THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEMS RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2016

141 BASIC APPLIED MATHEMATICS

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Contents

FOREWORD

The National Examinations Council of Tanzania has prepared this report on the analysis of the candidates' responses for Basic Applied Mathematics items for the ACSEE 2016 in order to provide feedback to students, teachers and other education stakeholders on how the candidates responded to the questions.

The analysis shows that, the candidates performed well in the questions that were set from the topics of Statistics, Linear Programming and Matrices; they performed averagely in the questions that were set from the topics of Probability, Functions and Calculating Devices and had weak performance in the questions that were set from the topics of Trigonometry, Integration, Differentiation and Algebra.

The candidates' weak performance was due to the following reasons: lack of knowledge on finding roots of a polynomial and solutions of simultaneous equations; lack of knowledge on implicit differentiation; inability to apply differentiation concepts in solving problems; inability to recall derivatives and anti-derivatives of common functions; inability to apply knowledge of integration in solving problems and lack of knowledge on the basic trigonometric identities.

It is the expectation of the Council that this report will be useful in improving the candidates' performance in future Basic Applied Mathematics examinations.

The Council would like to thank the Examiners, Examination Officers and all others who participated in preparing this report. The Council will also be grateful to receive constructive comments from the education stakeholders for improving future reports.

Dr. Charles Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report has been prepared from the analysis of the candidates' responses for each question, for the candidates who sat for the Basic Applied Mathematics examination in ACSEE 2016. Basic Applied Mathematics paper had a total of ten (10) compulsory questions, each carrying ten (10) marks. The report identifies the areas in which many candidates faced problems as well as the areas they performed well.

In 2016, a total of 26,787 candidates sat for Basic Applied Mathematics examination of which 12,753 (47.95%) candidates passed. The total number of candidates who sat for this examination in 2015 was 17,462; out of which 12,934 (74.10%) passed, indicating that in 2016 the performance dropped by 26.15 percent.

The analysis of the candidates' responses for each question is presented in the next section. In each question, the description of the requirements of the question and the performance of the candidates are provided. The performance of the candidates in each question is categorized as good, average or weak if the percentage of candidates who scored 35 percent or more of the marks in the questions, lie in the intervals 60 – 100, 35 – 59, 0 – 34 respectively.

The analysis also shows the topics with weak, average and good performance. Furthermore, the factors which have contributed to the overall average performance in 2016 Basic Applied Mathematics examination and the recommendations to improve the candidates' performance in this subject have been provided.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Calculating Devices

The question had parts (a) and (b). In both parts, the candidates were required to use a scientific calculator to evaluate:

a) (i) $\log_{0.75} 7.5 - \ln\left(5\sqrt{3}\right)$ correct to five significant figures.

(ii) $\left(\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix}\right)^{2.356}$ correct to four decimal places.

b) (i) mean weight, (ii) variance and (iii) standard deviation of the following weight of 37 members in a certain National Boxing Club.

62	78	40	70	58	65	54	69	71
67	74	64	65	59	68	70	66	80
54	62	83	77	51	72	79	66	83
63	67	61	71	64	59	76	67	58
64								

This question was attempted by 25,595 (95.5%) candidates, out of which 35 percent scored from 3.5 to 10 marks and 374 (1.5%) candidates scored full marks. This question was therefore averagely performed.

In part (a), few candidates were able to use correctly the scientific calculators to compute the answers in both parts (i) and (ii) and write them in the number of significant figures and decimal places as instructed. Also in part (b), few candidates were able to enter the given data in calculators and correctly compute the mean, variance and standard deviation. A sample answer from one of the candidates is shown in Extract 1.1.

Extract 1.1

1. (a) (i) $\log_{0.75} 7.5 - \ln(5\sqrt{3}) =$

$\frac{\log 7.5}{\log 0.75} - \ln 5\sqrt{3} = -9.1627.$

(ii) $\left(\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix}\right)^{2.356} = 906.6192.$

By using a calculator.

(b) (i) Mean weight = 66.4054 units

(ii) Variance = 80.9438 units

(iii) Standard deviation (σ) = 8.9969 units.

The responses in Extract 1.1 justify that the candidate used the correct functional keys of a calculator to compute the answers as required.

In this question, 65 percent of the candidates scored below 3.5 marks and 26.2 percent scored zero. These candidates lacked knowledge and skills in using scientific calculators to perform computations.

The analysis of the candidates' responses shows that, in item (a)(i), several candidates failed to change $\log_{0.75} 7.5$ into either $\frac{\log 7.5}{\log 0.75}$ or $\frac{\ln 7.5}{\ln 0.75}$, hence, ended up with incorrect answers. They did not recognize that they were supposed to change the first term of the given expression into either base 10 or *e* because the type of the calculators that were allowed to use cannot evaluate $\log_{0.75} 7.5$ directly.

Furthermore, in item (a)(ii), a number of candidates failed to use calculators to compute the determinant of the given matrix, as a result they got wrong values of the determinant which led into wrong answers. Some of the candidates used the concept of minors and cofactors to compute the determinant but could not obtain the correct value because of lack of arithmetic skills. These candidates could have entered the elements of the matrix in the calculator and computed the determinant directly.

It was also noted that, some of the candidates correctly computed the answers in parts (a)(i) and (ii) using calculators but failed to write them in the required number of significant figures or decimal places.

Furthermore, in part (b) some candidates failed to enter the given data correctly into the calculators and hence ended up with incorrect values for mean, variance and standard deviation. The analysis also shows that several candidates summarized the data in tabular form and then used the formulae for grouped data to compute the mean, variance and standard deviation. Most of them failed to obtain the correct answers because of either using incorrect formulae, grouping the data wrongly or performing wrong computations. Further analysis shows that, a number of candidates were able to enter the given data into the calculators and computed correct values but failed to distinguish variance from standard deviation as they interchanged the answers in parts (ii) and (iii). Extracts 1.2 (a) and (b) are sample answers showing how the candidates answered this question.

Extract 1.2 (a)

1. (a) (i) $\log_{0.75} 7.5 - \ln(5\sqrt{3})$.

$\ln 5\sqrt{3} = 2.787$

$\log_{0.75} 7.5 = \frac{\log 7.5}{\log 0.75} - \ln(5\sqrt{3})$.

$-7 - 2.787 = \underline{\underline{-9.7915}}$

$\therefore \log_{0.75} 7.5 - \ln(5\sqrt{3}) = \underline{\underline{-9.7915}}$

(ii) $\left(\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix}\right)^{2.356}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix} = 1\left|-4+3\right| - 1\left|-4+1\right| + 1\left|6+2\right|$$

$-1-(-3)+8$

$-1 - 1|-3| + 1|8|$

$-1 + 3 + 8 = \underline{\underline{10}}$

$\therefore (10)^{2.316} = \underline{226.9864}$

In Extract 1.2 (a), the candidate rounded the first term in the expression as -7 instead of -7.00392 and calculated the value of the second term as -2.787 instead of -2.15874 in part (a)(i). In part (a)(ii), the candidate got the determinant as 10 instead of 18, hence ended up with an incorrect answer.

Extract 1.2 (b)

B. Table of value.

weight.	weight value (f)	X	fX	$X-\bar{X}$	$(X-\bar{X})^2$
36 - 40	1	38	38	¯28.78	¯828.28
41 - 45	0	43	0	¯66.78	¯4459.6
46 - 50	0	48	0	¯66.78	¯4459.6
51 - 55	3	53	159	-13.78	¯189.89
56 - 60	4	58	232	-8.78	¯77.09
61 - 65	9	63	567	-3.78	¯204.16
66 - 70	8	68	544	1.22	1.489
71 - 75	4	73	292	6.22	38.69
76 - 80	5	78	390	11.22	125.89
81 - 85	3	83	249	16.22	263.09

ii. Variance:

$$\text{Variance} = \Sigma f\left(\frac{x-\bar{x}}{N}\right)^2$$

$$= \frac{+1,4438.951}{39}$$

$$\text{Variance} = +390.24.$$

(iii) S.D (standard deviation).

$$S.D = \sqrt{\Sigma f\left(\frac{x-\bar{x}^2}{N}\right)}$$

$$S.D = \sqrt{+390.24} = 19.75$$

Extract 1.2 (b) shows that, instead of entering the data directly in calculator, the candidate grouped the data and used incorrect formulae for the variance and standard deviation, hence ended up with wrong answers.

2.2 Question 2: Functions

The question had parts (a) and (b). In part (a), the candidates were given the function $f(x) = mx^2 + nx + k$ where $f(2) = 7$, $f(0) = -3$, $f(-1) = 2$ and were required to find (i) the values of m, n, k and (ii) the domain and range of $f(x)$. In part (b), the candidates were required to (i) sketch the graph of the rational function $g(x) = \dfrac{1}{4x-8}$ and (ii) find the values of x and y for which the function $g(x)$ is defined.

This question was answered by 22,617 (84.4%) candidates, out of which 32.6 percent scored from 3.5 to 6 marks, 9.8 percent scored from 6.5 to 10 marks indicating that the question was averagely performed.

The candidates who performed well in part (a) were able to substitute the given values in the quadratic function $f(x) = mx^2 + nx + k$ to get the correct values of m, n and k. They were also able to find the correct domain and range as illustrated in a sample answer from one of the candidates in Extract 2.1(a).

Extract 2.1(a)

Qn 2.

(a)(i) Solution.

from.

$f(x) = mx^2 + nx + k$.

given:

$f(2) = 7$.

thus:

$f(2) = m(2)^2 + n(2) + k = 7$.

$4m + 2n + k = 7$ --- eqn ①.

Also.

$f(0) = -3$.

Hence.

$f(0) = m(0)^2 + n(0) + k = -3$.

$k = -3$ ---- eqn (2)

2 $f(-1) = m(-1)^2 + n(-1) + k = 2$

$m + -n + k = 2$ --- eqn (3).

from eqn 2 $k = -3$.

Hence.

$m - n + -3 = 2$.

$m - n = 5$ --- ④.

Also.

$4m + 2n - 3 = 7$.

$4m + 2n = 10$ --- eqn 5.

Solve eqn 4 and 5 simultaneously

from $m = 5 + n$.

$4(5+n) + 2n = 10$.

$20 + 4n + 2n = 10$.

$20 + 6n = 10$.

$6n = 10 - 20$.

$6n = {}^{-}10$

$n = {}^{-}10/6$.

$n = -5/3$

Substitute the value of n on eqn 4

$m - \left(-5/3\right) = 5$.

$m = 5 - 5/3$

$m = 10/3$.

The values of $\underline{m = 10/3}$

$\underline{n = -5/3}$

$\underline{k = -3}$

(11.) Solution.

from the equation.

$$f(x) = \frac{10}{3}x^2 - \frac{5}{3}x - 3.$$

Domain = $\{x : x \in R\}$.

let $f(x) = y$

$$y = \frac{10}{3}x^2 - \frac{5}{3}x - 3.$$

$$\frac{dy}{dx} = \frac{20}{3}x - \frac{5}{3}$$

at minimum point $\frac{dy}{dx} = 0$.

$$\frac{20x}{3} - \frac{5}{3} = 0$$

$$\frac{20}{3}x = \frac{5}{3}.$$

$$x = \frac{5}{20}. \qquad x = \frac{1}{4}.$$

This $f(\frac{1}{4}) = \frac{10}{3}(\frac{1}{4})^2 - \frac{5}{3}(\frac{1}{4}) - 3$

$$y = \frac{-77}{24}.$$

Thus since point $y = \frac{-77}{24}$ is the minimum value.

Hence.

Range = $\{y : y \geq \frac{-77}{24}\}$.

Domain = $\{x : x \in R\}$.

In Extract 2.1(a), the candidate was able to formulate three equations from the given information and then solved them by substitution method to get the required values of m, n and k. The candidate was also able to state the domain and apply the concepts of differentiation to find the range of the function as required.

Part (b) was answered correctly by few candidates. The candidates were able to find y – intercept, and vertical and horizontal asymptotes which enabled them to sketch the graph of the rational function $g(x) = \dfrac{1}{4x-8}$ correctly. Extract 2.1(b) is a sample answer from one of the candidates illustrating this case.

Extract 2.1(b)

2(b)(i). Solution.

$f(x) = \frac{1}{4x-8}$

For vertical asymptote.

Equate the denominator with zero.

$4x - 8 = 0$.

$4x = 8$.

$x = 8/4$

$x = 2$.

Vertical asymptote $x = 2$.

for Horizontal asymptote.

$y = \frac{1}{4x-8}$.

$y = \dfrac{1/x}{\frac{4x}{x} - 8/x}$

$y = \dfrac{1/x}{4 - 8/x}$

~~As~~

$1/x \to 0$.

$y = \dfrac{0}{4-0}$

$y = 0$.

The Horizontal asymtote $y = 0$.

