

**THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**



**EXAMINERS' REPORT ON THE PERFORMANCE  
OF CANDIDATES**

**ACSEE, 2014**

**141 BASIC APPLIED MATHEMATICS**

**THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**



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ACSEE, 2014**

**141 BASIC APPLIED MATHEMATICS**

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## Table of Contents

FOREWORD .....	iv
1.0 INTRODUCTION .....	1
2.0 ANALYSIS ON INDIVIDUAL QUESTIONS .....	2
2.1 Question 1: Calculating Devices .....	2
2.2 Question 2: Functions .....	6
2.3 Question 3: Algebra .....	14
2.4 Question 4: Differentiation .....	21
2.5 Question 5: Integration .....	31
2.6 Question 6: Statistics .....	36
2.7 Question 7: Probability .....	44
2.8 Question 8: Trigonometry .....	52
2.9 Question 9: Matrices .....	59
2.10 Question 10: Linear Programming .....	66
3.0 SUMMARY OF THE CANDIDATES' PERFORMANCE .....	72
4.0 CONCLUSION AND RECOMMENDATIONS .....	76
4.1 CONCLUSION .....	76
4.2 RECOMMENDATION .....	78
APPENDIX .....	80
The Performance of Candidates in Basic Applied Mathematics Topic wise .....	80



## FOREWORD

The National Examinations Council of Tanzania is delighted to issue this Examiners' Report on the Performance of Candidates in Basic Applied Mathematics ACSEE 2014. This formal account was organized to deliver feedback to students, teachers, parents, policy makers and other education stakeholders on how the candidates responded to the examination questions. Basically, candidates' responses to the examination questions is one of the gauges that determine if the candidates were competent or incompetent in comprehending the Advanced Level Basic Applied Mathematics syllabus.

The analysis of candidates' responses indicates that, there were several factors which contributed to good performance of the candidates in the examination questions. Such factors include; the ability to use the appropriate rules, formulae and principles to comprehend the requirement of the questions. On the other hand, lack of understanding of the required concepts, failure to identify the requirements of the questions and incompetence in some of the topics contributed to slightly poor performance of some candidates.

The feedback given in this report will support various education stakeholders to find the proper measures to be taken in order to improve candidates' performance on Basic Applied Mathematics in forthcoming examinations administered by the Examinations Council.

The Council will highly appreciate remarks from scholars, teachers and the public that can be used to improve future Examiners' reports. Therefore, having a room for improvement, the Council will always address the suggestions and recommendations which are brought forward.

Lastly, the Council would like to acknowledge all the Examination Officers, Examiners and all others who contributed in the preparation of this report.



Charles E. Msonde

**EXECUTIVE SECRETAR**

## **1.0 INTRODUCTION**

This report provides an analysis on the performance of the candidates who sat for ACSEE 2014 Basic Applied Mathematics (BAM) examination. The report analyses the candidates' performance on each question basing on challenges experienced by candidates as observed during marking exercise. The purpose of the report is to enable education stakeholders especially teachers and students to learn from the observed challenges so as to improve the performance for the forthcoming examinations.

The ACSEE 2014 BAM paper consisted of ten (10) compulsory questions set according to the 2010 revised syllabus and its corresponding 2011 examination format. The scores for each question were ten (10) marks and the duration of the paper was three (3) hours.

In 2014, a total of 14,742 candidates sat for the BAM paper out of which 77.39 percent passed the examination; while 22.61 percent failed. In 2013, total of 15,849 candidates sat for the paper out of which 49.48 percent passed meanwhile 50.61 percent failed. As compared to last year, the performance of candidates in 2014 has increased by 45.3 %.

The analysis for each question is presented in the next section. It comprises of a brief description of demand of the question and the performance of the candidates. The possible factors that attributed to the good or poor performance of the candidates are pointed out, illustrated by the samples of the candidate's good responses and candidate's poor responses. In addition, the most common mistakes observed during marking are pointed out so as to help teachers and students to learn and rectify them.

The analysis has also shown a summary of the overall performance in tabular and graphical form where one can compare the candidates' performance more easily. In overall performance, three categories namely poor, average and good are identified to show the candidates' performance in each question including the topic tested. The analysis is concluded by putting the reasons for the performance and its recommendations.

## 2.0 ANALYSIS ON INDIVIDUAL QUESTIONS

### 2.1 Question 1: Calculating Devices

This question had three parts; (a), (b) and (c). In part (a) the candidates were required to solve the equation  $\ln(2x + 3) - 3 = \ln(x - 5)$  and write the answer correct to 2 decimal places. In part (b) the candidates were required to find the value of  $t$  correct to 4 decimal places given that  $3^{2t} = 5^{t+1}$ . In part (c) the candidates were required to solve the quadratic equation  $x^2 + 9x - 2.718282 = 0$ .

The question was attempted by 99.3 percent of the candidates of which 7,394 equivalent to 50.2% candidates scored 3 marks or above, indicating the good performance of the question. However, 7,347 (49.8%) candidates had their score in the range 0 to 2.5 marks, 4,242 (28.8%) in the range 3.0 to 4.5 marks and 3,152 (21.4%) in the range 5.0 to 10 marks.

This report has observed that candidates who attempted part (a) applied the laws of logarithms correctly and gave the answer in the required decimal places indicating that they had a good knowledge in approximation. In part (b) the candidates used the application of natural logarithm; although initially they overlooked to put  $t + 1$  in brackets then the next step had the correct approach. Extract 1.1 illustrates the case.

### Extract 1.1

a)  $\ln(2x+3) - 3 = \ln(x-5)$   
 $\ln(2x+3) - \ln(x-5) = 3$   
 $\ln\left(\frac{2x+3}{x-5}\right) = 3$   
 $\frac{2x+3}{x-5} = e^3$   
 $\frac{2x+3}{x-5} = 20.0855$   
 $2x+3 = 20.0855(x-5)$   
 $2x+3 = 20.0855x - 100.43$   
 $103.43 = 18.0855x$   
 $x = 5.7189 \approx \text{into two decimal places it will be } 5.72$   
 $\therefore 5.72$

b)  $3^{2t} = 5^{t+1}$   
soln  
 $3^{2t} = 5^{t+1}$   
Apply  $\ln$  both sides  
 $2t \ln 3 = (t+1) \ln 5$   
 $\frac{2t}{t+1} = \frac{\ln 5}{\ln 3}$   
 $2t = 1.465(t+1)$   
 $2t = 1.465t + 1.465$   
 $0.535t = 1.465$   
 $t = 2.738182 \approx 2.7382$   
In to 4 decimal places = 2.7382

Extract 1.1 demonstrates the best response from a candidate showing he/she was competent in using both common logarithms and natural logarithms.

