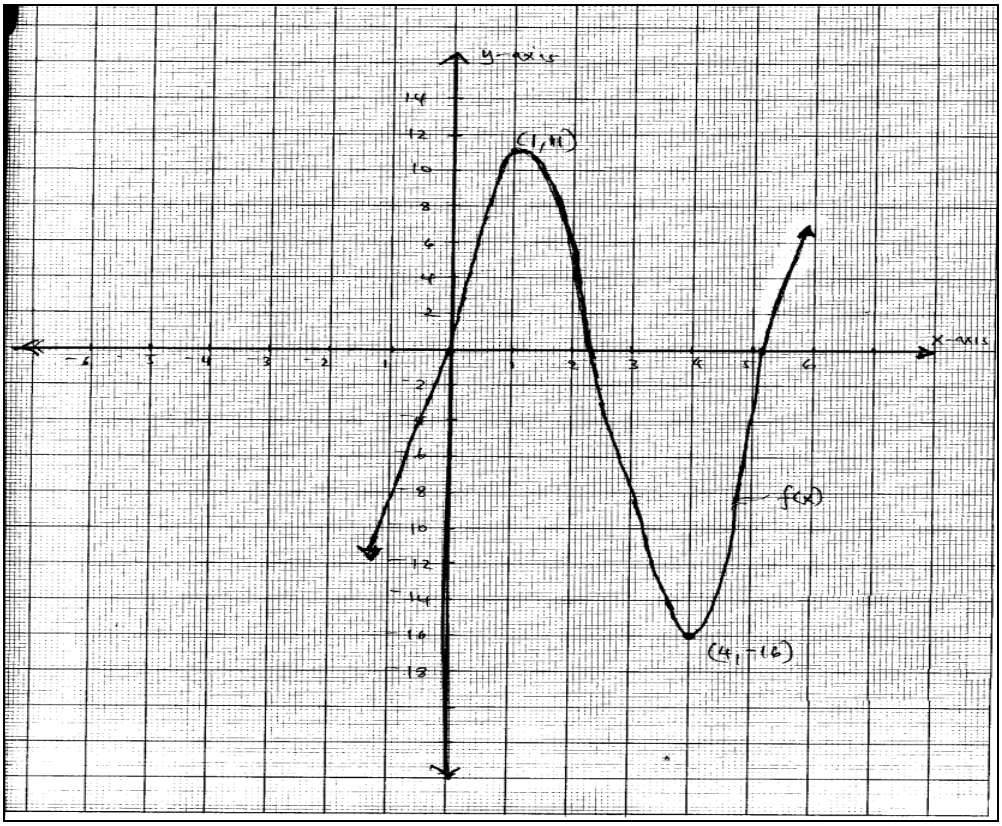
also for minimum point

$f''(x) > 0$

for $f''(4) = 12(4) - 30 = 18$

2b)(ii)

$\therefore$ The minimum stationary point is $(4, -16)$



Extract 2.2 shows that, the candidate answered the question correctly.

2.3 Question 3: Algebra

This question had parts (a) and (b). In part (a), the candidates were given a series defined by $S_n = \sum_{r=1}^{n} (2r - 3)$ and they were asked to (i) determine the value of S_{50} and (ii) find the value of n for which $S_n = 624$. In part (b), the candidates were required to solve the simultaneous equations $\log(x + y) = 1$ and $\log_2 x + 2\log_4 y = 4$.

This question was attempted by 73 percent of the candidates. Many of these candidates (76%) scored below 3.5 out of 10 marks. About half (49%) of the candidates who attempted this question scored zero. This question was therefore poorly performed.

In part (a), many candidates were unable to find the sum of the first fifty terms (S_{50}) of the given series and the number of terms for which the sum of the first n terms is 624. They were unable to realize that the given series could be represented as $S_n = -1+1+3+5+....$ which is an arithmetic progression (A.P) with the first term $a_1 = -1$ and the common difference $d = 2$. Although a few candidates were able to recognize that the given series forms an A.P, they could not correctly apply the formulae for finding the sum of n terms, to obtain $\mathrm{S}_{50} = \frac{50}{2}\{2(-1)+(50-1)(2)\} = 2{,}400$ or to solve for n in the equation $\frac{n}{2}\{2(-1)+(n-1)(2)\} = 624$. Several candidates worked out the solution wrongly, for example they found $\mathrm{S}_{50} = 2\times 50 - 3 = 9$ and solved the equation $2n-3 = 624$ to obtain the value of n.

In part (b), many candidates were unable to solve the given system of simultaneous equations. Most of the candidates were solving the equations without removing the logarithms and as a result could not obtain the required solution. Some candidates managed to apply the laws of logarithms to express the first equation as $x + y = 10$ but failed to remove the logarithms in the second equation. They did not realize they were supposed to change all the terms in the equation to the same base. For example they could have changed the equation to base 10 to get $\frac{\log x}{\log 2} + \frac{2\log y}{2\log 2} = 4$ and eventually $xy = 16$. Extract 3.1 is a sample answer illustrating how the candidates failed to answer this question.

Extract 3.1

3. (a) (i) $S_n = \sum_{r=1}^{n} (2r-3)$

$S_{50} = ((2\times 50)-3)$

$S_{50} = 100-3$

$S_{50} = 97$

$\therefore S_{50} = 97$

3 (a) (iii) $n = ?$

Solution.

From $S_n = \sum_{r=1}^{n} (2r - 3)$

$624 = (2n - 3)$

$624 + 3 = 2n$

$n = \frac{627}{2}$

$n = 313.5$

$\therefore n = 313.5$

3. (b) $\log(x+y) = 1$ — (i)

$\log_2 x + 2\log_4 y = 4$ — (ii)

From equation (i)

$\log(x+y) = \log x \times \log y$

$\log x \times \log y = 1$

also $\log_2 x + 2\log_4 y = 4.$

$= \log_2 x + 2\log_2(y+2) = 4.$

$= \frac{\log x}{\log 2} + \frac{2(\log(y+2))}{\log 2} = 4.$

$\log x + 2\log(y+2) = 4\log 2.$

$\log x + 2(\log y \times \log 2) = 4\log 2.$

From $\log x + \log y = 1$

$\log x = \frac{1}{\log y}$

$\frac{1}{\log y} + 2\log y + 2\log 2 = 4\log 2.$

$\frac{1 + 2\log y^2 + 2\log 2}{\log y} = 4\log 2$

$1 + 2\log y^2 + 2\log 2 = \log y \times 4\log 2$

$2\log y^2 - (4\log 2 \times \log y) + 2\log 2 + 1 = 0.$

let $\log y$ be x

3. (b) = $2x^2 - 1.2x + 1.6 = 0$

$x = 0.3$

but $x = \log y \quad = 0.3$

$y = \log^{-1}(0.3)$

$y = 2$.

from $\log(x+y) = 1$

$y = 2$

$\log(x+2) = 1$

$\log x \times \log 2 = 1$

$\log x = \frac{1}{\log 2}$

$x = \log^{-1}\left(\frac{1}{\log 2}\right)$

$x = 2.1$

$\therefore$ The value of $y = 2$ and the value of $x = 2.1$

In Extract 3.1, the candidate was unable to recall and apply the arithmetic progression formulae as well as the laws of exponents and logarithms in answering the question.

Despite the fact that many candidates had poor performance, 1.9 percent of the candidates answered this question correctly and hence scored all the 10 marks. A sample answer from one of these candidates is shown in Extract 3.2.

Extract 3.2

3 (a) (i) Given $S_n = \sum_{r=1}^{n} (2r-3)$

$S_{50} = \sum_{r=1}^{50} (2r-3)$

finding the formula for members.

for $r=1$ and $r=2$ and $r=3$

$S_n = 2(1)-3 + 2(2)-3 + 2(3)-3 + \cdots$

$= (2-3) + (4-3) + (6-3) + \cdots$

$S_n = -1 + 1 + 3 + \cdots$

Thus is arithmetic series with common difference $d = 1-(-1) = 3-1 = 2$

then from

$$S_n = \frac{n}{2}\left(2A_1 + (n-1)d\right) \text{ for}$$

arithmetic series where $n = 50$ $A_1 = -1$

$$S_{50} = \frac{50}{2}\left(2(-1) + (50-1)2\right)$$

$$= 25\left(-2 + 49(2)\right)$$

$$= 25(-2+98) = 25(96)$$

$$= 2400$$

$$\therefore S_{50} = 2400$$

3. (a) (ii) for $S_n = 624$

$$624 = \frac{n}{2}\left(2(-1) + (n-1)2\right)$$

$$624 = \frac{n}{2}\left(-2 + 2n - 2\right)$$

$$624 = \frac{n}{2}(2n-4)$$

$$624 = n^2 - 2n$$

$$n^2 - 2n - 624 = 0$$

On solving for n, $n = 26$

$$\therefore n = 26.$$

(b) $\begin{cases} \log(x+y) = 1 & \text{— (i)} \\ \log_2 x + 2\log_4 y = 4 & \text{— (ii)} \end{cases}$

for (ii)

$$\frac{\log x}{\log 2} + \frac{2\log y}{\log 4} = 4.$$

$$= \frac{\log x}{\log 2} + \frac{2\log y}{\log 2^2} = 4$$

$$= \frac{\log x}{\log 2} + \frac{2\log y}{2\log 2} = 4$$

$$= \frac{\log x}{\log 2} + \frac{\log y}{\log 2} = 4$$

$$= \log_2 x + \log_2 y = 4.$$

$$= \log_2(xy) = 4$$

and $\log(x+y) = 1$

writing in exponential form,

$$2^4 = xy \quad \text{—(i)}$$

$$10^1 = x+y \quad \text{—(ii)}$$

making y the subject of the formula.

$$10 - x = y \quad \text{—— (iii)}$$

putting (iii) into (i)

3. (b) $2^4 = x(10-x) = 10x - x^2$

$16 = 10x - x^2 \Rightarrow x^2 - 10x + 16 = 0$

on solving for x.

$x^2 - 8x - 2x + 16 = 0.$

$(x^2 - 8x) - (2x - 16) = 0.$

$x(x-8) - 2(x-8) = 0.$

$(x-2)(x-8) = 0$

either $x-2=0$ or $x-8=0.$

$x = 2$ or $x = 8$

Then $y = 10 - x$

$= 10-2$ or $10-8$

$= 8$ or 2

$\therefore$ $x = 2$ and $y = 8$

or $x = 8$ and $y = 2$

Extract 3.2 shows that, the candidate was able to answer the question correctly.