Y-interapt when $x = 0$.

$$y = \frac{1}{4(0) - 8}$$

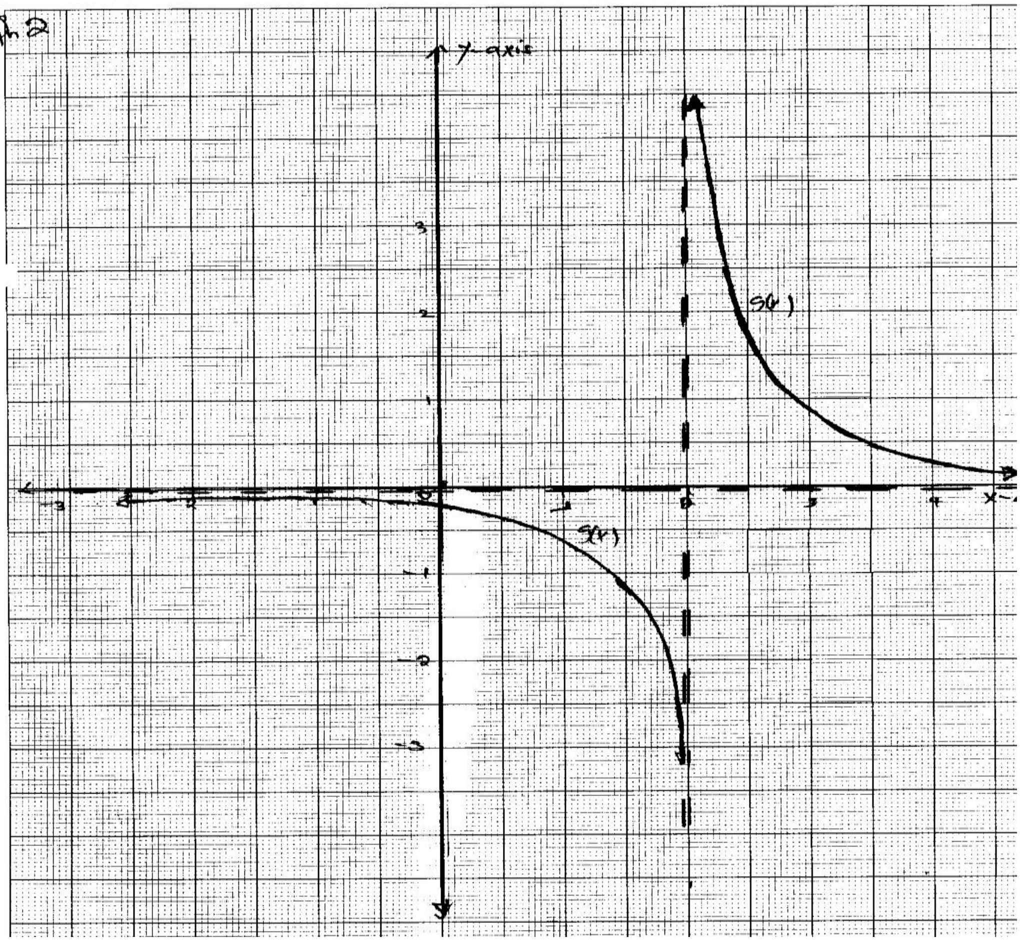
$$y = -\frac{1}{8}$$

~~ver~~ x-interapt when ~~y~~ = 0.

no x-interapt.

The sketching is on the graph paper.

Qn 2



2(b)

(ii) Value of x and y where g(x) is define.

for values of (x) {Domain}.

$= \{x : x \neq 2\}$

All values of x except 2.

for values of y {Range}.

$= \{y : y \neq 0\}$

All values of y except $y = 0$.

In Extract 2.1(b), the candidate was able to sketch the graph of the rational function and state the values of *x* and *y* for which the function is defined.

However, there were several candidates (50.2%) who scored low marks (from 0 to 2.5) in this question. In part (a)(i), some of the candidates were unable to find the values of m, n and k because they could not substitute the values correctly in the given equation. Most of them made arithmetical errors, as a result ended up with incorrect equations. The candidates were unable to obtain the equations $k = -3$, $m - n = 5$ and $2m + n = 5$ that were key requirement in answering this part. It was noted that, a number of candidates managed to formulate these equations but failed to apply Cramer's rule, substitution or inverse matrix method to get the required values. In part (a)(ii), many candidates had difficulties in finding the range as they were unable either to apply the concepts of differentiation or the turning point formula for quadratic function.

In part (b)(i), several candidates lacked the skills to draw graphs of rational functions. Some of the graphs crossed the horizontal asymptote (y = 0) and the vertical asymptote (x = 2). The axes were poorly drawn, incorrectly labeled and the graphs did not cross at the correct y – intercept. It was also noted that, some of the candidates completely lacked knowledge and skills on drawing graphs of rational functions, see Extract 2.2.

Extract 2.2

2.i Value of m, n, and k

solution

$f(x) = mx^2 + nx + k$ if

$f(2) = 7$ $f(0) = -3$ and $f(-1) = 2$

the Value of $m = 7$ $n = -3$ $k = 2$

ii To find domain and range

solution

Domain $= (-3, 7)$

range $= (0, 2)$

2.(b) $g(x) = \frac{1}{4x - 8}$

$g(x) = y$.

$y = \frac{1}{4x - 8}$.

The table of value.

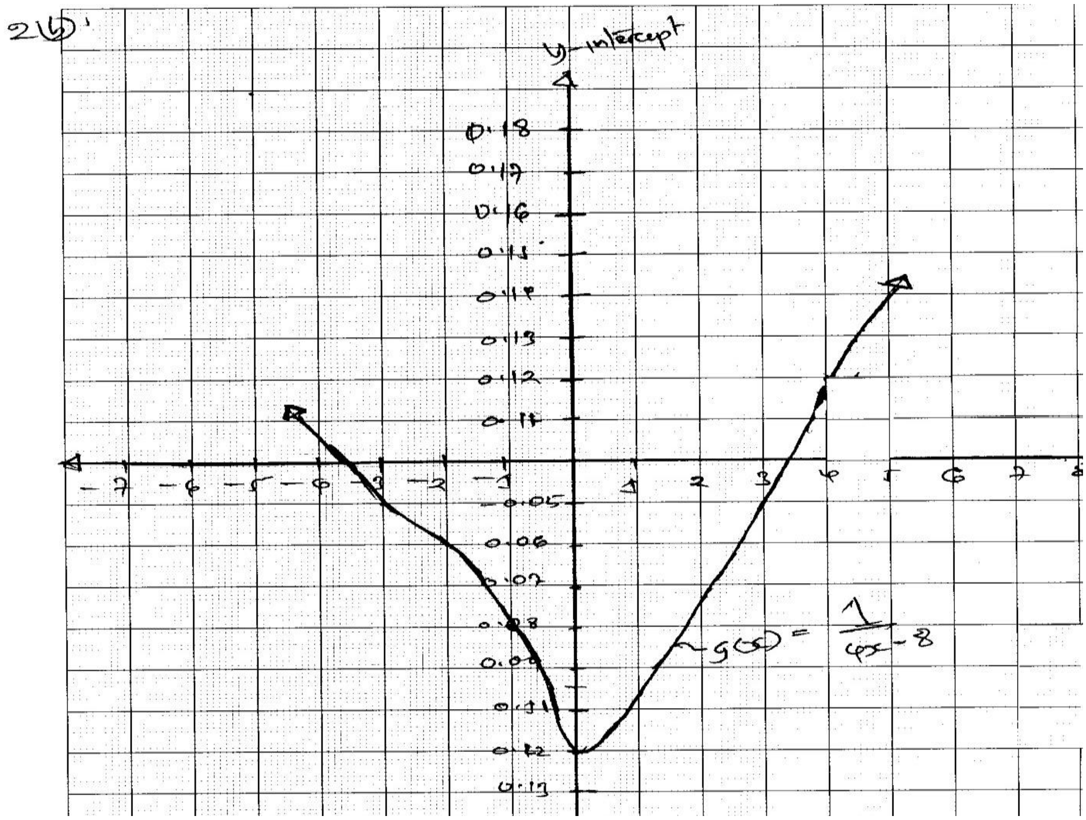
X	−3	−2	−1	0	1	2	3	4
y	−0.05	−0.0625	−0.08	−0.125	−0.25	∞	0.25	0.125

(ii) The value of X for the $y = -0.05$

$-0.05 = \frac{1}{4x - 8}$

$x = -0.16/0.2 = -3$

$X = -3$ and $y = -0.05$



In Extract 2.2, the candidate wrote down incorrect values of *m*, *n* and *k* without showing the workings. Moreover, the candidate did not find y – intercept and the asymptotes indicating that he/she lacked the skills to draw graphs of rational functions.

2.3 Question 3: Algebra

This question had parts (a) and (b). In part (a), the candidates were given that, the roots of the polynomial equation $p(x) = x^3 - 7x^2 + Ax - 8$ form a geometric progression and were then asked to find (i) the roots of the polynomial, (ii) the value of A and (iii) the abscissa at the turning points on the curve. In part (b), the candidates were required to use substitution method to solve the simultaneous equations $\begin{cases} xy = 16 \\ x^2 + y^2 = 32. \end{cases}$

This question was attempted by 20,370 (76%) of the candidates, out of which 99.9 percent scored from 0 to 3 marks, and 42.6 percent scored zero. There were only 8 candidates who scored above 5.5 out of the 10 marks that were allocated.

This was the worst performed question in this examination. Most of the candidates lacked knowledge on the tested concepts of finding roots of a polynomial equation, turning points and solutions of simultaneous equations.

Part (a) was poorly answered as many candidates were unable to understand the question and hence wrote answers that were not related to its demand. In part (a)(i), the candidates could not recognize that if m, n and l are the roots of the given polynomial, then $m + n + l = 7$ and $mnl = 8$. Since the roots form a geometric progression the candidates were supposed to work out the answer as follows: $\frac{n}{m} = \frac{l}{n} \rightarrow n^2 = ml;$ $n^3 = 8 \rightarrow n = 2;$ $4 = ml,$ $m + l = 5 \rightarrow m(5 - m) = 4$; and on solving this quadratic equation, the roots would be 1, 2 and 4 or 4, 2 and 1.

Failure of the candidates to answer part (a)(i) correctly, also led to incorrect answers in part (a)(ii) since $A = mn + ml + nl$. Part (a)(iii) was also poorly done since some of the candidates were unable to obtain the correct value of A while several candidates did not realize that they were supposed to differentiate $f(x) = x^3 - 7x^2 + 14x - 8,$ and then find the values of x when $f'(x) = 0;$ that is solving the quadratic equation $3x^2 - 14x + 14 = 0.$

In part (b), many candidates managed to substitute the equation $xy = 16$ into $x^2 + y^2 = 32$ to obtain $x^4 - 32x^2 + 256 = 0,$ but they were unable to solve it because they could not reduce this 4th order polnomial equation into a quadratic equation. The candidates were supposed to use the substitution $\mathrm{z} = x^2$ to reduce the polynomial into the quadratic equation $\mathrm{z}^2 - 32\mathrm{z} + 256 = 0,$ thereafter solve it for z and eventually obtain the values of x and y as required. Extract 3.1 is a sample answer showing some of the candidates' weakness in this question.

Extract 3.1

QUESTION NO 3

$p(x) = x^3 - 7x^2 + Ax - 8$.

(i) callucculate root

$p(x) = x^3 - 7x^2 + Ax - 8$.

$A_n = A_1 + (n-1)d$.

$A_8 = A_1 + (8-1)$.

$A_8 = 7 + (8-1)d$.

$A_8 = 7 + 7d$.

$A_8 = 14 + d$.

calculate A.

$\frac{A_n}{A-1\,d} = A_1 + \frac{(n-1)d}{n-1\,d}$

$A_1 = \frac{A_n}{(A-1)\,d}$.

$A_1 = \frac{A_n}{(A-1)\,d}$.

$A_1 = \frac{8}{(8-1)} = 7$

$A = 8/7$

iii) $G_n = G_1 r^{n-1}$.

$G_{\cancel{8}} = G_{\cancel{7}} r^{(8-2)}$

$G_7 = G_1 r^{8-1}$

$G_7 = G_1^{7}$

$= 14.$

The turn point is 14.

(b) $xy = 16$

$x^2 + y^2 = 32$

$x + y = 16$

$x^2 + y^2 = 32$

$y = 16 - x$

$x + 16 - x = 16.$

$\frac{\cancel{2}x}{\cancel{2}}$ $\frac{3\cancel{2}}{\cancel{2}}$

$x = 16$

$x^2 + y^2 = 32$

$16^2 + y^2 = 32.$

$256 + y^2 = 32$

$y^2 = 32 - 256$

$y^2 = -224$

$y = 14.9$

In Extract 3.1, the candidate could not find the roots of the polynomial equation and could not solve the given equations.