The candidates who failed part (a) and (b) lacked the knowledge of laws of logarithms completely and they made very early approximation that led to wrong final answer of the required decimal places. Also they could not use brackets and eventually they failed to proceed to the next step correctly. Extract 1.2 illustrates the typical case.

### Extract 1.2

1	@ $\sqrt[n]{(2x+3)-3} = \sqrt[n]{(x-5)}$
	$6x-9 = x-5$
	$6x-9+x+5=0$
	$5x-4=0$
	$5x = \frac{4}{5}$
	$x = 0.8$
	$x = 0.80$
	(b) $3^{2t} = 5^{t+1}$
	$3^{2t} = 3^{1.5(t+1)}$
	$2t = 1.5(t+1)$
	$2t = 1.5t + 1.5$
	$-1.5 = 1.5t - 2t$
	$-1.5 = \frac{0.5t}{0.5}$
	$t = -3.0000$

Extract 1.2 illustrates the work of a candidate in part (a) and (b) who had no knowledge of logarithms rule. He/She wrote a guesswork which was relatively illogical.

In part (c) which was an open question the candidates applied the calculator directly to solve or used the general formula for quadratic equations and wrote the solution correct to any number of decimal places.

Extract 1.3 shows a good solution of the candidate who applied the calculator to solve the given quadratic equation.

### Extract 1.3

01	c) $x^2 + 9x - 2.718282 = 0$
	By using the calculator
	$x^2 + 9x - 2.718282 = 0$
	$x_1 = -0.3129 \quad 0.2925$
	$x_2 = -8.687 \quad -9.2925$
	$\therefore$ The Values of $x$ are $0.2925$
	and $-9.2925$ .

Extract 1.3 indicates a good response of a candidate in part (c) showing the correct use of calculators to solve the quadratic problem.

Those who scored zero in part (c) lacked the important skills in using calculators to solve the quadratic equation which involves a decimal number on the independent term of  $x$ . For example, some did not know the formula while others could not even be able to apply the calculator. Extract 1.4 illustrates this case.

### Extract 1.4

1	(c) $x^2 + 9x - 2.718282 = 0$
	$x^2 + 9x = 0$
	$x^2 + 2.718282 = 0$
	$x^2 + 9x - 2.718282 = 0$
	$x = 9 - 2.718282 = 0$
	$x = 9 - 2.718282$
	$x = 6.2817$

Extract 1.4 illustrates a sample of response from a candidate who performed poorly because he/she lacked knowledge of using calculator to get the correct answer.

## 2.2 Question 2: Functions

This question had three parts namely (a), (b) and (c). In part (a) the candidates were required to find  $f\left(\frac{1}{8}\right)$ ,  $f(13)$  and  $f(-32)$  given that:

$$f(x) = \begin{cases} 2 & \text{for } x < -2 \\ x^2 + 2x & \text{for } -2 \leq x < 1 \\ 4 - x & \text{for } x \geq 1 \end{cases}$$

In part (b) the candidates were given the rational function  $f(x) = \frac{2x-1}{x+1}$  and

they were required to find asymptotes, sketch the graph then state domain and range of the function. In part 2(c) the candidates were required to find the set of values (x, y) that satisfies the equations  $x + y = 3$  and  $xy = 2$ .

The question was attempted by 99.3 percent of the candidates of which 7,212 equivalent to 48.9% candidates scored 3 marks or above, showing that the question was averagely performed. Furthermore, 7,530 (51.1%) candidates scored in the range 0 to 2.5 marks, 3,131 (21.2%) in the range 3.0 to 4.5 marks and 4,081 (27.7%) in the range 5.0 to 10 marks.

In part (a) the candidates who identified the appropriate indicator functions of the intervals managed to make the correct substitution of the respective function. This is illustrated by the extract 2.1.

## Extract 2.1

2(a) Given 
$$f(x) = \begin{cases} 2 & \text{for } x < -2 \\ x^2 + 2x & \text{for } -2 \leq x < 1 \\ 4 - x & \text{for } x \geq 1 \end{cases}$$

Then,

(i)  $f\left(\frac{1}{8}\right) = x^2 + 2x$ , where  $x = \frac{1}{8}$ ;  
$$= \left(\frac{1}{8}\right)^2 + 2\left(\frac{1}{8}\right)$$
$$f\left(\frac{1}{8}\right) = \frac{1}{64} + \frac{2}{8} = \underline{\underline{\frac{17}{64}}}$$

(ii)  $f(13) = 4 - x$   
where  $x = 13$   
Then,  $f(13) = 4 - 13$ 
$$= -9$$
$$f(13) = \underline{\underline{-9}}$$

(iii)  $f(-32) = 2$   
because  $-32 < -2$  and  $f(-32)$  corresponds to  $f(x) = 2$  for  $x < -2$ .

Extract 2.1 shows the good response in part (a) from a candidate's work who identified the indicator functions and he/she made a correct substitution.



However, the candidates who failed totally in part (a) could not identify the appropriate indicator functions of intervals which suite the domain on the given function. In connection to that, some candidates made trial and error during substitution of the whole function but ended with wrong answer. This is illustrated by Extract 2.2.