2.4 Question 4: Differentiation

This question had parts (a), (b) and (c). In part (a), the candidates were required to find $\frac{dy}{dx}$ given (i) $y = \frac{e^x\sqrt{\cos x}}{(2x+3)^2}$ when $x = 2\pi$ and (ii) $yx^2 - y^2x + 5y - 20x = 14$. In part (b), the candidates were required to differentiate the function $y = 4x^3 + 3x - 4$ from the first principles.

In part (c), the question was: A 13 m long ladder leans against a wall. The bottom of the ladder is pulled away from the wall at the rate of 6m/s. How fast does the height on the wall decrease when the foot of the ladder is 5 m away from the base of the wall?

Question 4 was attempted by 75.7 percent of the candidates. The majority of the candidates (91.7%) scored low marks (below 3.5 out of 10 marks) with 18.9 percent of them scoring 0. It was the second poorly performed question in this examination.

In part (a)(i), majority of the candidates were unable to apply the quotient rule: $\frac{dy}{dx}=\frac{vdu-udv}{v^2}$ where $u=e^x\sqrt{cos\,\mathrm{x}}$ and $v=(2x+3)^2$. Some interchanged these functions while others managed to identify them but failed to apply the product rule to differentiate the function *u*. Many candidates faced problems in differentiating $\sqrt{\mathrm{cosx}}$, as they failed to apply the chain rule $\frac{dw}{dx}=\frac{dw}{dz}\cdot\frac{dz}{dx}$, where $w=\sqrt{cosx}$ and $z=cosx,$ to obtain $\frac{dw}{dx}=\frac{1}{2}z^{-\frac{1}{2}}\cdot sinx=\frac{1}{2}(cosx)^{-\frac{1}{2}}sinx.$

In part (a)(ii), some candidates lacked knowledge of implicit differentiation while others had the knowledge of differentiating the function term by term with respect to *x* but then faced problems in finding the derivative of the terms like y and y^2 with respect to *x*. It was noted that, a number of candidates differentiated all the terms correctly to obtain $2xydx+x^2dy-y^2dx-2xydy+5dy-20dx=0$ but then failed to correctly express $\frac{dy}{dx}$ in terms of *x* and *y*.

In part (b), many candidates were able to apply the definition $\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ to obtain $\lim_{\Delta \mathrm{x}\to 0}\frac{4(\mathrm{x}+\Delta\mathrm{x})^3+3(\mathrm{x}+\Delta\mathrm{x})-4-(4\mathrm{x}^3+3\mathrm{x}-4)}{\Delta\mathrm{x}}$ but could not correctly expand and simplify the terms on the numerator of this expression. The candidates made mistakes in multiplying, adding or subtracting.

In part (c), some candidates were unable to identify the requirements of the question, as a result failed to translate it mathematically while others did not answer this part. Some of the candidates wrongly represented the given information diagrammatically while others used wrong concepts and formulae such as $s=ut+\frac{1}{2}at^2\ or\ \ v=u+at$. Extract 4.1 is a sample answer showing how the candidates failed to answer this question.

Extract 4.1

4\. a) i. required $\frac{dy}{dx}$ of

i. $y = \frac{e^{x}\sqrt{\cos x}}{(2x+3)^2}$. where $x = 2\pi$.

$y = \frac{e^{2\pi}\sqrt{\cos 2\pi}}{(2\times 2\pi+3)^2}$.

note that $(2\times 2\pi+3)^2 = 242.312$.

$y = \frac{e^{2\pi}(\cos 2\pi)^{\frac{1}{2}}}{242.312}$.

by using product rule.

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$. where $u = e^{2\pi}$ and $v = (\cos 2\pi)^{\frac{1}{2}}$

$\frac{dy}{dx} = e^{2\pi}\left(\frac{-2\sin 2\pi}{2}\right)^{-\frac{1}{2}} + (\cos 2\pi)^{\frac{1}{2}} \times 2\pi\, e^{2\pi}$.

$= \frac{-e^{2\pi}(\sin 2\pi)^{-\frac{1}{2}} + (\cos 2\pi)^{\frac{1}{2}} \times 2\pi e^{2\pi}}{242.312}$.

$\therefore \frac{dy}{dx} = \left(-\frac{e^{2\pi}}{\sqrt{\sin 2\pi}} + \sqrt{\cos 2\pi}\cdot 2\pi e^{2\pi}\right)\frac{1}{242.312}$.

4\. a) ii) soln.

given: $yx^2 - y^2x + 5y - 20x = 14$.

Applying "d" operator.

$d\,(yx^2 - y^2x + 5y - 20x = 14)$.

$2x\frac{dy}{dx} - 2y\frac{dy}{dx} + 5\frac{dy}{dx} - 20\frac{dx}{dx} = 0$.

$\frac{dy}{dx}(2x - 2y + 5) - 20 = 0$.

$\frac{dy}{dx}(2x - 2y + 5) = 20$.

$\frac{dy}{dx} = \frac{20}{2x - 2y + 5}$.

$\therefore \frac{dy}{dx} = \frac{20}{2x - 2y + 5}$.

4	b) $F(x) = 4x^3 + 3x - 4$
	From
	$\frac{dy}{dx} = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$
	$= \lim_{h\to} \frac{4(x+h)^3 + 3(x+h) - 4x^3 + 3x - 4}{h}$

4	b) $= \lim_{h\to 0} \frac{4(x^3 + 3x^2h + 2xh^2 + h^3) + 3(x+h) - 4x^3 + 3x - 4}{h}$
	$= \lim_{h\to 0} \frac{\cancel{4x^3} + 12x^2h + 8xh^2 + 4h^3 + 3x + 3h - \cancel{4x^3} + 3x - 4}{h}$
	$= \lim_{h\to 0} 12x^2 + 8xh + 4h^2 + 3x + 3 + 3x - 4$
	$= \lim 12x^2 + (8x \times 0) + (4 \times 0^2) + 3x + 3 + 3x - 4$
	$= \lim 12x^2 + 6x + 1.$
	$\frac{dy}{dx} = 12x^2 + 6x - 1.$

4	c. given initial height of the ladder = 13m.
	Rate pulled initially = 6 m/s
	Then Final height = 5m
	required Final rate.
	From $y = ut + \frac{1}{2}gt^2$
	$\left(\frac{dy}{dt}\right) = u + 2gt$
	$\frac{dy}{dx} = u + 5t$
	$6\frac{m}{s} = 0 + 9.8t.$
	$t = 0.612$
	$\frac{dy}{dx} = u + gt.$

In Extract 4.1, the candidate substituted the value of $x = 2\pi$ before differentiating in part (a)(i). In part (a)(ii), the candidate failed to differentiate the given implicit function. In part (b), he/she made mistakes while substituting the function in the definition of differentiating from the first principles and incorrectly expanded the brackets. In part (c), the candidate applied inappropriate concepts.

Only 4 out of 22,195 candidates managed to answer this question correctly and scored all the 10 marks. Extract 4.2 is a sample answer showing how the concepts of differentiation were correctly applied in answering parts (a), (b) and (c).

Extract 4.2

4. (a) (i) $y = \dfrac{e^x\sqrt{\cos x}}{(2x+3)^2}$

$y = \dfrac{e^x\sqrt{\cos x}}{(2x+3)(2x+3)}$

$y = \dfrac{e^x\sqrt{\cos x}}{(4x^2+12x+9)}$

Let

$u = e^x\sqrt{\cos x}$

$\dfrac{du}{dx} = e^x\sqrt{\cos x} + \dfrac{1}{2}(-\sin x)(\cos x)^{-\frac{1}{2}} e^x$

$\dfrac{du}{dx} = e^x\left((\cos x)^{\frac{1}{2}} - \dfrac{\sin x}{2\sqrt{\cos x}}\right)$

$\dfrac{du}{dx} = e^x\left(\sqrt{\cos x} - \dfrac{\sin x}{2\sqrt{\cos x}}\right)$

$v = (4x^2+12x+9)$

$\dfrac{dv}{dx} = (8x+12)$

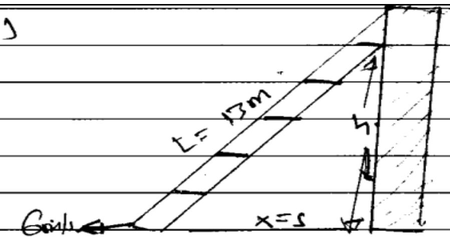
By quotient rule

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$= \dfrac{(2x+3)^2 e^x\left(\sqrt{\cos x} - \dfrac{\sin x}{2\sqrt{\cos x}}\right) - e^x\sqrt{\cos x}(8x+12)}{((2x+3)^2)^2}$

4 (a)(ii)	Soln
	Given $yx^2 - y^2x + 5y - 20x = 14.$
	~~2x~~ Differentiating each term with respect to x.
	$2xy + x^2\frac{dy}{dx} - \left(y^2 + 2xy\frac{dy}{dx}\right) + 5\frac{dy}{dx} - 20 = 0$
	$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} + 5\frac{dy}{dx} = y^2 - 2xy + 20$
	$\frac{dy}{dx}\left(x^2 - 2xy + 5\right) = y^2 - 2xy + 20$
	$\therefore \frac{dy}{dx} = \frac{y^2 - 2xy + 20}{x^2 - 2xy + 5}$
4. (b)	Soln
	Given $f(x) = 4x^3 + 3x - 4.$
	If h is small increment of x.
	$f(x+h) = 4(x+h)^3 + 3(x+h) - 4.$
	By first principle
	$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$
	$= \lim_{h\to 0} \frac{4(x+h)^3 + 3(x+h) - 4 - (4x^3 + 3x - 4)}{h}$
	$= \lim_{h\to 0} \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) + 3x + 3h - 4 - 4x^3 - 3x + 4}{h}$
	$= \lim_{h\to 0} \frac{\cancel{4x^3} + 12x^2h + 12xh^2 + 4h^3 + \cancel{3x} + 3h \cancel{- 4} \cancel{- 4x^3} \cancel{- 3x} \cancel{+ 4}}{h}$
	$= \lim_{h\to 0} \frac{12x^2h + 12xh^2 + 4h^3 + 3h}{h}$
	$= \lim_{h\to 0} 12x^2 + 12xh + 4h^2 + 3.$
	As $h \to 0$
	$12xh = 12 \times x \times 0 = 0.$
	$4h^2 = 4(0)^2 = 0.$
	$\therefore 12x^2 + 12xh + 4h^2 + 3 = 12x^2 + 3.$
	$\therefore f'(x) = 12x^2 + 3$

4 (c)



$h^2 = L^2 - x^2$

$h^2 = 13^2 - x^2$

$h = \sqrt{13^2 - x^2}$

$\frac{2h\,dh}{dt} = -2x\frac{dx}{dt}$

$2h\frac{dh}{dt} = -2x\frac{dx}{dt}$

but $\frac{dh}{dt} = \frac{-x}{h}\frac{dx}{dt}$

but $dx/dt = 6\,m/s$

$x = 5$

$h = \sqrt{13^2 - x^2}$

$= \sqrt{13^2 - 5^2}$

$= 12$

$\frac{dh}{dt} = -\frac{5}{12}(6)$

$\frac{dh}{dt} = -2.5\,m/s$

∴ the height of wall decreases by 2.5 m/s.

