THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEMS RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2016

142 ADVANCED MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEMS RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2016

142 ADVANCED MATHEMATICS

Published by
The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es salaam, Tanzania.

Contents

FOREWORD

The National Examinations Council of Tanzania is pleased to issue the report on Candidates' Item Response Analysis (CIRA) in Advanced Mathematics 2016 in order to inform teachers, parents, candidates, policy makers and other educational stakeholders, on how the candidates responded to the examination items. The report will enable the stakeholders to take appropriate measures to improve the performance of the candidates.

The analysis of the candidates' responses was done in order to identify the areas in which they faced problems, did well or averagely. Basically, the report highlighted the candidates' strengths and weaknesses in order to understand what the educational system managed to offer to the learners in their two years of Advanced Level Secondary Education.

The factors noted for the poor performance include failure of the candidates to identify the requirements of the questions, lack of knowledge, failure to apply correct formulae in answering the questions. Extracts of the candidates' responses from the scripts were used in this report to illustrate their performance on the items.

The National Examinations Council of Tanzania will highly appreciate comments and suggestions from teachers, students, educational stakeholders and the public in general that can highlight any area for improving the writing of future reports.

Finally, the Council would like to thank all Examination Officers, Subject Teachers and all others who were involved in the preparation and analysis of data used in this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The CIRA report in 142 Advanced Mathematics subject was prepared to provide feedback to stakeholders about the performance of the candidates and the challenges they faced in answering examination questions.

The section for analysis of the questions was organized into two categories. It started with the analysis of 10 questions in 142/1 Advanced Mathematics paper 1. Each question in paper 1 was compulsory and carried 10 marks. Then, it was followed by the analysis of 8 questions in 142/2 Advanced Mathematics paper 2 that was divided into two sections A and B. Section A had 4 compulsory questions and section B had 4 questions in which the candidates were required to answer two questions. Each question in section A carried 15 marks while in section B each carried 20 marks.

In 2016, a total of 9,700 candidates sat for the examination out of which 76.35 percent passed. In 2015 however, a total of 7,774 candidates sat for the examination out of which 85.02 percent passed. Therefore in 2016 there is 8.67 percentage drop in the number of candidates who passed the examination.

The decision on whether the performance of a question was good, average or poor was based on the percentage of candidates who scored 35% or more of the marks that were allocated to the question. The percentage boundaries that were used in the analysis were: 0 – 34, 35 – 59 and 60 – 100 to represent poor, average and good performance respectively. For instance, the percentage of candidates shall signify good performance if it is found in the interval 60 to 100.

The analysis of the individual questions is presented in section 2.0. It comprises of a short description of the requirements of the question and analysis on how the candidates responded to the questions. Extracts of well performed and poorly done questions were included to each analysis of a particular question. The factors that accounted for good or poor performance in each question have been indicated and illustrated using samples of candidates' responses. Therefore, the analysis of each question could be used as a practical guide to teachers and students to improve teaching and learning, and eventually candidates' performance.

The analysis of candidates' performance in the topics examined is also shown in appendix I and II whereby green, yellow and red colour represents good, average and poor performance respectively. Finally, the recommendations are included at the end of this report to help candidates, teachers and the government to improve candidates' performance in future Advanced Mathematics examination.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE

2.1 142/1 ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

This question had parts (a) and (b). In part (a), the candidates were instructed to use a scientific calculator to calculate (i) $\dfrac{\sqrt{(3.12\times\log 5)^3}}{\sqrt[3]{\left(\cos\dfrac{\pi}{9}+\sin 46^\circ\right)}}$ and (ii) $\left[\dfrac{\sqrt{e^3\log_2 6}\times\sinh^{-1}(0.6972)}{(\ln 3.5)\times(\cos 64.5^\circ)\times(\tan 46^\circ)}\right]\times(0.6467)^3$ correct to four decimal places:

In part (b), the candidates were given that "A rat has a mass 30 grams at birth. If it reaches maturity in 3 months, the rate of growth is modeled by the equation $\dfrac{dm}{dt}=120\,(2.1985t-3)^2$, where m is the mass of the rat, t months after birth." They were therefore required to find the mass of the rat when fully grown by using a scientific calculator.

The analysis of the data indicates that this was among the eight questions which had poor performance in this examination. Most candidates (72.1%) scored below 3.5 out of 10 marks and among them, 43.5 percent scored a zero mark.

The analysis of the candidates' responses shows that in part (a), some candidates with poor performance entered wrong data in their calculators. They used the given figures directly without changing the mode of the calculators such as changing from radian to degree and degree to radian. In part (b), they did not integrate the equation $\int_{30}^{m} dm = 120\int_0^3 (201985t-3)^2\,dt$ before attempting to find the required mass. In most cases, they computed the value of the right hand side and forgot the left hand side, ending up with 1336.93 grams instead of 1366.93 grams. Further analysis of the candidates' responses indicates that a significant number of them failed to round off their answers in the required number of decimal places. This is a proof that they lacked round-off techniques. Extract 1.1 is a sample answer from one of the candidates, showing some of these weaknesses.

Extract 1.1

1(a) (i) 2.6274

1(a) (ii) −17.8166

1(b) 1336.93444 g

In Extract 1.1, the candidate lacked the skills to use a scientific calculator in part (a). In part (b), he/she computed $120\int_0^3 (2.1985t-3)^2\, dt$ instead of $\int_0^{30} dm = 120\int_0^3 (2.1985t-3)^2\, dt$.

Despite all these weaknesses, a few candidates (3.5%) performed well in this question and scored all the 10 marks. Extract 1.2 shows a sample answer from one of the candidates who did well.

Extract 1.2

01 (a) (i) 2.7204

(ii) 2.2695

01 (b) Given:

$\frac{dm}{dt} = 120(2.1985t-3)^2$

To get mass of rat after 3 months the above equation is integrated within reasonable limits.

then:

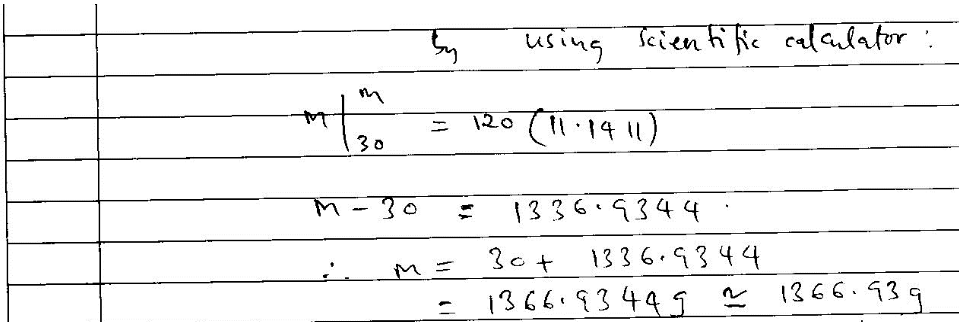
by separating variables

$dm = 120(2.1985t-3)^2\, dt$

applying $\int$ both sides

$\int_{30}^{m} dm = 120\int_0^3 (2.1985t-3)^2\, dt$

Where: m is final mass of rat at maturity.


By using scientific calculator:

$m\Big|_{30}^{m} = 120\,(11.1411)$

$m - 30 = 1336.9344$

$\therefore\ m = 30 + 1336.9344$

$= 1366.93449 \simeq 1366.939$

Extract 1.2 shows how the candidate was able to use a scientific calculator correctly to perform computations.

2.1.2 Question 2: Hyperbolic Functions

In this question, the candidates were required to; (a) express $\sinh x$ and $\cosh x$ in terms of t if $t = \tanh\frac{x}{2}$, (b) express $\sinh^{-1} x - \ln x$ in terms of natural logarithms, hence find the limit as $x \to \infty$ and (c) evaluate $\int_4^7 \frac{1}{\sqrt{(4x^2 - 8x + 7)}}\,dx$ correct to four decimal places.

The analysis of the data shows that 12,798 candidates attempted this question out of which 35.6 percent scored above 3 out of 10 marks with 1.2 percent scoring all the 10 marks. The question was therefore averagely performed.

The analysis of candidates' responses shows that the candidates with full marks realized that in hyperbolic functions the best way to tackle part (a) was by using the double angle formulae to write $\sinh x$ as $2\sinh\frac{x}{2}\cosh\frac{x}{2}$ and $\cosh x$ as $\cosh^2\frac{x}{2} + \sinh^2\frac{x}{2}$. Such candidates got $\frac{2t}{1-t^2}$ and $\frac{1+t^2}{1-t^2}$ as the required expressions for $\sinh x$ and $\cosh x$ in terms of t. Moreover, the candidates who answered part (b) correctly, used the formula $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$ and the laws of natural logarithms in working out their solution, hence ending up with the expression $\ln\left(1 + \sqrt{1 + \frac{1}{x^2}}\right)$ as required. The candidates were also able to find the value of the expression when $x \to \infty$ and got $\ln 2$ as required, see Extract 2.1. Further analysis of the candidates' responses indicates that the candidates were able to complete the square of $\sqrt{\left(4x^2-8x+7\right)}$ to get $\sqrt{3}\sqrt{\left(\frac{2x-2}{\sqrt{3}}\right)^2+1}$ and then use $\sinh u = \frac{2x-2}{\sqrt{3}}$ to evaluate $\int_4^7 \frac{1}{\sqrt{\left(4x^2-8x+7\right)}}\,dx$ correctly in part (c).

Extract 2.1

2. a) Given: $t = \tanh \frac{x}{2}$

Required: $\sinh x$ and $\cosh x$ in terms of t.

$\sinh x = \sinh\left(x/2 + x/2\right)$

$= 2\sinh x/2 \cosh x/2$

$= \frac{2\sinh x/2 \cosh x/2}{1}$

Dividing by $\cosh^2 x/2$ both to the numerator and denominator:

$\sinh x = \frac{2\sinh x/2 \cosh x/2}{\cosh^2 x/2}$

2. a) $\sinh x = \frac{2\tanh x/2}{\operatorname{sech}^2 x/2}$

But $\operatorname{sech}^2 x/2 = 1 - \tanh^2 x/2$

$\sinh x = \frac{2\tanh x/2}{1-\tanh^2 x/2}$

$= \frac{2t}{1-t^2}$

$\therefore \sinh x = \frac{2t}{1-t^2}$

$\cosh x = \cosh\left(x/2 + x/2\right)$

$= \cosh^2 x/2 + \sinh^2 x/2$

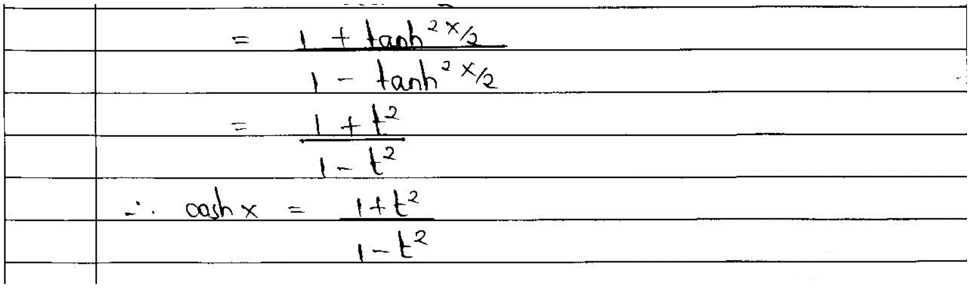
$= \frac{\cosh^2 x/2 + \sinh^2 x/2}{1}$

Dividing by $\cosh^2 x/2$ to both the numerator and denominator:

$\cosh x = \frac{\cosh^2 x/2 + \sinh^2 x/2}{\cosh^2 x/2}$

$\frac{1}{\cosh^2 x/2}$

$= \frac{1+\tanh^2 x/2}{\operatorname{sech}^2 x/2}$



$= \dfrac{1+\tanh^2 x/2}{1-\tanh^2 x/2}$

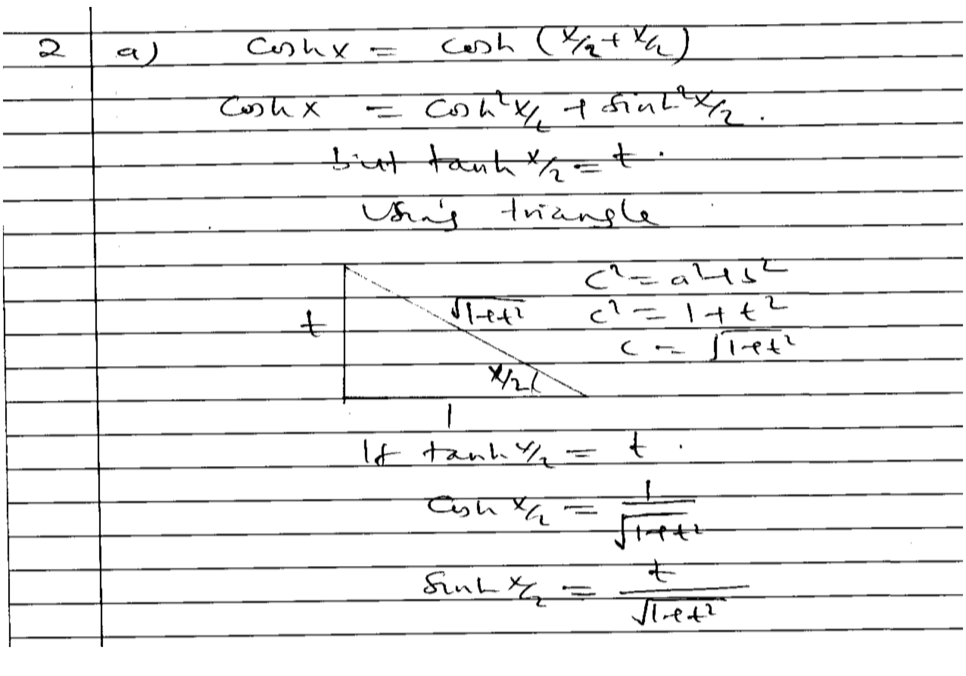
$= \dfrac{1+t^2}{1-t^2}$

$\therefore \cosh x = \dfrac{1+t^2}{1-t^2}$

In Extract 2.1, the candidate was able to apply the hyperbolic function identities and double angle formulae to express $\cosh x$ and $\sinh x$ in terms of t.

Nevertheless, some candidates (42.4%) performed poorly in this question and hence did not score any mark. Some of them were applying wrong concepts in part (a), for example using the concept of trigonometric ratios. Such candidates ended up with wrong expressions for $\sinh x$ and $\cosh x$. In part (b), many candidates wrongly expressed $\sinh^{-1} x - \ln x$ as $\sinh^{-1}(x - \ln x)$, while writing the given hyperbolic functions into natural logarithm. It was also noted that many candidates failed to identify a suitable substitution to evaluate $\int_4^7 \frac{1}{\sqrt{4x^2-8x+7}}\,dx$ correctly to four decimal places in part (c). A sample response from one of the scripts showing how the candidates provided wrong answers is shown in Extract 2.2.

Extract 2.2



2 a) $\cosh x = \cosh(x/2 + x/2)$

$\cosh x = \cosh^2 x/2 + \sinh^2 x/2.$

~~but $\tanh x/2 = t$.~~

Using triangle

$c^2 = a^2 + b^2$

$c^2 = 1 + t^2$

$c = \sqrt{1+t^2}$

If $\tanh x/2 = t$.

$\cosh x/2 = \dfrac{1}{\sqrt{1+t^2}}$

$\sinh x/2 = \dfrac{t}{\sqrt{1+t^2}}$

Then.

$$\cosh x = \cosh^2 \tfrac{x}{2} + \sinh^2 \tfrac{x}{2}.$$

$$= \left(\frac{1}{\sqrt{1+t^2}}\right)^2 + \left(\frac{t}{\sqrt{1+t^2}}\right)^2.$$

$$= \frac{1}{1+t^2} + \frac{t^2}{1+t^2}.$$

2 a) $\cosh x = \cosh \tfrac{x}{2} \cosh \tfrac{x}{2} + \sinh \tfrac{x}{2} \sinh \tfrac{x}{2}$

$$= \cosh^2 \tfrac{x}{2} + \sinh^2 \tfrac{x}{2}.$$

$$= \frac{1}{1+t^2} + \frac{t^2}{1+t^2}.$$

$$= \frac{1+t^2}{1+t^2}$$

$$\cosh x = 1$$

$$\sinh x = 2 \sinh \tfrac{x}{2} \cosh \tfrac{x}{2}.$$

$$= 2\left(\frac{1}{\sqrt{1+t^2}}\right)\left(\frac{t}{\sqrt{1+t^2}}\right)$$

In Extract 2.2, the candidate used the concept of trigonometric ratios instead of double angle formulae for hyperbolic sine and cosine showing lack of knowledge on the tested concept.

2.1.3 Question 3: Linear Programming

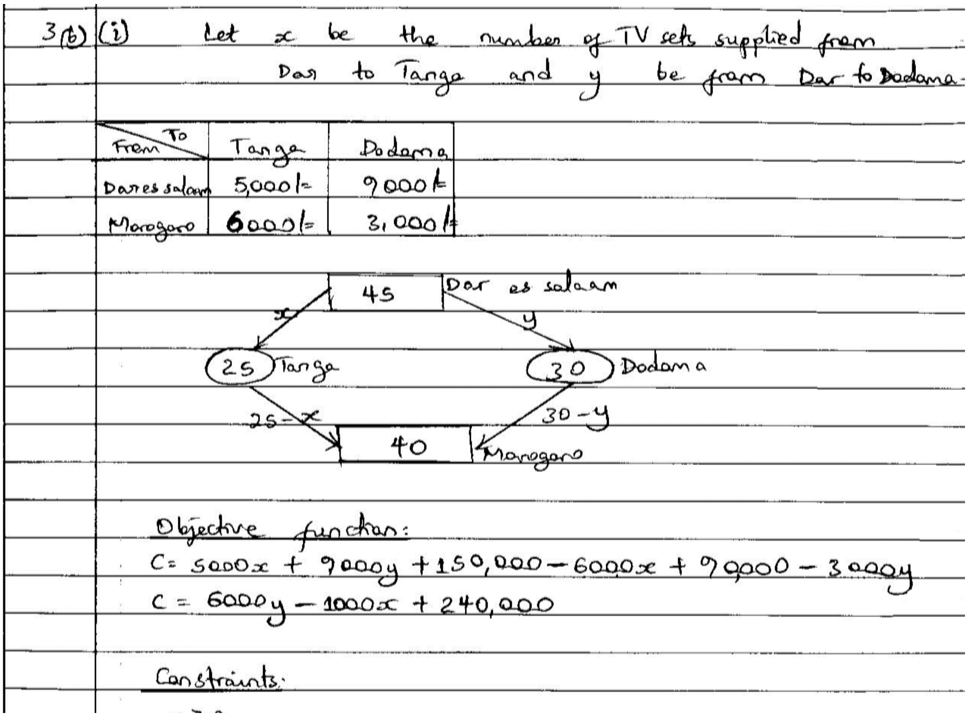
The question had parts (a) and (b). In part (a), the candidates were given that: “If Mr. Mutu takes two types of vitamin pills, he must have at least 16 units of vitamin A, 5 units of vitamin B and 20 units of vitamin C. From these vitamins, he can choose between pill M, which contains 8 units of A, 1 unit of B and 2 units of C; and pill N which contains 2 units of A, 1 unit of B and 7 units of C. Pill M costs 150 shillings and pill N costs 300 shillings”. They were required to find the number of pills for each type in order to minimize the cost.

In part (b), the candidates were given that "A TV dealer has stores in Dar es Salaam and Morogoro and retailers in Tanga and Dodoma. The stores have a stock of 45 and 40 TV sets respectively while the requirements of the retailers are 25 and 30 TV sets respectively. If the cost of transporting a TV set from Dar es Salaam to Tanga is Tsh 5,000/= and from Dar es Salaam to Dodoma is Tsh 9,000/=, from Morogoro to Dodoma is Tsh 3,000/= and Morogoro to Tanga is Tsh 6,000/=". They were required to (i) determine how the TV dealer should supply the requested number of TV sets at a minimum cost and (ii) find the cost.

This question was attempted by 12,798 candidates out of which more than two-thirds of the candidates (71.7%) scored above 3 out of 10 marks and among them 7.6 percent scored 10 marks. This question was among the five questions which were well performed.

The analysis of candidates' responses shows that the candidates who performed well were able to: choose the unknowns; write the objective functions; write the constraints as a system of linear inequalities and find the set of feasible solutions that graphically represent the constraints. Moreover, they showed competence in computing the coordinates of vertices from the compound of feasible solutions and managed to calculate the value of the objective function at each of the vertices to determine which one has the minimum value. A sample answer from one of the candidates who answered part (b) of this question correctly is shown in Extract 3.1.

Extract 3.1


3(b) (i) Let x be the number of TV sets supplied from Dar to Tanga and y be from Dar to Dodoma.