### Extract 2.2

Q.	(a). Given.
	$f(x) = \begin{cases} 2 & \text{for } x < -2 \\ x^2 + 2x & \text{for } -2 \leq x < 1 \\ 4 - x & \text{for } x \geq 1 \end{cases}$
	(i) $f\left(\frac{1}{8}\right)$
	$4 - x$
	$f(x) = 4 - x \geq 1$
	$f\left(\frac{1}{8}\right) = 4 - \frac{1}{8}$
	$f\left(\frac{1}{8}\right) = 3\frac{7}{8}$
	$f\left(\frac{1}{8}\right) = 3\frac{7}{8}$
	(ii) $f(13)$
	$f(13) = 2$
	(iii) $f(-32)$
	$x^2 + 2x = -2 \leq x < 1$
	$f(-32) = -1088$
	(b). Given $f(x) = \frac{2x-1}{x-1}$
	soln.
	$f(x) = \frac{2x-1}{x-1}$
	$f(x) = y$

Extract 2.2 illustrates the response of the candidate who lacked competence and knowledge to solve problems from the topic of functions. The candidate failed to identify the indicator functions

of the step function and failed to calculate both vertical and horizontal asymptotes.

Further analysis shows that, in part (b) the candidates who scored high marks had good knowledge of asymptotes and intercepts, so they managed to sketch the graph and stated correctly the domain and range of the function. This is illustrated by extract 2.3.

### Extract 2.3

2(b) Given  $f(x) = \frac{2x-1}{x+1}$

Soln:

(i) Let  $f(x) = y$   
Then,  
$$y = \frac{2x-1}{x+1}$$

For vertical asymptote (V.A)  
Let  $x+1=0$   
 $x = 0-1$   
 $x = -1$   
Then, V.A =  $-1$

For Horizontal asymptote (H.A)  
From;  
$$y = \frac{2x-1}{x+1}$$

Divide by "x" to each term in the right hand side

$$y = \frac{2x/\cancel{x} - \cancel{1}/\cancel{x}}{x/\cancel{x} + \cancel{1}/\cancel{x}}$$

$$y = \frac{2 - 1/\cancel{x}}{1 + 1/\cancel{x}}$$

Then, when  $x$  is very large ;

$$y = \frac{2 - 0}{1 + 0}$$

$$y = \frac{2}{1}$$

$$y = 2$$

therefore ;

Horizontal asymptote, H.A = 2.

From

$$y = \frac{2x-1}{x+1}$$

y-Intercept,  $x=0$

$$y = \frac{2(0)-1}{0+1}$$

$$= \frac{0-1}{1}$$

$$\underline{\underline{y = -1}}$$

x-Intercept,  $y=0$

$$0 = \frac{2x-1}{x+1}$$

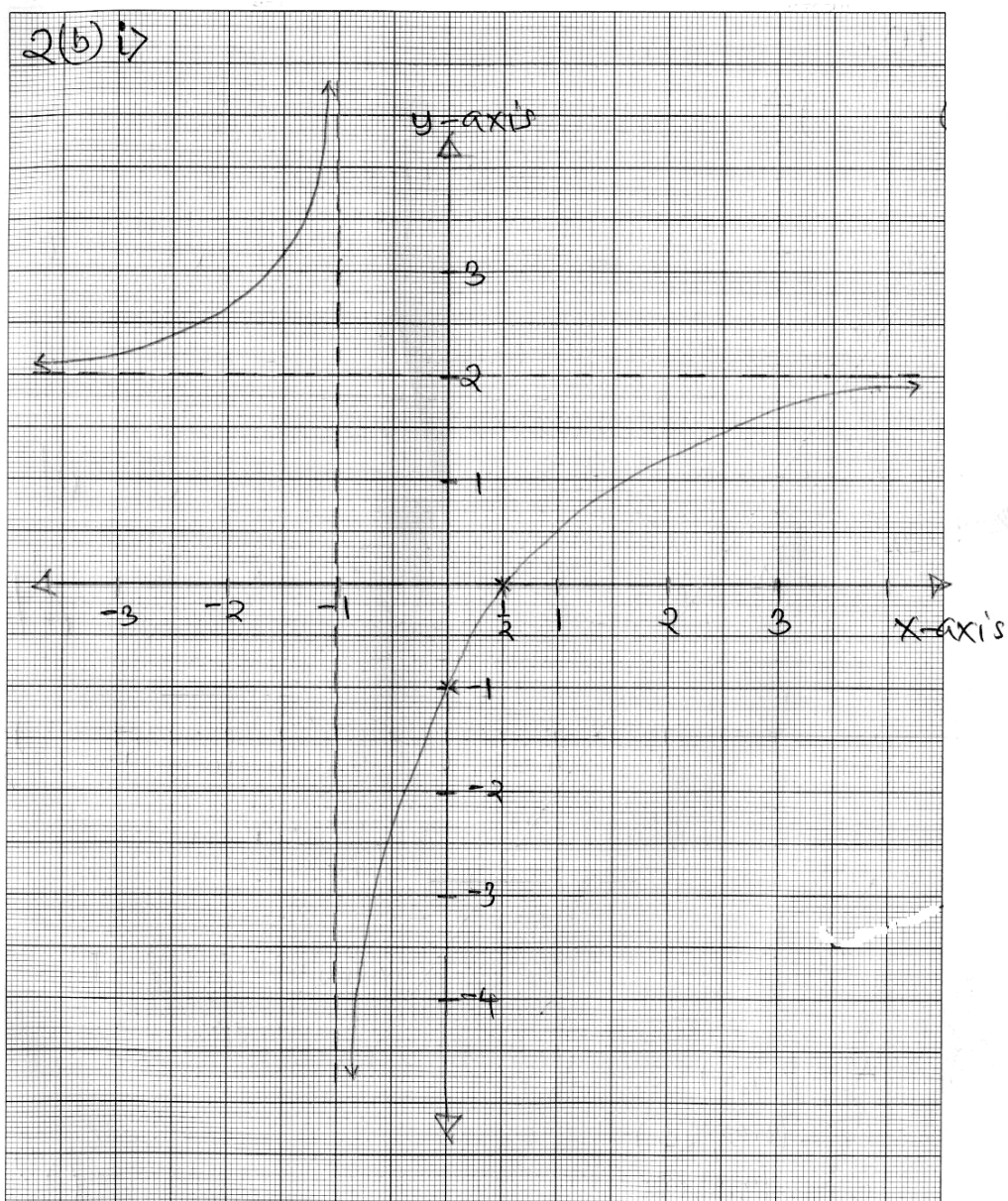
$$0 = 2x - 1$$

$$2x = 1$$

$$\underline{\underline{x = \frac{1}{2}}}$$

(ii) From the graph sketched ;  
Domain =  $\{x : x \neq -1\}$

$$\text{Range} = \{y : y \neq 2\}$$



Extract 2.3 illustrates one of the good answer of a candidate in part (b) (i) and (ii) of the question. He/she managed to find asymptotes and intercepts and sketched the graph. Finally, the domain and range of the function were stated.