Despite the poor performance in this question, there were few candidates who provided correct responses in parts (a) and (b). Extracts 3.2 (a) and (b) are sample answers from the scripts of the candidates.

Extract 3.2 (a)

3. (a) $f(x) = x^3 - 7x^2 + Ax - 8$

let a, b, and c be the roots of the equation.

3. (a) $x^3 - 7x^2 + Ax - 8 = (x-a)(x-b)(x-c)$

~~x−a~~ But $b/a = c/b = r$ for geometric progression

By expanding the root equation

$(x-a)(x-b)(x-c)$

$= (x^2 - bx - ax + ab)(x - c)$

$= x^3 - x^2c - bx^2 - ax^2 + acx + abx + bcx - abc$

from the equation given.

$-7x^2 = -x^2c - bx^2 - ax^2$

$7 = c + b + a$ — (i)

also

$Ax = acx + abx + bcx$

$A = ac + ab + bc$ — (ii)

also

$-8 = -abc$

$abc = 8$ — (iii)

But $ac = b^2$

$\therefore abc = 8$

$(ac)b = 8$

3. $(b^2)b = 8$

$b^3 = 8$

$b = 2$ — (i)

also $7 = c + 2 + a$

$5 = c + a$ — (iv)

But $ac = b^2$

$ac = (2)^2 = 4$

$a = 4/c$ — (v)

Substitute in (iv)

$5 = (4/c) + c$

$5 = \frac{4 + c^2}{c}$

$5c = c^2 + 4$

$c^2 - 5c + 4 = 0$

From calculator, $c = 4$ or 1

$a = \frac{4}{c} = \frac{4}{4} = 1$

or $\frac{4}{1} = 4$

$\therefore a = 1, b = 2, c = 4$ are the
or $a = 4, b = 2, c = 1$ roots

3. (a) $A = ac + ab + bc$

$A = (4 \times 1) + 4 \times 2 + 1 \times 2$

$= 4 + 8 + 2$

$A = 14$

(iii) The equation is

$y = x^3 - 7x^2 + 14x - 8$

Extract 3.2 (b)

3. (b) $xy = 16$

$x^2 + y^2 = 32$

$\left(\frac{16}{y}\right)^2 + y^2 = 32$

$\frac{16^2}{y^2} + y^2 = 32$

$16^2 + y^4 = 32y^2$.

Let:

$u = y^2$

$16^2 + u^2 = 32u$.

$u = 16$

Then

from:

$u = y^2$

$y^2 = 16$

$y = \pm\sqrt{16}$

$y = \pm 4$.

$xy = 16$

$x = \pm 4$.

$\therefore x = \pm 4$ and $y = \pm 4$.

In Extracts 3.2 (a) and (b), the candidates demonstrated a good understanding on finding the roots of the polynomial equation and solving the simultaneous equations.

2.4 Question 4: Differentiation

The question had parts (a), (b) and (c). In part (a), the candidates were required to show that $\frac{d}{dx}\left(\sin^{-1}(x-1)\right)=\frac{1}{\sqrt{2x-x^2}}$. In part (b), they were given a relation defined by the equation $y^2-4x^3-4x=0$ and were asked to find, (i) the slope of the curve at the point $x=2$ and, (ii) the equations of the tangents to the curve at the point $x=2$. In part (c), they were asked to find $\frac{dy}{dx}$ if $y=x^2\left(1-\frac{1}{\sqrt{x}}\right)e^{\tan x}$.

The question was attempted by 14,604 (54.5%) candidates of which 97 percent scored from 0 to 3 marks and 55.1 percent scored zero. Only 3 percent of the candidates managed to score above 3 marks, showing that the question was poorly performed.

Most of the candidates who attempted part (a) of this question did not perform well. They lacked knowledge of implicit differentiation and basic trigonometric identities that was needed in answering this part. Several candidates provided solutions that were not related to the requirement of the question. For example, some wrongly expressed $\sin^{-1}(x-1)$ as $\frac{1}{\sin(x-1)}$ and thereafter applied the quotient rule in differentiating this function. Other candidates differentiated the inverse sine function without expressing $y=\sin^{-1}(x-1)$; as a result, they could not obtain the correct answer (as shown in Extract 4.1 (a)).

In answering this part the candidates were supposed to express $y=\sin^{-1}(x-1)$ implying that $\sin y=(x-1)$ and then differentiate this function implicitly to get $\frac{dy}{dx}=\frac{1}{\cos y}$. In order to express the derivative in terms of x, the candidates were supposed to use the trigonometric identity $\cos y=\sqrt{1-\sin^2 y}$ to get $\cos y=\sqrt{1-(x-1)^2}=\sqrt{2x-x^2}$ implying that $\frac{dy}{dx}=\frac{1}{\sqrt{2x-x^2}}$. It was noted that, a number of candidates were able to find

$\frac{dy}{dx} = \frac{1}{\cos y}$ but they failed to complete the proof as required as it is seen in Extract 4.1 (b).

Extract 4.1 (a)

4. (a) $\frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{2x-x^2}}$

To Show:

let $y = \frac{1}{\sqrt{2x-x^2}}$

$\frac{d}{dx}\left(\sin^{-1}(x-1)\right) = y$

$y\,dx = \sin^{-1}(x-1)$

$\sin(y\,dx) = \sin(\sin^{-1}(x-1))$

$\sin(y\,dx) = x-1$

Extract 4.1 (a) shows that, the candidate lacked the skills to differentiate the inverse sine function.

Extract 4.1 (b)

4a $\frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{2x-x^2}}$

let $y = \sin^{-1}(x-1)$

$\frac{dy}{dx} = \frac{1}{\sqrt{2x-x^2}}$

but $(x-1) = \sin y$

$x = \sin y + 1$

$\frac{dx}{dy} = \cos y$

4a) $\frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{2x-x^2}}$

In Extract 4.1(b), the candidate was able to find the derivative $\frac{dx}{dy}$ correctly but failed to express it as a function of *x* and eventually failed to provide the required proof in part (a).

In part (b), many candidates lacked the basic concepts of differentiation and their application in finding the slope and equation of a tangent line at a specified point on a curve. Majority of the candidates were unable to carry out either implicit or explicit differentiation and hence scored zero in this part. Extract 4.1 (c) is a sample answer from one of the candidates illustrating this case. The analysis of candidates' responses also shows that, a number of candidates had the correct idea of differentiating the function $y^2 - 4x^3 - 4x = 0$ to obtain $\frac{dy}{dx} = \frac{6x^2}{y}$ but did not recognize that they were supposed to express this derivative as a function of x, that is, $\frac{dy}{dx} = \pm\frac{6x^2}{\sqrt{4x^3+4}}$. Extract 4.1 (d) illustrates this case. It was noted that among those few candidates who managed to find the derivative, there were some who ignored the $\pm$ symbol and thus missed some marks in this part as they were able to find the tangent equation $y = 4x - 2$ and the other one i.e. $y = -4x + 2$ was not taken into consideration.

Extract 4.1 (c)

4(b) (i) $y^2 - 4x^3 - 4 = 0$.

$y^2 = 4x^3 + 4$.

$\frac{dy}{dx} = 3 \times 4x^2 + 4$.

$\frac{dy}{dx} = 12x^2 + 4$

$\frac{dy}{dx} = 24x$.

$= 24(2)$.

$= 48$.

$\therefore$ Slope $= \underline{\underline{48}}$.

(ii) Slope $= \frac{\Delta y}{\Delta x}$.

$48 = \frac{y - 1}{x - 4}$

Extract 4.1 (c) shows that, the candidate completely lacked knowledge and skills of differentiation.

Extract 4.1 (d)

4 (b) $2y\frac{dy}{dx} - 12x^2 = 0$

$2y\frac{dy}{dx} = 12x^2$

$\frac{dy}{dx} = \frac{12x^2}{2y}$

$\frac{dy}{dx} = \frac{6x^2}{y}$

$x = 2$

$\frac{dy}{dx} = \frac{6 \times 2^2}{y}$

$\frac{dy}{dx} = \frac{6 \times 4}{y}$

$\therefore$ Slope $\left(\frac{dy}{dx}\right) = \frac{24}{y}$

In Extract 4.1 (d), the candidate did not express the derivative $\frac{dy}{dx}$ as a function of x and hence ended up with incorrect value of the slope.

Part (c) was also poorly performed as most of the candidates could not apply the product rule; that is, $\frac{dy}{dx} = \left[e^{\tan x}\right]\frac{d}{dx}\left(x^2 - x^{\frac{3}{2}}\right) + \left(x^2 - x^{\frac{3}{2}}\right)\frac{d}{dx}\left(e^{\tan x}\right)$ to obtain the required answer $\left(2x - \frac{3}{2}x^{\frac{1}{2}}\right)e^{\tan x} + \left(x^2 - x^{\frac{3}{2}}\right)\sec^2 x\, e^{\tan x}$. Very few got full marks as many candidates were unable to find the derivative of all the terms correctly. Extract 4.1 (e) is a sample answer showing how the candidates failed to answer this question.

Extract 4.1 (e)

4(c) $\frac{dy}{dx} = x^4\left(1 - \frac{1}{\sqrt{x}}\right)\left((1 - \tan^2 x)e^{\tan x}\right) +$

$e^{\tan x}\left(x^2 - \frac{1}{2x^{\frac{3}{2}}} + 2x - \frac{2x}{\sqrt{x}}\right)$

$\frac{dy}{dx} = \left(x^2 - \frac{\sqrt{x^5}}{x}\right)\left((1 - \tan^2 x)e^{\tan x}\right) + e^{\tan x}\left(x^2 - \frac{1}{2x^{\frac{3}{2}}} + 2x - \frac{2\sqrt{x^3}}{x}\right)$

Extract 4.1 (e) shows that, the candidate ended up with a wrong answer because he/she could not find the derivative of $e^{\tan x}$.

Only 4 candidates managed to answer all the three parts of this question correctly and scored the 10 marks that were allocated. Extract 4.2 is a sample answer from one of the candidates who answered the question correctly.

Extract 4.2

4 (a) Soln.

$$\frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{2x-x^2}}$$

let $y = \sin^{-1}(x-1)$

$x-1 = \sin y$

To find $\frac{dy}{dx}$

$$\frac{d}{dx}(x-1) = \frac{d}{dx}(\sin y)$$

$$1 = \cos y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y} \quad \text{But } \cos y = \sqrt{1-\sin^2 y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 y}}$$

$$= \frac{1}{\sqrt{1-(x-1)^2}}$$

$$= \frac{1}{\sqrt{1-(x^2-2x+1)}}$$

4 (a) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2+2x-1}}$

$\therefore \frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{2x-x^2}}$ hence shown.

4 (b) Soln.

(i) Given, $y^2 - 4x^3 - 4 = 0$

To find slope, $\frac{dy}{dx}$

$$\frac{d}{dx}\left(y^2 - 4x^3 - 4\right) = \frac{d}{dx}(0)$$

$$2y\frac{dy}{dx} - 4(3x^2) = 0$$

$$2y\frac{dy}{dx} - 12x^2 = 0.$$

$$\frac{dy}{dx} = \frac{12x^2}{2y}$$

when $x = 2$.

$$y^2 = 4x^3 + 4$$

$$= 4(2)^3 + 4$$

$$y = \pm 6$$

$$\therefore \frac{dy}{dx} = \frac{12(2)^2}{2\times 6} = \pm 4$$

(i) $\therefore \frac{dy}{dx} = \text{slope} = \pm 4$

4 (b) (ii) equations of tangent

From $\frac{dy}{dx} = \frac{12x^2}{2y}$

$$\frac{dy}{dx} = \frac{12x(2)^2}{12} = \pm 4$$

Slope $= \pm 4$

at $(x, y) = (2, \pm 6)$

$$\frac{y-6}{x-2} = 4$$

$$y - 6 = 4(x-2)$$

$$y - 6 = 4x - 8$$

$$y - 4x = -8 + 6$$

$$y - 4x = -2$$

$$\underline{y = 4x - 2}$$

Also when slope $= -4$

$$\frac{y-(6)}{x-2} = -4$$

$$\frac{y+6}{x-2} = -4$$

$$y + 6 = -4(x-2)$$

$$y + 6 = -4x + 8$$

$$y = -4x + 8 - 6$$

$$y = 2 - 4x$$

∴ equations of tangent are

$\underline{\underline{y = 4x - 2}}$ and $\underline{\underline{y = 2 - 4x}}$

4 (c) Soln.

$$y = x^2\left(1 - \frac{1}{\sqrt{x}}\right)e^{\tan x}$$

$$y = \left(x^2 - \frac{x^2}{x^{1/2}}\right)e^{\tan x}$$

$$y = \left[x^2 - \left(x^{2-\frac{1}{2}}\right)\right]e^{\tan x}$$

$$y = \left(x^2 - x^{3/2}\right)e^{\tan x}$$

Let $a = \tan x$

$$\frac{da}{dx} = \sec^2 x$$

$$b = e^{\tan x} = e^a$$

$$\frac{db}{da} = e^a$$

$$\frac{db}{dx} = \frac{db}{da} \cdot \frac{da}{dx}$$

$$\frac{db}{dx} = \sec^2 x \cdot e^{\tan x}$$

Also let $c = x^2 - x^{3/2}$

$$\frac{dc}{dx} = 2x - \frac{3}{2}x^{1/2}$$

From $y = \left(x^2 - x^{3/2}\right)e^{\tan x}$

$$y = c \cdot b$$

$$\frac{d(y)}{dx} = c\frac{db}{dx} + b\frac{dc}{dx}$$

4 (c) $\therefore \frac{d(y)}{dx} = \left(x^2 - x^{3/2}\right)\sec^2 x\, e^{\tan x} + e^{\tan x}\left(2x - \frac{3}{2}x^{1/2}\right)$

In Extract 4.2, the candidate managed to apply correctly the concepts of differentiation in answering the question.