From \ To	Tanga	Dodoma
Dar es salaam	5,000/=	9000/=
Morogoro	6000/=	3,000/=

Objective function:

$C = 5000x + 9000y + 150,000 - 6000x + 90000 - 3000y$

$C = 6000y - 1000x + 240,000$

Constraints:

$x \geq 0$

$y \geq 0$

$25 - x \geq 0 \Rightarrow x \leq 25$

$30 - y \geq 0 \Rightarrow y \leq 30$

$x + y \leq 45$

$25 - x + 30 - y \leq 40 \Rightarrow x + y \geq 15$

Corner points	$C = 6000y - 1000x + 240{,}000$
A (0, 30)	420,000/=
B (0, 15)	330,000/=
C (15, 0)	225,000/=
D (25, 0)	215,000/=
E (25, 20)	335,000/=
F (15, 30)	405,000/=

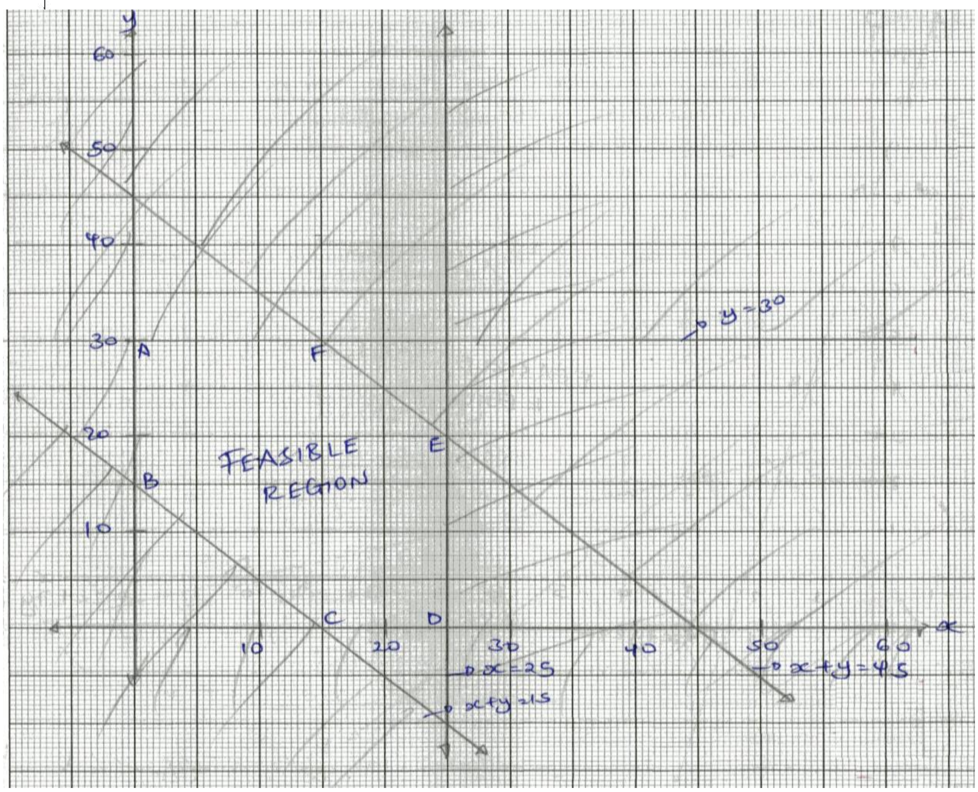
(b) (i) Optimum point is (25, 0)

∴ TV sets supplied from Dar es salaam to Tanga are 25

from Dar es salaam to Dodoma are 0

from Morogoro to Tanga are 0

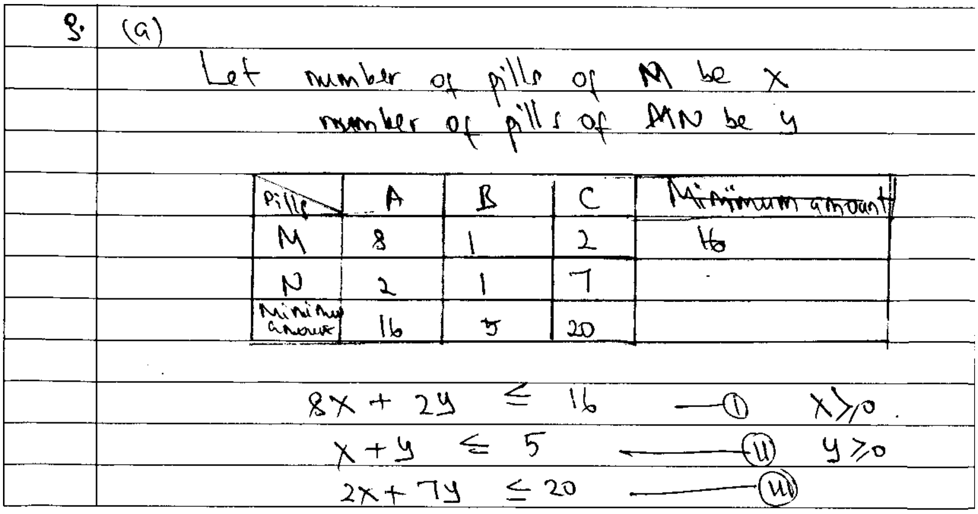
and from Morogoro to Dodoma are 30



In Extract 3.1, the candidate was able to transform the word problem mathematically, represent the inequality graphically and find the minimum cost correctly.

On the other hand, there were a few candidates (12.8%) who performed this question poorly. The analysis of the candidates' responses indicates that some candidates formulated the constraints wrongly, for instance using a wrong inequality sign. They used $\leq$ instead of using $\geq$, hence obtained incorrect feasible region while others failed to represent the inequalities graphically in part 3 (a). In part 3 (b), several candidates faced difficulties in writing the inequalities for the constraints. For example they wrote $25-x+30-y\geq 0$ instead of $25-x+30-y\leq 40$ and as a result ended up with wrong solution to the transportation problem. Extract 3.2 shows a sample answer from one among the candidates who answered the question incorrectly.

Extract 3.2



3. (a)

Let number of pills of M be x
number of pills of MN be y

Pills	A	B	C	Minimum amount
M	8	1	2	16
N	2	1	7	
Minimum amount	16	5	20	

$8x + 2y \leq 16$ —(i) $x \geq 0$

$x + y \leq 5$ —(ii) $y \geq 0$

$2x + 7y \leq 20$ —(iii)

In Extract 3.2, the candidate formulated the inequalities using wrong inequality signs, as a result ended up with incorrect graph and hence wrong feasible solution.

2.1.4 Question 4: Statistics

The question had parts (a), (b) and (c). Part (a) required the candidates to show that the standard deviation δ of the frequency distribution is given by $\delta^2 = C^2\left(\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2\right)$ if the frequency distribution of a variable X is classified in equal intervals of size C; the frequency in a class is denoted by *f*; the total frequency is *N* and the data is coded into a variable *u* by means of the relation $\bar{x} = a + C\bar{u}$, where X takes the central values of the class intervals.

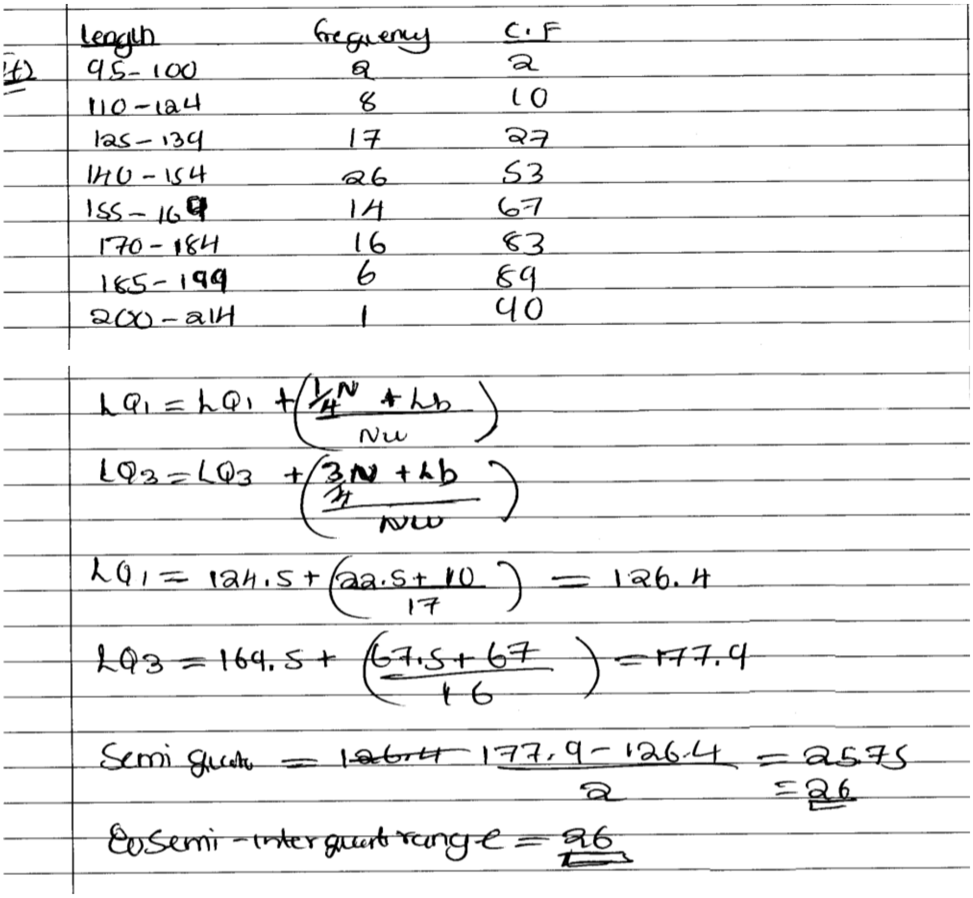
In part (b), the candidates were required to find the standard deviation of the whole group if the average heights of 20 boys and 30 girls are 160 cm and 155 cm and the corresponding standard deviations of boys and girls are 4 cm and 3.5 cm respectively.

In part (c), the candidates were given a table showing the length of 100 earth worms in millimeters and were required to obtain the semi-interquartile range correct to two significant figures.

The question was attempted by 12,798 candidates of which 88.9 percent scored from 0 to 3 marks, 8.1 percent scored from 3.5 to 5.5 marks and 3 percent scored from 6 to 10 marks, with only one candidate scoring all the 10 marks. The analysis shows that this question was among the eight questions which were worst performed in this examination.

The analysis of the candidates' responses indicated that, most candidates failed to derive the formula for standard deviation using coding method in part (a). In part (b), many of them failed to find the standard deviation of the combined data, that is they used $\bar{x} = \frac{\sum fx}{N}$ and $\delta = \sqrt{\left[\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2\right]}$ instead of $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$ and $\sigma^2 = \frac{1}{n_1 + n_2}\left[n_1\left(\sigma_1^{\ 2} + d_1^{\ 2}\right) + n_2\left(\sigma_2^{\ 2} + d_2^{\ 2}\right)\right]$ where $\bar{x}$ and δ are the mean standard deviation of the entire group respectively. Most candidates also used wrong formulae to compute the quartile (Q_i) and the semi-interquartile (SIQ_i). It was noted that they used $Q_i = Li + \left(\frac{\frac{iN}{4} + fbQi}{fwQi}\right)c$ instead of $Qi = Li + \left(\frac{i\frac{N}{4} - fbQi}{fwQi}\right)c$ to find the quartiles and $SIQ_r = \frac{Q_1 + Q_3}{2}$ instead of $SIQ_r = \frac{Q_3 - Q_1}{2}$ to find the semi-interquartile range, see Extract 4.1.

Extract 4.1



Length	Frequency	C.F
95-100	2	2
110-124	8	10
125-139	17	27
140-154	26	53
155-169	14	67
170-184	16	83
165-199	6	89
200-214	1	40

$LQ_1 = LQ_1 + \left(\frac{\frac{1}{4}N + Lb}{Nw}\right)$

$LQ_3 = LQ_3 + \left(\frac{\frac{3N}{4} + Lb}{Nw}\right)$

$LQ_1 = 124.5 + \left(\frac{22.5 + 10}{17}\right) = 126.4$

$LQ_3 = 169.5 + \left(\frac{67.5 + 67}{16}\right) = 177.4$

Semi quart $= \frac{177.9 - 126.4}{2} = 25.75$
$= 26$

Semi-inter quart range $= 26$

In Extract 4.1, the candidate used wrong formulae to calculate quartiles and semi-interquartile range and as a result ended up getting wrong answers.

On the other hand, there were a few candidates with good performance. They managed to derive the formula for standard deviation using the coding method. The candidates also used correct formulae to obtain the standard deviation of the entire group and semi-inter quartile range correct to two significant figures. Extract 4.2 shows one of the responses from a candidate who performed well in part (a).

Extract 4.2

04. a/. From

$$\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{N}}$$

but $N = \sum f$

Thus

$$\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

$$\sigma = \sqrt{\frac{\sum f(x^2 - 2x\bar{x} + \bar{x}^2)}{\sum f}}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} + \frac{\sum f\bar{x}^2}{\sum f} - \frac{2\bar{x}\sum fx}{\sum f}}$$

04. a/.

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} + \bar{x}^2 - 2\bar{x}^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

but $\bar{x} = \left(\frac{\sum fx}{\sum f}\right)$

Thus

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

but $\bar{x} = a + c\bar{u}$

Thus $x = a + cu$.

Hence:

$$\sigma = \sqrt{\frac{\sum f(a+cu)^2}{\sum f} - \left(\frac{\sum f(a+cu)}{\sum f}\right)^2}$$

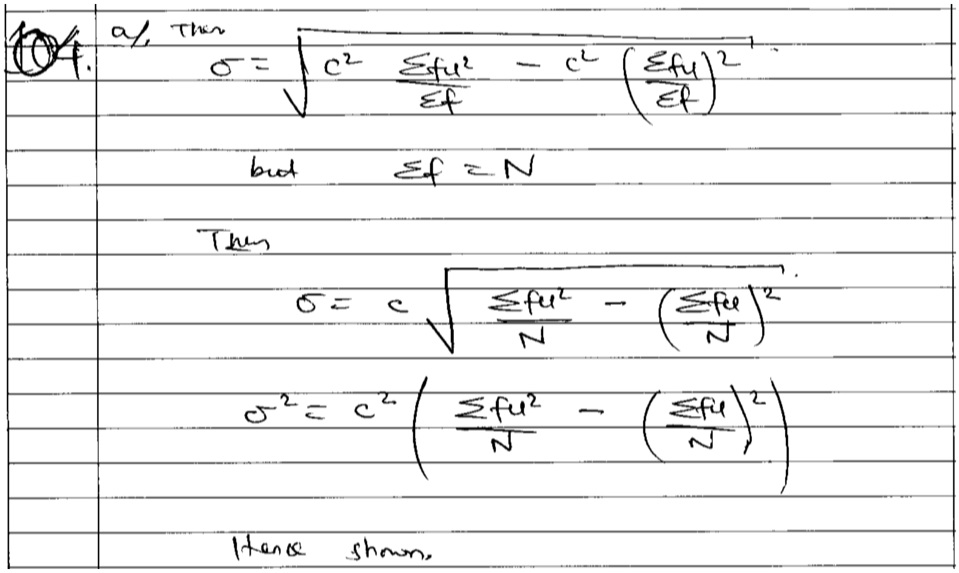
$$\sigma = \sqrt{\frac{\sum f(a^2 + 2acu + c^2u^2)}{\sum f} - \left(\frac{\sum fa}{\sum f} + \frac{\sum fcu}{\sum f}\right)^2}$$

but a = constant and c = constant

Thus $\frac{\sum fa}{\sum f} = a$ and $\frac{\sum fcu}{\sum f} = c\frac{\sum fu}{\sum f}$

$$\sigma = \sqrt{\frac{a^2\sum f}{\sum f} + 2ac\frac{\sum fu}{\sum f} + c^2\frac{\sum fu^2}{\sum f} - \left(a + c\frac{\sum fu}{\sum f}\right)^2}$$

$$\sigma = \sqrt{a^2 + 2ac\frac{\sum fu}{\sum f} + c^2\frac{\sum fu^2}{\sum f} - a^2 - 2ac\frac{\sum fu}{\sum f} - c^2\left(\frac{\sum fu}{\sum f}\right)^2}$$



04. a/ Then

$$\sigma = \sqrt{c^2 \frac{\Sigma fu^2}{\Sigma f} - c^2 \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$

but $\Sigma f = N$

Thus

$$\sigma = c\sqrt{\frac{\Sigma fu^2}{N} - \left(\frac{\Sigma fu}{N}\right)^2}$$

$$\sigma^2 = c^2\left(\frac{\Sigma fu^2}{N} - \left(\frac{\Sigma fu}{N}\right)^2\right)$$

Hence shown.

Extract 4.2 shows that the candidate was able to derive the formula for standard deviation using the coding method.

2.1.5 Question 5: Sets

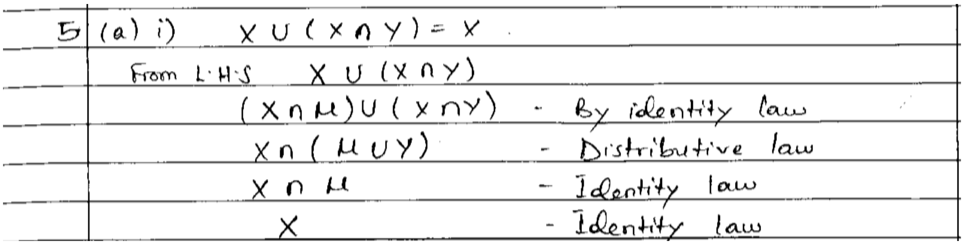
This question had parts (a), (b) and (c). In part (a), the question aimed to test the candidates' knowledge on how to use laws of algebra of sets to (i) verify that $X \cup (X \cap Y) = X$ and (ii) simplify $\left[A \cap (A \cup B)'\right]'$. Part (b) required the candidates to use Venn diagram to show whether $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$ when A, B and C are three non-empty sets.

In part (c), the candidates were given that "A class contains 15 boys and 15 girls. A survey of the class showed that: 20 pupils were studying Geography; 14 pupils were studying mathematics; 10 of the girls were studying Geography; 4 of the girls were studying mathematics; 3 of the girls were studying both Geography and mathematics and 3 of the boys were studying neither Geography nor mathematics." They were required to find the number of pupils who were studying both mathematics and Geography.

The analysis of data shows that many candidates (71.8%) scored from 3.5 to 10 marks, and among them, 909 candidates, equivalent to 7.1 percent, scored full marks. Based on this analysis, question 5 was among the five questions which had a good performance.

The analysis also shows that 38.0 percent of the candidates scored from 6 to 10 marks. Many of these candidates were able to use laws of algebra of sets to verify that $X \cup (X \cap Y) = X$ and simplify $\left[A \cap (A \cup B)'\right]'$ to μ as required. Most candidates were also able to prove the formula for the number of elements involving three sets using Venn diagram in part (b). Finally, most candidates were able to use Venn diagram to find the number of pupils who study both Mathematics and Geography in part (c) as instructed. This is an indication that they had sufficient knowledge in sets as demonstrated in Extract 5.1.

Extract 5.1



5 (a) i) $X \cup (X \cap Y) = X$

From L.H.S $X \cup (X \cap Y)$

$(X \cap \mu) \cup (X \cap Y)$ - By identity law

$X \cap (\mu \cup Y)$ - Distributive law

$X \cap \mu$ - Identity law

X - Identity law

Hence verified.

(ii) $[A \cap (A \cup B)']'$

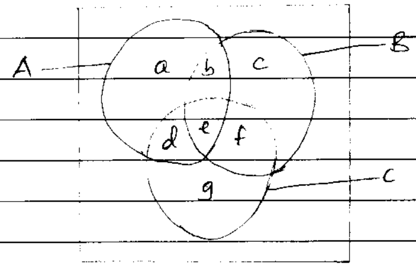
$A' \cup (A \cup B)$ - By demorgans law

$(A' \cup A) \cup B$ - Associative law

$\mu \cup B$ - Complement law

μ - Identity law

(b)



From the venn diagram: $n(A) = a + b + d + e$

$n(B) = b + c + e + f$

$n(C) = d + e + f + g$

$n(A \cap B) = b + e$

$n(A \cap C) = d + e$

$n(B \cap C) = e + f$

$n(A \cap B \cap C) = e$

$n(A \cup B \cup C) = a + b + c + d + e + f + g$

$= (a + b + d + e) + c + f + g$

$= n(A) + (b + c + e + f) + g - b - e$

$= n(A) + n(B) + (d + e + f + g) - b - e - d - e - f$

5 (b) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - (b + e) - d - e - f$

$= n(A) + n(B) + n(C) - n(A \cap B) - (d + e) - f$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - f - e + e$

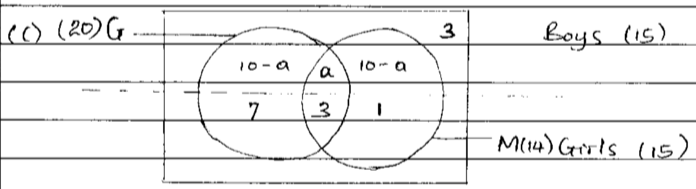
$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - (f + e) + e$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Hence shown that:

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

(c)



$15 - 3 = 12$ were studying geography or mathematics.

$10 - a + a + 10 - a = 12$

$20 - a = 12$

$a = 8$

$8 + 3 = 11$

$\therefore$ 11 students were studying both mathematics and geography.

In Extract 5.1, the candidate demonstrated good understanding of laws of Algebra. He/she managed to derive the formula for the number of elements of three sets in part (b) and used Venn diagram to solve the given problem in part (c) correctly.

A total of 692 (5.4%) candidates scored a zero mark in this question. These candidates showed several weaknesses such as indicating $n(A)$, $n(B)$, $n(C)$, $n(A \cap B)$, $n(A \cap C)$, $n(B \cap C)$ and $n(A \cap B \cap C)$ on Venn diagrams instead of writing letters like a, b, c, d, e, f and g; using logic symbols instead of sets symbols and applying wrong laws of algebra of sets. For example, idempotent instead of identity law and distributive instead of associative law see, Extract 5.2.

Extract 5.2

5 (a) (i) $X \cup (X \cap Y) = X$

Soln

$(X \cup X) \cap (X \cap Y)$ distributive

$(X \cap X) \cap (X \cap Y) \cup (X \cap X) \cap (X \cap Y)$ distributive

$(X \cap X) \cap (X \cap X) \cup (X \cap Y) \cap (X \cap Y)$ distributive

$X \cap U$ Identity law

X Identity law.

Hence verified

ii $(A \cap (A \cup B)')'$

Soln

$(A \cap (A \cup B)')'$

$(A \cup (A \cup B))$ de morgans law

$(A \cup A) \cup (A \cup B)$ distributive

$U \cup (A \cup B)$ Identity

$\therefore A \cup B$ Identity

Extract 5.2 shows how the candidate lacked skills in using laws of algebra of sets to prove the given identity.

2.1.6 Question 6: Functions

This question had parts (a), (b) and (c). In part (a), the candidates were required to (a) draw the graph of $f(x)=2+e^{2x}$ if $-3 \le x \le 1.2$ and $g(x)=1-e^{x}$ if $-3 \le x \le 2.7$ on the same xy plane by using a table of values. In part (b), they were required to (i) write down the composite function $g\,o\,f(x)$ in its simplest form and (ii) find the value of x if $g\,o\,f(x)=f\,o\,g(x)$ when $f(x)=x+1-\frac{1}{x}$ and $g(x)=\frac{1}{x}$. Part (c) required the candidates to find the equation of the asymptotes of the curve $y=\frac{x^2+3}{x-1}$ and then sketch it showing the coordinates of the turning points.

Further analysis of data shows that more than half of the candidates (53.9%) scored above 3 out of 10 marks and among them 40.5 percent scored from 3.5 to 5.5 marks. The analysis revealed that this was an averagely performed question.

The analysis of data shows that there were 8 candidates who scored full marks, an indication that they had adequate knowledge on functions. These candidates managed to construct tables of values for $f(x)=2+e^{2x}$ if $-3 \le x \le 1.2$ and $g(x)=1-e^{x}$ if $-3 \le x \le 2.7$ respectively, which were used in sketching the required graphs in part (a). In part (b), they worked out $gof(x)$ and $fog(x)$ correctly and hence managed to get $x=\pm 1$ when $gof(x)=fog(x)$. In part (c), it was noted that they were able to find the asymptotes $x=1$, $y=x+1$ and y –intercept $(0,3)$. In this part, the candidates also realized that the given curve never crosses the x-axis and thus they constructed a table of values which provided them with substantial information in sketching the curve of $y=\frac{x^2+1}{x-1}$. Extract 6.1 shows a sample of a response from a script of a candidate who answered this question correctly.