Similarly, the candidates who attempted part (c) correctly had the required knowledge of the topic. They solved the given equations simultaneously

and wrote the set of the required values. This response is illustrated by Extract 2.4.

#### Extract 2.4

$$\begin{cases} x + y = 3 & \text{--- (i)} \\ xy = 2 & \text{--- (ii)} \end{cases}$$

From eqn (ii)

$$xy = 2$$
$$y = \frac{2}{x}$$

Substituting  $y = \frac{2}{x}$  in eqn (i)

$$x + y = 3$$
$$x + \frac{2}{x} = 3$$
$$\frac{x}{1} + \frac{2}{x} = 3$$
$$\frac{x^2 + 2}{x} = 3$$
$$x^2 + 2 = 3x$$
$$x^2 - 3x + 2 = 0$$

From Calculator ;

$$x = 2 \text{ or } x = 1$$

$$\begin{array}{l}
 \text{Then, from eqn (i)} \\
 \text{When } x = 2, \\
 x + y = 3 \\
 2 + y = 3 \\
 y = 3 - 2 = 1.
 \end{array}$$

$$\begin{array}{l}
 \text{Also, when } x = 1 \\
 x + y = 3 \\
 1 + y = 3 \\
 y = 3 - 1 \\
 \underline{y = 2} \\
 \text{Then, when } x = 1, y = 2 \text{ and when} \\
 x = 2, y = 1 \\
 \text{Therefore ;} \\
 (x, y) = (1, 2) \text{ or } (2, 1).
 \end{array}$$

Extract 2.4 indicates the good answer in part (c) (ii) showing how the candidate obtained the actual values of  $x$  and  $y$  by substitution method. This shows the candidate's competence to solve system of simultaneous equations.

### 2.3 Question 3: Algebra

This question had parts (a) and (b). In part (a) there were three subparts where the candidates were required to; (i) write the first four terms of the progression, (ii) find the 20<sup>th</sup> term and (iii) find the sum to infinity of the series, given that the first term of a geometric progression was 2 and its common ratio was  $\frac{1}{2}$ . In part (b) the candidates were required to find the number of singers in the group given that the ages of certain group of singers form arithmetic progression whose common difference is 4 and the

youngest singer in the group is 8 years old and that the sum of the ages of all singers in the group is 168.

The question was attempted by 99.3 percent of the candidates of which 10,703 equivalents to 72.6% scored 3 marks or above showing a good performance of the question. The analysis also shows that 4,039 (27.4%) candidates scored in the range 0 to 2.5 marks, 2,344 (15.9%) in the range 3.0 to 4.5 marks and 8,359 (56.7%) in the range 5.0 to 10 marks.

The candidates who scored high marks in part (a) managed to respond well to all parts. For example in (a) (i) they used the appropriate formula  $G_n = G_1 r^{n-1}$  to calculate the first four terms of the progression. Likewise in part (a) (ii) they managed to calculate the 20<sup>th</sup> term while in part (a) (iii) the sum to infinity of the series was found by using the correct procedure. Extracts 3.1 illustrates a sample response from one of the candidates who managed to perform part (a) correctly.

### Extract 3.1

B. (a)	Data.
	$G_1 = 2.$
	$r = \frac{1}{2}.$
	i) To write down the first four terms.
	<u>Solution.</u>
	$G_n = G_1 r^{n-1}$
	for 1 <sup>st</sup> term
	$G_1 = 2.$
	for 2 <sup>nd</sup> term.
	$G_2 = G_1 \times \left(\frac{1}{2}\right)^{2-1}.$
	$= 2 \times \frac{1}{2} = 1.$
	for 3 <sup>rd</sup> term.
	$G_3 = G_1 \times \left(\frac{1}{2}\right)^{3-1}$
	$= 2 \times \left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}.$
	for 4 <sup>th</sup> term.
	$G_4 = G_1 \times \left(\frac{1}{2}\right)^{4-1}$
	$= 2 \times \left(\frac{1}{2}\right)^3 = 2 \times \frac{1}{8} = \frac{1}{4}.$
	$G_4 = \frac{1}{4}.$
	∴ The first four terms are 2, 1, $\frac{1}{2}$ , $\frac{1}{4}$ .



2a) (ii) To find 20<sup>th</sup> term.

Solution.

$$G_n = G_1 r^{n-1} \text{ but } G_1 = 2, r = \frac{1}{2}, n = 20$$

$$G_{20} = 2 \times \left(\frac{1}{2}\right)^{20-1} = 2 \times \left(\frac{1}{2}\right)^{19}$$

$$= 3.8 \times 10^{-6}$$

$\therefore$  The 20<sup>th</sup> term is  $3.8 \times 10^{-6}$ .

3. (a) (iii) The sum of the infinity of the series.

Solution.

$S_n$ , for  $r < 1$ .

$$S_n = \frac{G_1 (1 - r^n)}{1 - r}$$

$S_n = \frac{G_1 (1 - r^n)}{1 - \frac{1}{2}}$ , but for  $n \rightarrow \text{infinity}$   $r^n \Rightarrow 0$ .

$$S_n = \frac{G_1 (1 - 0)}{1 - \frac{1}{2}}$$

$$S_n = G_1 \div \frac{1}{2}$$

$$S_n = G_1 \times \frac{2}{1}$$

$$S_n = 2G_1, S_n = 2G_1 \text{ but } G_1 = 2.$$

$$S_n = 2 \times 2 = 4.$$

$\therefore$  The sum of the G.P. to infinity series is 4.

Extract 3.1 in part (a) illustrates the work of one of the candidate who achieved to find the first four terms, 20<sup>th</sup> term and the sum to infinity.