2.5 Question 5: Integration

The question had parts (a) and (b). In part (a), the candidates were asked to find $f(z)$ given that, $f'(z) = z\,e^{z^2}$ and $f(0) = \frac{9}{2}$. In part (b), they were asked to (i) calculate the area bounded by the curve $y = x^2 + 3x - 18$ and the line $y=0$ and, (ii) find the cost function given that the marginal cost of producing x units of a product was $c'(x)=0.6x^2+4x$ and the fixed cost was 30,000/=.

This question was poorly answered because 88.2 percent of the candidates who attempted it scored from 0 to 3 out of 10 marks. Only 10.3 percent of the candidates scored from 3.5 to 5.5 while 1.5 percent scored from 6 to 10 marks.

In part (a), many candidates could not find the required function $f(z)$ because they lacked knowledge of derivatives and anti-derivatives, see Extract 5.1. The candidates were unable to realize that the function will be obtained by integrating the given equation, i.e. $\int f'(z)dz=\int ze^{z^2}dz$ implying that $f(z)=\int ze^{z^2}dz$. It was noted that, some of the candidates managed to get this integral equation correctly but failed to obtain the final answer because they were unable to use the technique of integration by substitution. In obtaining the required solution the candidates were supposed to go through the following steps: let $k=z^2$, $dk=2zdz \rightarrow f(z)=\frac{1}{2}\int e^k dk$ and therefore $f(z)=\frac{1}{2}e^{z^2}+c$ where the constant of integration was to be determined using the given initial condition.

In part (b)(i), only few candidates were able to find the required area. Most of the candidates lacked knowledge and skills of integration. Some of the candidates were unable to integrate the terms of the given quadratic function correctly while others could not determine the limits of integration. It was also noted that, some candidates were applying formulae and concepts that were not relevant in this part. For example some applied the formula: $\text{Area}=\frac{1}{2}\times\text{base}\times\text{height}$ while other candidates were differentiating the function instead of integrating it.

The analysis of candidates’ responses shows that, part (b)(ii) was also poorly done as most of the candidates were unable to apply the concepts of integration to solve the given real life problem. It was observed that, some of the candidates were applying wrong concepts, for instance they differentiated the given marginal cost function instead of integrating it while several candidates provided inappropriate solutions; for example, they substituted the given fixed cost 30,000 into the marginal cost function to get $c(x)$. Extract 5.1 is a sample answer showing some of the candidates' weaknesses.

Extract 5.1

5(c) $f(u) = ze^{z^2}$

Let $z^2 = y$

$\frac{dy}{dt} = 2t$

$\cancel{\frac{dy}{2t} = dt}$

$y = ze^{y}$

$\frac{dy}{dt} = z\frac{d(e^{y})}{dz} + e^{y}\frac{dz}{dt}$

$\frac{dy}{dt} = ze^{y} + e^{y}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dy}{db}$

(5) $f(t) = ze^{t^2}$

$y = ze^{z^2}$

Let $u = z^2$

$dy/dt = \cancel{2-1}\ 2z$

$y = ze^{u}$

$dy/dt = z \times e^{u} + e^{u}$

$\cancel{dy/dt \times 1/(dz/dt)} = \cancel{ze^{u} + e^{u}}$

$dy/dt = ze^{u} + e^{u}$

5 (b)(i) $y = x^2 + 3x - 18$

$dy/dx = 2x + 3$

$0 = 2x + 3$

$x = -3/2$

$y = x^2 + 3x - 18$

$y = (-3/2)^2 + 3(-3/2) - 18$

$y = \frac{9}{4} - 9/2 - 18$

$y = \frac{9 - 18}{4} - 18$

$y = -\frac{9}{4} - 18$

$y = \frac{-9 - 72}{4}$

$y = -20.25$

5 (b)

$c(x) = 0.6x^2 + 4x$

$dy/dx = (0.6)2x + 4$

$c = 1.2x + 4$

sub $x = 30000$

$1.2(30000) + 4$

$= 36004$

The cost of the products

$36004/=$

In Extract 5.1, the candidate provided a solution that is not related to the question demand. This is an indicator of lack of knowledge and skills on the topic of integration.

Only 12 candidates attempted all parts of this question correctly and scored all the 10 marks. In part (a), they were able to recognize that, the given equation $f^{/}(z) = ze^{z^2}$ had to be integrated with respect to z and then use the initial condition $f(0) = \frac{9}{2}$ to obtain the value of the constant of integration and eventually the required function $f(z) = \frac{1}{2}e^{z^2} + 4$.

In part (b)(i), the candidates were able to find the limits of integration, either by solving simultaneously the equations $y = x^2 + 3x - 18$ and $y = 0$ or through sketching their graphs; which enabled them to find the area as required. Furthermore, in part (b)(ii), they were able to recognize that, the equation $c'(x) = 0.6x^2 + 4x$ had to be integrated with respect to x and the given fixed cost had to be substituted as the constant of integration so as to obtain the required cost function. Extract 5.2 shows a sample work of one of the candidates.

Extract 5.2

5 (a) Soln.

Given, $f'(z) = ze^{z^2}$ and $f(0) = 9/2$

$\int f'(z) = \int ze^{z^2} dz$

let $a = z^2$

$\frac{da}{dz} = 2z$

$\frac{da}{2} = zdz$

From $\int f'(z) = \int e^{z^2} \cdot zdz$

$= \int e^{a} \cdot \frac{da}{2}$

$= \frac{1}{2} \int e^{a} da$

$f(z) = \frac{1}{2} e^{a} + c$ But $a = z^2$

$f(z) = \frac{1}{2} e^{z^2} + c$

$f(0) = \frac{1}{2} e^{0^2} + c = 9/2$

$\frac{1}{2} e^{0} + c = 9/2$

$\frac{1}{2} + c = 9/2$

5 (a) $\frac{1}{2} + c = 9/2$

$c = 9/2 - 1/2$

$c = \frac{9-1}{2} = 8/2$

$c = 4$

$\therefore f(x) = \frac{1}{2}e^{x^2} + 4$

5 (b). The area of the region bounded by the curve.

$y = x^2 + 3x - 18$

$y = 0$ (x-axis).

To ~~draw~~ find the intercepts.

$y = x^2 + 3x - 18$

$x^2 + 3x - 18 = 0$

Solve by using calculator.

$x_1 = 3$, $x_2 = -6$.

5(b)(i) To sketch the graph.

The y - intercept.

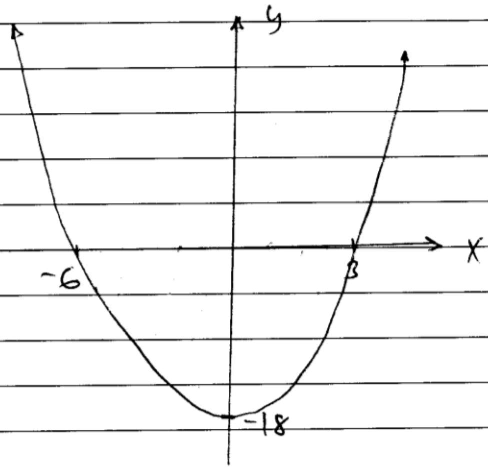
$x = 0$.

$y = x^2 + 3x - 18$

$x = 0$,

$y = 0^2 + 3(0) - 18$

$y = -18$



Then

$$A = \int_a^b f(x)\,dx$$

$$A = \int_{-6}^{3} (x^2 + 3x - 18)\,dx.$$

$$A = \left[\frac{x^3}{3} + \frac{3x^2}{2} - 18x\right]_{-6}^{3}$$

$$A = \left(\frac{3^3}{3} + \frac{3(3)^2}{2} - 18(3)\right) - \left(\frac{(-6)^3}{3} + \frac{3(-6)^2}{2} - 18(-6)\right)$$

5(b)(i) $A = \left(9 + 13.5 - 54\right) - \left(\frac{-216}{3} + 54 + 108\right).$

$A = (-31.5) - (-72 + 54 + 108).$

$A = (-31.5) - (90)$

$A = |-121.5|$

$A = 121.5$ Square units.

5 (b) (ii) From

$c'(x) = 0.6x^2 + 4x$

$\int c'(x) = \int (0.6x^2 + 4x)\,dx$

$c(x) = \frac{0.6x^3}{3} + \frac{4x^2}{2} + C$

$c(x) = \frac{0.6x^3}{3} + 2x^2 + C$

But $C = 30,000$

$\therefore c(x) = 0.2x^3 + 2x^2 + 30,000$

$\therefore$ Cost function $= \underline{\underline{0.2x^3 + 2x^2 + 30000}}$

Extract 5.2 shows that, the candidate had an adequate knowledge of derivatives and anti-derivatives and was able to apply it appropriately.

2.6 Question 6: Statistics

The question had two parts (a) and (b). In part (a), the candidates were given that, “the number of motorcycle accidents which were recorded in one region in Tanzania for seven weeks during November and December 2013, as 14, 2, 12, 4, 10, 6 and 8” and were then required to find (i) the mean number of accidents and (ii) the variance of the accidents.

In part (b), they were required to (i) draw the histogram and (ii) estimate the mode from the histogram using the heights of avocado trees in an Orchard that were summarized in a table as follows:

Height ($x10^{-1}$m)	2 - 6	7 - 11	12 - 16	17 - 21	22 - 26	27 - 31
Frequency	12	14	18	15	4	8

This was the best performed question in this examination paper. The analysis of data shows that 26,390 (98.5%) candidates attempted the question, out of which 19.5 percent scored from 0 to 3, 32.2 percent scored from 3.5 to 5.5 and 48.3 percent scored from 6 to 10 marks.

In part (a), many candidates scored good marks because they were able to use appropriate formulae to compute the mean and variance. In part (b), they were able to find the class marks and draw the histogram correctly. The class marks were correctly placed at the middle of the corresponding bars on the horizontal axis. The vertical axis was correctly drawn and labeled and the mode was correctly determined from the histogram. Extract 6.1 is a sample of good answer from one of the candidates.

Extract 6.1

6 a (i) To find mean.