Extract 6.1

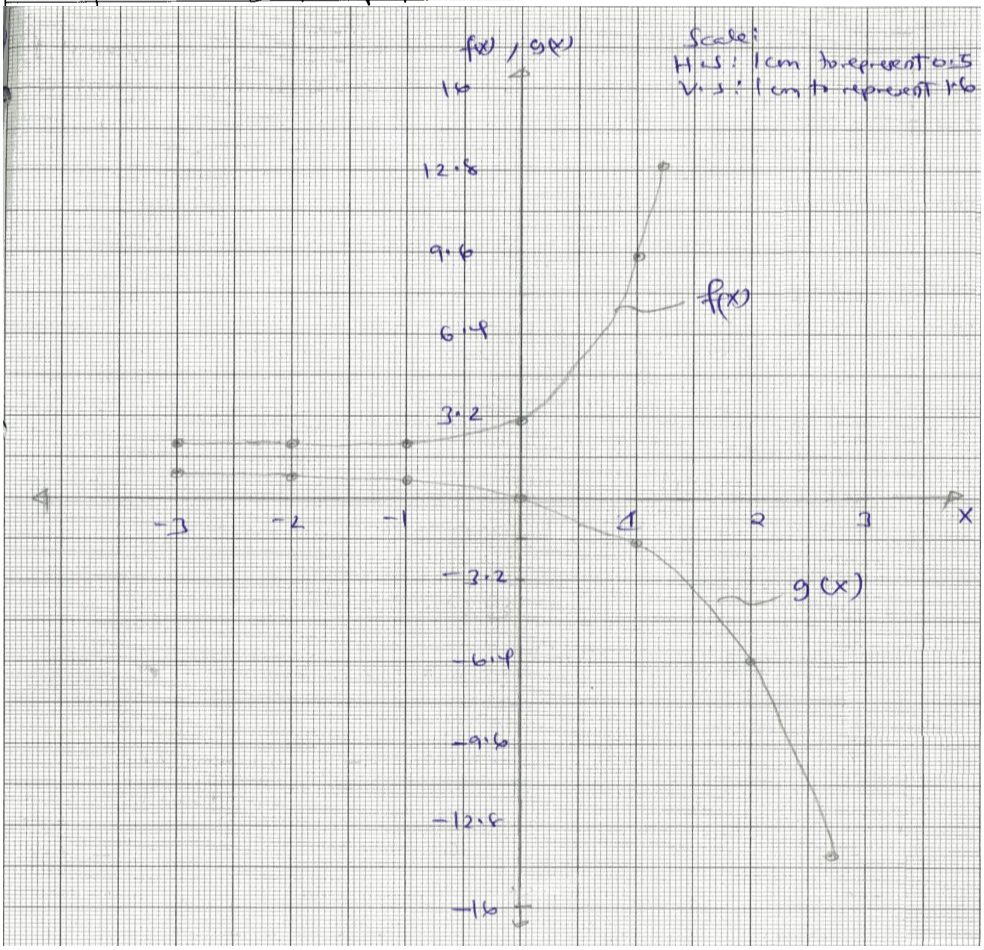
6 a) $f(x) = 2 + e^{2x}$ for $-3 \le x \le 1.2$
$g(x) = 1 - e^{x}$ if $-3 \le x \le 2.7$

Table of values

X	−3	−2	−1	0	1	1.2
fx	2	2.02	2.14	3	9.39	13.02

X	−3	−2	−1	0	1	2	2.7
g(x)	0.95	0.86	0.63	0	−1.72	−6.39	−13.88

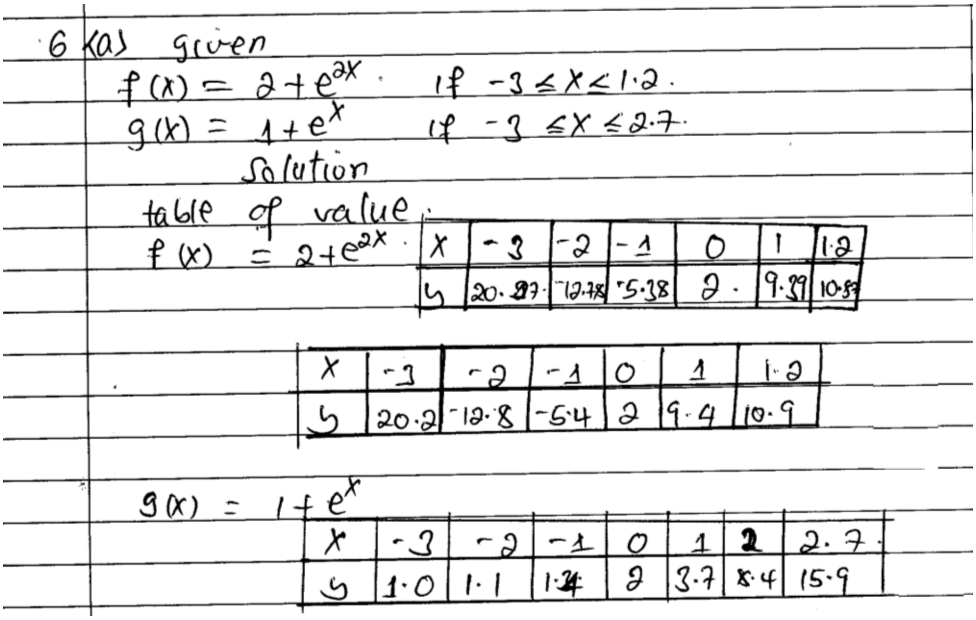
	The graph of f(x) and g(x) against x are drawn
	on the graph paper.



In Extract 6.1, the candidate succeeded to use the tables of values to draw the graph in part (a).

Conversely, a few candidates (5.1%) who scored a zero mark faced difficulties in constructing tables of values which was an essential step in sketching the required graphs in part (a). Extract 6.2 is a sample answer illustrating this case. In part (b) (i), some candidates substituted incorrectly $f(x)$ into $g(x)$, while others were able to perform the substitution but left the answers as $\dfrac{1}{x+1-\frac{1}{x}}$ instead of the expected final answer $\dfrac{x}{x^2+x-1}$. In part (c), the candidates failed to determine the y-axis, oblique and vertical asymptotes correctly, as a result ended up with sketching an incorrect curve for $y=\dfrac{x^2+1}{x-1}$.

Extract 6.2



6 (a) given

f(x) = 2 + e^{2x} if -3 ≤ x < 1.2.

g(x) = 1 + e^x if -3 ≤ x ≤ 2.7.

Solution

table of value

f(x) = 2 + e^{2x}.

x	-3	-2	-1	0	1	1.2
y	20.27	-12.78	-5.38	2	9.39	10.57

x	-3	-2	-1	0	1	1.2
y	20.2	-12.8	-5.4	2	9.4	10.9

g(x) = 1 + e^x

x	-3	-2	-1	0	1	2	2.7
y	1.0	1.1	1.4	2	3.7	8.4	15.9

In Extract 6.2, the candidate filled in incorrect values of y in tables of values for $f(x)$ and $g(x)$ which then affected the correctness of the graphs.

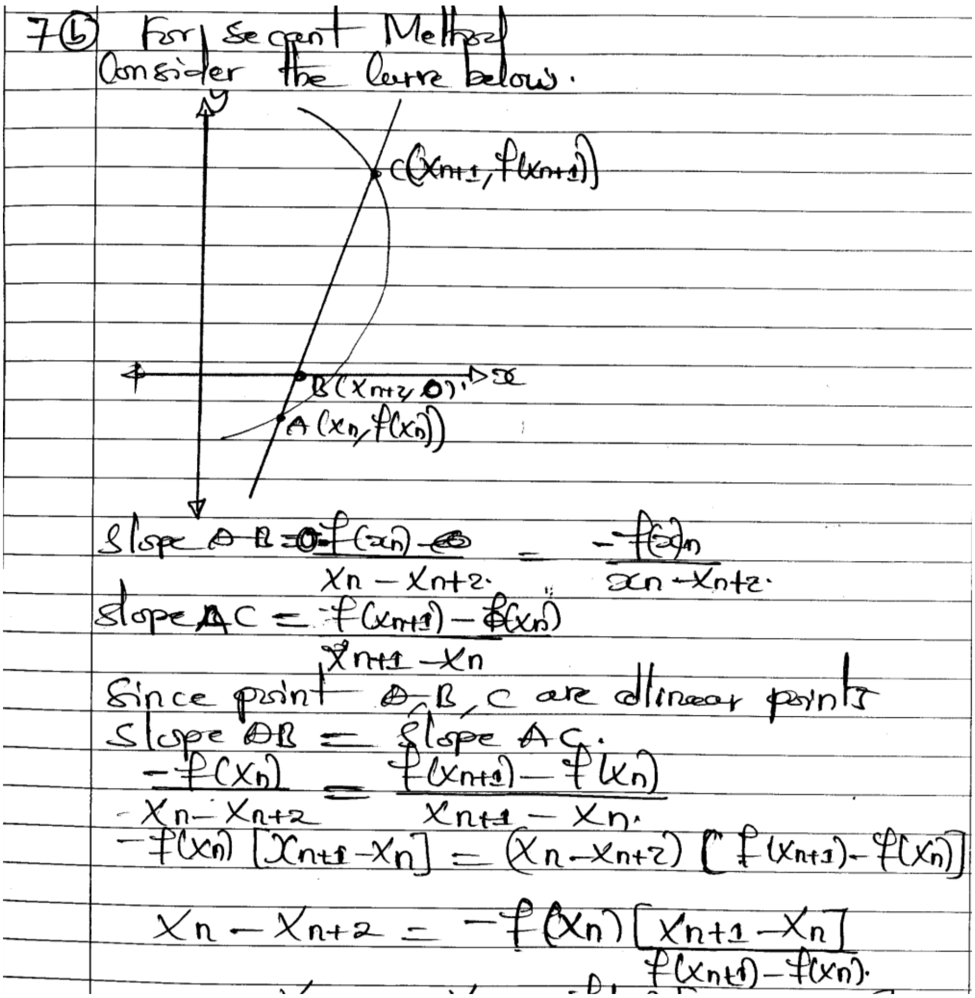
2.1.7 Question 7: Numerical Methods

This question had parts (a), (b) and (c). The question required the candidates to: (a) (i) write down four sources of errors in numerical computations; (a) (ii) derive the Newton-Raphson method for the function $f(x_n)$, if x_{n+1} is a better approximation to a root of the equation $f(x_n) = 0$; (b) use the Newton-Raphson formula obtained in (a) (ii), to derive the secant formula and comment why they would want to use it instead of the Newton-Raphson method and (c) perform three iterations to approximate the root of $x^2 - 2x - 1 = 0$ by using the secant method obtained in part (b), with $x_1 = 2$ and $x_2 = 3$ and hence compute the absolute error correct to four decimal places.

This question was attempted by 12,797 candidates whereby 72.2 percent scored from 0 to 3 marks, with 23.6 percent scoring a zero mark. The analysis of data shows that there was no candidate who scored full marks. Therefore, it was among the eight questions with poor performance.

In part (a), many candidates were unable to maneuvre the slope of the tangent line to graph of the function $f(x_n)$ that is $f'(x_n)=\dfrac{0-f(x_n)}{x_{n+1}-x_n}$ to yield $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$ as required. In part (b), most candidates could not use the results from the previous part of the question to derive the secant method as instructed. This indicates how the candidates failed to adhere to the instructions given in this question, see Extract 7.1. In part (c), some of them failed to realize that there was a root between $x_1=2$ and $x_2=3$ and thus, they were unable to use the obtained formula in part (b) to compute the absolute error as required, while others knew how to use the Secant Formula but made mistakes when working out the three iterations with the use of a calculator.

Extract 7.1


7(b) For Secant Method
Consider the curve below.

Slope AB = 0 − f(xn) − 0 / Xn − Xn+2 = −f(x)n / xn − Xn+2

Slope AC = f(Xn+1) − f(Xn) / Xn+1 − Xn

Since point A, B, C are collinear points
Slope AB = Slope AC.

−f(Xn) / Xn − Xn+2 = f(Xn+1) − f(Xn) / Xn+1 − Xn

−f(Xn) [Xn+1 − Xn] = (Xn − Xn+2) [f(Xn+1) − f(Xn)]

Xn − Xn+2 = −f(Xn) [Xn+1 − Xn] / f(Xn+1) − f(Xn).

$$X_{n+2} = X_n - f(x_n)\left[\frac{X_{n+1} - X_n}{f(x_{n+1}) - f(x_n)}\right]$$

$$X_{n+2} = X_n - f(x_n)\left[\frac{X_{n+1} - X_n}{f(x_{n+1}) - f(x_n)}\right]$$

is the Secant formular

In Extract 7.1, the candidate derived Secant formula without using the Newton Raphson formula, contrary to the requirements of the question.

Despite the fact that many candidates had poor performance, a total of 457 candidates, equivalent to 3.6 percent, scored from 6 to 9 out of 10 marks. Such candidates were able to apply the concepts of Numerical Methods in answering this question. A sample answer from one of those candidates is shown in Extract 7.2.

Extract 7.2

7 c) Soln,

Given:

$x_1 = 2$,

$x_2 = 3$

$x^2 - 2x - 1 = 0$

$f(x) = x^2 - 2x - 1$.

from Secant method;

$$x_{n+2} = x_n - \left(\frac{x_n - x_{n+1}}{fx_n - fx_{n+1}}\right) f x_n.$$

1st iteration, n = 1.

$$X_3 = x_1 - \left(\frac{x_1 - x_2}{fx_1 - fx_2}\right) f(x_1)$$

but; $f(x) = x^2 - 2x - 1$

for $x_1 = 2$;

$f(x_1) = 2^2 - 2(2) + 1$

$f(x_1) = -1$

$\therefore \quad (2, -1)$

for $x_2 = 3$; $\quad f(x_2) = 3^2 - 2(3) + 1$

$f(x_2) = 4 \cdot 2.$

thus;

$$x_3 = x_1 - \left(\frac{x_1 - x_2}{fx_1 - fx_2}\right) f(x_1)$$

$$= 2 \cdot 2 - \left(\frac{2 - 3}{-1 - 2}\right)(-1).$$

$x_3 = 2.3333.$

2nd iteration, n = 2.

$$x_4 = x_2 - \left(\frac{x_2 - x_3}{fx_2 - fx_3}\right) f(x_2).$$

$x_2 = 3, \quad f(x_2) = 2.$

$x_3 = 2.333 \quad f(x_3) = -0.2222.$

thus;

$$x_4 = x_2 - \left(\frac{x_2 - x_3}{fx_2 - fx_3}\right) fx_2.$$

$$= 3 - \left(\frac{3 - 2.3333}{2 - {}^{-}0.2222}\right)(2)$$

$x_4 = 2.40000.$

3rd iteration, n = 3

$x_5 = x_3 - \left(\frac{x_3 - x_4}{fx_3 - fx_4}\right) fx_3$.

$x_3 = 2.3333$, $f(x_3) = -0.2222$.

$x_4 = 2.4$ $f(x_4) = -0.04$.

$x_5 = 2.3333 - \left(\frac{2.3333 - 2.4}{-0.2222 - {}^{-}0.04}\right)(-0.2222)$

$x_5 = 2.4146$

∴ The approximate root is 2.4146.

from; $x^2 - 2x - 1 = 0$.

On solving; $x = 2.4142$.

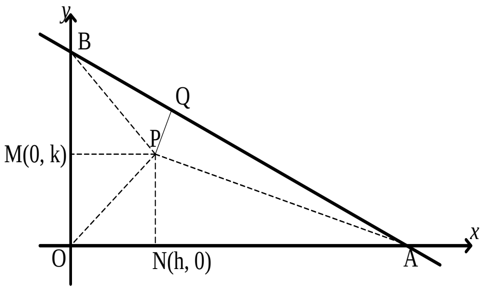
∴ Absolute error $= |2.4142 - 2.4146|$.

$= 0.0004$.

Extract 7.2 shows that the candidate was familiar with the tested concepts of Numerical Methods.

2.1.8 Question 8: Coordinate Geometry I

This question consisted of parts (a), (b) and (c). In part (a) (i), the candidates were given that “the line $Ax + By + C = 0$ meets the coordinates axes at A and B. Also P is a point (h, k) and $PQ = p$ is the perpendicular distance to AB”. They were required to use this information and the following figure to derive the perpendicular distance of the point P from the line AB.



In part (a) (ii), the candidates were required to find the value of *n*, given that the perpendicular distance from the point $(2, 5)$ to the line $ny = 2x - 4$

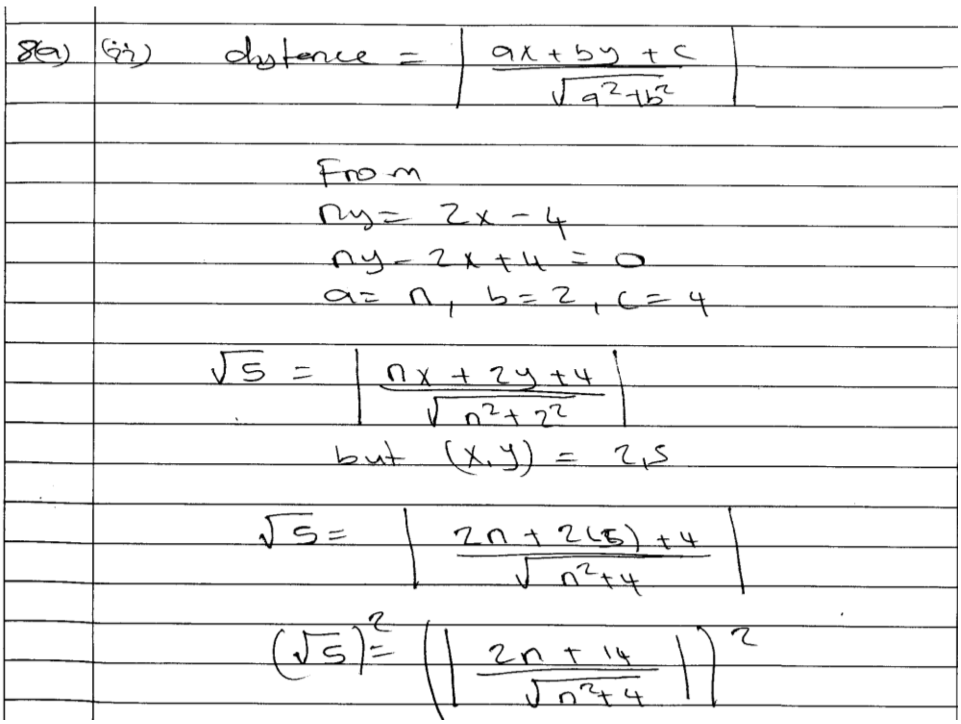
is $\sqrt{5}$. In part (b), the candidates were asked to write down the equation of the bisector of the acute angle between the line $3x+4y=1$ and $5x-12y+6=0$. Part (c) required the candidates to find the length of a tangent from the centre of the circle $x^2+y^2+6x+8y-1=0$ to the circle $x^2+y^2-2x+4y-3=0$.

This question was attempted by 12,798 candidates, of whom the majority (78.2%) scored below 3.5 out of 10 marks, and among them 29.5 percent scored a zero mark, indicating an overall poor performance in this question.

The analysis of the candidates' responses revealed that the candidates with low marks did not derive the formula for the perpendicular distance of a point from the line in part (a) (i). They did not recognize that they had to use the knowledge of area of triangle and Pythagoras theorem to obtain $p=\frac{Ah+Bh+C}{\sqrt{A^2+B^2}}$. However, in part (a) (ii) and (b), many of them could write the formulae correctly but made many arithmetic mistakes in solving them. These mistakes produced the wrong value of n and equations, see Extract 8.1.

In part (c), a considerable number of candidates could not realize that they were supposed to use the centres of the given circles, distance formula and Pythagoras theorem to find the required length, indicating a lack of understanding on the tested concepts.