In Extract 4.2, the candidate was able to apply correctly the concepts of differentiation in parts (a), (b) and (c).

2.5 Question 5: Integration

This question had parts (a), (b) and (c). In part (a), the candidates were required to evaluate the integral $\int_{0}^{0.5\pi} cos^3 x\, dx$. In part (b), they were required to find the equation of a curve whose slope at any point is defined by the equation $\frac{dy}{dx} = 3x - \frac{1}{x^2}$ where $x \neq 0$. In part (c), given a region bounded by the lines $y = mx$, $y = h$, $y = 0$ and $x = 0$, which is rotated about the y-axis, the candidates were required to find the volume of the region in terms of h and *r*, when $x = r$ and $y = h$.

This question was attempted by 37.6 percent of the candidates. The majority (97%) of these candidates scored from 0 to 3 out of 10 marks. Notably, 68.8 percent of the candidates scored 0 and only 0.4 percent managed to score all the 10 marks. It was the least attempted and worst performed question.

Most of the candidates lacked knowledge and skills of integration. In part (a), they were unable to use trigonometric identities to reduce the integrand into a function which could be easily integrated, for example $\cos^3 x = (1 - \sin^2 x)\cos x$ or $\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$. Instead some candidates evaluated the integral incorrectly, for example as $\int \cos^3 x dx = \sin^3 x$, or $\int \cos^3 x dx = \frac{\cos^4 x}{4}$, not back testing their solutions since integration is the inverse process of differentiation.

In part (b), majority of the candidates were unable to identify the requirement of the question and as a result applied incorrect formulae and concepts in finding the equation of the curve. For example some candidates found the horizontal and vertical asymptotes; the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ whereas other candidates wrongly considered the given equation as the equation of the curve. The candidates did not realize that the equation of the curve was to be obtained by integrating the given equation, that is, $y = \int dy = \int \left(3x - \frac{1}{x^2}\right) dx$.

Part (c) was also poorly done as most of the candidates were unable to correctly apply the formula $V = \int_a^b \pi f(y)dy$ to find the volume of the solid generated. The analysis of responses shows that the candidates performed poorly because most of them either used incorrect limits of integration or they integrated an incorrect function *f*(*y*), such as $V = \pi\int_r^h \left(\frac{y}{m}\right)^2 dy$, $V = \pi\int_0^h \left(\frac{y}{m}\right)^2 dy$ and $V = \pi\int \left(\frac{h}{r}\right)^2 dx$. This indicates that the tested concept was not well understood by the candidates. See Extract 5.1.

Extract 5.1

5 (a) $\int_0^{0.5\pi} \cos^2 x \, dx$.

= Solution.

$\int_0^{0.5\pi} \cos^2 x \, dx$

$= \sin^2 x \Big|_0^{0.5\pi}$

$= \sin^2 0.5\pi - \sin^2 0$.

$= 9.0598 \times 10^{-5}$

$\therefore$ 9.0598×10^{-5} Answer.

5(B) Given Slope = $\frac{dy}{dx} = 3x - \frac{1}{x^2}$. $x \neq 0$. —— ①

Equation of the curve.
From $y = mx + c$.
But m = slope = $3x - \frac{1}{x^2}$. So

Substituting the slope into eqn ①.

$$y = \left(3x - \frac{1}{x^2}\right)x + c$$

$$y = 3x^2 - \frac{x}{x^2} + c$$

$$y = 3x^2 - \frac{1}{x} + c$$

∴ The equation of the line was $y = 3x^2 - \frac{1}{x} + c$.

5. (c) $y = mx$

$y = mx$

$h = mr$

$m = \frac{h}{r}$

Volume at y-axis

$\pi \int x^2 dx$

$\pi \int \left(h/r\right)^2 dx$

$$\left.\frac{h/r^{2+1}}{2+1}\right]\pi$$

$$\left.\frac{h/r^{3}}{3}\right|\pi$$

Volume $= \pi/3 \, (h/r)^3$

Extract 5.1 shows a solution from a candidate who lacked the basic knowledge and skills of integration.

On the other hand, the candidates who did well in this question demonstrated a good understanding of the concept of integration. In part (a), they were able to use substitution of trigonometric identities correctly in carrying out the integration. In part (b), the candidates were able to re-write the given equation as $dy = \left(3x - x^{-2}\right)dx$ and thereafter integrate to obtain the required equation of the curve. In part (c), the candidates were able to translate the given word problem mathematically and then correctly integrate to find the volume. Extract 5.2 is a sample solution of a candidate who performed well in this question.

Extract 5.2

5a) $\int_0^{0.5\pi} \cos^3 x \, dx$

$= \int_0^{0.5\pi} \cos^2 x \cdot \cos x \, dx$ from $\sin^2 x + \cos^2 x = 1$.

$\cos^2 x = 1 - \sin^2 x$.

$= \int_0^{0.5\pi} (1 - \sin^2 x) \cos x \, dx$ let $\sin x = u$

$du/dx = \cos x$.

$du/\cos x = dx$.

$= \int_0^{0.5\pi} (1 - \sin^2 x) \frac{du}{\cancel{\cos x}} \cdot \cancel{\cos x}$

but $\sin x = u$

$= \int (1 - u^2) \, du = u - \frac{u^3}{3} + c$.

$= \sin x - \frac{\sin^3 x}{3} \Big|_0^{0.5\pi}$ $0.5\pi = 90°$.

$= \left[\sin 90° - \frac{\sin^3 90}{3}\right] - \left[\sin 0 - \frac{\sin^3 0}{3}\right]$

$= \left[1 - \frac{1}{3}\right] - 0 = \underline{\underline{2/3}}$.

9) $\frac{dy}{dx} = 3x - \frac{1}{x^2}$ when $x \neq 0$

$y = \int (3x - \frac{1}{x^2}) dx.$

$= \int (3x\,dx - \frac{dx}{x^2})$

$= \left[\frac{3x^2}{2} + \frac{1}{x}\right] + c.$

$\therefore y = \frac{3x^2}{2} + \frac{1}{x} + c.$

$\therefore$ equation of the curve is $y = \frac{3x^2}{2} + \frac{1}{x} + c.$

1(e)

$V = \pi \int_a^b x^2 dy$

where $a = 0$, $b = h$.

$x^2 = \frac{y^2}{M^2}.$

$\therefore$

$V = \pi \int_0^h \frac{y^2}{M^2} dy.$

$= \frac{\pi}{M^2}\left[\frac{y^3}{3}\right]_0^h$

$= \frac{\pi}{3M^2}(h^3 - 0^3)$

$= \frac{\pi h^3}{3M^2}.$

but $h = Mx$, $x = r$

$\Rightarrow M = \frac{h}{x} = \frac{h}{r}.$

$V = \frac{\pi h^3}{3(\frac{h^2}{r^2})} = \frac{\pi h^3}{3h^2} r^2 = \frac{\pi h r^2}{3}$

$\therefore$ The volume generated is

$V = \frac{\pi h r^2}{3}$

Extract 5.2 shows a solution from a candidate who was able to apply the concept of integration correctly.

2.6 Question 6: Statistics

This question had parts (a), (b) and (c). In part (a), the candidates were required to give the definitions of (i) range and (ii) class size as applied in statistics.

In part (b), they were given a frequency distribution table which showed the number of absent workers of a certain Gold Mining Company and the corresponding frequencies and they were required to construct a cumulative frequency.

In part (c), the candidates were given the time in seconds for a students swimming pool competition and were required to (i) prepare the frequency distribution using the class intervals of 0 - 4, 5 - 9, etc. and (ii) determine the standard deviation of the data.

This question was attempted by 98.9 percent of the candidates; 77.3 percent of these candidates scored from 3.5 to 10 marks and 2.2 percent of them scored all the 10 marks. This was the most attempted and best performed question in this examination.

The candidates who performed well in this question were able to define the given terms correctly in part (a). Some of the definitions they gave include; *a range is the difference between the largest and the smallest number in given data* whereas *class size is the difference between the upper real limit and lower real limit of a class or it is the difference between two successive upper real limits.*

In part (b), the candidates were able to prepare a table which showed the cumulative frequencies with their corresponding upper real limits and correctly used it to construct the cumulative frequency curve.

In part (c), they were able to correctly count the number of absent workers in each class and prepare the frequency distribution as required. The candidates also managed to correctly determine the standard deviation using the formulae $\delta = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$ and $\delta = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$. Extract 6.1 is a sample solution of a candidate who performed well in this question.

Extract 6.1

6 (a) i/ Range
This is the difference between the highest value and the lowest Value of given data.
Range = H − L
where H - Highest Value
L - Lowest Value.

ii/. Class size.
This is the difference between the upper Boundary and the Lower Boundary of a given Class Interval.
It is the difference of upper Real limit and the lower real limit.

(b) Table of workers.

Number of Absent wks	f	x	C.F	Upper Boundary
5 − 9	6	7	6	9,5
10 − 14	9	12	15	14.5
15 − 19	18	17	33	19.5
20 − 24	16	22	49	24,5
25 − 29	3	27	52	29.5

6 (c) Table of Values (Distribution Table).