However, the candidates who failed part (a) some used common ratio  $r = \frac{1}{2}$  as a common difference in computation of the first four terms of the progression in part (i) something which was illogical while others used a

wrong formula  $s_n = \frac{n}{2}(n-1)d$  instead of  $G_n = G_1 r^{n-1}$  to find the 20<sup>th</sup> term in

part (ii) and also used wrong formula  $G_n = \frac{G_1 r^{n-1}}{1 - \frac{1}{2}}$  instead of  $S_\infty = \frac{G_1}{1-r}$  to

find the sum to infinity of the series in part (iii). Generally the candidates had problem in interpreting the problem into mathematical modal. This is illustrated by Extracts 3.2.

### Extract 3.2

3.0)

Q) At  $G_1 = 2$   
 $r = \frac{1}{2}$ .  
 from the formula;  $G_n = \frac{G_1 r^{n-1}}{1-r}$

i)  $\therefore G_n = \frac{G_1 r^{n-1}}{1-r}$   
 $G_1 = \frac{2(\frac{1}{2})^{2-1}}{1-\frac{1}{2}}$   
 $G_2 = \frac{2(\frac{1}{2})^1}{\frac{1}{2}}$   
 $G_2 = 2$   
 $G_3 = \frac{G_1 r^{3-1}}{1-\frac{1}{2}} = \frac{2 \times (\frac{1}{2})^2}{\frac{1}{2}} = 1$   
 $G_4 = \frac{G_1 r^{4-1}}{1-\frac{1}{2}} = \frac{2 \times (\frac{1}{2})^3}{1-\frac{1}{2}} = 0.5$   
 $G_5 = \frac{G_1 r^{5-1}}{1-\frac{1}{2}} = \frac{2 \times (\frac{1}{2})^4}{\frac{1}{2}} = 0.25$   
 $G_6 = \frac{G_1 r^{6-1}}{1-\frac{1}{2}} = \frac{2 \times (\frac{1}{2})^5}{\frac{1}{2}} = 0.125$   
 Therefore the first four terms of the progression are; 2, 1, 0.5, 0.25, 0.125.

ii) The 20<sup>th</sup> term  
 from  $G_n = \frac{G_1 r^{n-1}}{1-r} = \frac{G_1 r^{20-1}}{1-r} = \frac{2 \times (\frac{1}{2})^{19}}{\frac{1}{2}} =$   
 $G_{20} = 7.629394531 \times 10^{-6}$   
 $\therefore$  sum the 20<sup>th</sup> term  $= 7.6294 \times 10^{-6}$ .

3 (d) (ii)	2	- - - (i)
	$2 + \frac{1}{2} = 2\frac{1}{2}$	(ii)
	$2\frac{1}{2} + \frac{1}{2} = 3$	(iii)
	$3 + \frac{1}{2} = 3\frac{1}{2}$	(iv)
	$3\frac{1}{2} + \frac{1}{2} = 4$	(v)
	$4 + \frac{1}{2} = 4\frac{1}{2}$	(vi)
	$4\frac{1}{2} + \frac{1}{2} = 5$	(vii)
	$5 + \frac{1}{2} = 5\frac{1}{2}$	(viii)
	$5\frac{1}{2} + \frac{1}{2} = 6$	(ix)
	$6 + \frac{1}{2} = 6\frac{1}{2}$	(x)
	$6\frac{1}{2} + \frac{1}{2} = 7$	(xi)
	$7 + \frac{1}{2} = 7\frac{1}{2}$	(xii)

3	(c) (ii)	<u>Solution</u>
	2	- - - (i)
	$2 + \frac{1}{2} = 2\frac{1}{2}$	$= 2.5$ (ii)
	$2\frac{1}{2} + \frac{1}{2} = 3$	- - - (iii)
	$3 + \frac{1}{2} = 3\frac{1}{2}$	$= 3.5$ - - (iv)
	<del><math>3\frac{1}{2} + \frac{1}{2} = 4</math></del>	
	(1) The first four terms of progression is	
	2, $2\frac{1}{2}$ , 3, and $3\frac{1}{2}$	

$$\begin{aligned}
 S_{20} &= \frac{n}{2} (n-1) d \\
 &= \frac{20}{2} (20-1) \frac{1}{2} \\
 &= \frac{10(19) \frac{1}{2}}{190} \\
 \underline{S_{20} = 95.}
 \end{aligned}$$

3 a)

iii Sum to infinity series  $= G_n = \frac{G_1 r^{n-1}}{1-r}$

$G_\infty = \frac{G_1 r^{\infty-1}}{1-r}$

$\therefore \text{Sum to infinity} = G_\infty = \frac{G_1 r^{\infty-1}}{\frac{1}{2}}$

Extract 3.2 shows a sample of poor responses from one of the candidate who used wrong formula. The candidate showed the trial and error in the procedure and the whole work ended with unrelated solution indicating insufficiency knowledge of the topic.

In part (b), the candidates who scored good marks were able to demonstrate the application of skills underlying the topic of sequence and series to solve the given problem as it is illustrated by extract 3.3.

### Extract 3.3

3(b)	<u>Data.</u>
	$n = \text{number of singers form A.P.}$
	$d = 4, A_1 = 8.$
	$S_n = 168.$ To find $n.$
	from $S_n = \frac{n}{2} (A_1 + A_1 + d(n-1)).$ $S_n = \frac{n}{2} (2A_1 + (n-1)d)$
	$168 = \frac{n}{2} ((2 \times 8) + (n-1)4).$
	$336 = n(16 + 4n - 4)$
	$336 = n(16 - 4 + 4n), 336 = n(12 + 4n)$
	$336 = 12n + 4n^2.$
	$4n^2 + 12n - 336 = 0,$ Solving Quadratic equation
	$n = 7.78 \approx 8, n_2 = -10.78 \approx -11,$ but the number
	of singers must be positive, $n = 8 = n = 8.$
	$\therefore$ The number of singers is 8.

Extract 3.3 illustrates a good answer from a candidate who did accurately the application of the knowledge of the the topic of finite series for Arithmetic Progression (AP).

On the other hand, the candidates who scored low marks in part (b), wrote irrelevant formulae to find the number of singers and could not proceed further. One of these approaches is illustrated by Extract 3.4.