Extract 8.1



8(a) (ii) distance $= \left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right|$

From

$ny = 2x - 4$

$ny - 2x + 4 = 0$

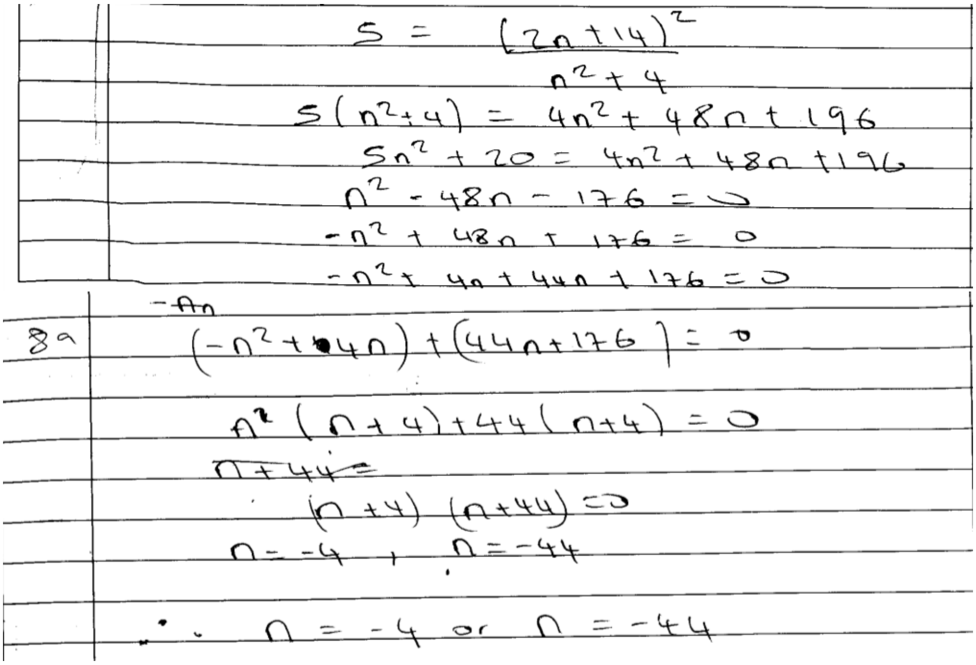
$a = n,\ b = 2,\ c = 4$

$\sqrt{5} = \left|\frac{nx+2y+4}{\sqrt{n^2+2^2}}\right|$

but $(x, y) = 2, 5$

$\sqrt{5} = \left|\frac{2n+2(5)+4}{\sqrt{n^2+4}}\right|$

$(\sqrt{5})^2 = \left(\left|\frac{2n+14}{\sqrt{n^2+4}}\right|\right)^2$



$S = \frac{(2n+14)^2}{n^2+4}$

$5(n^2+4) = 4n^2+48n+196$

$5n^2+20 = 4n^2+48n+196$

$n^2 - 48n - 176 = 0$

$-n^2 + 48n + 176 = 0$

$-n^2 + 4n + 44n + 176 = 0$

8a

$(-n^2+4n)+(44n+176) = 0$

$n^2(n+4)+44(n+4) = 0$

~~$n+44=$~~

$(n+4)(n+44) = 0$

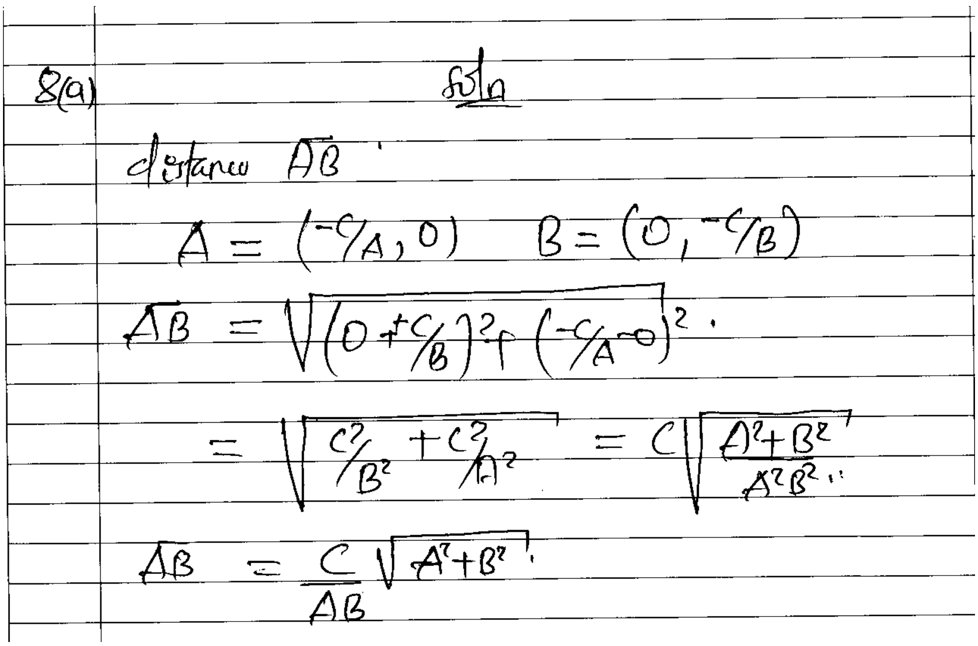
$n = -4$, $n = -44$

$\therefore n = -4$ or $n = -44$

In Extract 8.1, the candidate failed to manipulate $\sqrt{5} = \left|\frac{2\times 2 - n\times 5 - 4}{\sqrt{2^2+n^2}}\right|$, as a result ended up getting $n = -4$ or $n = -44$, instead of $n = \pm 1$.

Despite the fact that question 8 was among the eight questions with poor performance, there were 26 (0.2%) candidates who scored all 10 marks. Extract 8.2 provides a sample answer from one of these candidates, showing how they answered part (a) correctly.

Extract 8.2



8(a) soln

distance AB

$A = (-C/A, 0)$ $B = (0, -C/B)$

$AB = \sqrt{(0 + C/B)^2 + (-C/A - 0)^2}$

$= \sqrt{\frac{C^2}{B^2} + \frac{C^2}{A^2}} = C\sqrt{\frac{A^2+B^2}{A^2B^2}}$

$AB = \frac{C}{AB}\sqrt{A^2+B^2}$

$$\text{Area of } \Delta APB = \frac{1}{2}\begin{vmatrix} h & k & 1 \\ -\frac{c}{A} & 0 & 1 \\ 0 & -\frac{c}{B} & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left| h\left(\frac{c}{B}\right) - k\left(\frac{-c}{A}\right) + \frac{c^2}{AB} \right|$$

8(a) $$A = \frac{c}{2AB}(Ah + Bk + c) \quad \text{— (i).}$$

(i) Also.

$$A = \frac{1}{2} bh = \frac{1}{2} AB\, p.$$

$$A = \frac{1}{2} \frac{c}{AB} \sqrt{A^2+B^2} \cdot p \quad \text{— (ii).}$$

eqn (i) = eqn (ii).

$$\frac{c}{2AB}(Ah+Bk+c) = \frac{c}{2AB}\sqrt{A^2+B^2}\, p$$

$$p = \left| \frac{Ah + Bk + C}{\sqrt{A^2+B^2}} \right|.$$

∴ perpendicular distance is $p = \left| \frac{Ah+Bk+C}{\sqrt{A^2+B^2}} \right|$

hence derived.

Extract 8.2 shows how the candidate derived the perpendicular distance formula in part (a) (i) correctly.

2.1.9 Question 9: Integration

This question consisted of parts (a), (b), (c) and (d). The question required the candidates to; (a) (i) show whether $\int \frac{f'(x)}{f(x)}dx = \ln A\, f(x)$, where A is a constant, (a) (ii) find $\int \cos 2x \cos 4x \cos 6x\, dx$, (b) evaluate $\int_0^{\frac{\pi}{2}} x \sin x \cos x\, dx$, (c) find the area of the region bounded by the curve $y = 3x^2 - 2x + 1$, the lines $x + 1 = 0$, $x - 2 = 0$ and $y = 0$ and (d) find the volume generated when the area between the curve $3x^2 + y^2 = 9$ and the y –axis from $y = -3$ to $y = 3$ is rotated about the y-axis.

Most candidates (81.6%) scored from 0 to 3 marks out of the 10 marks and the minority (18.4%) scored more than 3 marks. Also, more than half of the candidates (53.5%) scored a zero mark indicating lack of knowledge and skills for the majority of the candidates on the tested topic of Integration.

The analysis done in the scripts of the candidates shows that the candidates with low marks had the following weaknesses: One, in part (a), most candidates failed to use the substitution $u = f(x)$ and $du = f'(x)\, dx$ in $\int \frac{f'(x)}{f(x)} dx$, which could be further manipulated to give $\ln Af(x)$ as required. Moreover, in part (a) (ii) the majority of the candidates could realize that they should apply both factor formula $2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \cos A + \cos B$ and double angle formula $\cos 2A = 2\cos^2 A - 1$ in rewriting the integral $\int \cos 2x \cos 4x \cos 6x\, dx$ as $\frac{1}{4}\int (1 + \cos 12x + \cos 8x + \cos 4x)\, dx$. This integral could be evaluated to give the required answer $\frac{1}{4}x + \frac{1}{48}\sin 12x + \frac{1}{32}\sin 8x + \frac{1}{16}\sin 4x + C$. Two; in part (b), many candidates could not express $\sin x \cos x$ as $\frac{\sin 2x}{2}$ and thereafter use the method of integration by parts to evaluate $\frac{1}{2}\int_0^{\frac{\pi}{2}} x \sin 2x\, dx$ to obtain the intended answer $\frac{\pi}{8}$. In part (c), many candidates failed to represent the given information in a drawing which would have enabled them to identify the limits of integration that were essential for proceeding further with the evaluation of the required area. Four; failure of candidates to sketch the graph of $3x^2 + y^2 = 9$ so as to determine the limits of integration in part (d). Also, several candidates used incorrect formulae such as $2\pi\int y^2$ or $\pi\int y^2$ or even $\int x^2 dx$ to find the volume of the solid generated about the $y-axis$. Extract 9.1 is a sample answer illustrating one of these cases.

Extract 9.1

09 (a) Solution.

Given:

$3X^2 + y^2 = 9.$

$y = \sqrt{9 - 3X^2}$

from:

$$\text{Volume} = \int_{-3}^{3} \pi\,(f(x))^2\,dx.$$

09 (a)

$$\text{Volume} = \pi\int_{-3}^{3}(9 - 3X^2)\,dx.$$

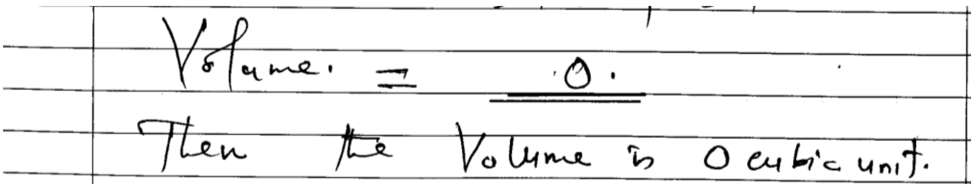
$$= \pi\int_{-3}^{3} 9\,dx - \pi\int_{-3}^{3} 3X^2\,dx.$$

$$= 9\pi\int_{-3}^{3} dx - 3\pi\int_{-3}^{3} X^2\,dx$$

$$= 9\pi\Big[X\Big|_{-3}^{3} - 3\pi\Big[\frac{X^3}{3}\Big|_{-3}^{3}$$

$$= 9\pi\Big[-3 - 3\Big] - 3\pi\Big[-\frac{27}{3} - \frac{27}{3}\Big]$$

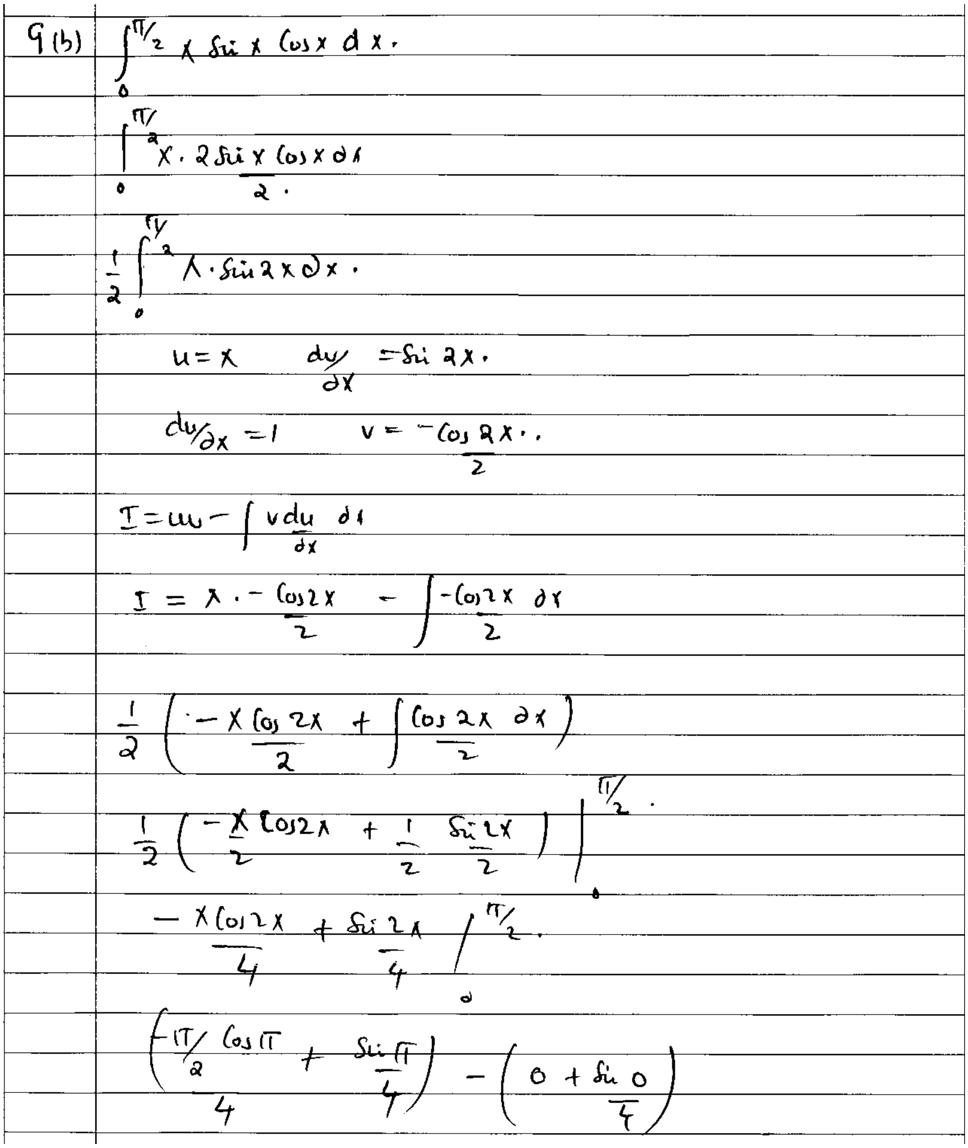
$$= -54\pi + 54\pi$$


Volume = 0.
Then the Volume is 0 cubic unit.

In Extract 9.1, the candidate used a wrong formula to find the volume of the solid generated.

Despite these frequent mistakes, there were 14 candidates who scored full marks. Extract 9.2 is a sample solution from one of the candidates showing how he or she was able to apply correctly his/her knowledge and skills on the topics of Integration.

Extract 9.2


9 (b) $\int_0^{\pi/2} x \sin x \cos x \, dx$.

$\int_0^{\pi/2} x \cdot \frac{2\sin x \cos x}{2} \, dx$

$\frac{1}{2}\int_0^{\pi/2} x \cdot \sin 2x \, dx$.

$u = x \qquad \frac{dv}{dx} = \sin 2x$.

$\frac{du}{dx} = 1 \qquad v = \frac{-\cos 2x}{2}$.

$I = uv - \int v \frac{du}{dx} \, dx$

$I = x \cdot \frac{-\cos 2x}{2} - \int \frac{-\cos 2x}{2} \, dx$

$\frac{1}{2}\left(\frac{-x\cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx\right)$

$\frac{1}{2}\left(\frac{-x\cos 2x}{2} + \frac{1}{2} \cdot \frac{\sin 2x}{2}\right)\Big|_0^{\pi/2}$.

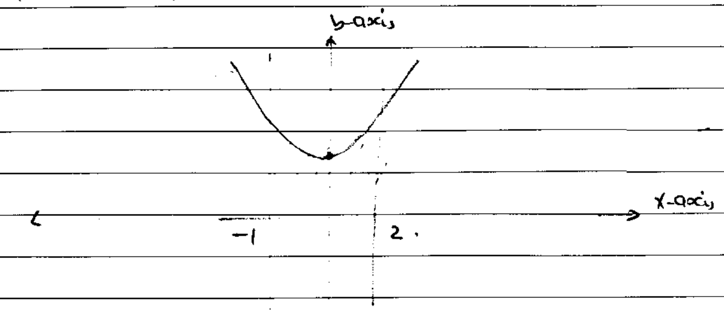
$\frac{-x\cos 2x}{4} + \frac{\sin 2x}{4}\Big|_0^{\pi/2}$.

$\left(\frac{-\frac{\pi}{2}\cos \pi}{4} + \frac{\sin \pi}{4}\right) - \left(0 + \frac{\sin 0}{4}\right)$

$\frac{\pi}{8} + 0 = \frac{\pi}{8}$

9(c) $y = 3x^2 - 2x + 1$

$x = -1 \quad x = 2 \quad y = 0$



$A = \int y\,dx.$

$A = \int_{-1}^{0} 3x^2 - 2x + 1 \; dx + \int_{0}^{2} (3x^2 - 2x + 1)\,dx$

$A = \left.\frac{3x^3}{3} - \frac{2x^2}{2} + x \right|_{-1}^{0} + \left.\frac{3x^3}{3} - \frac{2x^2}{2} + x \right|_{0}^{2}$

$A = 0 - (-1 - 1 - 1) + (8 - 4 + 2) - (0)$

$A = (3) + (6)$

$A = 9$ units square

Extract 9.1 shows that the candidate was able to evaluate the definite integral of trigonometrical in part (b). The candidate also managed to find the area bounded by the given curve and the lines in part (c).

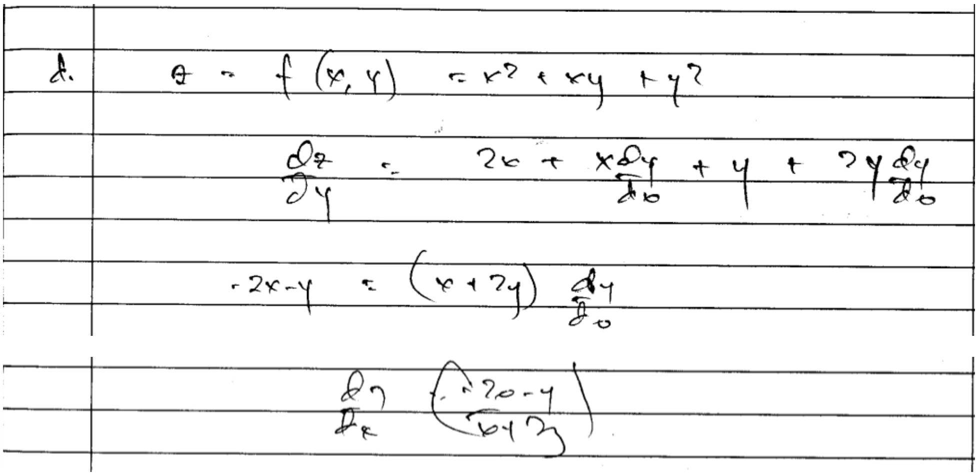
2.1.10 Question 10: Differentiation

The question required the candidates to: (a) find the derivative of $\frac{1}{x} + \cos 3x$ from first principles; (b) use the Taylor theorem to obtain the series expansion for $\cos\left(x + \frac{\pi}{3}\right)$, stating the terms including x^3 and hence obtaining a value for $\cos 61^\circ$ correct to five decimal places; (c) show whether the line $2x - y = 0$ and the curve $4x^2 - 4xy + y^2 4x - 8y + 10 = 0$ intersect at a right angle and (d) find $\frac{\partial z}{\partial y}$ at $(1,1,1)$ given that a two variable functions f is defined as $z = f(x, y) = x^2 + xy + y^2$.

This question was generally poorly done, with many candidates (82.1%) scoring from 0 to 3 out of 10 marks, and among them, 51.7 percent scored a zero mark, while 2,292 (17.9%) candidates scored from 3 to 10 marks. The analysis has shown that there was no candidate who scored full marks.

The candidates with low marks wrote correctly the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ but they could not use it to find the derivative in part (a). In part (b), the majority of the candidates were unable to establish the expansion for $\cos\left(x + \frac{\pi}{3}\right)$. They failed to identify that Taylor's theorem of f with center $x = a$ is given by $f(x+a) = f(a) + xf'(a) + \frac{x^2}{2} f''(a) + \frac{x^3}{6} f'''(a) + ...$ and not $f(x+a) = xf(a) + \frac{x^2}{2} f'(a) + \frac{x^3}{6} f''(a) + ...$ which then affected the answers to the rest of this part. In part (c), some candidates failed to find where the given line intersects with the given curve, while others struggled to differentiate $4x^2 - 4xy + 4xy^2 - 8y + 10 = 0$ with the expression such as $\frac{dy}{dx} = \frac{-2x + y^2 - y}{-x + 2xy - 2}$ being encountered. Such candidates ended up there and did not proceed to the required conclusion. In part (d), the candidates were not aware of the meaning of the term "partial derivative", thus many of them wrote $\frac{\partial z}{\partial y}$ as $2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx}$ instead of $x + 2y$, see Extract 10.1.

Extract 10.1



d. $z = f(x, y) = x^2 + xy + y^2$

$\frac{\partial z}{\partial y} = 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx}$

$-2x - y = (x + 2y)\frac{dy}{dx}$

$\frac{dy}{dx} = \left(\frac{-2x - y}{x + 2y}\right)$

In Extract 10.1, the candidate differentiated the given function partially with respect to y to get $\frac{dy}{dx} = \frac{2x - y}{x + 2y}$ instead of $\frac{\partial z}{\partial y} = x + 2y$, showing insufficient knowledge on the tested concept.

Only 3 out of the 12,797 candidates who attempted this question managed to score 9.5 marks. A sample answer from one of those candidates showing how they were able to apply correctly the knowledge and skills on the topics of differentiation, to find the derivative of $\frac{1}{x} + \cos 3x$ from first principles, is shown in Extract 10.2.

Extract 10.2

10. (a) Soln

Given

$$y = \frac{1}{x} + \cos 3x.$$

Then

$$y + \Delta y = \frac{1}{(x+\Delta x)} + \cos 3(x+\Delta x).$$

$$\therefore \Delta y = \frac{1}{(x+\Delta x)} + \cos(3(x+\Delta x)) - y$$

$$\therefore \Delta y = \frac{1}{(x+\Delta x)} + \cos(3x+3\Delta x) - \frac{1}{x} - \cos 3x$$

$$\Delta y = \frac{x - x - \Delta x}{x(x+\Delta x} - 2\sin\left(\frac{3\Delta x}{2}\right)\sin\left(3x + \frac{3\Delta x}{2}\right)$$

$$\Delta y = \frac{-\Delta x}{x(x+\Delta x)} - 2\sin\left(\frac{3\Delta x}{2}\right)\sin\left(3x + \frac{3\Delta x}{2}\right)$$

but as $\Delta x \to 0$ $\sin \Delta x \simeq \tan \Delta x \simeq \Delta x$

also

$$\therefore \frac{\Delta y}{\Delta x} = \frac{-1}{x(x+\Delta x)} - 2\left(\frac{3}{2}\right)\sin\left(3x + \frac{\Delta x}{2}\right)$$

then since $\Delta x \simeq$ very small

$$\frac{dy}{dx} \simeq \frac{\Delta y}{\Delta x} \quad \Delta x \to 0$$

$$\therefore \frac{dy}{dx} = \frac{-1}{x(x)} - 3\sin(3x+0)$$

$$= \frac{-1}{x^2} - 3\sin 3x$$

$$\therefore dy/dx = -1/x^2 - 3\sin 3x$$

Extract 10.2 shows how well the candidate differentiated the given function from first principles. He/she applied the factor formula $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ on $\cos(3x+3\Delta x) - \cos 3x$ to get $\frac{dy}{dx} = -\frac{1}{x^2} - 3\sin 3x$.