C Interval	f	x	fx	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
20-24	3	22	66	−6.625	43.891	131.673
25-29	21	27	567	−1.625	2.641	55.461
30-34	16	32	512	3.375	11.391	182.256
	$\Sigma f = 40$		$\Sigma fx = 1145$			$\Sigma f(x-\bar{x})^2 = 369.39$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1145}{40} = 28.625.$$

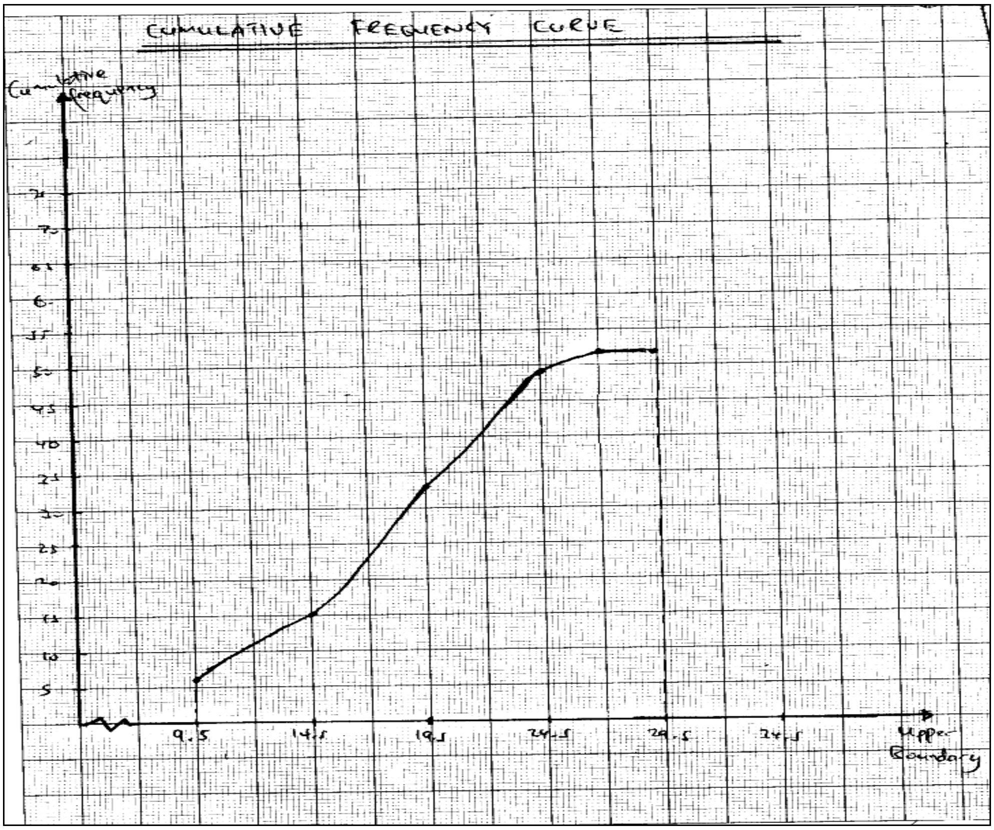
$$\text{S.Deviation} = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{369.39}{40}}$$

$$= 3.0389$$

$\therefore$ Standard Deviation $= 3.0389$.



Extract 6.1 is a solution from a candidate who demonstrated good understanding on the topic of Statistics.

However, some candidates (22.7%) scored from 0 to 3 out 10 marks and 1.6 percent of these candidates scored zero. In part (a), most of these candidates were unable to define the basic Statistics terms. Some of the definitions they gave for range include: *range is the difference of highest and lowest; range is the difference between highest class and lowest class; range is the outcome of taking upper real limit and lower real limit.* The definitions they gave for class size include: *class size is the difference between two adjacent class marks; class size is the total values within a class interval; class size is the interval from one class mark to another*.

In part (b), the candidates drew incorrect cumulative frequency curves. Most of them plotted cumulative frequency on the vertical axis correctly but plotted class marks on the horizontal axis instead of the upper real limits. Some candidates used both incorrect cumulative frequencies and upper real limits. Other candidates incorrectly plotted the frequencies on the vertical axis and the cumulative frequencies on the horizontal axis. Bar charts, histograms and frequency polygons are also examples of graphs that were drawn by the candidates who performed poorly in this part.

In part (c), the candidates computed the standard deviation using either incorrect data or incorrect formulae or both incorrect data and formulae. Some of the incorrect formulae that were picked from the candidates scripts include: $\delta = \sqrt{\frac{\sum(f-\bar{x})^2}{\sum f}}$, $\delta = \sqrt{\frac{\sum(x_i - x)^2}{N}}$ and $\delta = \sqrt{\frac{f(x-\bar{x})^2}{\sum N}}$. Extract 6.2 is a sample answer showing how the candidates performed poorly in this question.

Extract 6.2

06. (a) (i) Range - Is the highest class minus lowest class in the data.

(ii) Class size - Is the class mark of the highest frequency in the data.

(c) (i) Distribution table.

Class int	Frequency	X	Fx	$(\bar{x}-x)$	$(\bar{x}-x)^2$
20-25	7	22.5	157.5	~~-129.25~~ 5.75	~~16705.56~~ 33.0625
25-30	20	27.5	550	1	1
30-35	13	32.5	422.5	-4	16
	N = 40		$\Sigma Fx = 1130$		Σf 50.0625

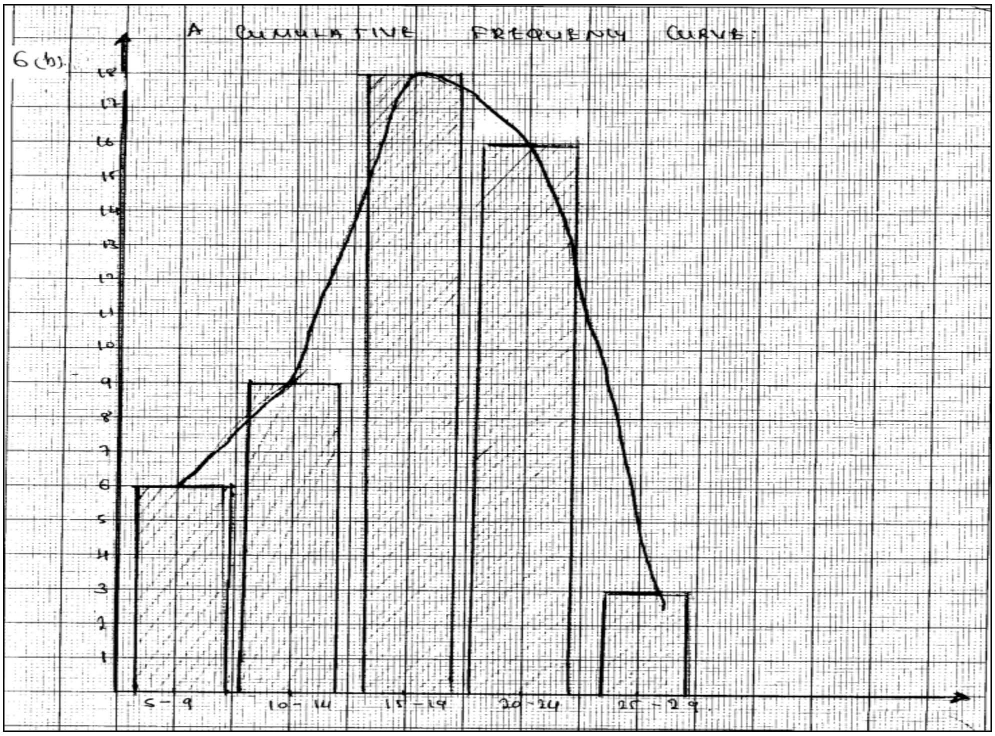
(ii) Standard deviation

$$S.D = \sqrt{\frac{\Sigma f(\bar{x}-x)^2}{N}}$$

06 (c) (ii)

$$S.D \sqrt{\frac{50.0625}{40}}$$

$\therefore$ Standard deviation $= 1.118$



In Extract 6.2, the candidate was not able to apply the tested concepts of Statistics correctly.

2.7 Question 7: Probability

The question had parts (a) and (b). In part (a), the candidates were given that $P(n, 4) = 42P(n, 2)$ and were required to (i) find the value of n and (ii) evaluate $P(n, 2)$ and $P(n, 4)$. In part (b), the candidates were supposed to find (i) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$, (ii) $P(A \cup B)$ and (iii) $P(A \cup C)$ given that $P(A) = \frac{1}{2}$, $\mathrm{P}(\mathrm{B}) = \frac{1}{4}$, $P(C) = \frac{1}{3}$ and that A and B are independent events while B and C are mutually exclusive events.

This question was attempted by 76.3 percent of the candidates, where 86.2 percent scored below 3.5 out of 10 marks and 14.1 percent of them scored zero. Only 13.8 percent scored from 3.5 to 10 marks. The question was poorly performed.

The analysis of responses shows that in part (a), some of the candidates failed to recall the correct permutation formula and hence failed to answer this question. The analysis also shows that several candidates managed to express the given permutation equation as $\frac{n!}{(n-4)!} = 42\frac{n!}{(n-2)!}$ but then failed to find the value of *n*. These candidates did not realize that they could replace $(n-2)!$ by $(n-2)(n-3)(n-4)!$ in order to obtain the quadratic equation $n^2 - 5n - 36 = 0$, an essential step in finding the value of *n*.

In part (b), many candidates showed no understanding of the rules of probability. The candidates were using incorrect rules such as $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B})$, $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{C})$ and $\mathrm{P}(\mathrm{A} \cup \mathrm{C}) = \mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{C})$. The candidates were supposed to apply the following rules in answering this part: (i) $P(A \cap B) = P(A)P(B)$ (ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and (iii) $P(A \cup C) = P(A) + P(C)$ or $P(A \cup C) = P(A) + P(C) - P(A \cap C)$. Extract 7.1 is a sample solution showing some of the difficulties the candidates faced in answering this question.