Given, data,

14, 2, 12, 4, 10, 6, 8

From

Mean $(\bar{x}) = \frac{\Sigma x}{N}$

$\bar{x} = \frac{14+2+4+12+10+6+8}{7}$

$\bar{x} = 8$

$\therefore$ Mean $= 8$

(ii) Variance

From

$$\text{Variance} = \frac{\sum (x-\bar{x})^2}{N}$$

To prepare the table of values of $x, \bar{x}, x-\bar{x}$ and N

x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
14	8	6	36
2	8	-6	36
12	8	4	16
4	8	-4	16
10	8	2	4
6	8	-2	4
8	8	0	0
$\sum$			112

$$\therefore \text{Variance} = \frac{\sum (x-\bar{x})^2}{N}$$

$$= \frac{112}{7}$$

$$\therefore \text{Variance} = 16$$

6b (i) To draw the histogram

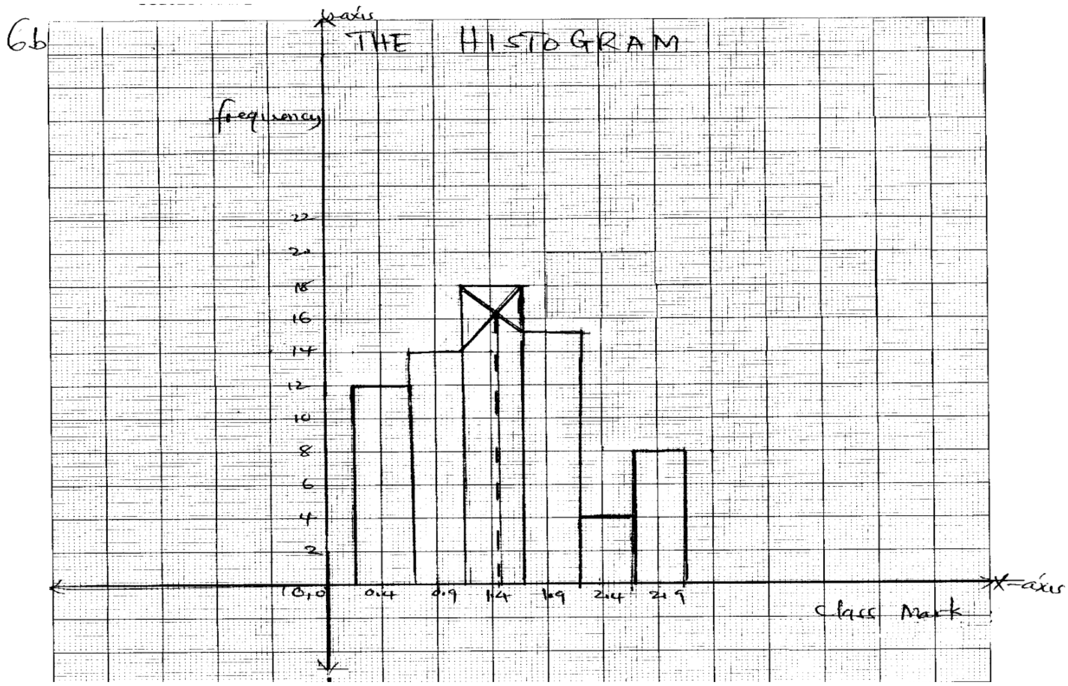
Histogram - Is the graph drawn between frequency against class Mark

∴ Consider the table below

Height	class Mark (x)	frequency
0.2 – 0.6	0.4	12
0.7 – 1.1	0.9	14
1.2 – 1.6	1.4	18
1.7 – 2.1	1.9	15
2.2 – 2.6	2.4	4
2.7 – 3.1	2.9	8

(ii) From the histogram

Mode = 1.43



In Extract 6.1, the candidate answered all parts of the question correctly showing all important steps.

The analysis of the candidates' responses shows that, the candidates who scored low marks in this question showed the following weaknesses:

- Poor computational skills; an aspect that was reflected in providing incorrect answers for the mean and variance.
- Inability to use correct formulae for mean and variance. Some of the incorrect formulae for mean that were used include $\bar{x}=\frac{\sum f}{N}$ and $\bar{x}=\frac{\sum N}{N}$ instead of $\bar{x}=\frac{\sum f\,x}{N}$ and for the variance they used: $variance=\frac{\sum(x-\bar{x})}{\mathrm{N}}$ and $variance=\frac{\sum(\mathrm{x}-\bar{\mathrm{x}}^{2})}{N}$ instead of $variance=\frac{\sum(x-\bar{x})^{2}}{\mathrm{N}}$.
- Inability to follow instructions; as some of the candidates used a formula to calculate the mode instead of reading the value from the histogram as instructed.

- Failure to locate correctly the position of the mode on the histogram; as a result they ended up with an incorrect value.

- Lack of skills to draw histogram; an aspect that was reflected in: drawing bars of different width, incorrect labeling of the axes, placing the class marks at the beginning or at the end of the bars instead of placing them at the middle.

- Lack of knowledge about histograms. This was observed in drawing bar graphs, frequency polygons, pie charts and other graphs.

Extract 6.2 is a sample response from one of the candidates illustrating how the candidates failed to answer this question.

Extract 6.2

6a Mean $= \frac{\Sigma F}{\Sigma N}$

Mean $= 14 + 24 + 12 + 4 + 10 + 6 + 8 = \frac{52}{7}$

$\frac{52}{7} = 7.428$

∴ Mean $= 7.43$

Variance of accidents

Variance $= \frac{\Sigma f(x - F\bar{x})^2}{\Sigma N} = \frac{55.184}{52} = 1.06$

Variance $= \frac{\Sigma f(\bar{x})^2}{\Sigma N}$

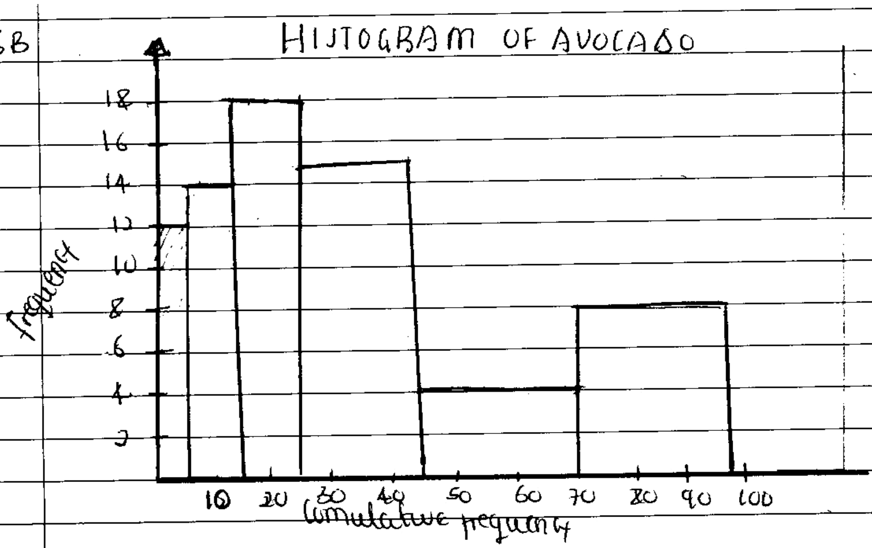
Variance $= \frac{55.184}{52} = 1.06$

∴ Variance $= 1.06$

6B

Height	Frequency	x	Cumulative frequency
2 – 6	12	4	4
7 – 11	14	9	13
12 – 16	18	14	27
17 – 21	15	19	46
22 – 26	4	24	70
27 – 31	8	29	99

6B



$$Mode: L + \frac{t_1}{t_1 + t_2})i$$

$L = 11 \cdot 5$

$t_1 = 4$

$t_2 = 3$

$i = 5$

$$Mode = L + \frac{t_1}{t_1 + t_2})i$$

$$= 11 \cdot 5 + \frac{4}{3+4})5$$

$$= 11'5 + \frac{4}{7})5$$

$$Mode = 11 \cdot 5 + 0{,}571)5$$

$$= 11 \cdot 5 + 2 \cdot 857$$

$$= 14 \cdot 357$$

$$\therefore Mode = 14{,}357$$

In Extract 6.2, the candidate used incorrect formulae in finding the mean and variance. The candidate drew a histogram with different widths and incorrectly labeled the horizontal axis.

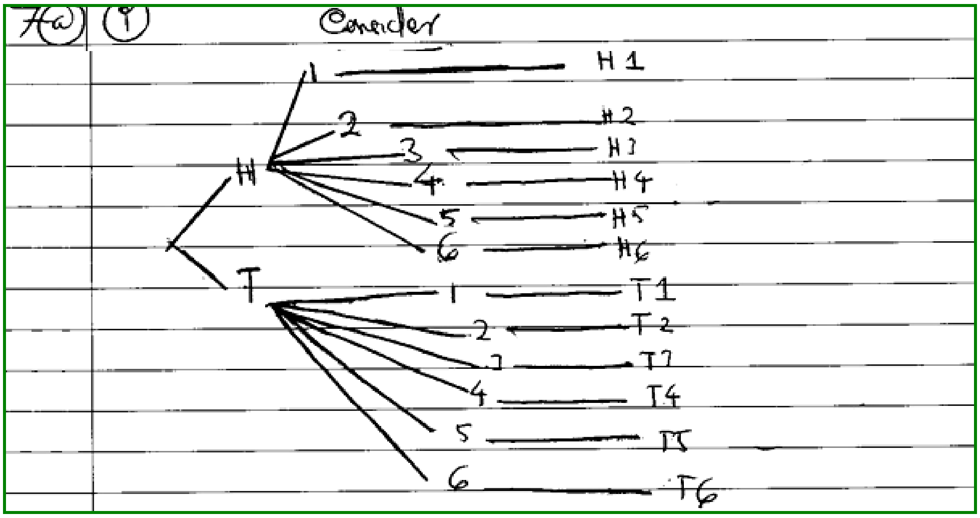
2.7 Question 7: Probability

This question had two parts, (a) and (b). In part (a), the candidates were required to draw a probability tree diagram to show the possible outcomes, and find the probability of the event which contains a head and an even number when a fair coin is tossed once followed by a fair die. In part (b), the candidates were required to find $P(X/Y)$ and $P(X \cup Y)$ if $P(X) = \frac{2}{3}$ and $P(X \cap Y)' = \frac{3}{4}$ given that events X and Y are independent.

This question was attempted by 20,781 (77.6%) candidates, of which 50 percent scored from 0 to 3 marks, 33.8 percent scored from 3.5 to 5.5 marks and 16.2 percent scored from 6 to 10 marks. This implies that the performance in this question was average.

The candidates who answered this question correctly had adequate knowledge and skills on the tested concepts of probability. In part (a), they were able to draw the tree diagram and use the formula $\text{P(E)} = \frac{\text{n(E)}}{\text{n(S)}}$ to compute the probability of obtaining a head and an even number as required. In part (b), they were also able to use the formulae $\text{P}(\text{X} \cap \text{Y}) = \text{P(X)P(Y)}$ and $\text{P(X/Y)} = \frac{\text{P}(\text{X} \cap \text{Y})}{\text{P(Y)}}$ to find the required probabilities. Extracts 7.1 (a) and (b) are sample answers from one of the candidates.

Extract 7.1 (a)



$\quad 6 \qquad\qquad \overline{76}$

(ii) $P(\text{H and even number}) = \dfrac{n(\text{H and even number})}{n(S)}$

But,
$n(\text{H and even number}) = \{2H, 4H, 6H\}$

$n(S) \Rightarrow \underline{12}$.

$P(\text{H and even number}) = 3/12 = 1/4$

Extract 7.1 (a) shows that, the candidate was able to determine the sample space and used it to find the required probability.

Extract 7.1 (b)

7b) Solution..
For independent event

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)}$$

But
$$P(X \cap Y) = 1 - P(X \cap Y)'$$
$$= 1 - 3/4 = \underline{1/4}.$$

$$P(X/Y) = \frac{1/4}{\underline{P(Y)}}$$

$$P(X \cap Y) = P(X) \cdot P(Y)$$
$$P(Y) = \frac{P(X \cap Y)}{P(X)} \Rightarrow \frac{1/4}{(2/3)} \Rightarrow \underline{3/8}.$$

$$P(X/Y) = \frac{1/4}{(3/8)} \Rightarrow \underline{2/3}$$

(ii) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{2}{3} + \frac{3}{8} - \frac{1}{4}$

$\Rightarrow \frac{19}{24}$

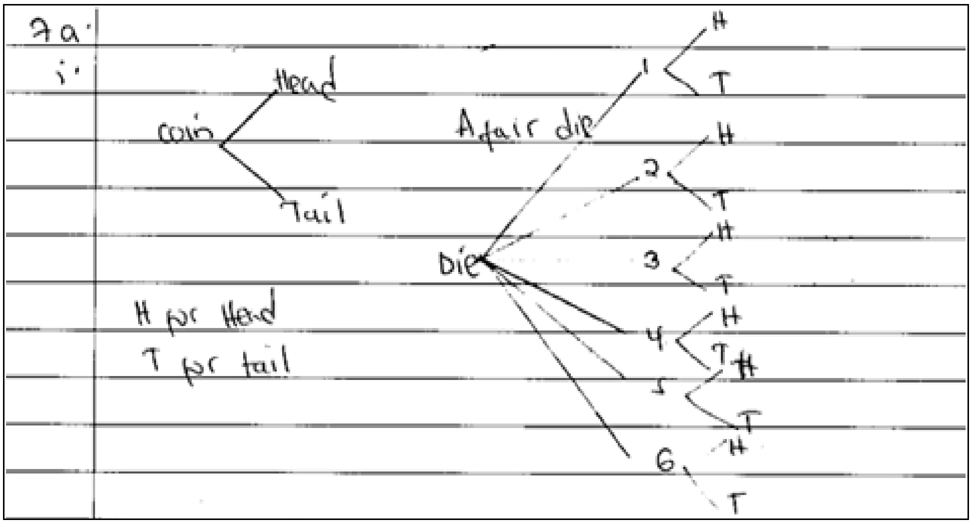
In Extract 7.1 (b), the candidate correctly applied the formulae for conditional probability and independent events to answer part (b).

The candidates who scored low marks had the following weaknesses: In part (a), some of them drew the tree diagram for a die followed by a tree diagram for a coin, contrary to the instructions of the question. Furthermore, some of them drew the tree diagrams for coin and a die separately which led to failure in identifying the sample space and determining the probability of individual outcomes within the sample space. Others drew correct tree diagrams but were unable to use them in determining the required probability.