2.2 142/2 ADVANCED MATHEMATICS 2

2.2.1 Question 1: Complex Numbers

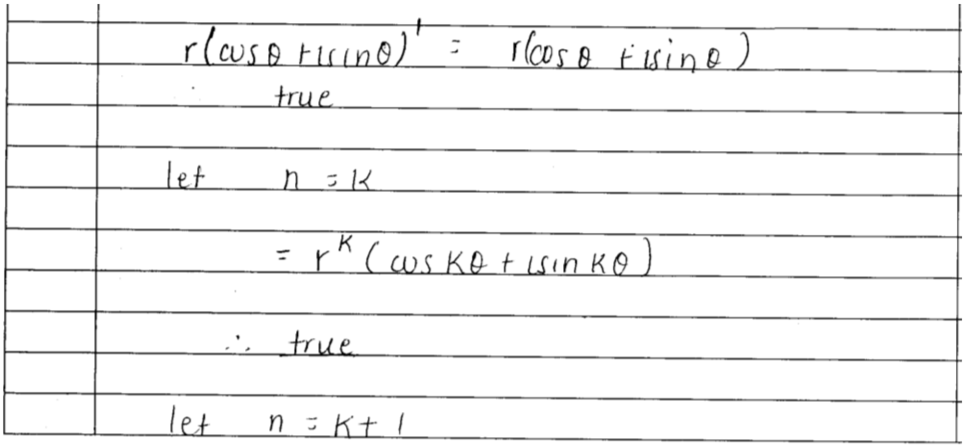
This question had parts (a), (b) and (c). In part (a) (i), the candidates were required to use the De Moivre's theorem to find the value of $(1+i)^8$ while in part (a) (ii), they were required to use mathematical induction in order to prove that $(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$. In part (b) (i), they were required to find the locus of the point representing z in an argand diagram when $\arg\left(\frac{z-1}{z+i}\right) = \frac{\pi}{4}$ whereas $z = x + iy$. In part (b) (ii), they were required to solve the system of equations $\begin{cases} iz - w = 2i \\ iz + iw = i \end{cases}$ where z and w are complex numbers. Finally, in part (c) they were required to find the other roots of the equation $z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$, when one of the roots is $1+i$.

A total of 12,791 (100%) candidates responded to the question, out of which 6,387 (49.9 %) candidates got their scores from 5.5 to 15 marks, 27.9 percent had their scores below 3 marks and 8.7 percent scored a zero mark. Generally, the performance in this question was average.

The analysis of the candidates' response shows that the candidates who scored high marks had adequate knowledge on complex numbers. They correctly applied the De Moivre's theorem to find the value of the given expression in part (a) (i). They also managed to prove that if $\left(r\left(\cos n\theta + i\sin n\theta\right)\right)^n = r^n\left(\cos n\theta + i\sin n\theta\right)$ is true for $n=1$ and $n=k$ then it also holds for $n=k+1$ in part (a) (ii). Furthermore, they were able to get $x^2 + y^2 = 1$, which is a circle with the radius of 1 unit and centre at the origin as the required locus equation in part (b) (i). In part (b) (ii), they correctly solved the given system of simultaneous equations to get z and w as $\frac{3}{2} + \frac{1}{2}i$ and $-\frac{1}{2} - \frac{1}{2}i$ respectively. Finally, they managed to find the correct roots of the given equation in part (c). Extract 11.1 is a sample answer from one of the candidates who attempted part (a) (ii) correctly.

Extract 11.1

(ii)	$r(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + \sin n\theta)$
	let $n = 1$



$r(\cos\theta + i\sin\theta)^1 = r(\cos\theta + i\sin\theta)$

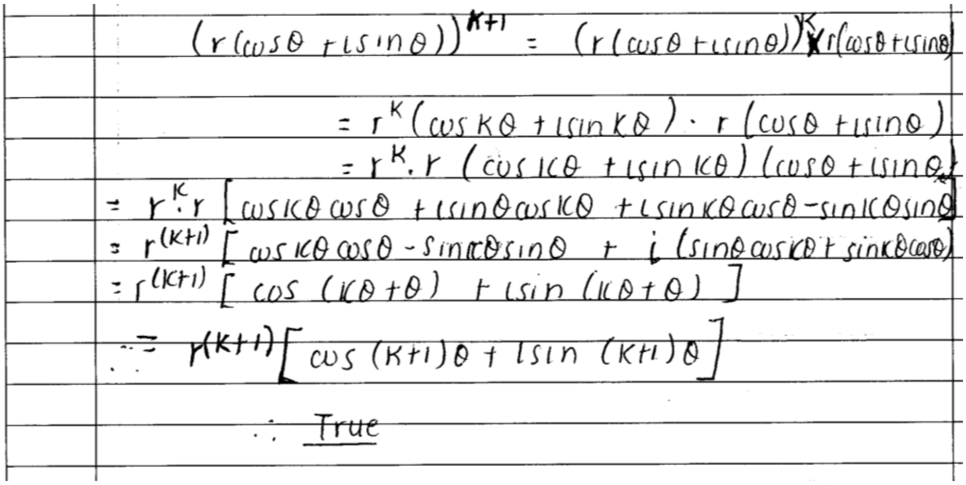
true

let $n = k$

$= r^k(\cos k\theta + i\sin k\theta)$

$\therefore$ true

let $n = k+1$



$(r(\cos\theta + i\sin\theta))^{k+1} = (r(\cos\theta + i\sin\theta))^k \times r(\cos\theta + i\sin\theta)$

$= r^k(\cos k\theta + i\sin k\theta) \cdot r(\cos\theta + i\sin\theta)$

$= r^k . r(\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$

$= r^k . r[\cos k\theta\cos\theta + i\sin\theta\cos k\theta + i\sin k\theta\cos\theta - \sin k\theta\sin\theta]$

$= r^{(k+1)}[\cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\sin\theta\cos k\theta + \sin k\theta\cos\theta)]$

$= r^{(k+1)}[\cos(k\theta+\theta) + i\sin(k\theta+\theta)]$

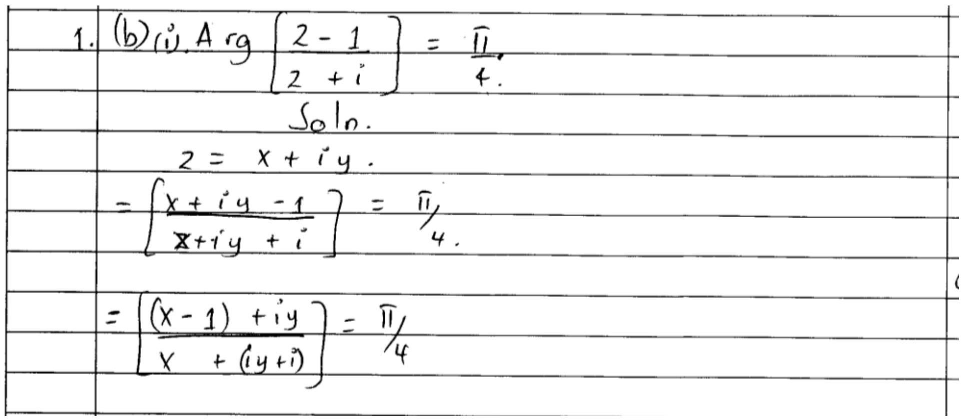
$\therefore = r^{(k+1)}[\cos(k+1)\theta + i\sin(k+1)\theta]$

$\therefore$ True

In Extract 11.1, the candidate managed to prove the given identity by mathematical induction, showing that he/she had adequate knowledge on the tested concept.

It was also noted that the candidates (27.8%) who scored from 5.5 to 8.5 marks faced difficulties in answering parts (a) (ii), (b) (i) and (c). In part (a) (ii), some candidates did not apply the mathematical induction, while others managed to prove that the given identity is true for $n=1$ and $n=k$ but failed to prove it when $n=k+1$ because they could not maneuvre the equation $\left(r\left(\cos\theta+i\sin\theta\right)\right)^k r\left(\cos\theta+i\sin\theta\right)$ to get $\left(r\left(\cos\theta+i\sin\theta\right)\right)^{k+1} = r^{k+1}\left[\cos(k+1)\theta+i\sin(k+1)\theta\right]$. In part (b) (i), they failed to recall the formula $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$ which was essential in answering this part. Wrong interpretations of $\arg\left(\frac{z-1}{z+i}\right)=\frac{\pi}{4}$ such as $\frac{x-1+iy}{x+1+iy}=\frac{\pi}{4}$, $\frac{(x-1)+iy}{x+(iy+i)}=\frac{\pi}{4}$, $\tan^{-1}\left(\frac{(x-1)^2+y^2}{x^2+(1+y)^2}\right)=\frac{\pi}{4}$ e.t.c were normally seen in the scripts of the candidates. Such candidates did not recognize that "if $\arg\left(\frac{z-1}{z+i}\right)=\frac{\pi}{4}$ and $z=x+iy$ then $\tan^{-1}\left(\frac{y}{x-1}\right)-\tan^{-1}\left(\frac{y+1}{x}\right)=\frac{\pi}{4}$" as a result they were unable to continue with the process of obtaining the required locus, see Extract 11.2. In part (c), the candidates failed to realize that if $z=1+i$ then $\bar{z}=1-i$ is another root of the equation $z^4-6z^3+23z^2-34z+26=0$. Such an error resulted in the failure of the candidates to formulate a quadratic factor $z^2-4z+13$ which could then be solved to get $2+3i$ and $2-3i$ as other roots.

Extract 11.2

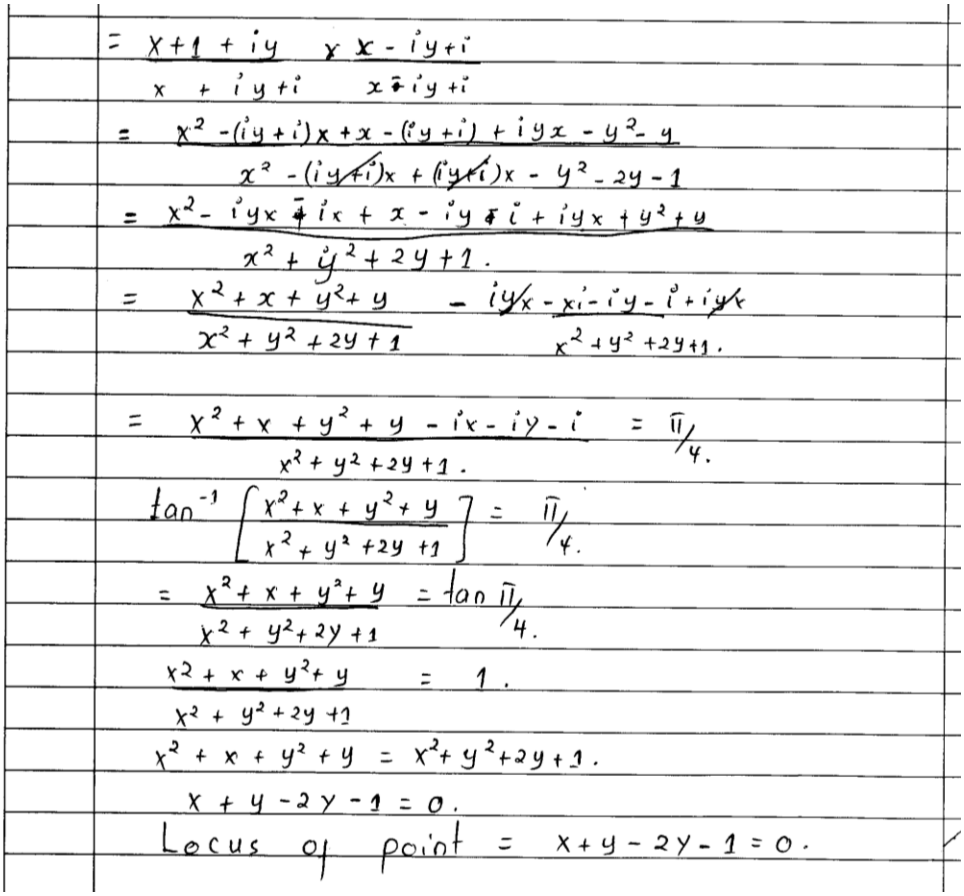

1. (b)(i) Arg $\left[\frac{z-1}{z+i}\right]=\frac{\pi}{4}$.

Soln.

$z = x+iy$.

$=\left[\frac{x+iy-1}{x+iy+i}\right]=\pi/4$.

$=\left[\frac{(x-1)+iy}{x+(iy+i)}\right]=\pi/4$



$$= \frac{x+1+iy}{x+iy+i} \times \frac{x-iy+i}{x\mp iy+i}$$

$$= \frac{x^2-(iy+i)x+x-(iy+i)+iyx-y^2-y}{x^2-(iy+i)x+(iy+i)x-y^2-2y-1}$$

$$= \frac{x^2-iyx+ix+x-iy+i+iyx+y^2+y}{x^2+y^2+2y+1.}$$

$$= \frac{x^2+x+y^2+y}{x^2+y^2+2y+1} - \frac{iyx-xi-iy-i+iyx}{x^2+y^2+2y+1.}$$

$$= \frac{x^2+x+y^2+y-ix-iy-i}{x^2+y^2+2y+1.} = \pi/4.$$

$$\tan^{-1}\left[\frac{x^2+x+y^2+y}{x^2+y^2+2y+1}\right] = \pi/4.$$

$$= \frac{x^2+x+y^2+y}{x^2+y^2+2y+1} = \tan \pi/4.$$

$$\frac{x^2+x+y^2+y}{x^2+y^2+2y+1} = 1.$$

$$x^2+x+y^2+y = x^2+y^2+2y+1.$$

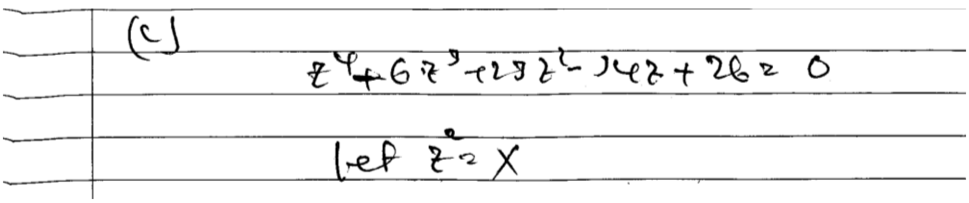
$$x+y-2y-1=0.$$

Locus of point $= x+y-2y-1=0.$

What is written in Extract 11.2 could be an indication that the candidate did not have an idea on how to deal with the expression involving the argument of a complex number**.**

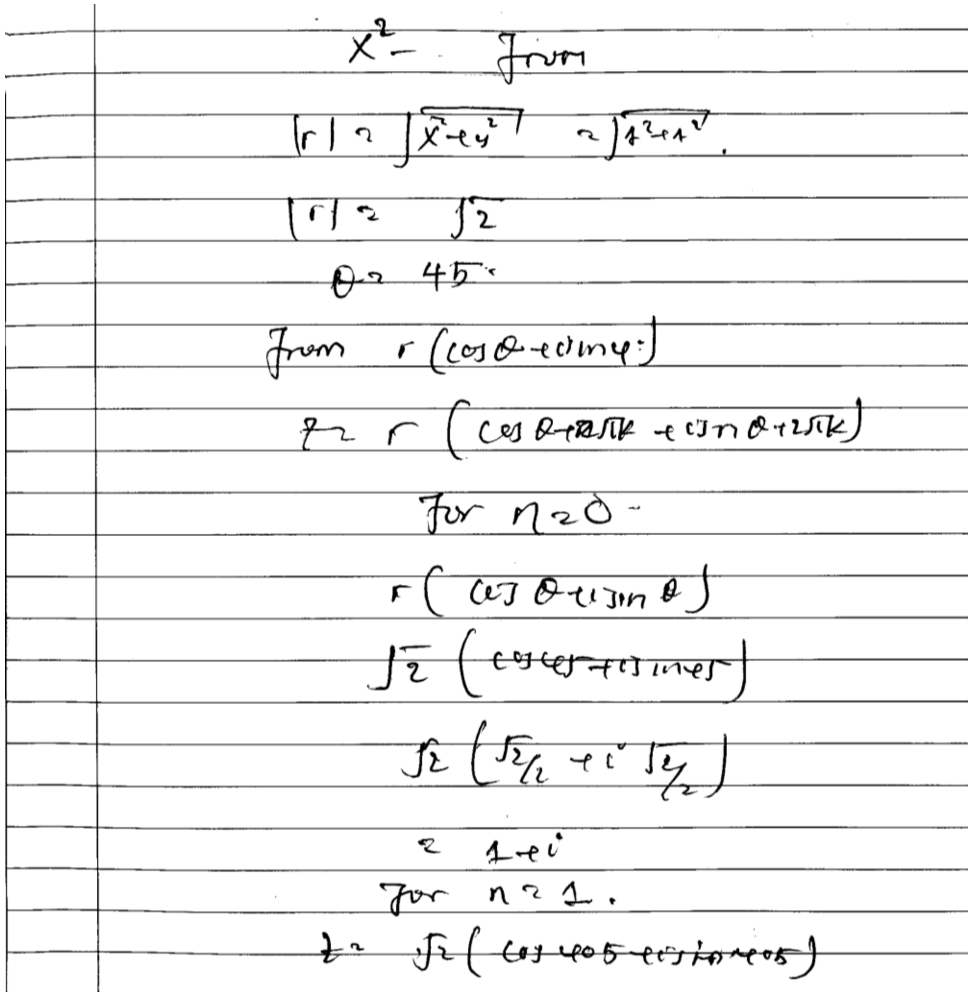
The candidates who scored low marks failed to write correct answers. The analysis of the candidates' responses shows those who scored a zero mark were unable to use De Moivre's theorem to find the the value of $(1+i)^8$ in part (a) (i), and also failed to prove the given identity by using mathematical induction in part (a) (ii). Most of them also responded wrongly in part (a) and (b), indicating that they lacked knowledge on the tested concept of Complex Numbers. Extract 11.2 shows a sample answer from one of the candidates who performed poorly in this question.

Extract 11.2



(c) $z^4+6z^3+29z^2-34z+26=0$

let $z^2 = x$


x^2 - from

$|r| = \sqrt{x^2+y^2} = \sqrt{1^2+1^2}$.

$|r| = \sqrt{2}$

$\theta = 45°$

from $r(\cos\theta + i\sin\varphi)$

$z = r(\cos\theta+2\pi k + i\sin\theta+2\pi k)$

for $n = 0$

$r(\cos\theta + i\sin\theta)$

$\sqrt{2}(\cos 45 + i\sin 45)$

$\sqrt{2}(\sqrt{2}/2 + i\sqrt{2}/2)$

$= 1+i$

for $n = 1$.

$z = \sqrt{2}(\cos 405 + i\sin 405)$

In Extract 11.2, the candidate used a wrong approach to find the roots of the given equation in part (c).

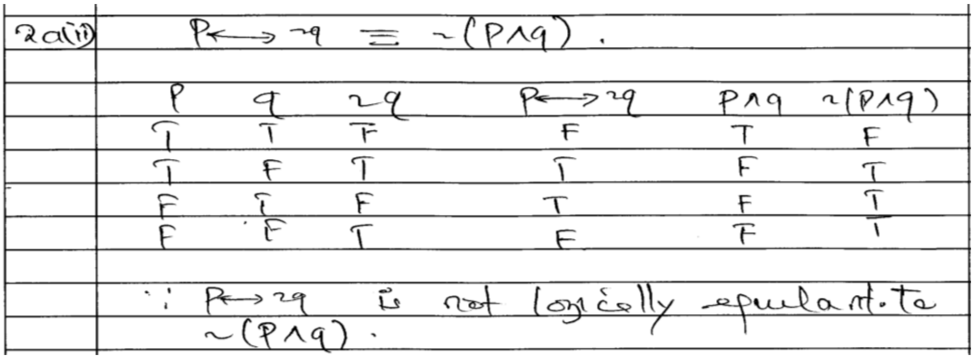
2.2.2 Question 2: Logic

This question had parts (a) and (b). In part (a) (i), the question demanded the candidate to draw the simplified electrical circuit for the argument $[p\wedge(p\vee q)]\vee[q\wedge\sim(p\wedge q)]$, while in part (a) (ii), it demanded the candidates to use a truth table to show whether or not $p\leftrightarrow\sim q$ and $\sim(p\wedge q)$ are logically equivalent. In part (b) (i), the candidates were required to use a truth table to test the validity of the following argument: "If I am intelligent, then I will pass this examination. I am intelligent. Therefore, I will pass this examination". In part b (ii), they were required to write the converse, inverse and contrapositive of the statement "If Mathematics is interesting then Biology is boring and tough".

The question was answered by all the candidates of which 74.2 percent had scores from 5.5 to 15 marks, 8.1 percent had their scores below 3 marks and 2.2 percent scored a zero mark. In this question, 267 candidates scored all the 15 marks that were allocated. The analysis has shown that question 2 was the best performed question in this examination.

The analysis of candidates' responses shows that the majority of the candidates had average performance. The few candidates (2.1%) who scored full marks were able to apply the algebraic laws for logic expressions to simplify $\left[p\wedge(p\vee q)\right]\vee\left[q\wedge(p\wedge q)\right]$ to get $p\vee q$ and then construct the required electrical circuit in part (a) (i). In part (b), the candidates managed to prepare a table of values with columns for p, q, ~q, $p\wedge q$, $\sim(p\wedge q)$ and $p\leftrightarrow\sim q$. From the table, they were able to identify that $p\leftrightarrow\sim q$ and $\sim(p\wedge q)$ do not have the same truth values and thus they are not logically equivalent, see Extract 12.1. Furthermore, they were able to test the validity of the given argument in part (b) (i). In part (b) (ii), most candidates wrote the converse, inverse and contrapositive of the given argument correctly as *If Biology is boring and tough then Mathematics is interesting*; *If Mathematics is not interesting then Biology is neither boring nor tough* and *If Biology is neither boring nor tough then Mathematics is not interesting* respectively.

Extract 12.1


2a(ii) $p\leftrightarrow\sim q \equiv \sim(p\wedge q)$.

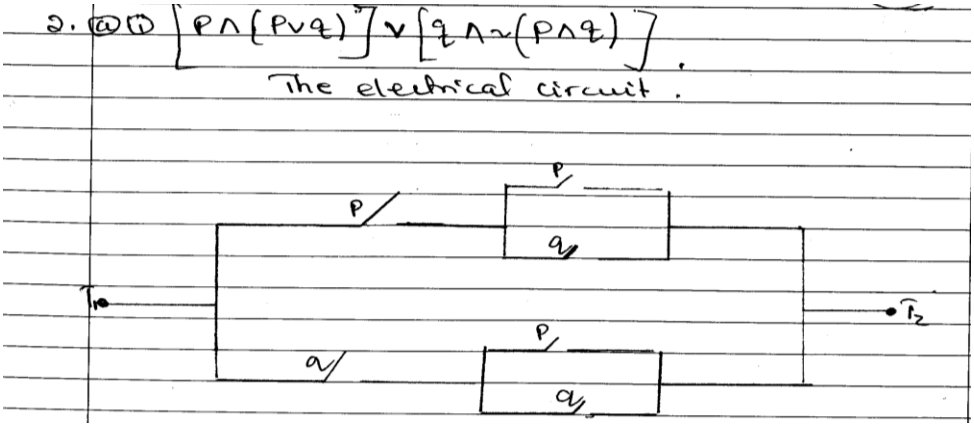
p	q	~q	p↔~q	p∧q	~(p∧q)
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	F	T

∴ $p\leftrightarrow\sim q$ is not logically equivalant to $\sim(p\wedge q)$.