Extract 7.1

7	a/: $P(n, 4) = 42\,P(n,2)$
	i/ Value of n;
	Soln;
	$P(n, 4) = 42\,P(n,2)$
	$\Rightarrow \frac{\cancel{P}(n+4)}{\cancel{P}} = 42\frac{\cancel{P}(n+2)}{\cancel{P}}$
	$n+4 = 42(n+2)$
	$n+4 = 42n + 84$
	$n - 42n = 84 - 4$
	$-41n = 80$
	$n = \frac{-80}{41}$
	$n = -1{\cdot}95$
	$\therefore$ Value of $n = -1{\cdot}95$

ii/ $P(n, 2)$

$\Rightarrow P(n+2)$

but $n = -1.95$

$P(n+2) = P(-1.95+2)$

$\Rightarrow \underline{P(n+2) = P(0.05)}$

$P(n+4)$

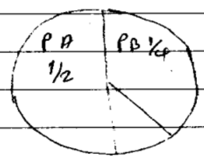
$P(-1.95+4)$

$P(2.05)$

$\Rightarrow \underline{P(n+4) = P(2.05)}$

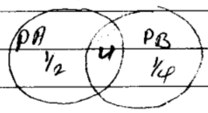
7 (b) (i) $P(A \cap B)$

$\frac{1}{2} \cap \frac{1}{4}$



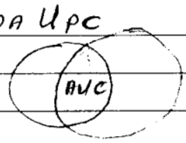
(ii) $P(A \cup B)$

$\frac{1}{2} \cup \frac{1}{4}$



(iii) $P(A \cup C) = \frac{1}{2} \cup \frac{1}{3}$

$PA \cup PC$



Extract 7.1 shows that the candidate lacked knowledge of permutations and probability.

However 1.6 percent of the candidates managed to answer this question correctly and scored all the 10 marks. Extract 7.2 shows a sample answer from one of the candidates.

Extract 7.2

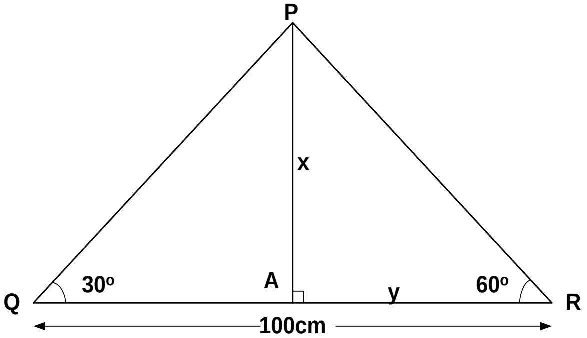
7 (a)	
(i)	$P(n,4) = 42P(n,2)$.
	$^nP_4 = 42(^nP_2)$
	$\frac{n!}{(n-4)!} = 42 \times \frac{n!}{(n-2)!}$
	$\frac{n!}{(n-4)!} = 42 \times \frac{n!}{(n-2)(n-3)(n-4)!}$
	$\frac{1}{1} = \frac{42.}{(n-2)(n-3)}$
	$(n-2)(n-3) = 42.$
	$n^2-3n-2n+6 = 42.$
	$n^2-5n+6-42 = 0.$
	$n^2-5n-36 = 0,$
	$n^2-9n+4n-36 = 0.$
	$n(n-9)+4(n-9) = 0.$
	$(n-9)(n+4) = 0.$
	$n-9 = 0.$ OR $n+4 = 0$
	$n = 9.$ $n = -4$
	but n cant be negative
	$\therefore n = 9$
7(a)(ii)	$= \frac{9!}{(9-2)!} = \frac{9!}{7!}$
	$= \frac{9 \times 8 \times 7!}{7!}$
	$= 9 \times 8 = 72$
	$\therefore P(n,2) = 72.$
	$P(n,4)$
	$= {^nP_4}$
	$= \frac{n!}{(n-4)!}$
	$= \frac{9!}{(9-4)!}$
	$= \frac{9!}{5!}$
	$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!}$
	$= 3024.$
	$\therefore P(n,4) = 3024$

7(b)	$P(A) = 1/2 \quad P(B) = 1/4 \quad P(C) = 1/3.$
(i)	$P(A \cap B) = P(A) \times P(B)$
	$= \frac{1}{2} \times \frac{1}{4}. \quad = \frac{1}{8}.$
	$P(A \cap B) = 1/8$
7(b)	
(ii)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	$= \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$
	$P(A \cup B) = \frac{5}{8}.$
(iii)	$P(A \cup C) = P(A) + P(C)$
	$= \frac{1}{2} + \frac{1}{3}.$
	$P(A \cup C) = \frac{5}{6}.$

In Extract 7.2, the candidate was able to correctly apply the permutation formula and probability concepts in answering this question.

2.8 Question 8: Trigonometry

This question had parts (a) and (b). In part (a), the candidates were required to (i) express $\sin 3\theta$ in terms of $\sin\theta$ and (ii) show that $\sqrt{\frac{1-\cos\phi}{1+\cos\phi}} = \operatorname{cosec}\phi - \cot\phi$. In part (b), the candidates were required to (i) determine the values of *x*, *y* and (ii) find $\sin(Q\hat{P}A)$ from the following figure:



This question was attempted by 49.2 percent of the candidates; 84.4 percent of these candidates scored from 0 to 3 marks and 47.8 percent of them scored zero. The question was poorly performed because only 15.6 percent scored above 3 marks. Furthermore, it was the second least attempted question.

In part (a)(i), many candidates were unable to recall and apply the trigonometric identities $\sin(A+B) = \sin A\cos B + \cos A\sin B$, $\sin^2 A + \cos^2 A = 1$, $\sin 2A = 2\sin A\cos A$ and $\cos 2A = \sin^2 A - \cos^2 A$ in expanding $\sin 3\theta$. Likewise in part (a)(ii), the candidates could not use the reciprocals of the three main trigonometric functions, that is, $\operatorname{cosec} A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$ and $\cot A = \frac{1}{\tan A}$ in proving the identity. Extract 8.1 illustrates this case.

In part (b), the candidates were unable to use the definitions of the trigonometric ratios to determine the values of x, y and $sin(Q\hat{P}A)$. They could not consider the right angled triangles PAQ and PAR to obtain equations such as $\sin 30^\circ = \frac{x}{QP}$, $\sin 60^\circ = \frac{100-y}{QP}$ and $\tan 60^\circ = \frac{x}{QP}$ that were to be solved to get the required values. The candidates were performing irrelevant calculations.

Extract 8.1

8.	a) i) Express sin 3θ in term of sin θ.
	solution
	$\sin 3\theta = (\theta + \theta + \theta)$
	$\sin 3\theta = \sin 3\theta \cos 3\theta + \cancel{\sin\theta} \cos\theta \cdot \sin\theta + \sin\theta\cos\theta$
	$= 2\sin\theta\cos\theta + \cos\theta\sin\theta.$
	$= \cos\theta\,(2\sin\theta + \sin\theta)$
	$= \cos\theta\,(3\sin\theta)$
	$= 3\cos\theta\,(\sin\theta)$
	$= 3\cos\theta\,(\sin\theta)$
	$\frac{\theta}{3\cos\theta} = \sin\theta$
	$= \sin\theta.$
	$\therefore \sin 3\theta = \sin(\theta + \theta + \theta).$

8(a)(ii) required to show

$$\sqrt{\frac{(1-\cos\phi)}{(1+\cos\phi)}} = \operatorname{cosec}\phi - \cot\phi$$

consider L.H.S

$$= \sqrt{\frac{(1-\cos\phi)}{(1+\cos\phi)}} \quad ---(i)$$

from $\sin^2\phi + \cos^2\phi = 1$

$$\sin^2\phi = 1-\cos^2\phi$$

$$\cos\phi = \sqrt{1-\sin^2\phi} \quad ---(ii)$$

substitute (ii) into (i)

$$= \sqrt{\frac{1-(1-\sin^2\phi)^{1/2}}{1+(1-\sin^2\phi)^{1/2}}} = \sqrt{\frac{1^{1/2}-(1-\sin^2\phi)^{1/2}}{1^{1/2}+(1-\sin^2\phi)^{1/2}}}$$

$$= \sqrt{\frac{(\sin^2\phi)^{1/2}}{(2-\sin^2\phi)^{1/2}}}$$

$$= \sqrt{\frac{(\sin^2\phi)^{1/2}}{(1+1-\sin^2\phi)^{1/2}}} \quad ---(iii)$$

but $1 = \sin^2\phi + \cos^2\phi \quad ---(iv)$

substitute eqn (iv) into (iii)

$$= \sqrt{\frac{(\sin^2\phi)^{1/2}}{(\sin^2\phi + \cos^2\phi - \sin^2\phi + \sin^2\phi + \cos^2\phi)^{1/2}}}$$

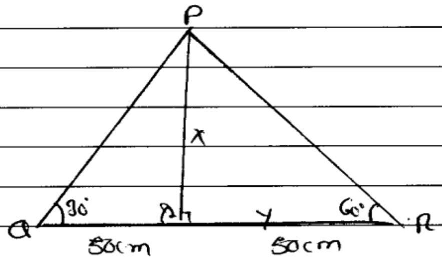
8(a) (ii)

$$= \sqrt{\frac{(\sin^2\phi)^{1/2}}{(\sin^2\phi + 2\cos^2\phi)^{1/2}}}$$

$$= \sqrt{\left(\frac{(\sin^2\phi)}{\sin^2\phi + 2\cos^2\phi}\right)^{1/2}}$$

$$= \sqrt{\left(\frac{\sin^2\phi + 2\cos^2\phi}{\sin^2\phi}\right)^{1/2}}$$

8 (b) i)



Determine x, and y

$y = 50cm$

from $\tan\theta = \frac{opp}{Ady} = \frac{PD}{DR} = \frac{x}{y}$

$\tan\theta = x/y = x/50cm$

$x = 50\tan\theta$

$= 50\tan 60$

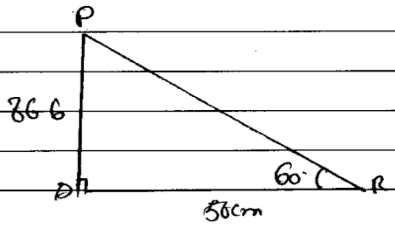
$= 86.6cm$

Then $x = 86.6cm$

$y = 50cm.$

$\therefore$ $x = 86.6cm$ $y = 50cm$

8 b(ii)



From phythagoras theorem

$$\overline{PR}^2 = \overline{PQ}^2 + \overline{QR}^2$$
$$= (86.6)^2 + (50)^2$$
$$=$$
$$PR = \sqrt{(86.6)^2 + (50)^2}$$
$$= 99.9 \simeq 100\text{cm}$$
$$= \cancel{99.9\text{cm}}\ 100\text{cm}$$

then $$\frac{\sin P\hat{Q}R}{PR} = \frac{\sin Q\hat{P}R}{QR}$$

$$\frac{\sin 30}{\cancel{100.9}} = \frac{\sin Q\hat{P}R}{100}$$

$$\frac{\sin 30}{100} = \frac{\sin Q\hat{P}R}{100}$$

$$\sin 30 = \sin Q\hat{P}R$$

$$\sin Q\hat{P}R = 0.5$$

But $$\sin Q\hat{P}A = \frac{\sin Q\hat{P}R}{2}$$

$$\sin (Q\hat{P}A) = \frac{0.5}{2} = 0.25.$$

$$\therefore \sin (Q\hat{P}A) = 0.25$$

In Extract 8.1, the candidate was unable to use trigonometric identities and ratios in answering the question.