In part (b), many candidates used incorrect formulae, indicating that the concepts of conditional probability and independent events were not familiar to them. Some of the incorrect formulae the candidates used were: $P(X/Y) = \frac{P(X)}{P(Y)}$, $P(X \cap Y)' + P(X \cup Y) = 1$, $P(X \cup Y) = P(X) + P(Y) + P(X \cap Y)$ e.t.c.

Extracts 7.2 (a) and (b) illustrate how the candidates failed to provide the required solution in this question.

Extract 7.2 (a)



1{HT} 2{HT} 3{HT} 4{HT} 5{HT} 6{HT}

∴ outcome 1{HT} 2{HT} 3{HT} 4{HT} 5{HT} 6{HT}

ii. Outcome contain a head and an even number

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{n\{2, 4, 6\}}{6}$

$P(E) = \frac{3}{6} = \frac{1}{2} = \frac{1}{2}$ or 0.5

∴ Probability is $\frac{1}{2}$

In Extract 7.2 (a), the candidate drew separate tree diagrams for a coin and die instead of combining them, which led to incorrect sample space and probability.

Extract 7.2 (b)

8. $P[x] = 2/3$ and $P(x \cap y)' = 3/4$.

(i) $P(x/y)$

$P(x/y = P(x] - P(x \cap y)'$

$= 2/3 - 3/4 \quad \frac{8-9}{12} = -1/12$

$P(x/y) = -1/12$

(ii) $P(x \cup y) =$

$P(x \cup y) = P(x] + P(y) + P(x \cap y)$.

$= 2/3 - 1/12 + 3/4$.

In Extract 7.2 (b) the candidate used incorrect formulae in determining $P(X/Y)$ and $P(X \cup Y)$.

2.8 Question 8: Trigonometry

This question had three parts on which the candidates were required to:

a) define the terms “sine” and “tangent” of an angle.

b) evaluate $\tan 15^\circ + \cot 75^\circ$ giving the answer in simplest surd form.

c) prove that $(1 - \cos A)(1 + \sec A) = \sin A \tan A$.

This question was attempted by 16,408 (61.8%) candidates of whom 79.9 percent scored from 0 to 3 marks, 14.3 percent scored from 3.5 to 5.5 marks and 5.8 percent scored from 6 to 10 marks. These data show that the general performance of the the candidates was poor in this question.

Most of the candidates lacked knowledge of trigonometry. In part (a), they were unable to define the given terms correctly. Most of the definitions were not related to ratios of sides in a right angled triangle, see Extract 8.1. In part (b), several candidates used calculators to evaluate $\tan 15^o + \cot 75^o$ which led to the answers that were not in surd form. The analysis also indicates that, some used incorrect formula while others made incorrect substitution of values. The candidates were supposed to express 15^{o} and 75^{o} as sum or difference of degrees of special angles and then apply the identity $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$. For example, they could have used $15^{\circ} = 45^{\circ} - 30^{\circ}$ and $75^{\circ} = 45^{\circ} + 30^{\circ}$, but instead some of the candidates used degrees of non special angles. In part (c), several candidates were able to expand the brackets but could not apply relevant trigonometric identities to complete the proof. However, there were some candidates who faced difficulties in opening the brackets while others copied down the question and could not proceed. Extracts 8.1 (a) and (b) illustrate these cases.

Extract 8.1 (a)

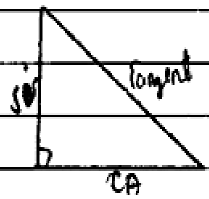
8. Define the following terms

i) sine

is the form of Mathematics which can be used to solve different equation of the trigonometric equation.

ii) Tangent

is the form of mathematics which can be used to present the heights in the equation



Extract 8.1 (a) shows that, the candidate failed to give the definitions of sine and tangent, and used a calculator to compute tan 15^{o} + tan 75^{o} instead of using compound angle formula.

b) Evaluate

$\tan 15^\circ + \cot 75^\circ$

$= 0.267949192 + 0.258819045$

$= 0.526768237$

$= 0.5268$

8c Prove that $(1 - \cos A)(1 + \sec A) = \sin A \tan A$

sin

$= (1 - \cos A)(1 + \sec A)$

$= (1 - \sin A)(1 + \tan A)$

then oppen the brakets

$= 1 - \sin A \times 1 + \tan A$

$= \sin A \times \tan A$

$= \sin A \tan A$ hence proved

Extract 8.1 (b) shows that, the candidate wrongly expanded the expression (1 – cos A) (1 + sec A).

On the other hand, some candidates (0.3%) were able to answer all parts of this question correctly. This shows that, they had sufficient knowledge and skills in the topic of Trigonometry. Extracts 8.2 (a), (b) and (c) provide a sample answer from one of the candidates who answered this question correctly.

Extract 8.2 (a)

Solution.

8. (a)(i). Sine – Is the trigonometric ratio which is given as the ratio of opposite side to hypotenuse side for a given angle in the right angled triangle

(a) (ii) Tangent – Is the trigonometric ratio which is given as the ratio opposite side to adjacent side for a given angle in a right angled triangle.

$$\tan = \frac{\text{Opposite}}{\text{Adjacent}}.$$

In Extract 8.2 (a), the candidate wrote correct definitions of sine and tangent as it was required.

Extract 8.2 (b)

8 (b) Solution

$$\tan 15^\circ + \cot 75 = \tan(45-30) + \frac{1}{\tan(45+30)}$$

$$\tan 15 + \cot 75 = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} + \frac{1 - \tan 45 \tan 30}{\cancel{1 + \tan}\ \tan 45 + \tan 30}$$

$$\tan 15 + \cot 75 = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} + \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}}$$

$$\tan 15 + \cot 75 = \frac{2 - 2/\sqrt{3}}{1 + 1/\sqrt{3}}$$

$$\tan 15 + \cot 75 = \frac{2\sqrt{3} - 2}{\sqrt{3} + 1}$$

Rationalizing the denominator

$$\tan 15 + \cot 75 = \frac{2\sqrt{3} - 2}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$\tan 15 + \cot 75 = \frac{2\times 3 - 2\sqrt{3} - 2\sqrt{3} + 2}{3 - \sqrt{3} + \sqrt{3} - 1}$$

$$\tan 15 + \cot 75 = \frac{6 - 4\sqrt{3} + 2}{2}$$

$$\tan 15 + \cot 75 = 4 - 2\sqrt{3}$$

In Extract 8.2 (b), the candidate correctly applied the compound angle formulae and substituted the correct values of special angles to find the required answer.

Extract 8.2 (c)

8 (c) $(1 - \cos A)(1 + \sec A) = (1 - \cos A)\left(1 + \frac{1}{\cos A}\right)$

$$(1 - \cos A)(1 + \sec A) = \frac{\cancel{\cos A} + 1 - \cos^2 A - \cancel{\cos A}}{\cos A}$$

$$(1 - \cos A)(1 + \sec A) = \frac{1 - \cos^2 A}{\cos A}$$

$$(1 - \cos A)(1 + \sec A) = \frac{\sin^2 A}{\cos A} \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$(1 - \cos A)(1 + \sec A) = \frac{\sin A \cdot \sin A}{\cos A}$$

$$(1 - \cos A)(1 + \sec A) = \sin A \cdot \tan A$$

In Extract 8.2 (c), the candidate used correct trigonometric identities to prove that $(1-\cos A)(1+\sec A) = \sin A \tan A$.

2.9 Question 9: Matrices

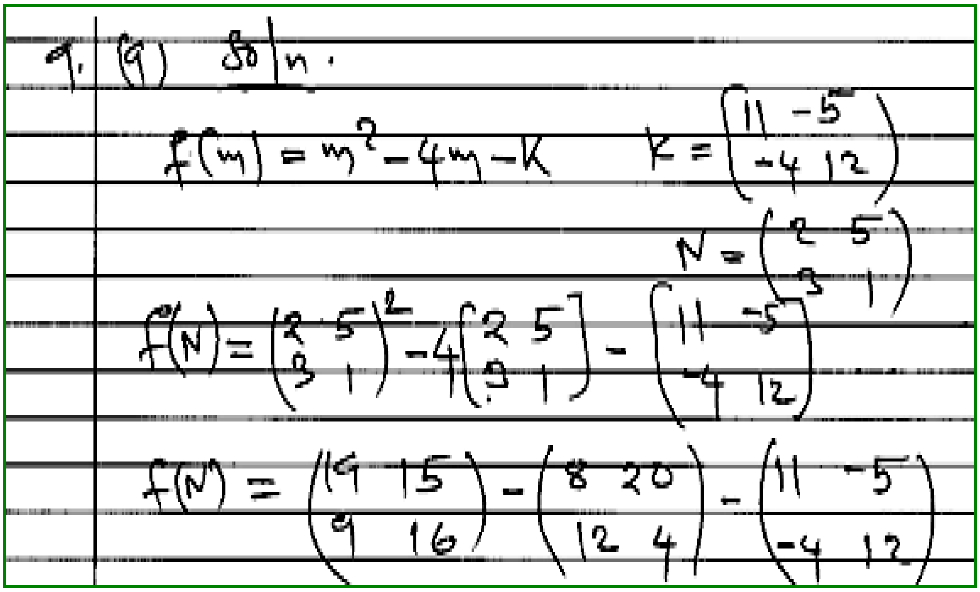
This question had two parts (a) and (b). In part (a), the candidates were required to find $f(N)$ given that, $f(m) = m^2 - 4m - k$ when $k = \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$. In part (b), the candidates were required to use Cramer's rule to solve the system of simultaneous equations:

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

This question was attempted by 25,139 (93.8%) candidates of whom 52.6 percent scored from 6 to 10 marks, 33.3 percent scored from 0 to 3 marks and 14.2 percent scored from 3.5 to 5.5 marks. These data imply that, the performance in this question was good.

In part (a), many candidates were able to substitute the given data correctly in the given function to get correct matrix for $f(N)$. In part (b), they were able to use Cramer's rule to solve the given simultaneous equations correctly. Extracts 9.1 (a) and (b) are samples of a good response from one of the candidates.

Extract 9.1 (a)



9. (a) Soln.

$f(m) = m^2 - 4m - K \qquad K = \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

$N = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$

$f(N) = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}^2 - 4\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

$f(N) = \begin{pmatrix} 19 & 15 \\ 9 & 16 \end{pmatrix} - \begin{pmatrix} 8 & 20 \\ 12 & 4 \end{pmatrix} - \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

$$= \begin{pmatrix} 11 & -5 \\ -3 & 12 \end{pmatrix} - \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$$

9(a) $\therefore f(N) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

In Extract 9.1 (a), the candidate made correct substitution of the values of k and N in the given equation and managed to obtain the required matrix for $f(N)$.

Extract 9.1 (b)

9 (b) Soln.

$5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7$

$$\begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \\ 7 \end{pmatrix}$$

let $A = \begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}$

$$|A| = 5\begin{vmatrix} -8 & -1 \\ 2 & -6 \end{vmatrix} + 7\begin{vmatrix} 6 & -1 \\ 3 & -6 \end{vmatrix} + 1\begin{vmatrix} 6 & -8 \\ 3 & 2 \end{vmatrix}$$

$|A| = 55$

9 (b) Soln.

$5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7$

$$\begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \\ 7 \end{pmatrix}$$

let $A = \begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}$

$$|A| = 5\begin{vmatrix} -8 & -1 \\ 2 & -6 \end{vmatrix} + 7\begin{vmatrix} 6 & -1 \\ 3 & -6 \end{vmatrix} + 1\begin{vmatrix} 6 & -8 \\ 3 & 2 \end{vmatrix}$$

$$|A| = 55$$

$$\Delta x = \begin{pmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{pmatrix}$$

$$|\Delta x| = 11\begin{vmatrix} -8 & -1 \\ 2 & -6 \end{vmatrix} + 7\begin{vmatrix} 15 & -1 \\ 7 & -6 \end{vmatrix} + 1\begin{vmatrix} 15 & -8 \\ 7 & 2 \end{vmatrix}$$

$$|\Delta x| = 55$$

9 (b)

$$\Delta y = \begin{pmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{pmatrix}$$

$$|\Delta y| = 5\begin{vmatrix} 15 & -1 \\ 7 & -6 \end{vmatrix} - 11\begin{vmatrix} 6 & -1 \\ 3 & -6 \end{vmatrix} + 1\begin{vmatrix} 6 & 15 \\ 3 & 7 \end{vmatrix}$$

$$|\Delta y| = -55$$

$$\Delta z = \begin{pmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{pmatrix}$$

$$|\Delta z| = 5\begin{vmatrix} -8 & 15 \\ 2 & 7 \end{vmatrix} + 7\begin{vmatrix} 6 & 15 \\ 3 & 7 \end{vmatrix} + 11\begin{vmatrix} 6 & -8 \\ 3 & 2 \end{vmatrix}$$

$$\Delta z = -55$$

$$x = \frac{|\Delta x|}{|A|} = \frac{55}{55} = 1$$

$$y = \frac{|\Delta y|}{|A|} = \frac{-55}{55} = -1$$

$$z = \frac{|\Delta z|}{|A|} = \frac{-55}{55} = -1$$

$\therefore$ $x = 1$, $y = -1$ and $z = -1$.