In Extract 12.1, the candidate was able to apply the basic skills on the topic of logic to verify the equivalence of the given symbolic statements.

Inspite of good performance, there were a few candidates (25.8%) who scored low marks. Specifically those who scored a zero could not simplify the expression in order to draw the required electrical circuit in part (a) (i). For instance, some candidates used the argument $[p\wedge(p\vee q)]\vee[q\wedge\sim(p\wedge q)]$ directly to draw a network before simplifying it, see Extract 12.2. In part (a) (ii), they were unable to test the equivalence of logical statements using truth table. Also in part (b) (i), they were unable to symbolize the given compound statement as $[(p\to q)\wedge p]\to q$, thus failing to test its validity. In part (b) (ii), they failed to realize that to form the converse, inverse and contrapostive of the given conditional statement, they were supposed to interchange the hypothesis and conclusion; take the negation of the hypothesis and the conclusion and interchange the hypothesis and the conclusion of the inverse statement respectively, see Extract 12.3.

Extract 12.2



In Extract 12.2, the candidate drew the electrical circuit for $[p \wedge (p \vee q)] \vee [q \wedge \sim (p \wedge q)]$ instead of $p \vee q$, contrary to the requirements of the question.

Extract 12.3

ii) Soln

Let

P = Mathematics is interesting

q = Biology is boring and tough

∴ Original statement $p \to q$

Converse; $q \to p$

(Biology is boring and tough if mathematics is interesting)

Contrapositive; $\sim q \to \sim p$

(Biology is not boring and tough if mathematics is not interesting)

In Extract 12.3, the candidate failed to write the converse and contrapositive of the given conditional statement in words, showing inadequate knowledge on the tested concept.

2.2.3 Question 3.Vectors

The question comprised parts (a), (b) and (c). In part (a), the candidates were required to (i) determine the cross product $\overrightarrow{AB} \times \overrightarrow{BC}$ and (ii) find the the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$ when the position vectors; $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are defined by $\overrightarrow{OA} = 2\underline{i} - \underline{j} + 3\underline{k}$, $\overrightarrow{OB} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ and $\overrightarrow{OC} = -\underline{i} + 3\underline{j} - 2\underline{k}$ respectively. In part (b), the candidates were required to use vectors defined by $\underline{a} = 3\underline{i} + 2\underline{j} + 9\underline{k}$ and $\underline{b} = \underline{i} + \lambda \underline{j} + 3\underline{k}$ to find (i) the value of λ in surd form when $\underline{a} + \underline{b}$ is perpendicular to $\underline{a} - \underline{b}$ and (ii) find the projection of $\underline{a}$ onto $\underline{b}$. Finally in part (c), the candidates were required to derive the cosine rule using the vectors $\underline{u}$ and $\underline{v}$.

The question was answered by all the candidates of which 47.7 percent scored above 5 marks, 52.3 percent scored from 0 to 5 marks and 12.4 percent scored a zero mark. This question had average performance as 34.9 percent scored between 5.5 and 9 marks, while only 17 percent scored above 9 out of 15 marks.

The analysis of candidates' responses shows that a few candidates who scored high marks were able to: determine the vector cross product and the

angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in part (a). The candidates were also able to use the fact that $\underline{a} + \underline{b}$ is perpendicular to $\underline{a} - \underline{b}$ if and only if $(\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b}) = 0$ to get the correct value of λ in part (b) (i). In part (b) (ii), they used the formula $\dfrac{\underline{a} \cdot \underline{b}}{|\underline{b}|}$ to get the required value for the projection of $\underline{a}$ onto $\underline{b}$. Moreover, the candidates, managed to prove the cosine rule using the concept of vector. Extract 13.1 illustrates this case.

Extract 13.1

3. (a) (i) Soln

Given;

$\overrightarrow{OA} = 2i - j + 3k$

$\overrightarrow{OB} = 3i + 2j - 4k$

$\overrightarrow{OC} = -i + 3j - 2k$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = (3i + 2j - 4k) - (2i - j + 3k)$.

$= i + 3j - 7k$

$\overrightarrow{AB} = i + 3j - 7k$

Also,

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{BC} = (-i + 3j - 2k) - (3i + 2j - 4k)$

$\overrightarrow{BC} = -4i + j + 2k$

3 (a) (i) $\overrightarrow{AB} = i + 3j - 7k$

$\overrightarrow{BC} = -4i + j + 2k$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} i & j & k \\ 1 & 3 & -7 \\ -4 & 1 & 2 \end{vmatrix}$$

$= i(6 + 7) - j(2 - 28) + k(1 + 12)$

$= 13i - j(-26) + 13k$

$= 13i + 26j + 13k$

$\overrightarrow{AB} \times \overrightarrow{BC} = 13i + 26j + 13k$

(ii) Soln;

From;

$a \cdot b = |a||b| \cos\theta$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = |\overrightarrow{AB}||\overrightarrow{BC}| \cos\theta$

$|\overrightarrow{AB}| = \sqrt{x^2 + y^2 + z^2}$

$|\overrightarrow{AB}| = \sqrt{1 + 9 + 49}$

$|\overrightarrow{AB}| = \sqrt{59}$

Also $|\overrightarrow{BC}| = \sqrt{16 + 1 + 4}$

$= \sqrt{21} = \sqrt{21}$

3 (a) (ii) $\overrightarrow{AB} \cdot \overrightarrow{BC} = -4 + 3 - 14$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = 3 - 18$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = -15$

From;

$\overrightarrow{AB} \cdot \overrightarrow{BC} = |\overrightarrow{AB}||\overrightarrow{BC}| \cos\theta$

$-15 = \sqrt{59}\sqrt{21} \cos\theta$

$\frac{-15}{\sqrt{1239}} = \cos\theta$

$\theta = \cos^{-1}\left[\frac{-15}{\sqrt{1239}}\right]$

$\theta = 115.22°$

∴ The exact value of angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$ is $115.22°$

(b) (i) Soln

Given;

$\underset{\sim}{a} = 3i + 2j + 9k$

$\underset{\sim}{b} = i + \lambda j + 3k$

$\underset{\sim}{a} + \underset{\sim}{b} = 4i + (2+\lambda)j + 12k$

$\underset{\sim}{a} - \underset{\sim}{b} = 2i + (2-\lambda)j + 6k$

$(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = 0$

3 (b) (i)

$(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = 0$

$8 + (2+\lambda)(2-\lambda) + (12)(6) = 0$

$8 + 4 - \lambda^2 + 72 = 0$

$-\lambda^2 = -84$

$\lambda^2 = 84$

$\lambda = \sqrt{84}$

The value of λ is $\sqrt{84}$

(b) (ii) $\underset{\sim}{a} = 3i + 2j + 9k$

$\underset{\sim}{b} = i + \sqrt{84}\, j + 3k$

$$Proj_a^b = a \cdot \frac{b}{|\underset{\sim}{b}|}$$

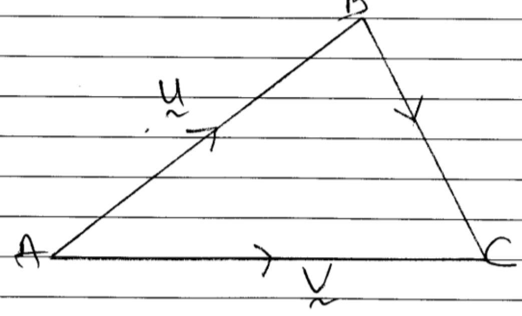
$$= (3i + 2j + 9k) \cdot \left(\frac{i + \sqrt{84}\, j + 3k}{\sqrt{1 + 84 + 9}} \right)$$

$$= (3i + 2j + 9k) \cdot \left(\frac{i + \sqrt{84}\, j + 3k}{\sqrt{94}} \right)$$

$$= \frac{3}{\sqrt{94}} + \frac{2\sqrt{84}}{\sqrt{94}} + \frac{9(3)}{\sqrt{94}}$$

$$= \frac{30 + 2\sqrt{84}}{\sqrt{94}} = \frac{\sqrt{94}\,(30 + 2\sqrt{84})}{94}$$

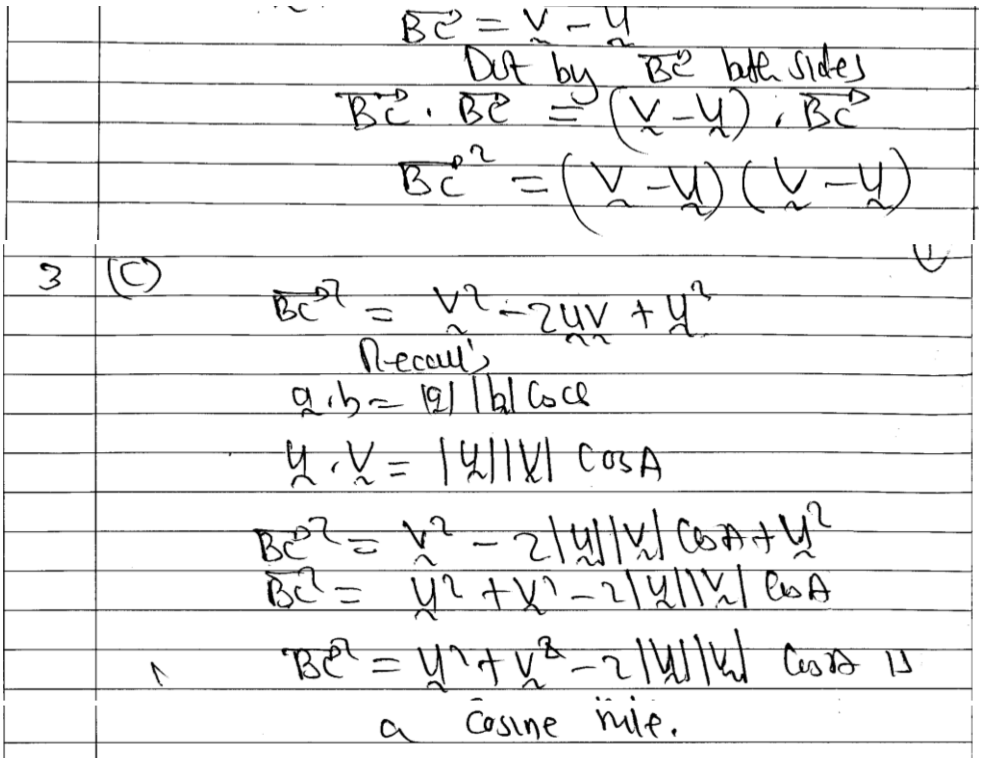
(c) Consider the Figure below



let: Vector BC = c where: c is the resultant vector ($\overrightarrow{BC}$)

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\underset{\sim}{u} + \overrightarrow{BC} = \underset{\sim}{v}$



$\overrightarrow{BC} = \underset{\sim}{v} - \underset{\sim}{u}$

Dot by $\overrightarrow{BC}$ both sides

$\overrightarrow{BC} \cdot \overrightarrow{BC} = (\underset{\sim}{v} - \underset{\sim}{u}) \cdot \overrightarrow{BC}$

$\overrightarrow{BC}^2 = (\underset{\sim}{v} - \underset{\sim}{u})(\underset{\sim}{v} - \underset{\sim}{u})$

3 (c)

$\overrightarrow{BC}^2 = \underset{\sim}{v}^2 - 2\underset{\sim}{u}\underset{\sim}{v} + \underset{\sim}{u}^2$

Recall;

$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}| \cos A$

$\overrightarrow{BC}^2 = \underset{\sim}{v}^2 - 2|\underset{\sim}{u}||\underset{\sim}{v}| \cos A + \underset{\sim}{u}^2$

$\overrightarrow{BC}^2 = \underset{\sim}{u}^2 + \underset{\sim}{v}^2 - 2|\underset{\sim}{u}||\underset{\sim}{v}| \cos A$

$\overrightarrow{BC}^2 = \underset{\sim}{u}^2 + \underset{\sim}{v}^2 - 2|\underset{\sim}{u}||\underset{\sim}{v}| \cos A$ is a cosine rule.

In Extract 13.1, the candidate was able to apply the knowledge of vectors to prove the cosine rule and answer other parts of this question.

However, the candidates who scored low marks showed the following weaknesses: In part (a) (i), most candidates managed to find correctly the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ as $\overrightarrow{AB} = \underline{i} + 3\underline{j} - 7\underline{k}$ and $\overrightarrow{BC} = -4\underline{i} + \underline{j} + 2\underline{k}$ but could not determine the cross product $\overrightarrow{AB} \times \overrightarrow{BC}$. In part (a) (ii), most of them used wrong formulae to find the angle between two vectors. The most commonly used formulae include $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}|^2 |\overrightarrow{BC}|^2}$, $\sin\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|}$ etc. Such candidates ended up getting the value of $\theta = 101.5°$ instead of $\theta = 115.2°$. Also, in part (b) (i), some of the candidates proved to have no idea that if "two vectors are perpendicular then their dot product is zero", which was a prerequisite to get the correct value of λ. For instance, some candidates used the wrong concepts such as $(\underline{a} + \underline{b}).(\underline{a} - \underline{b}) = -1$ and $\underline{a} + \underline{b} = \underline{a} - \underline{b}$ to get $\lambda = 9.2$ and $\lambda = -4$ instead of $\lambda = \pm 2\sqrt{21}$ as required, see Extract 13.2. Moreover, in part (b) (ii), it was noted that many candidates used the formula $\dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}|}$ for the vector projection of a vector

$\underset{\sim}{b}$ onto a vector $\underline{a}$ contrary to the requirements of the question. Finally, in part (c), they failed to use vectors $\underline{u}$ and $\underline{v}$ to derive the cosine rule $w^2 = u^2 + v^2 - 2uv\cos\theta$.

Extract 13.2

3b i) Solution:

$\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 9\underset{\sim}{k} = [3, 2, 9]$

$\underset{\sim}{b} = \underset{\sim}{i} + \lambda\underset{\sim}{j} + 3\underset{\sim}{k}. = [1, \lambda, 3]$

$\therefore \underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{a} - \underset{\sim}{b}.$

$\Leftrightarrow [3, 2, 9] + [1, \lambda, 3] = [3, 2, 9] - [1, \lambda, 3]$

$\Leftrightarrow [3+1, 2+\lambda, 9+3] = [3-1, 2-\lambda, 9-3]$

$\Leftrightarrow [4, 2+\lambda, 12] = [2, 2-\lambda, 6]$

$\Leftrightarrow 4+2+\lambda+12 = 2+2-\lambda+6$

$\Leftrightarrow 18+\lambda = 4-\lambda+6$

$\Leftrightarrow 18+\lambda = 10-\lambda$

$\Leftrightarrow 18-10 = -\lambda-\lambda$

$\Leftrightarrow 8 = -2\lambda$

$\Rightarrow \lambda = -4$

In Extract 13.2, the candidate failed to realize that if the given vectors are perpendicular, then $(\underline{a}+\underline{b}).(\underline{a}-\underline{b}) = 0$, showing lack of knowledge on the tested concept.

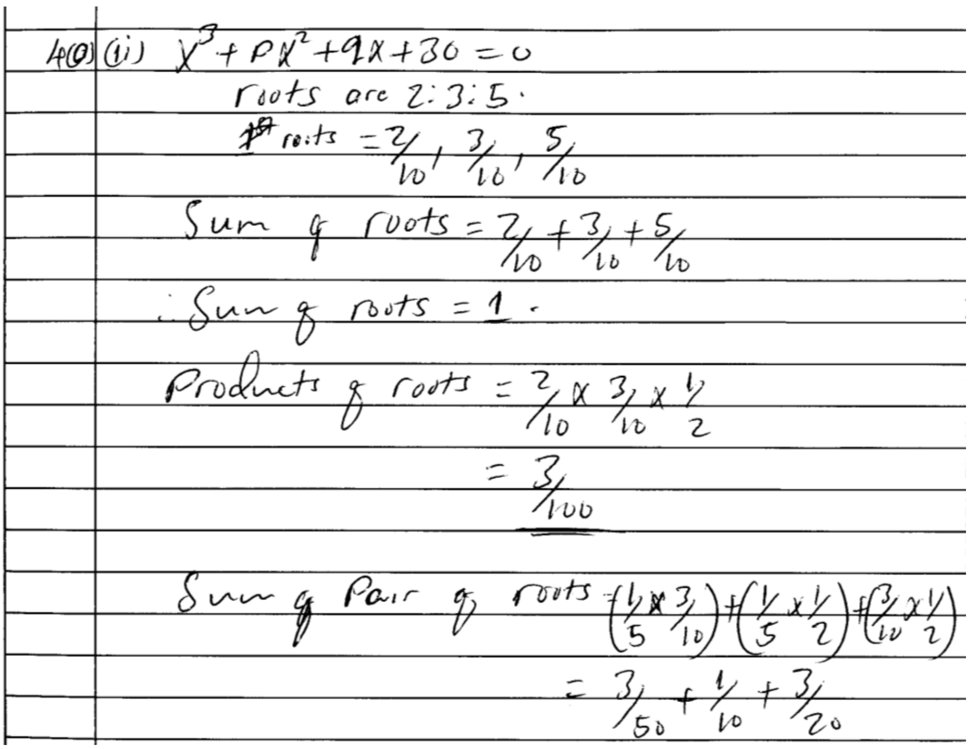
2.2.4 Question 4: Algebra

The question had parts (a) and (b). In part (a), the candidates were required to (i) find the value of *a* if the 17th and 18th terms of the expansion $(2+a)^{50}$ are equal, while in part (ii) they were required to find the value of *p* and *q* when the roots of the equation $x^3 + px^2 + qx + 30 = 0$ are in the ratio 2:3:5. In part (b), the candidates were required to (i) state the principle of Mathematical Induction and (ii) use the principle of Mathematical Induction to prove that $\sum_{r=1}^{n} 3r - 1 = \frac{n}{2}(3n+1)$.

This question was attempted by 12,790 (100%) candidates whereas only 9.5 percent had their scores above 5 marks. It was the worst performed in this examination because 90.5 percent scored below 5.5 marks and among them 39.9 percent of candidates scored zero.

The candidates who scored low marks in this question were unable to recall the general formula $t_{r+1} = {}^nC_r x^{n-r} y^r$ for $(r+1)^{th}$ term of the expansion $(x+y)^n$ which could have been used to solve for a when t_{17} and t_{18} are equal in part (a) (i). It was noted that many candidates did not attempt part (a) (ii). Some of the candidates who answered this part were unable to use the ratio 2:3:5 to establish the relationship between the coefficients of the given polynomial equation and cubic equation, whose roots are α, β and γ i.e $x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\alpha\gamma)x - \alpha\beta\gamma = 0$. The analysis done in their script shows that many of them considered the given ratio as the roots of the polynomial $x^3 + px^2 + qx + 30 = 0$, which resulted in getting wrong values for p and q, see Extract 14.1. Moreover, in part (b) many candidates failed to use the principle of mathematical induction to show that if $\sum_{r=1}^{n} 3r - 1 = \frac{n}{2}(3n+1)$ is true for $n=1$ then the assumption is true for $n=k$ and $n=k+1$.

Extract 14.1



4(a) (ii) $x^3 + px^2 + qx + 30 = 0$

roots are 2:3:5.

roots $= \frac{2}{10}, \frac{3}{10}, \frac{5}{10}$

Sum of roots $= \frac{2}{10} + \frac{3}{10} + \frac{5}{10}$

∴ Sum of roots $= 1$.

Products of roots $= \frac{2}{10} \times \frac{3}{10} \times \frac{1}{2}$

$= \frac{3}{100}$

Sum of Pair of roots $= \left(\frac{1}{5} \times \frac{3}{10}\right) + \left(\frac{1}{5} \times \frac{1}{2}\right) + \left(\frac{3}{10} \times \frac{1}{2}\right)$

$= \frac{3}{50} + \frac{1}{10} + \frac{3}{20}$

$= 31/100$

from the equation ~~$x^3 - (\alpha\beta+\alpha\gamma+\beta\gamma)x^2 +$~~

$x^2 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\alpha\gamma+\beta\gamma)x - \alpha\beta\gamma = 0$

$-p = 1$

$p = -1.$

$q = 31/20.$

In Extract 14.1, the candidate failed to answer question 4 (a), indicating lack of knowledge on how to determine the values of p and q.

Despite the weaknesses shown, there were 265 (2.1%) candidates who scored high marks. Such candidates applied correctly the concepts of Algebra in answering this question. A sample answer from one of those candidates is shown in Extract 14.2

Extract 14.2

04. (a) (i) given

$(2+a)^{50}$

From :.

term, $U_{r+1} = {}^{n}C_r\, a^{n-r}\, b^{r}.$

For

$17 = r+1 \Rightarrow r = 16$

$18 = r+1 \Rightarrow r = 17.$

$U_{17} = {}^{50}C_{16} \cdot (2)^{50-16} (a)^{16}$

$U_{18} = {}^{50}C_{17} \cdot (2)^{50-17} (a)^{17}.$

but.

$U_{17} = U_{18}$

${}^{50}C_{16} \cdot 2^{34} \cdot a^{16} = {}^{50}C_{17} \cdot 2^{33} \cdot a^{17}.$

$a = \frac{{}^{50}C_{16} \cdot 2^{34}}{{}^{50}C_{17} \cdot 2^{33}}$

$a = 2 \times \frac{1}{2}$

$a = 1$

Hence;

The value of $a = 1$

(ii) For the equation $x^3 + px^2 + qx + 30 = 0$

$\alpha : \beta : \gamma = 2 : 3 : 5$

$\alpha/\beta = 2/3$ and $\beta/\gamma = 3/5$

04. (a)(ii)

$$\begin{cases} \alpha + \beta + \gamma = -p \\ \alpha\beta + \alpha\gamma + \beta\gamma = q \\ \alpha\beta\gamma = -30 \end{cases}$$ From the cubic equ given

now: $\frac{\alpha}{\beta} = \frac{2}{3} \Rightarrow \alpha = \frac{2}{3}\beta$

$\frac{\beta}{\gamma} = \frac{3}{5} \Rightarrow \gamma = \frac{5}{3}\beta$

From:

$\Rightarrow \alpha + \beta + \gamma = -p$

$\frac{2}{3}\beta + \beta + \frac{5}{3}\beta = -p$

$\frac{10}{3}\beta = -p$

$\beta = -\frac{3}{10}p$ ---- (i)

$\Rightarrow$ Again:

$\alpha\beta\gamma = -30$

$\left(\frac{2}{3}\beta\right)(\beta)\left(\frac{5}{3}\beta\right) = -30$

$\frac{10}{9}\beta^3 = -30$

$\beta^3 = -27$

$\beta^3 = (-3)^3$

$\beta = -3$

But

$\beta = -\frac{3}{10} p$

$p = -\frac{10}{3} \times -3 = 10$

04. (a) (ii) $\beta\left(\frac{2}{3}\beta\right) + \left(\frac{2}{3}\beta\right)\left(\frac{5}{3}\beta\right) + \beta\left(\frac{5}{3}\beta\right) = q$

$q = \beta^2\left[\frac{2}{3} + \frac{10}{9} + \frac{5}{3}\right]$

$q = \frac{31}{9} \times \beta^2$ but $\beta = -3$

$q = 31$

Hence;

The value of $p = 10$ and $q = 31$.