### Extract 3.4

b/	$S = \frac{n}{2}(n-1)d$
	$S_{168} = \frac{n(n-1)}{2}4$
	$S_{168} = \frac{n^2 - n \times 4}{2}$
	$= \frac{4n^2 - 4n}{2}$

Extract 3.4 shows a sample of a candidate's response who used irrelevant formula when finding the number of singers in the group.

## 2.4 Question 4: Differentiation

This question had three parts; (a), (b) and (c). In part (a) there were two subparts where the candidates were required to differentiate with respect to

$x$  the functions (i)  $y = \frac{2e^{5x}}{3 \sin x}$  and (ii)  $y = x^2 \sin 3x$ . In part (b) the

candidates were required to find the slope of the curves

(i)  $f(x) = x^3 - 5x^2$  at the point  $x = 2$  and (ii)  $x^2 - 3xy + 2y^2 - 2x = 4$  at the point  $(1, 2)$ . In part (c) the candidates were required to find the rate of

change of the radius when radius is 6 cm given that the volume of air which is pumped into a rubber ball every second is  $4\text{cm}^3$  and that the volume of

the ball as  $v = \frac{4}{3}\pi r^3$  for  $r$  which changes with the increase of air.

The question was attempted by 99.3 percent of the candidates of which 4,738 equivalent to 32.2% scored 3 marks or above showing that the question was averagely performed. On the other hand, 5,480 (37.2%) candidates scored 0 marks indicating that they lacked knowledge in the topic. Moreover, the analysis has indicated that 4,524 (30.7%) candidates

scored in the range 0.5 to 2.5 marks, 2,308 (15.7%) in the range 3.0 to 4.5 marks and 2,430 (16.5%) in the range 5.0 to 10 marks.

The candidates who performed well in part (a) applied both quotient and product rules appropriately to differentiate the given functions, showing that they met the requirement of the question and had adequate knowledge in differentiation. Extract 4.1 show good answers from one of the candidates.

#### Extract 4.1

4 @ ii)  $y = \frac{2e^{5x}}{3\sin x}$

for

$y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$u = 2e^{5x}$

$v = 3\sin x$

$\frac{dy}{dx} = \frac{3\sin x \frac{d}{dx}(2e^{5x}) - 2e^{5x} \frac{d}{dx}(3\sin x)}{(3\sin x)^2}$

$$\frac{d}{dx} (2e^{5x})$$

$$\text{let } u = 5x$$

$$\frac{du}{dx} = 5$$

$$y = 2e^u$$

$$\frac{dy}{du} = 2e^u$$

$$\frac{dy}{dx} = 2e^u \times 5$$

$$\frac{dy}{dx} = 10e^{5x}$$

$$\frac{d}{dx} (2e^{5x}) = 10e^{5x}$$

$$\frac{d}{dx} (3\sin x) = \sin x \frac{d(3)}{dx} + 3 \frac{d(\sin x)}{dx}$$

$$\sin x(0) + 3(\cos x)$$

$$\frac{d}{dx} (3\sin x) = 3\cos x$$

$$\therefore \frac{dy}{dx} = 3\sin x (10e^{5x}) - 2e^{5x} (3\cos x)$$

$$= 9\sin^2 x$$



$$\frac{dy}{dx} = \frac{30 \sin x e^{5x} - 6 \cos x e^{5x}}{9 \sin^3 x}$$

$$\frac{dy}{dx} = \frac{e^{5x} (30 \sin x - 6 \cos x)}{9 \sin^3 x}$$

$$\frac{dy}{dx} = \frac{6 e^{5x} (5 \sin x - \cos x)}{9 \sin^3 x}$$

$$\frac{dy}{dx} = \frac{2 e^{5x} (5 \sin x - \cos x)}{3 \sin^3 x}$$

4 (iii)  $y = x^2 \sin 3x$

$$y = u \cdot v$$

$$u = x^2$$

$$v = \sin 3x$$

$$f'(x) = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$f'(x) = \sin 3x \left( \frac{d(x^2)}{dx} \right) + x^2 \left( \frac{d \sin 3x}{dx} \right)$$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} (\sin 3x)$$

$$\text{let } u = 3x$$

$$\frac{du}{dx} = 3$$

$$y = \sin 3x$$

$$y = \sin u$$

$$\frac{dy}{du} = \cos u$$

$$\frac{d}{dx} (\sin 3x) = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} (\sin 3x) = \cos(3x) \cdot 3$$

$$\frac{d}{dx} (\sin 3x) = 3 \cos 3x$$

$$\therefore f'(x) = 2x \sin(3x) + x^2 (3 \cos 3x)$$

Extract 4.1 shows a sample of a good response from one of the candidates' work who used both product rule and quotient rule to differentiate the functions, indicating that they were knowledgeable and had the required concept.

However, the candidates who failed in part (a) had a mess in using the quotient rule and product rule to differentiate the given functions. For example, some candidates mixed up the formulae for the quotient rule versus the product rule. Furthermore, some failed to apply the chain rule in trigonometric expressions and could not identify when the sign is supposed to change for differentiation of cosine versus sine. Therefore candidates used wrong procedures and ended with wrong solutions like one which is indicated by Extract 4.2.

## Extract 4.2

4	(a)(i) $y = \frac{2e^{5x}}{3 \sin x}$
	$\frac{dy}{dx} = \frac{2e^{5x}}{3 \cos x}$
	$\therefore \frac{dy}{dx} = \frac{2e^{5x}}{3 \cos x}$
	(ii) $y = x^2 \sin 3x$
	let $u = \sin 3x$
	$\frac{du}{dx} = \cos 3x$
	$\frac{dy}{dx} = 2x \cos 3x$
	$\frac{dy}{dx} = 2x \cos 3x$
	$\frac{dy}{dx} = 6x \cos 3x$
	$\therefore \frac{dy}{dx} = 6x \cos 3x$

Extract 4.2 indicate a response of one of the candidates who used both the quotient rule and product rule incorrectly to differentiate the given functions.

The candidates who did well in part (b) managed to apply differentiation method in finding the slope of the curves at a given point. This competence is illustrated by Extract 4.3.