There were few candidates (2%) who managed to score from 8 to 10 marks and some of these (0.2%) scored all the 10 marks. Most of the candidates were able to use appropriate trigonometric identities and ratios in answering the question. Extract 8.2 is a sample answer from one of these candidates.

Extract 8.2

8. a) (i) Sin 3θ interms of $\sin\theta$

$\sin 3\theta = \sin(2\theta + \theta)$

$\sin(2\theta + \theta) = \sin 2\theta \cos\theta + \sin\theta\cos 2\theta$

from relation

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\therefore \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

But

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin(2\theta+\theta) = 2\sin\theta\cos\theta \times \cos\theta + \sin\theta(\cos^2\theta - \sin^2\theta)$

$\sin(2\theta+\theta) = 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

But $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin(2\theta+\theta) = 2\sin\theta(1-\sin^2\theta) + \sin\theta(1-\sin^2\theta) - \sin^3\theta$

$\sin(2\theta+\theta) = 2\sin\theta - 2\sin^3\theta + \sin\theta - \sin^3\theta - \sin^3\theta$

$\sin(2\theta+\theta) = 2\sin\theta + \sin\theta - 2\sin^3\theta - 2\sin^3\theta$

$\sin(2\theta+\theta) = 3\sin\theta - 4\sin^3\theta$

$\sin(2\theta+\theta) = \sin(3\theta)$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(ii)

$\sqrt{\frac{(1-\cos\phi)}{1+\cos\phi}} = \operatorname{cosec}\phi - \cot\phi$

Given $\operatorname{cosec}\phi = \frac{1}{\sin\phi}$

$\cot\phi = \frac{1}{\tan\phi}$

8. a) (ii) $\text{cosec}\,\phi - \cot\phi = \dfrac{1}{\sin\phi} - \dfrac{1}{\tan\phi}$

But $\tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$1/\tan\phi = \dfrac{\cos\phi}{\sin\phi}$

$$\text{cosec}\,\phi - \cot\phi = \frac{1}{\sin\phi} - \frac{\cos\phi}{\sin\phi}$$

$$= \frac{1-\cos\phi}{\sin\phi}$$

$$\sin^2\phi + \cos^2\phi = 1$$
$$\sin^2\phi = 1 - \cos^2\phi$$
$$\sin\phi = \sqrt{1-\cos^2\phi}$$

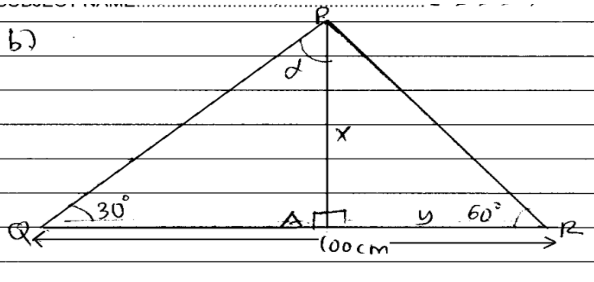
$$\text{cosec}\,\phi - \cot\phi = \frac{1-\cos\phi}{\sqrt{1-\cos^2\phi}}$$

$$\text{cosec}\,\phi - \cot\phi = \sqrt{\frac{(1-\cos\phi)^2}{((1-\cos\phi)(1+\cos\phi))}}$$

$$= \sqrt{\frac{1-\cos\phi}{1+\cos\phi}}$$

Hence shown

8. b)



$\sin = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$, $\tan = \dfrac{\text{Opposite}}{\text{Adjacent}}$

$$\tan 30° = \frac{x}{(100-y)}$$

$$x = \tan 30\,(100-y)$$
$$x = 0.577\,(100-y)$$
$$x = 57.74 - 0.5774y \quad \text{——— (i)}$$

$$\tan 60 = \frac{x}{y}$$

8. b) $\text{Tan } 60 = \dfrac{x}{y}$

$x = y \text{ Tan } 60^\circ$ ——— (ii)

$x = x.$

$y \text{ Tan } 60^\circ = \text{Tan } 30 (100 - y)$

$y \text{Tan} 60 = 100 \text{Tan} 30 - y \text{ Tan} 30$

$y \text{Tan} 60 + y \text{ Tan} 30 = 100 \text{ Tan} 30$

$\dfrac{y(\text{Tan} 60 + \text{Tan} 30)}{\text{Tan} 60 + \text{Tan} 30} = \dfrac{100 \text{ Tan } 30}{\text{Tan} 60 + \text{Tan} 30}$

$y = \dfrac{100 \text{ Tan } 30}{\text{Tan } 60 + \text{Tan } 30}$

$y = \dfrac{57.74}{1.732 + 0.5774}$

$y = 25$

$x = y \text{ Tan } 60$

$x = 25 \times \text{Tan} 60$

$x = 43.30127019 \approx 43.3$

$\therefore \quad x = 43.3 \text{ cm}$

$y = 25 \text{ cm}$

ii) $\sin (Q\hat{P}A)$

let $Q\hat{P}A = \alpha$

$\sin \alpha = \dfrac{(100 - y)}{\text{Hypotenuse}}$

8. b) ii) $100 - y = 100 - 25$

$= 75$

$\sin \alpha = \frac{75}{\text{Hypotenuse (QP)}}$

$QP = \sqrt{(x^2) + (QA)^2}$

$QP = \sqrt{(43.3)^2 + (75)^2}$

$QP = \sqrt{1875 + 5625}$

$QP = \sqrt{7500}$

$QP = 86.60254038 \approx 86.6$ cm

$\sin \alpha = \frac{75}{86.6}$

$\sin \alpha = \sin(Q\hat{P}A)$

$\sin(Q\hat{P}A) = 0.866025403 \approx 0.866$

$\therefore \sin(Q\hat{P}A) = 0.866$

Extract 8.2 shows a solution from a candidate who demonstrated good understanding of the topic of Trigonometry.

2.9 Question 9: Matrices, Exponential and Logarithmic Functions

This question had parts (a) and (b). In part (a), the candidates were required to find the value of (i) a if $2^{2a+8} - 32(2^a) + 1 = 0$ and (ii) N if $2\log_8 N = p,\ \log_2 2N = q$ and $q - p = 4$. In part (b), they were given the system of linear equations: $x + y + z = 7,\ x - y + 2z = 9,\ 2x + y - z = 1$ and they were asked to (i) write it in matrix form, (ii) find the determinant and the inverse of the coefficient matrix and (iii) determine the values of x, y and z.

This question was attempted by 97.7 percent of the candidates; 50.6 percent of these candidates scored from 0 to 3 marks and 49.4 percent scored from 3.5 to 10 marks. The question was therefore averagely performed.

In part (a)(i), only a small number of candidates were able to apply the laws of exponents to express the given equation as a quadratic equation, that is,

$256m^2 - 32m + 1 = 0$ where $m = 2^a$ and then solve it to get $m = \frac{1}{16}$ and finally $a = -4$. Similarly, in part (a)(ii), only few candidates managed to correctly apply the laws of exponents and logarithms to find the value of N.

In part (b), many candidates managed to write the given system of equations in matrix form, find the determinant and the inverse of the coefficient matrix and hence got the correct solution. Extract 9.1 is a sample solution of a candidate who performed well in this question.

Extract 9.1

9. a)

i) Required to find a.

soln

$2^{2a+8} - 32(2^a) + 1 = 0$

$2^{2a+8} - (2^5)(2^a) + 2^0 = 0$

$(2^{2a}) \times 2^8 - (2^5)(2^a) + 1 = 0$

$(2^a)^2 \times 2^8 - (2^5)(2^a) + 1 = 0$

let $2^a = y$

then

$y^2 \times 2^8 - 2^5 y + 1 = 0$

$256y^2 - 32y + 1 = 0$

$y = 0.0625$

but

$2^a = y$

$2^a = 0.0625$

applying logarithm both side

$\log 2^a = \log 0.0625$

$a \log 2 = \log 0.0625$

$a = \frac{\log 0.0625}{\log 2}$

$a = -4$

9	a) Given
	(ii) ii $2\log_8 N = P$, $\log_2 2N = q$, $q - P = 4$
	Required to find N.
	soln
	for $2\log_8 N = P$
	$= \log_8 N^2 = P$
	$N^2 = 8^P$
	from. $q - P = 4$
	$\log_2 2N - 2\log_8 N = 4$
	$\log_2 2N - \log_8 N^2 = 4$
	from $\log_2 2 + \log_2 N - \log_8 N^2 = 4$
	$\log_2 2 + \log_2 N - 2\log_8 N = 4.$
	$1 + \log_2 N - 2\left[\frac{1}{\log_N 8}\right] = 4.$
	$1 + \log_2 N - 2\left[\frac{1}{\log_N 2^3}\right] = 4.$
	$1 + \log_2 N - 2\left[\frac{1}{3\log_N 2}\right] = 4$
	$1 + \log_2 N - \frac{2}{3}\left[\frac{1}{\log_N 2}\right] = 4$
9.	$1 + \log_2 N - \frac{2}{3}\log_2 N = 4.$
	let $\log_2 N = x$
	$1 + x - \frac{2}{3}x = 4$
	$1 + \frac{1}{3}x = 4$
	$\frac{1}{3}x = 4 - 1$
	$\frac{1}{3}x = 3$
	$x = 9$

$\log_2 N = x$

$\log_2 N = 9$

$N = 2^{9}$

$N = 512$

b) Given

$x + y + z = 7$

$x - y + 2z = 9$

$2x + y - z = 1$

i) In matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$$

9(ii) determinant of matrix

$$\text{determinant} = 1 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 1\left((-1 \times -1) - 2\right) - 1\left((1 \times -1) - 4\right) + 1\left(1 - -2\right)$$

determinant $= 7$

Inverse of matrix

let it be matrix A

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Minor of

$$A = \begin{bmatrix} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \end{bmatrix}$$

$$\text{Minor of } A = \begin{bmatrix} -1 & -5 & 3 \\ -2 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

Cofactor of A $= \begin{bmatrix} -1 & 5 & 3 \\ 2 & -3 & 1 \\ 3 & -1 & -2 \end{bmatrix}$

Adjoint of A $= \begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$

Inverse $A^{-1} = \frac{1}{\det(A)} \times Adj A$

$= \frac{1}{7}\begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$

(ii) Values of x, y, z

By multiplying A^{-1} both sides of the equation

$\frac{1}{7}\begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = +\frac{1}{7}\begin{bmatrix} -1\times 7 + 2\times 9 + 3\times 1 \\ 5\times 7 + -3\times 9 + -1\times 1 \\ 3\times 7 + 1\times 9 + -2\times 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ 7 \\ 28 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14/7 \\ 7/7 \\ 28/7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

Thus $x = 2$

$y = 1$

$z = 4$.