Extract 14.2 shows that the candidate had adequate knowledge on the concepts of finding coefficients of the terms in the expansion $(x+y)^n$ and roots of cubic equation.

2.2.5 Question 5: Trigonometry

The question had parts (a), (b), (c) and (d). In part (a), the candidates were required to (i) solve the equation $\tan^{-1}\left(\frac{x-1}{x+2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{2}$, leaving the answer in surd form and (ii) prove the identity $\frac{1+\sin x}{1-\sin x} \cong (\tan x + \sec x)^2$. In part (b), the candidates were required to use t-formulae to find the value of θ when $2\sin\theta + \cos\theta = 1$ in the interval $0^\circ \le \theta \le 180^\circ$, while in part (c) they were required to (i) show that $\frac{\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot\left(\frac{5\theta}{2}\right)$ and (ii) verify that $\frac{\sin(A+B+C) + \sin(A-B-C)}{\cos(A+B+C) - \cos(A-B-C)} = \frac{\tan B \tan C - 1}{\tan B + \tan C}$. Finally, part (d) required the candidates to express $3\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, giving values of R and α.

A total of 9,223 (72.1%) candidates responded to this question, out of which 34.6 percent scored from 7 to 12 marks and 40.7 percent scored from 12.5 to 20 marks. The analysis has shown that this question was best performed in this examination because 72.3 percent scored from 7 to 20 marks.

Despite the fact that the question was optional, the majority of the candidates attempted it. Many of those who attempted it did reasonably well. In part (a) (i), it was clear from the scripts that the candidates were able to replace $\tan^{-1}\left(\frac{x-1}{x+2}\right)$ and $\tan^{-1}\left(\frac{x+1}{x+2}\right)$ with A and B respectively to get $\frac{\tan A+\tan B}{1-\tan A\tan B}=1$. Then, they substituted $\frac{x-1}{x+2}$ for $\tan A$ and $\frac{x+1}{x+2}$ for $\tan B$ to get the equation $2x^2=5$ that was solved to obtain $x=\sqrt{\frac{5}{2}}$. In part (a) (ii), the candidates who proved the given identity from the right hand side to the left hand side, substituted $\frac{\sin x}{\cos x}$ for $\tan x$ and $\frac{1}{\cos x}$ for $\sec x$ in $(\tan x+\sec x)^2$ and then simplified the resulting equation to get $\frac{1+\sin x}{1-\sin x}$ as required. For those who proved it from the left hand side to the right hand side, they were able to rationalize the denominator of $\frac{1+\sin x}{1-\sin x}$ and then replace $1-\sin^2 x$ with $\cos^2 x$ to obtain $(\tan x+\sec x)^2$. In part (b), the candidates were also able to use the t-formulae ($\sin\theta=\frac{2t}{1+t^2}$ and $\cos\theta=\frac{1-t^2}{1+t^2}$) correctly to change the equation $2\sin\theta+\cos\theta=1$ into $2t^2-4t=0$, which was then solved to obtain the values of θ as $0°$ and $126.8°$. Moreover, a number of candidates were able to use the factor formulae for sine and cosine to prove the trigonometric identites in part (c) (i) and (ii) as required, see Extract 15.1. Finally, in part (d), most candidates applied the compound angle formula ($\sin(\theta-\alpha)=\cos\alpha\sin\theta-\cos\theta\sin\alpha$) to express $3\sin\theta-4\cos\theta$ as $5\sin(\theta-53.1°)$ correctly.

Extract 15.1

5 c) i) $\frac{\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot\left(\frac{5\theta}{2}\right)$

Soln

Consider L.H.S

$= \frac{\cos\theta + \cos 4\theta + \cos 2\theta + \cos 3\theta}{\sin\theta + \sin 4\theta + \sin 2\theta + \sin 3\theta}$

$= $ ~~$2\cos 5\theta/2$~~ By factor formula.

$= \frac{2\cos 5\theta/2 \cos 3\theta/2 + 2\cos 5\theta/2 \cos \theta/2}{2\sin 5\theta/2 \cos 3\theta/2 + 2\sin 5\theta/2 \cos \theta/2}$

5 c) i) $= \frac{2\cos 5\theta/2 \left(\cos 3\theta/2 + \cos \theta/2\right)}{2\sin 5\theta/2 \left(\cos 3\theta/2 + \cos \theta/2\right)}$

$= \frac{\cos 5\theta/2}{\sin 5\theta/2}$

$= \cot 5\theta/2$

Hence Shown L.H.S = R.H.S

5 c) ii) $\frac{\sin(A+B+C) + \sin(A-B-C)}{\cos(A+B+C) - \cos(A-B-C)} = \frac{\tan B \tan C - 1}{\tan B + \tan C}$

Consider L.H.S

$= \frac{\sin(A+B+C) + \sin(A-B-C)}{\cos(A+B+C) - \cos(A-B-C)}$

By factor formula

$$= \frac{2\sin\left(\frac{A+B+C+A-B-C}{2}\right)\cos\left(\frac{A+B+C-(A-B-C)}{2}\right)}{-2\sin\left(\frac{A+B+C+A-B-C}{2}\right)\sin\left(\frac{A+B+C-(A-B-C)}{2}\right)}$$

$$= \frac{2\sin A\cos(B+C)}{-2\sin A\sin(B+C)}$$

5 c) ii)

$$= \frac{2\sin A\cos(B+C)}{-2\sin A\sin(B+C)}$$

$$= \frac{\cos(B+C)}{-\sin(B+C)}$$

$$= \frac{\cos B\cos C + \sin B\sin C}{-(\sin B\cos C + \sin C\cos B)}$$

$$= \frac{\sin B\sin C - \cos B\cos C}{\sin B\cos C + \sin C\cos B}$$

Divide by $\cos B\cos C$ throughout each term in numerator and denominator

$$= \frac{\frac{\sin B\sin C}{\cos B\cos C} - \frac{\cos B\cos C}{\cos B\cos C}}{\frac{\sin B\cos C}{\cos B\cos C} + \frac{\sin C\cos B}{\cos B\cos C}}$$

$$= \frac{\tan B\tan C - 1}{\tan B + \tan C}$$

Hence verified L.H.S = R.H.S.

Extract 15.1 shows the solution of a candidate who managed to answer part (c) (i) and (ii) correctly.

Further analysis of the responses from the candidates shows that a few candidates who scored low marks failed to recall the formula for sum of inverse tangent function, t-formulae, factor formulae and the compound angle formulae correctly which were the prerequisite in answering the question. For example, the candidates who failed to show that $\frac{\cos\theta+\cos2\theta+\cos3\theta+\cos4\theta}{\sin\theta+\sin2\theta+\sin3\theta+\sin4\theta}=\cot\left(\frac{5\theta}{2}\right)$, had no skills on how to convert expressions such as $\sin A+\sin B$ and $\cos A+\cos B$ into $2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, see Extract 15.2.

Extract 15.2

(C)(ii) $\frac{\sin(A+B+C)+\sin(A-B-C)}{\cos(A+B+C)-\cos(A-B-C)} = \frac{\tan B\tan C-1}{\tan B+\tan C}$

Consider the Left hand side:

From;

$A+B+C=180$

$\Rightarrow A=180-B-C$

$\Rightarrow \frac{\sin 180+\sin(180-B-C-B-C)}{\cos 180-\cos(180-B-C-B-C)}$

$= \frac{0+\sin(180-2B-2C)}{-1-\cos(180-2B-2C)}$

$= \frac{\sin 2(90-B-C)}{-1-\cos 2(90-B-C)}$

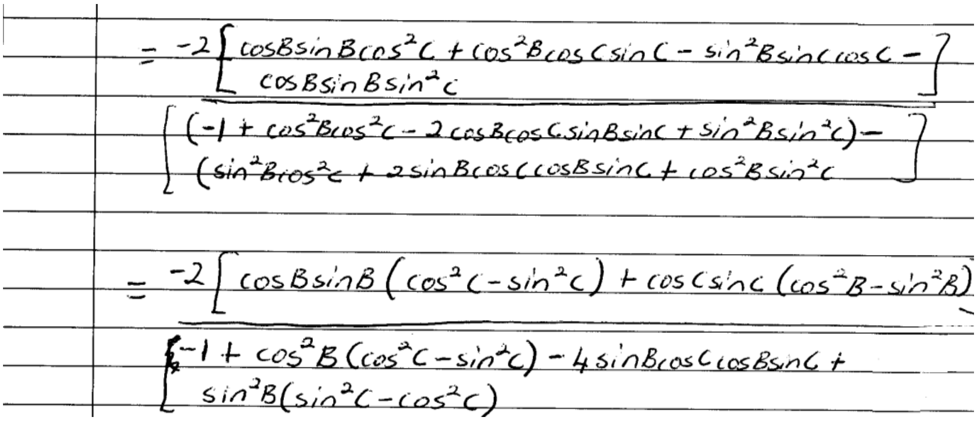
5 (C)(ii) $= \frac{2\sin(90-B-C)\cos(90-B-C)}{-1-(\cos^2(90-B-C)-\sin^2(90-B-C))}$

$= \frac{-2\cos(B+C)\sin(B+C)}{-1-(\sin(B+C))^2+(\cos(B+C))^2}$

$= \frac{-2\cos(B+C)\sin(B+C)}{-1-\sin^2(B+C)+\cos^2(B+C)}$

$= \frac{-2\cos(B+C)\sin(B+C)}{-1+(\cos^2(B+C)-\sin^2(B+C))}$

$= \frac{-2[(\cos B\cos C-\sin B\sin C)(\sin B\cos C+\cos B\sin C)}{-1+(\cos B\cos C-\sin B\sin C)^2-(\sin B\cos C+\cos B\sin C)^2}$



In Extract 15.2, the candidate failed to use the factor formula to verify the given trigonometric identity in part (c) (ii).

2.2.6 Question 6: Probability

This question consisted of parts (a), (b), (c) and (d). In part (a), the candidates were required to use the given information that "the probability that a keyboard picked at random from the assembly line in a factory to be defective is 0.01, whereas a sample of three is to be selected" and then asked to (i) construct the probability distribution of the defective keyboards and in (ii) find the mean and standard deviation (leaving the answers correct to 2 decimal places). In part (b), the candidates were provided with the information that "the bag R contains 5 red and 3 green balls and bag P contains 3 red and 5 green balls with the condition that one ball is drawn from bag R and two from bag P". Then, the candidates were supposed to find the probability that out of the three balls drawn, two are red and one is green. In part (c), the candidates were provided with the information that "the random variable X has a probability distribution $P(x)$ and k is a certain number such that $P(X)=\begin{cases} k & \text{if } x=0 \\ 2k & \text{if } x=1 \\ 3k & \text{if } x=2 \\ 0 & \text{otherwise} \end{cases}$

Then, from the given information, the candidates were required to (i) determine the value of k and (ii) find $P(x<2)$, $P(x\le 2)$ and $P(x\ge 2)$.

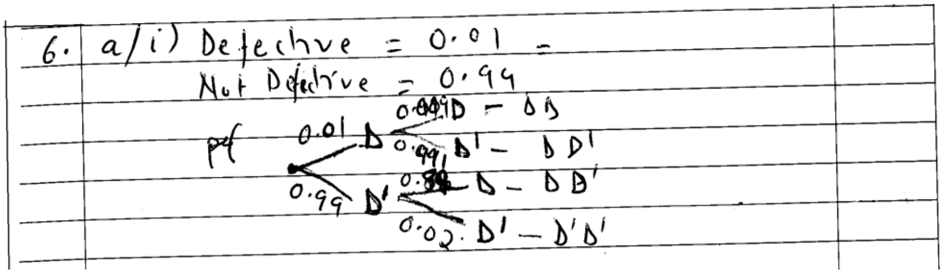
Finally, in part (d) the candidates were given the information that if X is a discrete random variable, where $E(X)$ is the expected value of X, they were required to (i) show that $E(Ax+b)=AE(x)+b$, where A and b

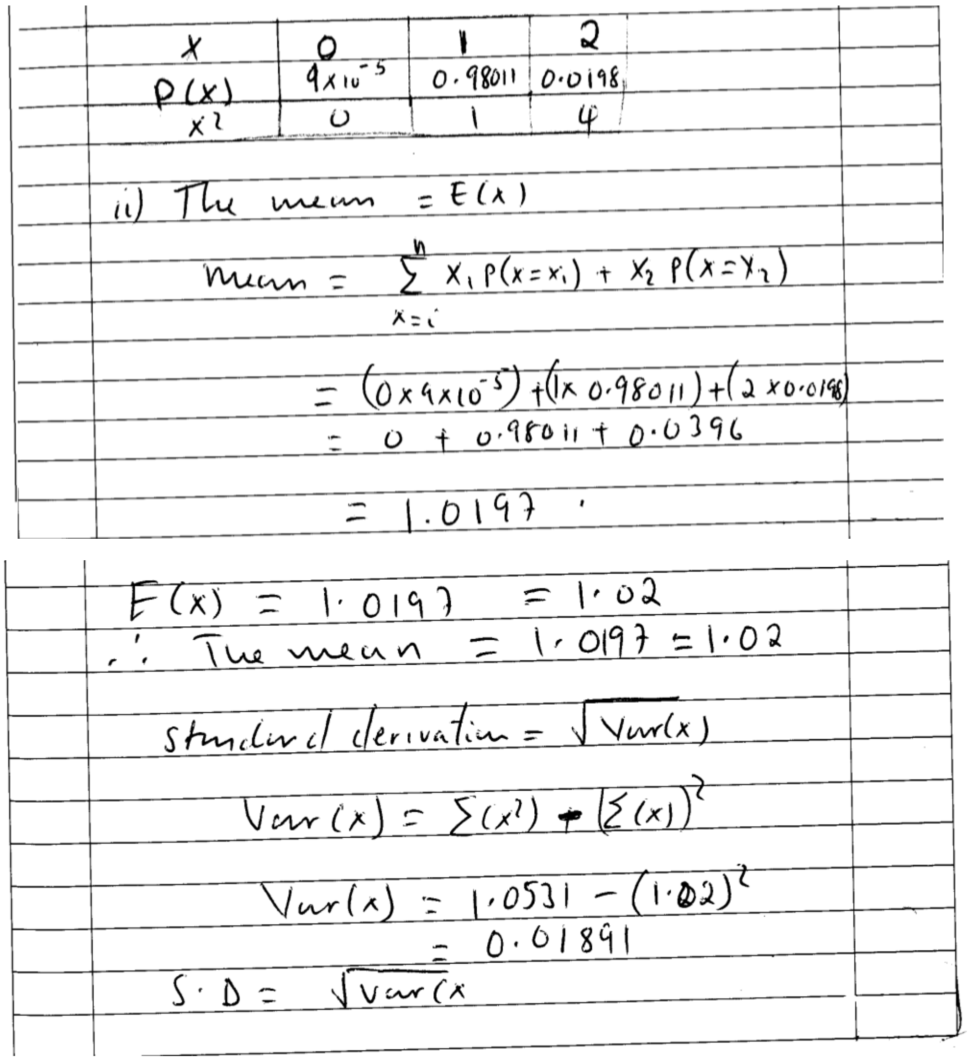
are constants. Then, part (d) (ii) required the candidates to find the probability of at most 5 seeds germinating if given the modern seeds of a certain crop, having the probability of germinating of 0.9 and if only six seeds are sown.

The question was attempted by 1,666 (13 %) candidates, out of which only 202 (12.1%) were able to score from 7 to 20 marks. The analysis shows that 9.8 percent scored from 7.0 to 11.5, while 87.9 percent of the candidates scored from 0 to 6.5. Also the analysis indicates that 380 (22.8%) candidates scored a zero mark, whereas only 1.9 percent scored from 12 to 20 marks in this question. Therefore, the general performance of the question is poor.

A good number of candidates (87.9%) out of those who chose this question performed poorly. They failed because they were unable to construct the probability distribution table, use binomial distribution, and to apply basic concepts in probability to prove and find mean and standard deviation. For instance, in part a(i) there were candidates who used the tree diagram method which was wrong because that method could not account for the the number of trials X = 0, 1, 2 or 3. At the same time, even the computation for P(X) did not involve the binomial distribution as the necessary strategy in solving part (a) (i). Also, in part (a) (ii) they used wrong formulae to find the mean and standard deviation. For example, some used the formula $\text{Mean} = \sum_{x=i}^{n} x_1 p(x = x_1) + x_2 p(x = x_2)$ instead of $\text{Mean} = \text{np}$ and to get the standard deviation, they used the formula $\sigma = \sqrt{E(x^2) - [E(x)]^2}$ instead of $\sigma = \sqrt{npq}$. Also, many struggled to use wrong notion of Continuous Probability Distribution instead of Discrete Random Variable. Extract 16.1 illustrates the case.

Extract 16.1





x	0	1	2
$P(x)$	9×10^{-5}	0.98011	0.0198
x^2	0	1	4

ii) The mean $= E(x)$

Mean $= \sum_{x=i}^{n} x_1 P(x=x_1) + x_2 P(x=x_2)$

$= (0\times9\times10^{-5}) + (1\times0.98011) + (2\times0.0198)$

$= 0 + 0.98011 + 0.0396$

$= 1.0197$

$E(x) = 1.0197 = 1.02$

$\therefore$ The mean $= 1.0197 \approx 1.02$

standard deviation $= \sqrt{Var(x)}$

$Var(x) = \sum(x^2) + [\sum(x)]^2$

$Var(x) = 1.0531 - (1.02)^2$

$= 0.01891$

$S.D = \sqrt{Var(x}$

In Extract 16.1, the candidate failed to compute the required mean and standard deviation, indicating lack of knowledge on the tested concept of probability distributions.

On the other hand, a few candidates (12.1%) managed to score high marks, from 7.0 to 20.0 and only one candidate managed to score full marks. Therefore, basing on the analysis, it implies that very few candidates had knowledge to answer correctly, this question from the topic of probability. The few candidates, who performed well, were able to construct the probability distribution table. Also, they used the correct formula $\text{Mean} = \text{np}$ and standard deviation $\sigma = \sqrt{npq}$ as required. They used the concept of tree diagram to solve part (b) of the question and managed to determine the value of k in part (c) from $k + 2k + 3k = 1$, imply that $k = \frac{1}{6}$

of which they used it to come up with the required probability. The definition of expectation and the Binomial Distribution formulae were both used correctly to solve part d (i) and d (ii) respectively. Extract 16.2 illustrates the case.

Extract 16.2

6 (a)(i)

X	0	1	2	3
$P(X=x)$	0.970	0.029	2.97×10^{-4}	1×10^{-6}

$P(X=x) = {}^nC_x p^x q^{n-x}$.

$p = 0.01$, $q = 0.99$.

(ii) Soln

$p = 0.01$ $q = 1 - 0.01 = 0.99$.
$n = 3$

Mean $= np = 0.01 \times 3 = 0.03$

$= 0.017$.

Standard deviation $= \sqrt{npq}$

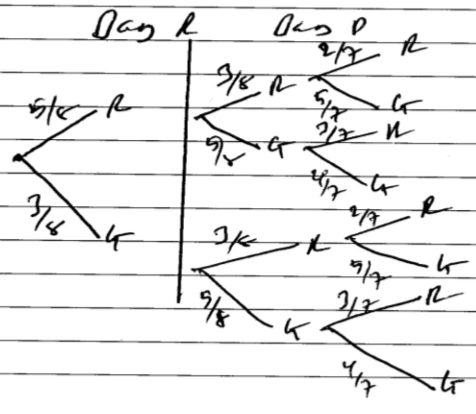
$= \sqrt{3 \times 0.01 \times 0.99}$

Standard deviation $= 0.172 \approx 0.17$.

6 (b). Soln

Bag R: 5 − red, 3 − Green

Bag P: 3 − red, 5 − Green.



Probability that two are Red and one is green
$R_1R_2G_1$, $R_1G_2R_2$, $G_1R_1R_2$

$$\frac{5}{8}\times\frac{3}{8}\times\frac{5}{7}+\frac{5}{8}\times\frac{7}{8}\times\frac{3}{7}+\frac{7}{8}\times\frac{3}{8}\times\frac{2}{7}$$

$$=\frac{75}{448}+\frac{75}{448}+\frac{18}{448}=\frac{75+75+18}{448}$$

$$=\frac{168}{448}=\frac{3}{8}$$

Probability that two are Red and one is green 5 $\frac{3}{8}$

6 (c) Soln

$$P(x)=\begin{cases}K & \text{if } X=0\\ 2K & \text{if } X=1\\ 3K & \text{if } X=2\\ 0 & \text{otherwise}\end{cases}$$

(i) The value of K

$P(x)=K$ if $X=0$
$P(x)=2K$ if $X=1$
$P(x)=3K$ if $X=2$

Take $K+2K+3K=1$

$6K=1$

$K=\frac{1}{6}$.

$$P(x)=\begin{cases}\frac{1}{6} & \text{if } X=0\\ \frac{1}{3} & \text{if } X=1\\ \frac{1}{2} & \text{if } X=2\end{cases}$$

(ii) $P(X<2)=\frac{1}{6}+\frac{1}{3}$

$=\frac{1+2}{6}=\frac{3}{6}=\frac{1}{2}$

$P(X<2)=\frac{1}{2}$

6 (c) (ii)

$$P(X \leq 2) = \tfrac{1}{6} + \tfrac{1}{3} + \tfrac{1}{2}$$

$$P(X \leq 2) = 1$$

$$P(X \geq 2) = \tfrac{1}{2}$$

(d) (i) Soln

$$E(aX + b) = \sum (ax+b) P(x)$$

$$= \sum ax P(x) + \sum b P(x)$$

$$= a \sum x P(x) + b \sum P(x)$$

but $\sum P(x) = 1$

$$= a E(X) + b(1)$$

$$= a E(x) + b$$

(ii) Soln

Probability of germinating; $p = 0.9$

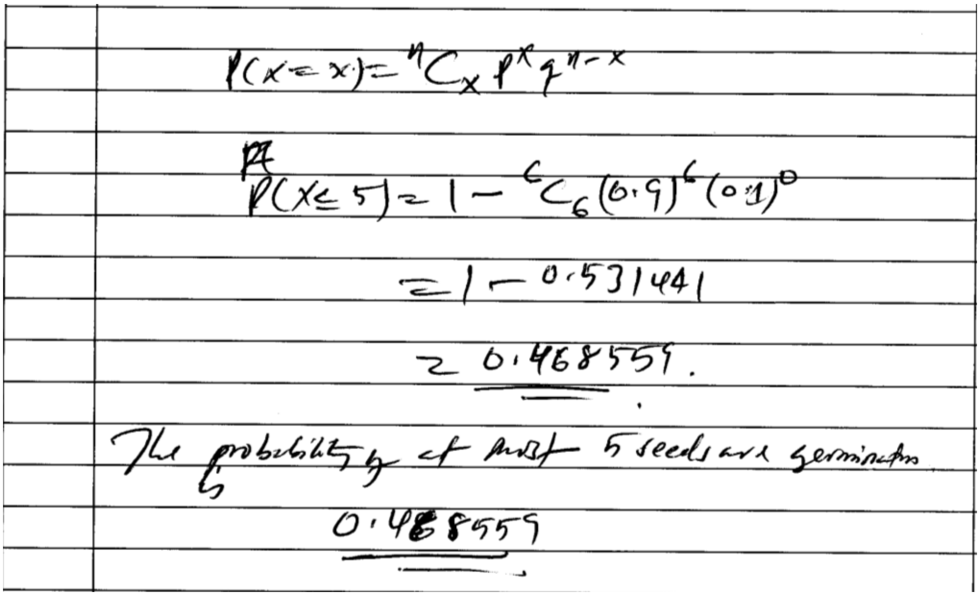
$q = 1 - 0.9 = 0.1$

6 (d) (ii). Soln

$n = 6$.