### Extract 4.3

4 (b) iv  $f(x) = x^3 - 5x^2$

$$f'(x) = 3x^2 - 10x$$

Slope when  $x = 2$

$$\text{Slope} = 3(2)^2 - 10(2)$$

$$\text{Slope} = 3 \times 4 - 20$$

$$\text{Slope} = 12 - 20$$

$$\text{Slope} = -8$$

4 (b) iii  $x^2 - 3xy + 2y^2 - 2x = 4$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(3xy) + \frac{d}{dx}(2y^2) - \frac{d}{dx}(2x) = \frac{d}{dx}(4)$$

$$2x - \left( 3x \frac{dy}{dx} + 3y \frac{dx}{dx} \right) + \frac{4y}{dx} - 2 = 0$$

$$2x - 3x \frac{dy}{dx} - 3y + \frac{4y}{dx} - 2 = 0$$

$$\begin{aligned}
 4b) \quad 2x - 3y - 2 &= 3x \frac{dy}{dx} - 4y \frac{dy}{dx} \\
 \frac{dy}{dx} (3x - 4y) &= 2x - 3y - 2 \\
 \frac{dy}{dx} &= \frac{2x - 3y - 2}{3x - 4y} \\
 \text{slope } \left( \frac{dy}{dx} \right) \text{ when } x=1, y=3 \\
 \text{slope} &= \frac{2(1) - 3(3) - 2}{3(1) - 4(3)} \\
 \text{slope} &= \frac{2 - 9 - 2}{3 - 12} \\
 \text{slope} &= \frac{-9}{-9} \\
 \text{slope} &= 1
 \end{aligned}$$

Extract 4.3 shows one of the good responses from the candidate who applied differentiation method to determine the slope of the curves at a given point.

On the other hand, the candidates who had very poor response in part (b) they could not apply the rule of BODMAS especially on negative sign when applying the product rule for implicit differentiation of the term  $-3xy$ .

Additionally, other candidates lacked the knowledge of implicit differentiation. Extract 4.4 is one of the example of poor responses provided by the candidates.

#### Extract 4.4

4	(b) (i) $f(x) = x^3 - 5x^2$ at $x=2$
	$f(x) = 2^3 - 5x^2$
	$8 - 20$
	$f(x) = -12$
	$\therefore \text{slope} = -12$
	(ii) $x^2 - 3xy + 2y^2 - 2x = 4$ (1, 3)
	when $x = 1$
	$x^2 - 3xy + 2y^2 - 2x = 4$
	$1^2 - 3(1)y + 2y^2 - 2(1) = 4$
	$1 - 3y + 2y^2 - 2 = 4$
	$-3y + 2y^2 - 1 = 4$
	$-3y + 2y^2 = 5$
	$2y^2 - 3y - 5 = 0$
	$a \quad b \quad c$
	fm $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
	$x = \frac{-3 \pm \sqrt{9 - 4(2)(-5)}}{2(2)}$
	$x = 2.5$
	slope is 2.5, or -1
	when $x = 3$ ,
	fm
	$x^2 - 3xy + 2y^2 - 2x = 4$
	$3^2 - (3x)y + 2y^2 - 2(3) = 4$
	$9 - 9y + 2y^2 - 6 = 4$
	$-9y + 2y^2 - 3 = 4$
	$-9y + 2y^2 = 7$
	$-9y + 2y^2 - 7 = 0$
	$2y^2 - 9y - 7 = 0$
	$a \quad b \quad c$
	fm $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4	(b) (ii) $y = \frac{49 \pm \sqrt{92 - 4 \times 2 \times 7}}{2 \times 2}$
	$y = 5.17, \text{ or } -0.676$
	slope = 5.17 or -0.676.

Extract 4.4 shows a response of one of the candidates who substituted the points in a polynomial to find the slope at a given point before differentiation. This indicates lack of adequate knowledge in the topic.

Candidates who attempted part (c) correctly, showed good understanding of the concepts in the application of differentiation and did it appropriately. Extract 4.5 illustrates this case.

#### Extract 4.5

4C.	$v = \frac{4}{3} \pi r^3$
	$\frac{dv}{dr} = \frac{4 \times 3}{3} \pi r^2$
	$\frac{dv}{dr} = 4\pi r^2$
	$\frac{dr}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt}$
	$\frac{dr}{dt} = \frac{1}{4\pi r^2} \times 4\pi r^2/s$
	$\frac{dr}{dt} = \frac{1}{4\pi (6)^2} \times 4$
	$\frac{dr}{dt} = \frac{1}{36\pi} \text{ cm/sec.}$

In Extract 4.5 demonstrates the work of a good response from one of the candidates who applied correctly the differentiation method to calculate the rates of change of the radius in the given problem.

However, the candidates who failed part (c) had inadequacy knowledge in application of differentiation. The candidates copied the data given in the question but could not proceed as is shown by extract 4.6.

#### Extract 4.6

$$(c) \quad V = \frac{4}{3} \pi r^3$$

$$V = 4 \text{ cm}^3 \text{ per second.}$$

$$\text{Radius} = 6.$$

$$\frac{4}{3} \pi 6^3$$

$$\frac{4}{3} \pi 216$$

$$\text{Rate change} = \underline{\underline{904.32}}$$

Extract 4.6 indicates the work of the candidate who just copied the information given to the question but could not proceed to answer the question. This implies that the candidate had no concept on the applications of differentiation.

## 2.5 Question 5: Integration

This question had three parts; (a), (b) and (c). In part (a) there were two sub-parts where the candidates were required to; (i) evaluate the definite integral  $\int_2^5 (3x^2 - 5x) dx$  and (ii) find the indefinite integral  $\int x^4 \sqrt{x^5 + 3} dx$ .

In part (b) the candidates were required to evaluate  $\int_1^5 (f(x) + 3) dx$  given



that  $\int_1^5 f(x)dx = 4$ . In part (c) the candidates were required to sketch the graph of the curve  $f(x) = x(x - 4)$  and hence find the area between the x-axis and the curve.

The question was attempted by 99.3 percent of the candidates of which 5,826 equivalent to 39.5% scored 3 marks or above showing that the question was averagely performed. Additionally, the analysis has also shown that 8,916 (60.5%) candidates scored in the range 0 to 2.5 marks, 2,408 (16.3%) in the range 3.0 to 4.5 marks and 3,418 (23.2%) in the range 5.0 to 10 marks.