In Extract 9.1 (b), the candidate correctly showed all the important steps of solving simultaneous equations by Cramer's rule.

On the other hand, the candidates who failed to answer part (a) of this question had inadequate knowledge and skills in performing operations with matrices. They were unable to multiply matrices, perform scalar multiplication and subtract matrices. This case is illustrated in Extract 9.2 (a).

In part (b), some of the candidates were unable to obtain the required solution because they were unable to correctly write the given equations in matrix form $(AX = B)$ and compute the denominator and numerator determinants; that is, $|A|$, $|A_x|, |A_y|$ and $|A_z|$. The analysis shows that, there were some errors in the calculations of the determinants and also some errors in replacing the respective *x*, *y* and *z* coefficients in matrix *A* with the elements of matrix *B*. It was also noted that, other candidates lacked knowledge of finding determinants of three by three matrices, see Extract 9.2 (b). Furthermore, it was noted that, some candidates used the inverse matrix method to find the solution, which was contrary to the requirement of the question.

Extract 9.2 (a)

9B $\quad m^2 - 4m - k$

$$\begin{pmatrix}2&5\\3&1\end{pmatrix}^2 - 4\begin{pmatrix}2&5\\3&1\end{pmatrix} - \begin{pmatrix}11&-5\\-4&12\end{pmatrix}$$

$$\begin{pmatrix}2&5\\3&1\end{pmatrix}\begin{pmatrix}2&5\\3&1\end{pmatrix}$$

$$\begin{pmatrix}4&6\\15&1\end{pmatrix} - 4\begin{pmatrix}2&5\\3&1\end{pmatrix} - \begin{pmatrix}11&-5\\-4&12\end{pmatrix}$$

$$\begin{pmatrix}4&6\\15&1\end{pmatrix} - \begin{pmatrix}8&20\\12&4\end{pmatrix} - \begin{pmatrix}11&-5\\-4&12\end{pmatrix}$$

In Extract 9.2 (a), the candidate failed to carry out the matrix multiplication operation, which led to incorrect final answer of f(N).

Extract 9.2 (b)

9b) Soln.

$$\begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \\ 7 \end{pmatrix}.$$

First, required $|R|$.

let $R = \begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Then, find the determinant.

$$5\begin{pmatrix} -8 & -1 \\ 2 & -6 \end{pmatrix} = 5(+48+^{+}2).$$

$$= 250$$

$$-7\begin{pmatrix} 6 & -1 \\ 3 & -6 \end{pmatrix}$$

$$= -7(-36+^{+}3)$$

$$= 231.$$

$$1\begin{pmatrix} 6 & -8 \\ 3 & 2 \end{pmatrix}$$

$$1(12 + ^{+}24)$$

$$= 36.$$

$$|R| = 250+231-36$$

$$|R| = 445.$$

Now required.

$$R_x = \begin{pmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & +2 & -6 \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix} = \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}$$

9b)

$$R_x = 11\begin{pmatrix} -8 & -1 \\ 2 & -6 \end{pmatrix}$$

$$= 11(48 + {}^{+}2)$$

$$= 11 \times 50$$

$$= 550$$

$$R_y = \begin{pmatrix} 5 & 11 & +1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -7 \\ -8 \\ 2 \end{pmatrix}.$$

$$R_y = 11\begin{pmatrix} 6 & -1 \\ 3 & -6 \end{pmatrix}$$

$$= 11(-36 + {}^{+}3)$$

$$= 11 \times 33$$

$$= 363$$

$$R_z = \begin{pmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & +2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}.$$

$$R_z = 11\begin{pmatrix} 6 & -8 \\ 3 & +2 \end{pmatrix}$$

$$R_z = 11(12 + {}^{+}24)$$

$$= 11 \times 36$$

$$= 396$$

Now

$$R = \frac{R_x}{|R|}.$$

9b) $R_1 = \frac{550}{445} = \frac{110}{89}$.

also $R_2 = \frac{R_y}{|R|}$

$= \frac{363}{445} = 121$

$R_3 = \frac{R_z}{|R|}$.

$= \frac{396}{445}$.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 110/89 \\ 363/445 \\ 396/445 \end{pmatrix}$$

$x = \frac{110}{89}$.

$y = \frac{363}{445}$

$z = \frac{396}{445}$.

In Extract 9.2 (b), the candidate wrongly computed the determinants, hence ended up with incorrect solution.

2.10 Question 10: Linear Programming

This question had two parts (a) and (b). In part (a), the candidates were required to draw the graph and list the corner points of the feasible region for the inequalities $2y < 4x,\ x \leq 6,\ \ y > 2$ and $2x + 3y \leq 30$. In part (b), they were required to (i) represent the linear programming problem graphically and (ii) determine the minimum and maximum profit obtained by Fruits Beverage Company, if the daily profit in the business was given by the objective function $f(x, y) = 250x + 350y - 2200$ and the constraints: $x + y \geq 5.5,\ \ 4x + 2y \geq 16$ and $x + 2.5y \geq 9$.

The analysis of data shows that, 25,500 (95.2%) candidates attempted this question, of which 46.3 percent scored from 6 to 10 marks, 29.7 percent scored from 3.5 to 5.5 marks and 24 percent scored from 0 to 3 marks. The statistics show that, this was the second best performed question.

The analysis of candidates' responses shows that, most of the candidates had adequate knowledge of Linear Programming. Majority of them obtained the correct solution as illustrated in Extracts 10.1 (a) and (b). They were able to draw the graphs correctly in both parts, which enabled them to identify the correct corner points of the feasible region and the corresponding values of the objective function.

Extract 10.1 (a)

10. a/. i) Given.

$2y \leq 4x\ ,\ x \leq 6,\ y \geq 2,\ 2x+3y \leq 30$

Let $2y = 4x$.

$y = 2x$.

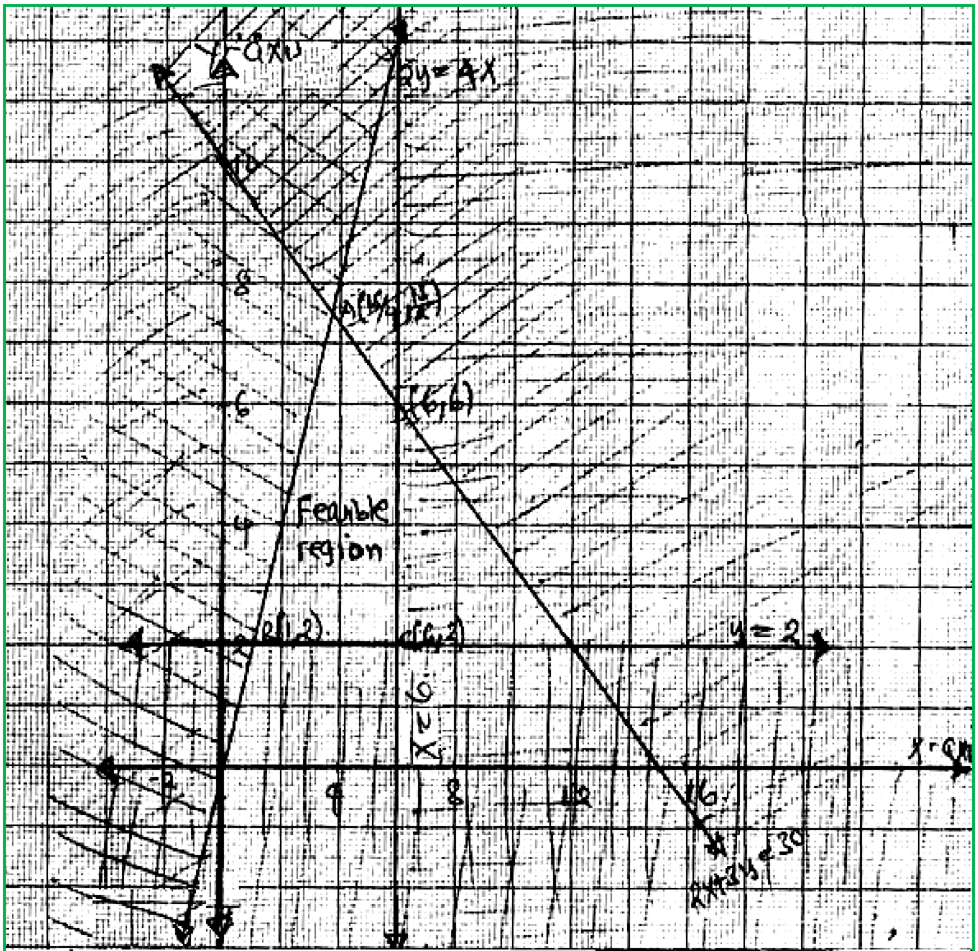
x- and y-intercept for $y=2x$.

x	0	0
y	0	0

Let $x=6$, $y=2$. and $2x+3y = 30$.

x-and y-intercepts of $2x+3y=30$.

x	0	15
y	10	0



ii/ From the graph drawn.
corner points A, B, C and D are
A = (15/4, 15/2).
B = (1, 2)
C = (6, 2)
D = (6, 6).

Extract 10.1 (a) shows that, the candidate was able to represent the linear inequalities graphically, identify the feasible region and indicate the corner points as it was required.

Extract 10.1 (b)

10. b/. Given.

$x+y \geq 5.5$

$4x+2y \geq 16$

$x+2.5y \geq 9.$

objective function: $f(x,y) = 250x + 350y = 220c$

Required inequalities

$x+y \geq 5.5$

$4x+2y \geq 16.$

$x+2.5y \geq 9.$

$x \geq 0$

$y \geq 0.$

let $x+y = 5.5$

x- and y-intercept for $x+y=5.5$

x	0	5.5
y	5.5	0

let $4x+2y = 16,$

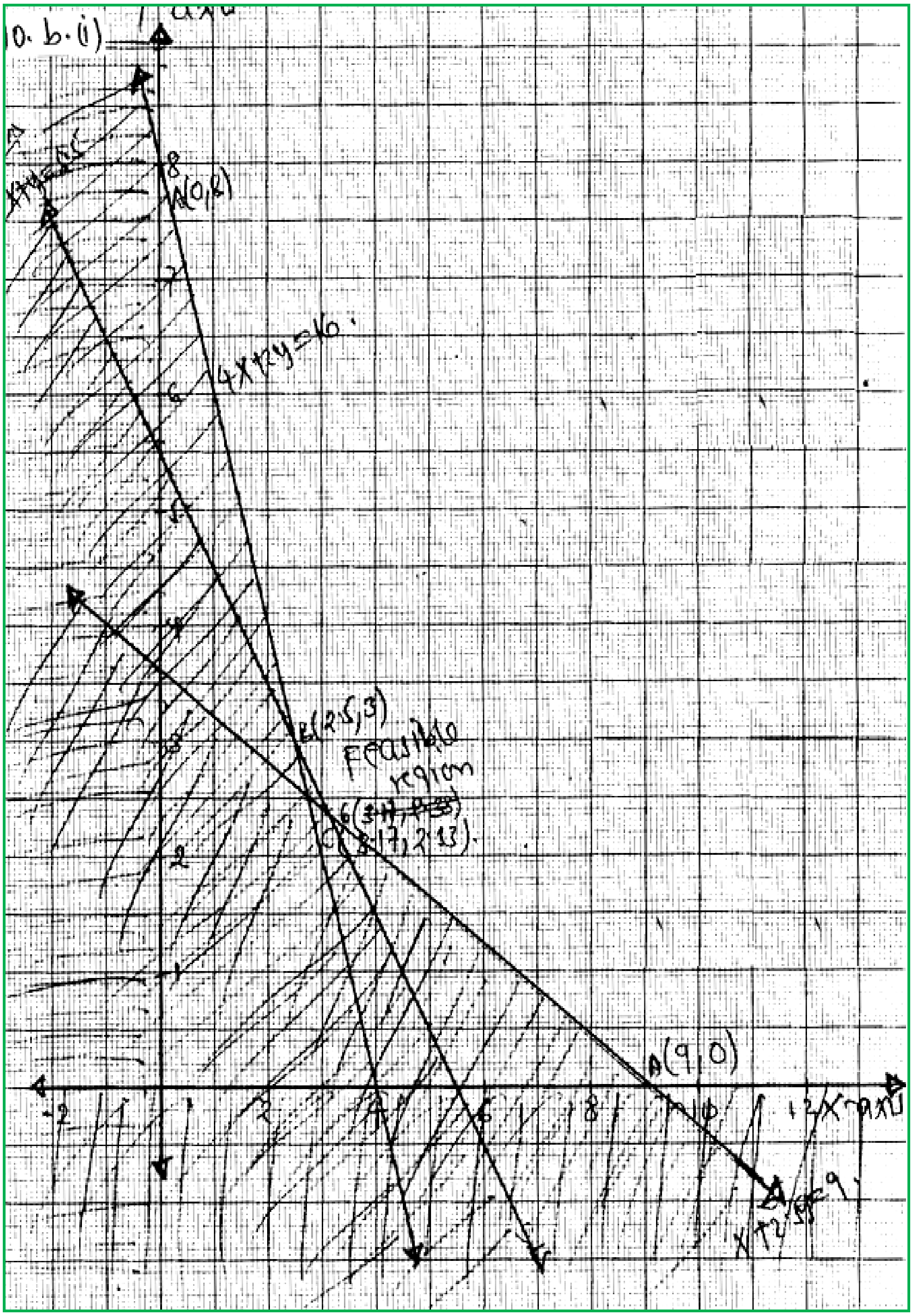
x-y- intercept. for $4x+2y=16.$

x	0	4
y	8	0

let $x+2.5y = 9.$

x and y-intercept for $x+2.5y=9.$

x	0	9.
y	3.6	0


10. b. (i)
4x+y=16
x+y=5.5
B(2.5,3)
Feasible region
A(9,0)
x+y=9
A(0,8)

ii/. From the graph plotted, corner points of the feasible region are:

A(0,8), B(2.5,3), C(3.17, 2.33), D(9,0)

consider table

Corner points	$f(x,y) = 250x + 350y - 2200$	Total.
A(0,8)	$250(0) + 350(8) - 2200$	600
B(2.5,3)	$250(2.5) + 350(3) - 2200$	−525
C(3.17, 2.33)	$250(3.17) + 350(2.33) - 2200$	−592
D(9,0)	$250(9) + 350(0) - 2200$	50

(ii) Minimum profit is 50, and maximum profit is 600

Extract 10.1 (b) shows that, the candidate was able to represent the linear inequalities graphically and determine the maximum and minimum profit of the company as it was asked.

Conversely, the candidates, who scored low marks in this question, lacked knowledge and skills for drawing the graphs of linear inequalities. Some of the candidates were unable to use correct *x* and *y* intercepts to draw the graphs of the corresponding linear graphs. Other candidates were able to draw the graphs of the linear inequalities but were unable to shade the region represented by the inequalities and find the corner points of the feasible region. An example of a poor response is illustrated in Extract 10.2.

Extract 10.2

10(a) i/ Corresponding graph.

$2y \leq 4x$, $x < 6$, $y > 2$ and $2x + 3y \leq 30$

$2y \leq 4x$

$\frac{2y}{2} = \frac{4x}{2}$

$y = 2x$

X	4	5	6
y=2x	8	10	12

$x \le 6$

$x \le 6$ or $x = 6$

$y > 2$

$2x + 3y \le 30$

$2x + 3y = 30$

$2x - 2x + 3y = 30 - 2x$

$\frac{3y}{} = \frac{30 - 2x}{}$

10a) i/ Corresponding graph.

$2y \le 4x$, $x \le 6$, $y > 2$ and $2x + 3y \le 30$

$2y \le 4x$

$\frac{2y}{2} = \frac{4x}{2}$

$y = 2x$

X	4	5	6
y=2x	8	10	12

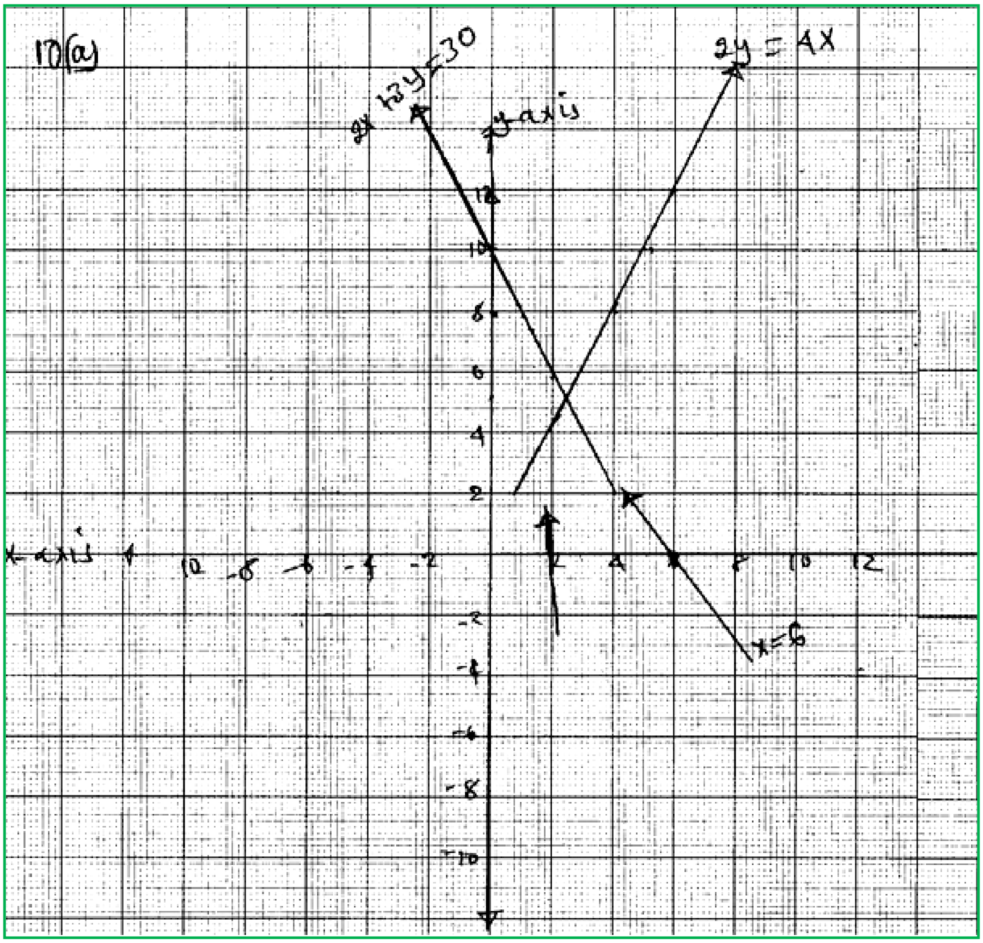
$x \le 6$

$x \le 6$ or $x = 6$

$y > 2$

$2x + 3y \le 30$

$2x + 3y = 30$



In Extract 10.2, the candidate drew incorrect lines, hence failed to get the feasible region.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE TOPIC - WISE

In Basic Applied Mathematics, ten (10) topics were examined. The analysis shows that, the candidates had good performance in the topics of Statistics, Matrices and Linear Programming; average performance in the topics of Probability, Functions and Calculating Devices and weak performance in the topics of Trigonometry, Integration, differentiation and Algebra. The performance was highest in the topic of Statistics and lowest in Algebra. The percentage of candidates who passed in each topic is shown in the following Figure and in the Appendix.

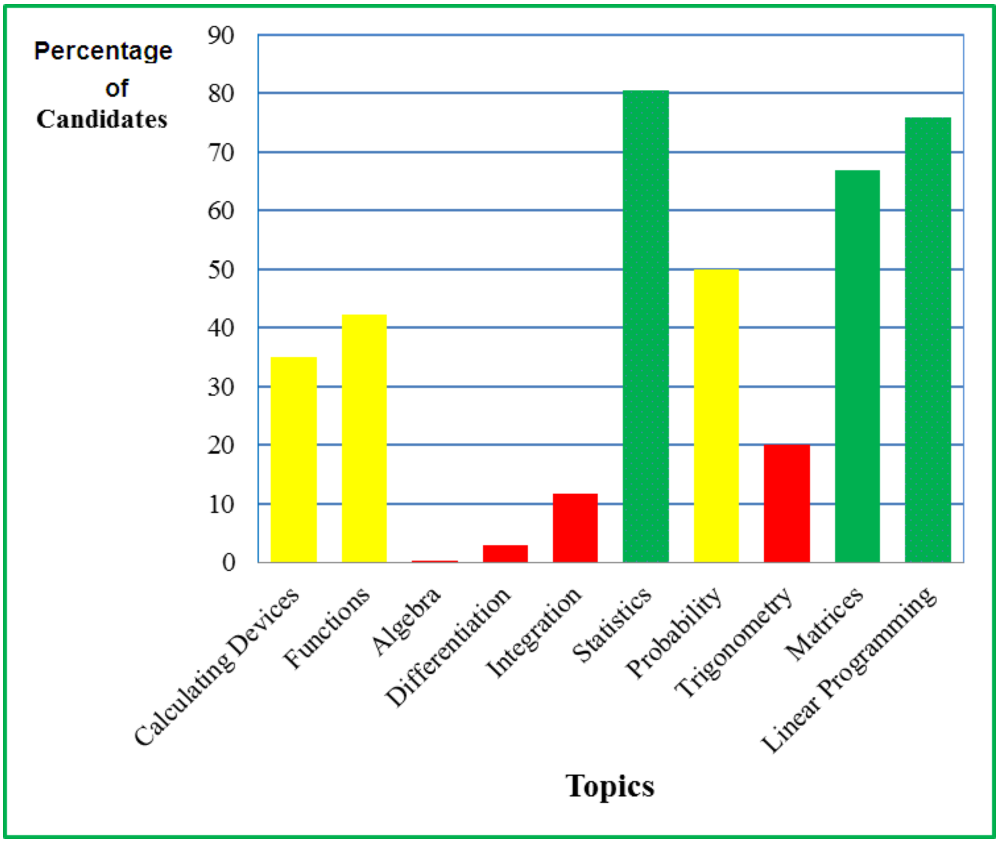


Figure: The percentages of candidates who passed in each topic.

It is evident from the Appendix and the Figure that, 3 out of the 10 topics that were examined had average performance. The main reasons behind the average performance in the topics of Probability, Functions and Calculating Devices were that, the candidates did not have adequate knowledge and skills on: drawing and using probability tree diagrams, applying probability concepts and formulae, finding domains, range and using calculators to do computations.

It is also evident from the Appendix and the Figure that, 4 out of the 10 topics that were tested had weak performance. The reasons behind the weak performance includes: lack of knowledge of implicit differentiation; inability to apply differentiation concepts in solving problems; inability to recall derivatives and anti-derivatives of common functions; inability to apply knowledge integration in solving problems and lack of knowledge on the basic trigonometric identities.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The general performance of the candidates in Basic Applied Mathematics for the ACSEE 2016 was average. The analysis of the candidates' performance showed that the candidates performed well in questions 6, 9 and 10; averagely in questions 1, 2 and 7 and had weak performance in questions 3, 4, 5 and 8.

The analysis of candidates' performance topic-wise revealed that, the candidates had good performance in the topics of Statistics, Matrices and Linear Programming; average performance in the topics of Probability, Functions and Calculating Devices and weak performance in the topics of Trigonometry, Integration, Differentiation and Algebra, see the Appendix.

The reasons behind the weak performance includes: lack of knowledge of implicit differentiation; inability to apply differentiation concepts in solving problems; inability to recall common derivatives and anti-derivatives; inability to apply knowledge of integration in solving problems and lack of knowledge on the basic trigonometric identities.

4.2 Recommendations

In order to improve future candidates' performance in this subject it is recommended that; students should put more effort on the topics of Probability, Functions and Calculating Devices which had average performance and on the topics of Trigonometry, Integration, Differentiation and Algebra which had weak performance. The candidates are advised to study these topics in order to have thorough understanding of all the concepts covered in these topics. They are also advised to do many exercises in order to be able to apply the concepts correctly.

Furthermore, the candidates are advised to show all necessary workings when answering the questions since marking of National Examination items does not only rely on the final answer, but on the entire work of the candidate.

Finally, the Ministry of Education, Science and Technology is advised to make use of this report to influence their policies and operations and to make a follow up on the teaching and learning in order to raise the standard of performance in this subject. In particular, the Ministry is advised to set a policy that will ensure that the subject is taken seriously as any other subject in advanced level since Basic Applied Mathematics will enable the students to master the content in their subject combinations.

APPENDIX

S/N	Topic	Number of Questions	Percentage of candidates who Scored 35% or more	Remarks
1	Statistics	1	80.5	Good
2	Linear Programming	1	76.0	Good
3	Matrices	1	66.7	Good
4	Probability	1	50.0	Average
5	Functions	1	42.4	Average
6	Calculating Devices	1	35.0	Average
7	Trigonometry	1	20.1	Poor
8	Integration	1	11.8	Poor
9	Differentiation	1	3.0	Poor
10	Algebra	1	0.1	Poor
Overall Average Performance			38.56	Average

In this table, green, yellow and red colours indicate good, average and poor performance respectively.