In Extract 9.1, the candidate correctly applied the laws of exponents and logarithms in answering part (a). In part (b) this candidate was able to solve the system of linear equations by using the inverse matrix method.

In this question, about half of the candidates who attempted it scored below 3 out of 10 marks and notably 4.7 percent scored 0. The analysis of responses shows that the following were the reasons that made the candidates perform poorly.

- Lack of skills to use suitable substitutions to re-write the given equation as a quadratic equation. Many candidates were solving the equation in part (a) (i) directly, as a result ended up with incorrect solutions.
- Lack of knowledge on the laws of logarithms and exponents, how they are related and how to apply them. Many candidates were unable to convert the given logarithmic equations into the exponential equations $N = 8^{\frac{p}{2}}$ and $N = \frac{1}{2}\left(2^q\right) = 2^{q-1}$. When combined, these equations would give the equation $3p = 2q - 2$ that was to be solved together with the given equation $q - p = 4$ to get the required value of N.
- Lack of understanding on how to find minors and cofactors, resulting in wrong values of x, y and z.

Extract 9.2 is a sample answer from one of the candidates who performed poorly in this question.

Extract 9.2

9. (a) (i) Given expression

$$2^{2a+8} - 32(2^a) + 1 = 0$$

required to find the value of a

soln

$$2^{2a+8} - 32(2^a) + 1 = 0$$

$$2^{2a+8} - 32(2^a) = -1$$

apply logaithm

$$\log\left(\frac{2^{2a+8}}{32(2)^a}\right) = -1$$

so

$$\frac{\log 2^{2a+8}}{\log 32(2)^a} = -1$$

9. (a)(i) $(2a+8)0.30103 = -1(1.53148a)$

$0.60206a + 2.40824 = -1.53148a$

collect like terms

$0.60206a + 1.53148a = -2.40824$

$\frac{2.13354a}{2.13354} = \frac{-2.40824}{2.13354}$

$a = -1.128$

$a = -1.13$

$\therefore$ The value of $a = -1.13$

(ii) Given data

$2\log_8 N = P$ and $\log_2 2N = a$

also $a - P = 4$

required to find N

so

$a - P = 4$

$2N - 2\log_8 N = 4$

$2N - \log_8 N^2 = 4$

$\frac{-\log_8 N^2}{-\log_8} = \frac{4-2N}{-\log_8}$

$N^2 = -\log_8^{-1}(4-2N)$

$N^2 = -\log_8^{-1} 4 - -\log_8^{-1} 2N$

9. (a)(ii) $N^2 = -\log_8^{-1}\left(\frac{4}{2N}\right)$

$N^2 = \frac{-\log_8^{-1} 4}{-\log_8^{-1} 2N}$

$N^2 = \frac{-10000}{-\log_8^{-1} 2N}$

(b) (ii) (i) In matrix form

$$\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}7\\ 9\\ 1\end{pmatrix}$$

(ii) determinant $|A|$

$$\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}=1(1-2)-1(-1-4)+1(-2-1)$$

$$|A| = 1$$

(iii) Given matrix

$$|A|=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}7\\ 9\\ 1\end{pmatrix}$$

$$|A| = 1(+1-2)-1(-1-4)+1(-2-1)$$

$$|A| = 1$$

$$x=\begin{pmatrix}7 & 1 & 1\\ 9 & -1 & 2\\ 1 & 1 & -1\end{pmatrix}=\frac{7(1-2)-1(-9-2)+1(+1)}{1}=12$$

$$y=\begin{pmatrix}1 & 7 & 1\\ 1 & 9 & -1\\ 2 & 1 & 1\end{pmatrix}=\frac{1(9-1)-7(1+2)+1(1-18)}{1}=-2$$

$$z=\begin{pmatrix}1 & 1 & 7\\ 1 & -1 & 9\\ 2 & 1 & 1\end{pmatrix}=\frac{1(-1-9)-1(1-18)+7(-2-1)}{1}=-14$$

$\therefore x = 12,\ y = -2,\ z = -14$

Extract 9.2 shows a solution of a candidate who lacked understanding of the topic of Matrices, Exponential and Logarithmic functions.

2.10 Question 10: Linear Programming

This question had parts (a) and (b). In part (a), the candidates were required to give the definitions of (i) linear programming and (ii) constraints. In part (b), the candidates were required to use the information about production of cakes and loaves of bread that was given in the linear programming problem to (i) sketch a graph to illustrate the information, (ii) find the maximum amount of money to be obtained if both cakes and loaves of bread must be prepared and (iii) state how the maximum profit could be obtained.

The question was attempted by 93.4 percent of the candidates; 66.4 percent of them scored from 3.5 to 10 marks and 1.6 percent of these candidates scored full marks. This was the second best performed question in this examination.

The candidates who performed well in this question defined correctly the terms that were given in part (a). For example one of the candidates defined *linear programming as a branch of mathematics which deals with organization of activities and costs in order to minimize loss and maximize profit in a project* and *constraints as the rules which guide the relationship of the activities in a project.*

In part (b), the candidates were able to use the given information to define the decision variables and formulate the objective function and the constraints. They also managed to represent the constraints graphically so as to identify the feasible region and the corner points. Finally, they managed to find the value of the objective function at each of the corner points in order to determine which one gives the maximum profit. Extract 10.1 is a sample answer from one of these candidates.

Extract 10.1

10 (a)(i)	Linear programming is the branch of mathematics which deals with linear inequalities so as to maximise profits. and minimise costs.
10(a) (ii)	Constraints are linear inequalities used in solving the linear programming problems.
10 (b)	Soln
	Let x represents quantities of loaf bread
	y represents quantities of cake
	Constraints.
	$25x + 15y \leq 15000$
	$10x + 18y \leq 9000$
	$30x + 30y \leq 1920$
	$x \geq 0$,
	$y \geq 0$.
	Objective function.
	$f(x,y) = 4200x + 2000y$.
	Thus solving $f(x,y) = 4200x + 2000y$
	under constraints
	$25x + 15y \leq 15000$
	$10x + 18y \leq 9000$
	$30x + 30y \leq 1920$.
	$x \geq 0$
	$y \geq 0$

10 (b) Let $25x + 15y \leq 15000$ be $25x + 15y = 15000$

x	0	600
y	1000	0

Let $10x + 18y \leq 9000$ be $10x + 18y = 9000$

x	0	900
y	500	0

Let $30x + 30y \leq 1920$ be $30x + 30y = 1920$

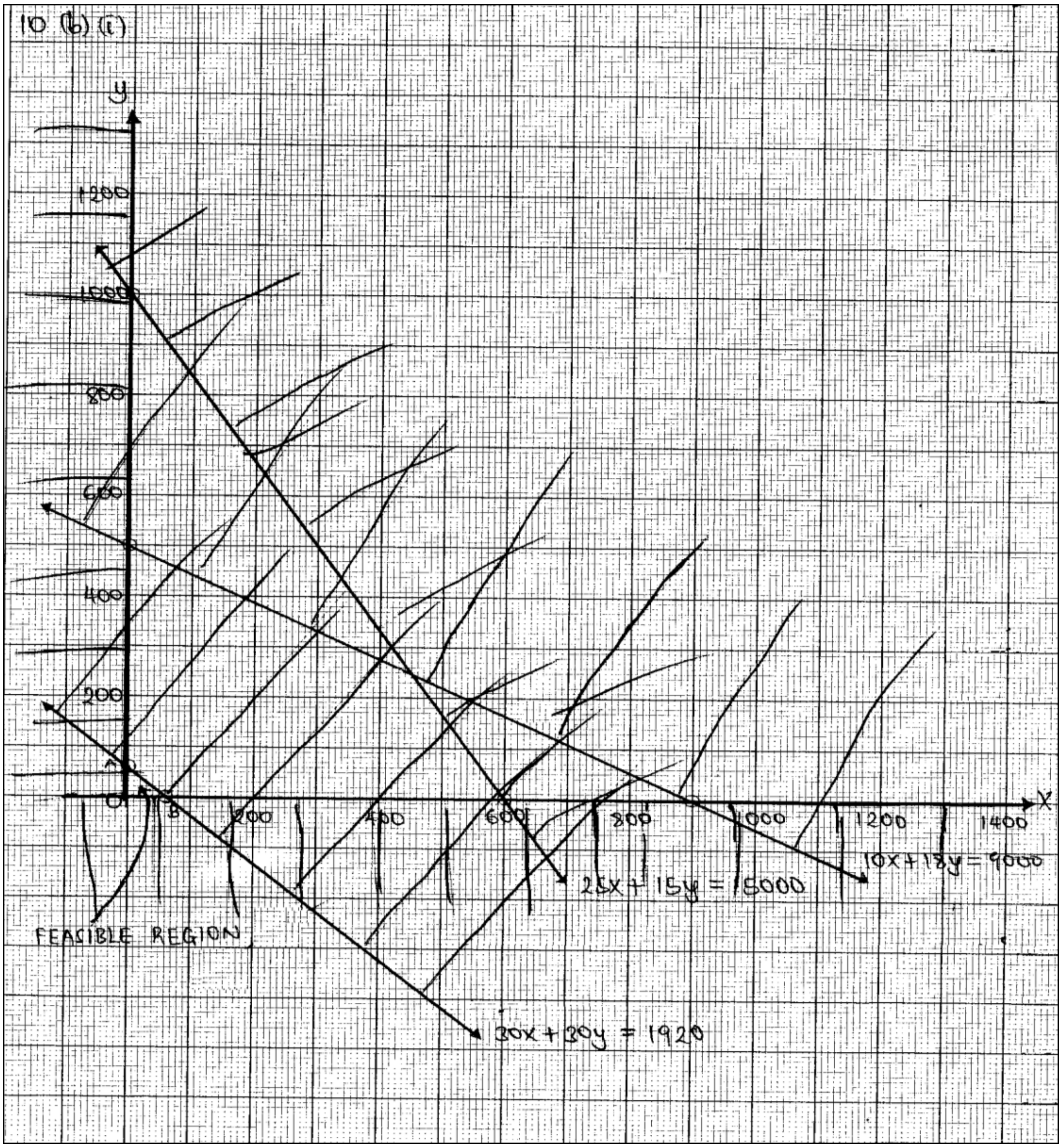
x	0	64
y	64	0

10(b)(ii) From the graph.