Required $P(X \leq 5)$.

$$P(X \leq 5) = 1 - P(X = 6)$$


$P(X=x) = {}^{n}C_x\, p^x q^{n-x}$

$P(X \le 5) = 1 - {}^{6}C_6 (0.9)^6 (0.1)^0$

$= 1 - 0.531441$

$= 0.468559.$

The probability of at most 5 seeds are germination is 0.468559

In Extract 16.2, the candidate demonstrated good understanding of the basic concepts of probability and used them correctly in answering this question.

2.2.7 Question 7: Differential Equations

This question consisted of parts (a), (b), (c) and (d). In part (a), the candidates were required to (i) show that $x^2 ye^{-y} = 2$ when $x(1-y)\frac{dy}{dx} + 2y = 0$ and $y = 2$ when $x = e$ and (ii) solve the differential $(2x - y)\frac{dy}{dx} = 2x - y + 2$ when $y = 1$ and $x = 2$. Part (b) required the candidates to form a differential equation whose solution is the function $y = Ae^{2x} + Be^{-3x}$ where A and B are arbitrary constants. In part (c), the candidates were given “A tank which initially contains 1000 litres of water with 10 kg of salt dissolved in it. The mixture was poured off at a rate of 20 litres per minute, and simultaneously pure water was added at a rate of 20 litres per minute. All the time the tank was stirred to keep the mixture uniform”. From this information they were required to find (i) the mass of the salt in the tank after 5 minutes and (ii) the time in which the mass in the tank falls to 5 kg. Finally, in part (d) the candidates were required to find the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 10e^{-2x}$.

A total of 7,426 (58.1%) candidates responded to this question out of which 41.6 percent scored from 7 to 11.5 marks and 16.4 percent scored above 12 out of 20 marks. Therefore, basing on these data the question had an average performance.

The analysis of candidates' responses shows that many of those who opted for this question showed the following strengths: In part (a) (i), the candidates were able to re-arrange the differential equation $x(1-y)\frac{dy}{dx}+2y=0$ into the form $\frac{1-y}{y}dy=\frac{-2}{x}dx$ that was integrated to obtain the required solution of $x^2ye^{-y}=2$ when $y=2$ and $x=e$. In part (a) (ii), the candidates used the substitution $z=2x-y$ to transform the differential equation $(2x-y)\frac{dy}{dx}=2x-y+2$ into a separable equation $\frac{z}{z-2}dz=dx$. Thereafter they evaluated the integral $\int\frac{z}{z-2}dz=\int dx$ to obtain the general solution $2x-y+2+2\ln(2x-y-2)=x+c$. Finally, they substituted $x=2$ and $y=1$ to find the constant c and thus ended up with the solution $2\ln(2x-y-2)=y-x+1$ as required. In part (b), most candidates differentiated the function $y=Ae^{2x}+Be^{-3x}$ twice to get $\frac{dy}{dx}=2Ae^{2x}-3Be^{-3x}$ and $\frac{d^2y}{dx^2}=4Ae^{2x}+9Be^{-3x}$ respectively, which were then solved simultaneously to form the equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$ as required. Moreover, in part (c), many candidates represented the given information into a differential equation $\frac{dx}{dt}=-kx$ that was solved to find the amount of salt which remained and the time spent for the mass of salt to fall to 5 kg. Finally, the candidates were able to find the general solution $y=Ae^{x}+Be^{3x}+\frac{2}{3}e^{-2x}$ in part (d) by using the auxiliary equation $m^2-4m+3=0$ to get the complementary function and $y=pe^{-2x}$ to get the particular solution. Extract 17.1 is a sample answer from one of the candidates showing how they answered correctly part (d) of this question.

Extract 17.1

(d) given:

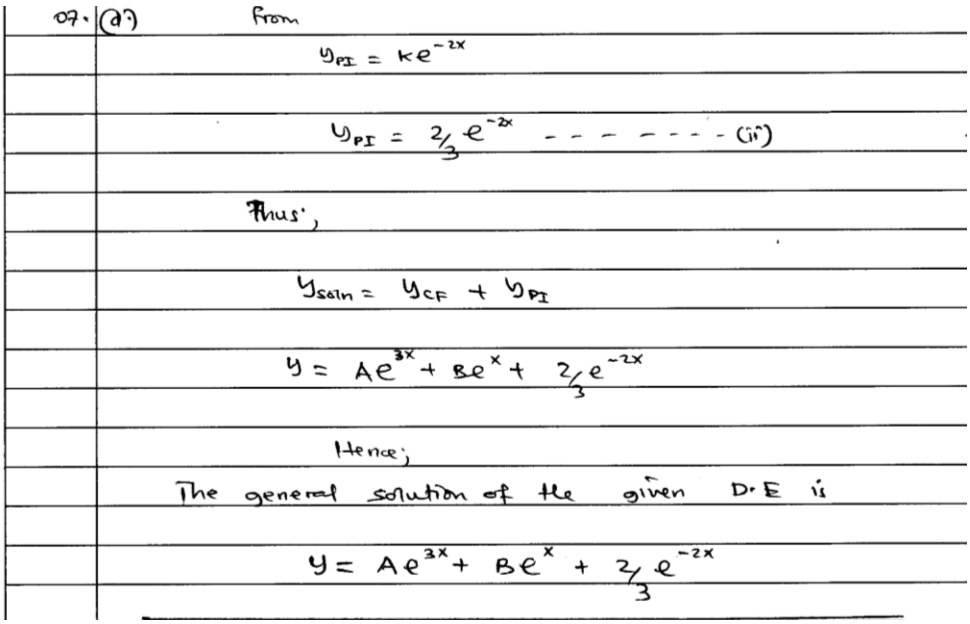
$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=10e^{-2x}$$

For C.F,

taking the a.q.e.

$$m^2-4m+3=0$$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$


07. (a) From

$y_{PI} = ke^{-2x}$

$y_{PI} = \frac{2}{3}e^{-2x}$ - - - - - - - (ii)

Thus,

$y_{soln} = y_{CF} + y_{PI}$

$y = Ae^{3x} + Be^{x} + \frac{2}{3}e^{-2x}$

Hence;

The general solution of the given D.E is

$y = Ae^{3x} + Be^{x} + \frac{2}{3}e^{-2x}$

In Extract 17.1, the candidate worked out correctly the complementary function and the particular solution and added them together to obtain the required general solution.

However, there were several candidates (42%) who scored low marks. The candidates were unable to use the knowledge and skills of differential equations to solve the first order differential equation $x(1-y)\frac{dy}{dx}+2y=0$ and non-homogeneous differential equation $(2x-y)\frac{dy}{dx}=2x-y+2$. The analysis of the candidates' responses shows the following were the most frequent weaknesses:

- Using inappropriate integrating factor $e^{\int 2y\,dy}$ instead of re-arranging the given ordinary differential equation into the variable separable equation $\frac{dy}{dx}=f(x)g(y)$ in part (a) (i);
- Using the incorrect substitution $y=vx$ instead of the obvious substitution $z=2x-y$ in part (a) (ii);
- Failure to eliminate the constants A and B from a solution of differential equation which was an important step to formulate a differential equation in part (b);
- Failure to formulate a first order differential equation to solve the real life problem in part (c);
- Failure to identify the particular integral in part (d).

Extract 17.2 is a sample answer from one of the candidates to illustrate one of these weaknesses.

Extract 17.2

7 (ii)

$$\frac{dy}{dx} = \frac{2x-y}{2x-y} + \frac{2}{2x-y}$$

$$\frac{dy}{dx} = 1 + \frac{2}{2x-y}$$

$$y = xv$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$(2x-y)\left(v + x\frac{dv}{dx}\right)$$

$$(2x-xv)\left(v + x\frac{dv}{dx}\right) = 2x - xv + 2$$

$$2xv + 2x^2\frac{dv}{dx} - xv^2 - x^2v\frac{dv}{dx} = 2x - xv + 2$$

$$2xv - xv^2 + (2x^2 - x^2v)\frac{dv}{dx} = 2x - xv + 2$$

$$(2x^2 - x^2v)\frac{dv}{dx} = 2x - xv + 2 + xv^2 - 2xv$$

$$(2x^2 - x^2v)\frac{dv}{dx} = 2x - 3xv + xv^2 + 2$$

$$\frac{dv}{dx} = \frac{2x - 3x + xv^2 + 2}{2x^2 - x^2v}$$

$$\frac{dv}{dx} = \frac{x(2 - 3 + v^2) + 2}{x(2 - v)}$$

In Extract 17.2, the candidate used wrong substitution in solving the given non-homogenous differential equation in part (a) (ii).

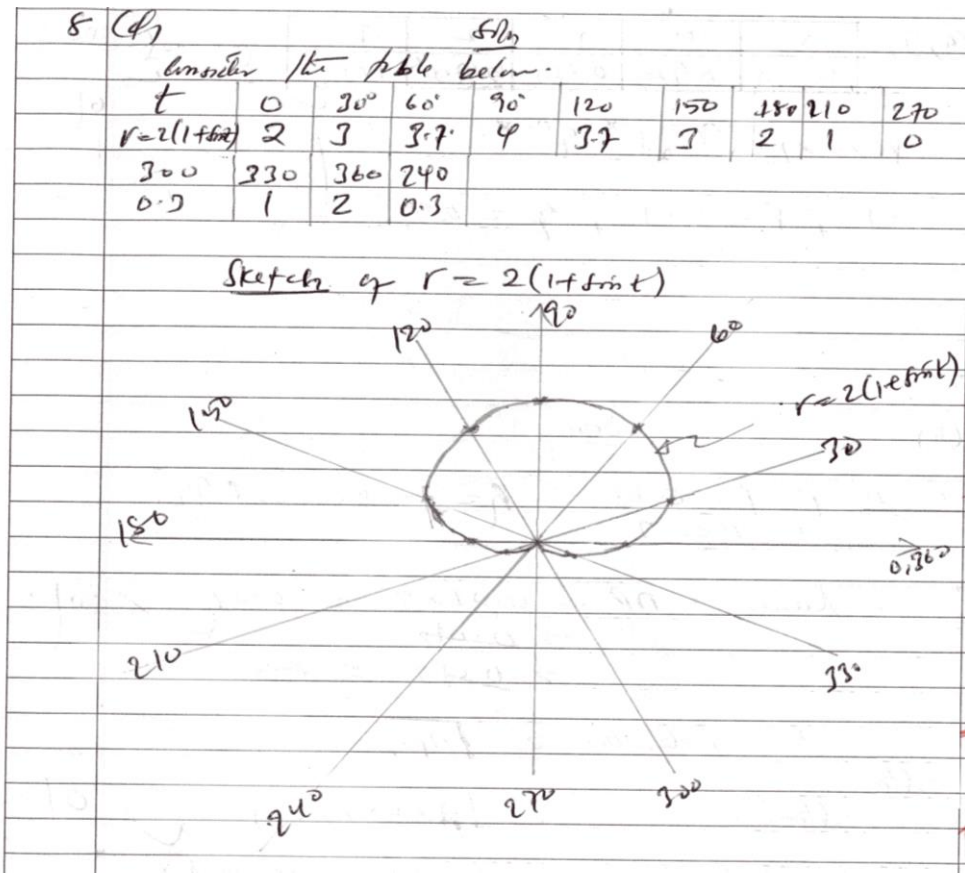
2.2.8 Question 8: Coordinate Geometry II

The question had parts (a), (b), (c) and (d). In part (a), the candidates were required to (i) find the center, vertices, foci and equation of the asymptotes from the equation of the hyperbola $9y^2-54y-25x^2+200x-544=0$, (ii) convert cartesian equation $x^2+y^2=4x$ into polar equation and in (iii) convert $(1,-1)$ into polar coordinates. In part (b), the candidates were required to find the equation of the tangent and normal at $P(a\cos\alpha, b\sin\alpha)$ to the ellipse $b^2x^2+a^2y^2=a^2b^2$. Part (c) required the candidates to (i) define conic section and (ii) use the given information that "a man running a race- course notes that the sum of the distances from the two flag posts to him is always 10 meters and the distance between the flag posts is 8 meters", to find the equation of the path traced. Finally, part (d) required the candidates to sketch the polar graph of $r=2(1+\sin t)$.

This question was attempted by 6,543 (51.2%) candidates out of which 41.6 percent scored from 7 to 11.5 marks, 19.8 percent scored from 12 to 20 marks, while 38.6 percent scored from 0 to 6.5 marks, with 251 (3.8%) candidates scoring a zero mark in this question. Generally, the performance of this question was good.

The analysis of the candidates' responses shows that the majority of those candidates who attempted this question had good performance. In part (a) (i), they were able to write down the given equation into a translated hyperbola $\frac{(y-3)^2}{5^2}-\frac{(x-4)^2}{3^2}=1$ which was compared with $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ to get the required values of centre, vertices, foci and equation of asymptotes. In part (a) (ii), the candidates substituted correctly $x=r\cos\theta$ and $y=r\sin\theta$ into the equation $x^2+y^2=4x$, $x^2+y^2=4x$ which was maneuvered to get the simplified polar equation $r=4\cos\theta$ as required. In part (a) (iii), most candidates correctly substituted the x and y-coordinates of the given point into the formulae $r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$ to obtain $r=\sqrt{2}$ and $\theta=45°$ or $315°$ which were later combined to form the polar coordinates as either $(\sqrt{2}, 315°)$ or $(\sqrt{2}, 45°)$. In part (b), the candidates were able to find the equation of the tangent and normal line to the ellipse through the following four main steps: One, differentiate the equation $b^2x^2+a^2y^2=a^2b^2$ to calculate the slope $m_T = \frac{-b}{a}\cot\alpha$. Two, use the slope from step 1 and the point $P(a\cos\alpha, b\sin\alpha)$ to find the equation $bx\cos\alpha + ay\sin\alpha - ab = 0$ of the tangent line to the ellipse. Three, find the negative reciprocal of $m_T = \frac{-b}{a}\cot\alpha$ to get the slope $m_N = \frac{a}{b}\tan\alpha$ for the normal line to the ellipse and four, use the given point to find the equation $ax\sin\alpha - by\cos\alpha - (b^2 - a^2)\sin\alpha\cos\alpha = 0$ of the normal line. In part (c) (i), the majority of the candidates correctly defined conic section as 'any section obtained from a cone when it is cut by a plane'. In part (c) (ii), they managed to represent the given information mathematically and got the equation of the path correctly as $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Finally, in part (d) most candidates were able to use of values with $0° \le t \le 360°$ to sketch the correct shape for the graph of $2(1+\sin t)$. Extract 18.1 is a sample answer from the script of one of the candidates showing how they answered part (d) of this question correctly.

Extract 18.1



In Extract 18.1, the candidate prepared a table of values that enabled him/her to correctly sketch the graph of the polar curve**.**

Despite this good performance, there were a few candidates (38%) who scored low marks. The candidates failed to complete the square of $9y^2 - 54 - 25x^2 + 200x - 544$ which was the prerequisite to obtain the standard equation of the hyperbola. Such weakness resulted into getting incorrect values for the center, vertices and the equations of asymptotes in part (a) (i). In part (a) (iii), several candidates substituted incorrectly $x = 1$ and $y = -1$ into the equation $x^2 + y^2 = 4x$ in part (a) (ii). Such candidates failed to understand that parts (ii) and (iii) were not related. In part (b), most candidates were unsuccessful in differentiating the given equation as a prior step in solving this question. The analysis of candidates' responses shows that incorrect equations for the tangent to the ellipse such as $x^2 + y^2 - ax\cos\alpha - by\sin\alpha = 0$ were often encountered in the scripts of the candidates, see Extract 18.2. In part (c), most of them could recall the meaning of conic section in part (i) but in part (ii) they could not identify that the information provided in this question were about conic section, particularly the ellipse and therefore they were unable to find the equation of the path as required. In part (d), the candidates had difficulties in creating the table of values which was a necessary step for sketching the polar graph and as a result ended up sketching incorrect graphs.

Extract 18.2

8(b) soln:

$$2x + 2y\frac{dy}{dx} = 0.$$

$$2y\frac{dy}{dx} = -2x.$$

$$\frac{dy}{dx} = \frac{-2x}{2y}.$$

$$\frac{dy}{dx} = \frac{-x}{y}.$$

Equation of the tangent is given by:

$$y - y_1 = m(x - x_1)$$

$$m = \frac{-x}{y} \qquad p(a\cos\alpha, b\sin\alpha)$$

$$y - b\sin\alpha = \frac{-x}{y}(x - a\cos\alpha)$$

$$y - b\sin\alpha = \frac{-x^2}{y} + \frac{ax\cos\alpha}{y}$$

$$y^2 - by\sin\alpha = -x^2 + ax\cos\alpha.$$

$$y^2 + x^2 - by\sin\alpha - ax\cos\alpha = 0.$$

$$\Rightarrow x^2 + y^2 - ax\cos\alpha - by\sin\alpha = 0.$$

$\therefore$ The equation is $x^2 + y^2 - ax\cos\alpha - by\sin\alpha = 0.$

In Extract 18.2, the candidate failed to differentiate the given equation to get the required equation of the tangent to the ellipse.

3.0 A SUMMARY OF THE CANDIDATES' PERFORMANCE

The Advanced Mathematics examination consisted of two question papers with a total of 18 questions from 18 topics as follows: *Calculating Devices, Hyperbolic Functions*; *Linear Programming*; *Statistics; Sets; Functions; Numerical Methods, Coordinate Geometry I*; *Differentiation, Integration*; *Complex Numbers; Logic*; *Vectors*; *Algebra; Trigonometry*; *Probability; Differential Equations* and *Coordinate Geometry II*. The analysis of the candidates' performance topic-wise in 2016 indicated that five (5) topics were well performed, five topics (5) were averagely performed and the remaining eight (8) topics were poorly performed. The topics with good performance were *Logic, Trigonometry, Sets, Linear Programming* and *Coordinate Geometry,* while the topics which had average performance were *Differential Equations, Vectors, Functions, Complex Numbers* and *Hyperbolic Functions*. The factors that accounted for good performance in those topics include: the ability of the candidates to apply the concepts, formula/theorems and mathematical strategies correctly.

On the other hand, the topics which had poor performance were *Calculating Devices (27.9%), Numerical Methods (27.8%), Coordinate Geometry I (21.8%), Integration (18.4%), Differentiation (17.9%), Statistics (16.6%), Probability (12.1%)* and *Algebra (9.5%).* The poor performance among the candidates was due to failure of the candidates to identify the fundamental concepts of the questions, failure to apply the correct formulae, lack of skills to perform computations and manipulations.

4.0 CONCLUSION

In general, the overall performance of the candidates for the ACSEE 2016 in Advanced Mathematics has recorded a slight drop of 8.67 percent of candidates who passed the examination as compared to the performance in 2015. The question-wise analysis of the candidates' response indicates that they performed well in the following topics: Logic, Trigonometry, Sets, Linear Programming and Coordinate Geometry II. There were several factors that made the candidates to score good marks in the questions that were set from these topics, which includes ability to use basic concepts and applying the correct formula and theorems.

However, many candidates performed poorly in the questions which were tested from the topics of Algebra, Probability, Statistics, Differentiation, Integration, Coordinate Geometry I, Numerical Methods and Calculating Devices. The reasons that have contributed to the poor performance in these topics include, failure to use the basic concepts and failure to apply the correct formulae and theorems. Also, insufficient knowledge and skills to manipulate the equations and poor computation skills contributed to the poor performance.

It is hoped that this report can help to support the teachers and students in improving the teaching of various concepts and learning of those topics which had poor performance. In addition, the government and education stakeholders should ensure that the teaching and learning environment is conducive for improving performance.

5.0 RECOMMENDATIONS

In order to improve the performance of candidates, it is recommended that:

(i) Students should learn all the topics to make sure that they understand thoroughly the underlined concepts and that they are able to apply them.

(ii) The future candidates should be encouraged to do many exercises to get experience in applying various formulas/concepts in answering questions.

(iii) Students should use the school subject clubs to conduct discussions on Mathematics topics that were poorly performed.

(iv) Teachers are advised to pinpoint students with learning difficulties so that they can provide them with remedial teaching.

(v) The Education Quality Assurers should make close monitoring on the teaching and learning processes in Advanced Mathematics subject in order to improve the performance of the subject.

(vi) The Ministry of Education, Science and technology should conduct seminars to the stake holders, especially teachers, on the topics with poor performance in this examination.

Appendix I

Analysis of the Candidates' Performance per Topic in Advanced Mathematics

S/N	Topic	Number of Question	The % of Candidates Who Scored 35 or above	Remarks
1	Logic	1	74.2	Good
2	Trigonometry	1	72.3	Good
3	Sets	1	71.8	Good
4	Linear Programming	1	71.7	Good
5	Coordinate Geometry II	1	61.4	Good
6	Differential Equations	1	58	Average
7	Vectors	1	56.1	Average
8	Functions	1	53.9	Average
9	Complex Numbers	1	50	Average
10	Hyperbolic Functions	1	35.6	Average
11	Calculating Devices	1	27.9	Weak
12	Numerical Methods	1	27.8	Weak
13	Coordinate Geometry I	1	21.8	Weak
14	Integration	1	18.4	Weak
15	Differentiation	1	17.9	Weak
16	Statistics	1	16.6	Weak
17	Probability	1	12.1	Weak
18	Algebra	1	9.5	Weak

Appendix II

Analysis of the Candidates' Performance Topic-wise