Corner points of the feasible region	Objective function $f(x,y) = 4200x + 2000y$	Value
O(0,0)	4200(0) + 2000(0)	0
A(0,64)	4200(0) + 2000(64)	128,000
B(64,0)	4200(64) + 2000(0)	268,800

10: (b) (ii) The trader obtain only MAXIMUM amount of money when only loaves of bread are prepared
= 268,200

(iii) The trader should prepare only 64 loaves of bread to maximize the profit.



Extract 10.1 shows that, the candidate was able to solve the given linear programming problem correctly.

On the other hand, 33.6 percent of the candidates who attempted this question scored from 0 to 3 out of 10 marks and among them 6.5 percent scored zero. The analysis of responses shows that in part (a), most of the candidates who scored low marks gave incorrect definitions for the given basic Linear Programming terms showing poor understanding. One of the candidates for example defined *linear programming as the mathematical expression of various data to graphical form* and *constraints as the unit that used to construct linear programming*. Another candidate defined *linear programming as the branch of mathematics which deals with solving of uncertainty problems* and *constraints as the linear equations used to solve uncertainty problems*.

In part (b), some of the candidates managed to formulate the objective function and the constraints but failed to graph the inequalities because they lacked the skills to draw graphs. The graphs were not passing at the correct *x* and *y* intercepts and resulting in incorrect feasible regions.

It was noted that, other candidates could not correctly interpret the given information mathematically and as a result lost all the marks in this part. For example one of the candidates wrote the inequalities for the constraints as $25x + 10y \geq 30,\ \ 15x + 18y \geq 30$ while another candidate wrote the inequalities as $25x + 15y \geq 1500,\ \ 10x + 18y \geq 9000,\ 30x + 30y \geq 1920$ instead of $25x + 15y \leq 15000$, $5x + 3y \leq 3000$ and $10x + 18y \leq 9000$. Extract 10.2 is an example of a solution from a candidate who performed poorly in this question.

Extract 10.2

10a) i Linear programming: Is the way of using farious method and data in programming busness, and constructing budgets and making budget with differencies eshmations. It is always used to businesses activities.

ii Constraints: These are details which constracted and prepared during programming the values. Those valueused they are called constraints.

10. (b)

Let X — Bread.

y — cake.

For ingredients X.

$$25x + 15y \geq 15000 \quad \text{------(i)}$$

For ingredients Y.

$$10x + 18y \geq 9000 \quad \text{------(ii)}$$

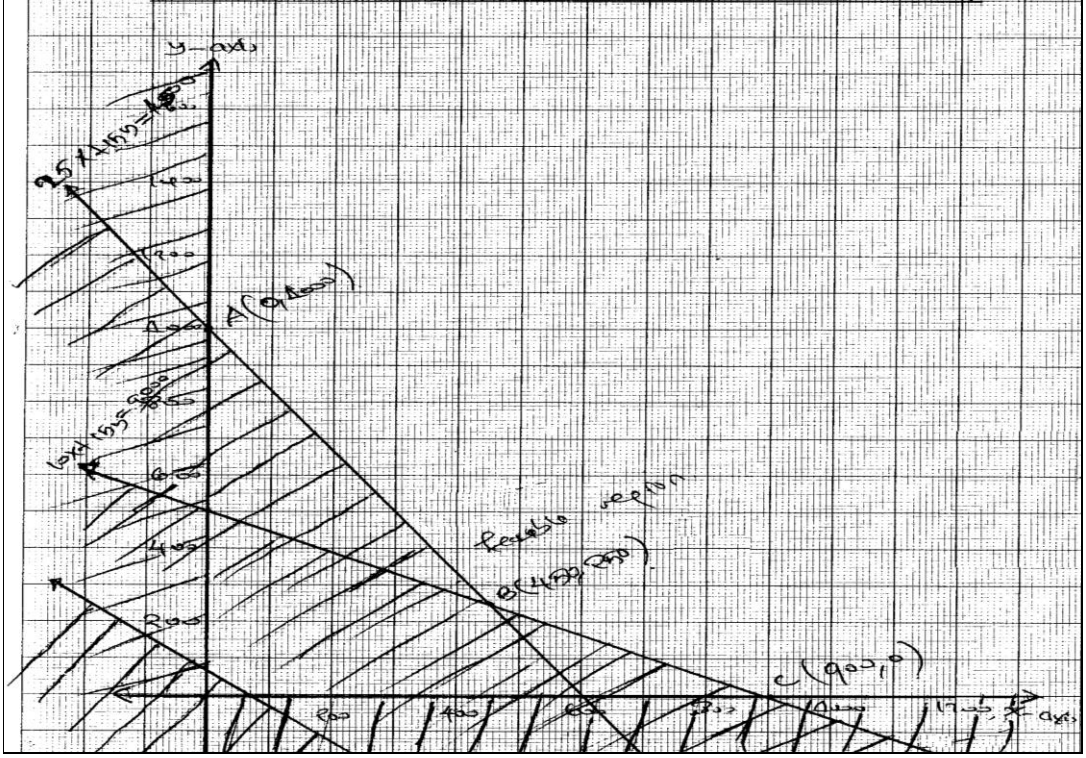
For ingredients Z.

$$30x + 30y \geq 1920 \quad \text{-----(iii)}$$

The equations obtained from the inequalities

$$25x + 15y = 15000 \quad \text{---(i)}$$
$$10x + 18y = 9000 \quad \text{---(ii)}$$
$$30x \; 30y = 1920 \quad \text{--(iii)}$$

10 b(ii) THE GRAPH TO ILLUSTRATE THE INFORMATION



10. A (0, 1000)

B (450, 250)

C (900, 0)

f (x+y) = 4200x + 2000y.

A (0, 1000)

f(x, y) = 4200 × (0) + 2000 × (1000)

= 2000000/=.

B (450, 250)

f (x, y) = 4200 × (450) + 2000(250)

= 2390000/=

C (900, 0)

f (x, y) = 4200 × (900) + 2000 × (0)

= 3780000/=

∴ The maximum amount of money is

10 (b) 2390000/= shilings.

(b)(iii) A trader should make a production of 450 bread and 250 cake in order to obtain a maximum profit.

Extract 10.2 shows that, the candidate was unable to define the terms given in part (a). In part (b), the candidate was unable to correctly define the decision variables, formulate the objective function and the constraints.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE TOPIC - WISE

The Basic Applied Mathematics examination had 10 questions that were set from 10 topics. The analysis shows that, the candidates had good performance in 2 questions that were set from the topics of *Statistics* and *Linear Programming*. The analysis also shows that the candidates had average performance in 2 questions that were set from the topics of *Calculating Devices*, *Matrices*, *Exponential* and *Logarithmic Functions*. Further analysis shows that the candidates had weak performance in the remaining 6 questions that were set from the topics of *Algebra, Functions, Trigonometry, Probability, Differentiation* and *Integration*. The percentage of candidates who passed in each topic is shown in Appendix I and II.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 CONCLUSION

The analysis of candidates' performance topic-wise shows that, the candidates had weak performance in 6 out of 11 topics that were examined. These topics were: *Algebra, Functions, Trigonometry, Probability, Differentiation* and *Integration*. The factors that contributed to the weak performance include: inability to solve simultaneous equations; lack of knowledge on the sigma notation; lack of understanding of composite functions; inability to use the remainder theorem; inability to identify stationary points of a polynomial; inability to use the quotient and chain rule to differentiate functions; lack of knowledge of implicit differentiation; inability to apply knowledge of integration in answering questions; lack of knowledge on the basic trigonometric identities, ratios and rules of probability.

4.2 RECOMMENDATIONS

In order to improve future candidates' performance in this subject, it is recommended that both teachers and students should put more effort in the topics of *Algebra, Functions, Trigonometry, Probability, Differentiation* and *Integration* that had weak performance. The candidates are advised to do many exercises in order to be able to apply various concepts and formulae in solving questions. The teachers are advised to provide more support to the students when solving questions particularly from these topics so that students understand how to apply various concepts and formulae.

APPENDICES

APPENDIX I

CANDIDATES' PERFORMANCE TOPIC – WISE
ACSEE 2017

S/N	Topic	Question Number	Percentage of candidates who Passed	Remarks
1.	Statistics	6	77.3	Good
2.	Linear Programming	10	66.4	Good
3.	Matrices, Exponential and Logarithimic Functions	9	49.4	Average
4.	Calculating Devices	1	39.0	Average
5.	Algebra	3	24.0	Weak
6.	Functions	2	19.3	Weak
7.	Trigonometry	8	15.6	Weak
8.	Probability	7	13.8	Weak
9.	Differentiation	4	8.3	Weak
10.	Integration	5	3.0	Weak

In this table, green, yellow and red colors indicate good, average and weak performance respectively.

APPENDIX II

CANDIDATES' PERFORMANCE TOPIC – WISE
ACSEE 2017