The candidates who answered part (a) (i) very well, used the right procedure to integrate the problem appropriately. Likewise, candidates who did well in part (b) managed to make a specific substitution and proper interpretation of  $\int_1^5 f(x)dx = 4$ . Extract 5.1 (a) shows a work of one candidate who performed well in both part (a) (i) and (b).

### Extract 5.1

Handwritten work for Extract 5.1 (a) showing the integration of a function from  $x=2$  to  $x=5$ :

$$\begin{aligned}
 & \text{5 a i) } \int_2^5 (3x^2 - 5x) dx \\
 &= \int_2^5 3x^2 dx - \int_2^5 5x dx \\
 &= \left[ \frac{3x^3}{3} \right]_2^5 - \left[ \frac{5x^2}{2} \right]_2^5 \\
 &= (5^3 - 2^3) - \frac{5}{2} (5^2 - 2^2) \\
 &= 117 - 52.5 \\
 &= 64.5
 \end{aligned}$$

5 b	<u>Soln</u>
	$= \int_1^5 (F(x) + 3) dx.$
	$= \int_1^5 F(x) dx + \int_1^5 3 dx.$
	given $\int_1^5 F(x) dx = 4.$
	$= 4 + \left[ 3x \right]_1^5$
	$= 4 + (3 \times 5) - (3 \times 1).$
	$= 4 + 12$
	$= 16$

Extract 5.1 illustrates the work of one of the candidates who answered correctly (a) (i) and (b). The candidates made the correct integration and substitution of the given limits.

However, the candidates who failed part (a) used wrong techniques in integration and ended up with a poor attempt. For example some candidates removed the integral and the square root sign of  $\int x^4 \sqrt{x^5 + 3} dx$  without following the proper techniques of substitution. Other candidates had illogical procedures such as  $x^6 + 3dx^2 = 1 \Rightarrow 3dx^8 = 1$ . This indicates that the candidates had no concept on how to integrate the problem. This misconception is shown by Extract 5.2.

## Extract 5.2

	$\int x^4 \sqrt{x^5 + 3} dx$
sol	remove Integral.
	$x^4 \sqrt{x^5 + 3} dx$
	$x^4 \sqrt{x^5 + 3} dx = 1$
	remove the square root.
	$\left( x^4 \sqrt{x^5 + 3} dx \right)^4 = (1)^4$
	$x(x^5 + 3 dx) = 1$
	$x^6 + 3 dx^2 = 1$
	$3 dx^8 = 1$
	$\frac{3 dx^8}{3 dx} = \frac{1}{3 dx}$
	$x^8 = \frac{1}{3 dx}$
	remove d.
	$x^8 = \frac{1}{3}$
	$x^8 = \frac{1}{3}$

Extract 5.2 illustrates a sample of a poor response from one of the candidates who lacked the knowledge and techniques of integration to solve the given problem.

The candidates who obtained high marks in part (c) were able to find the integration limits using table of values and managed to evaluate the required area. Extract 5.3 is one of the example of good response provided by the candidates.

### Extract 5.3

Sc  $y = x(x-4) = x^2 - 4x$

x	-2	-1	0	1	2	3
y	12	5	0	-3	-4	-3

area with x axis  $y = 0$

$$x(x-4) = 0$$

$$x = 0$$

$$x = 4$$

area  $\int_0^4 (x^2 - 4x) dx$

$$\int_0^4 x^2 dx - \int_0^4 4x dx$$

$$= \left[ \frac{x^3}{3} \right]_0^4 - \left[ \frac{4x^2}{2} \right]_0^4$$

$$= \frac{1}{3} (4^3 - 0^3) - 2(4^2 - 0^2)$$

$$= \frac{1}{3} (4^3 - 0^3) - 2(4^2 - 0^2)$$

$$= \frac{64}{3} - 32$$

$$= \frac{-32}{3} \text{ units}$$

Area between the curve and x axis =  $\frac{32}{3}$  units

Extract 5.3 illustrates the work of one of the candidates who managed to find the area between the x-axis and the curve but did not sketch the graph.

## 2.6 Question 6: Statistics

The candidates were provided with the following information: During a Biology practical, a random sample of 20 grasshoppers was selected and the length of each grasshopper recorded in centimeters as follows: 1.0, 1.0, 5.0, 4.0, 5.0, 5.0, 4.0, 2.0, 4.0, 2.0, 4.0, 2.0, 2.0, 3.0, 2.0, 3.0, 2.0, 3.0, 3.0, 3.0. From the information candidates were required to (a) (i) Prepare a frequency distribution table and a histogram for the length distribution and (ii) find the range, mode, median, mean and standard deviation without grouping the data. In part (b) candidates were required to indicate from part (ii), the measures of central tendency and the measures of dispersion.

The question was attempted by 99.3 percent of the candidates of which 10,763 equivalent to 73.0 % scored 3 marks or above which is a good performance. In addition to that, the analysis has also indicated that 3,979 (27.0%) candidates performed in the range 0 to 2.5 marks, 3,595 (24.4%) in the range 3.0 to 4.5 marks and 7,168 (48.6%) in the range 5.0 to 10 marks.

The candidates who scored high marks in part (a) showed clearly that they understood the requirement of the question and gave appropriate solution. They managed to organize the ungrouped data in frequency distribution table and sketched the histogram. Furthermore, in part (b) they indicated the required procedures to calculate the range, mode, median, and standard deviation. Extract 6.1 is a sample example of good response provided by a candidate in part (a) while extract 6.2 presents good response for part (b).

# Extract 6.1

6. a) (i)	Length (cm) (x)	frequency (f)
	1.0	2
	2.0	6
	3.0	5
	4.0	4
	5.0	3
- a frequency distribution table		
(ii) - Range = maximum length - minimum length		
= 5 cm - 1 cm		
= 4 cm		
- Mode = length with highest frequency		
= 2 cm		
- Median		
arranging in ascending order:		
1.0, 1.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 3.0, 3.0, 3.0, 3.0, 3.0, 4.0, 4.0, 4.0, 4.0, 5.0, 5.0, 5.0		
$\therefore$ Median = 3 cm		
- Mean = $\frac{\sum fx}{N}$		
where N = total frequency		
fx = product of frequency and data (x)		

Length (cm) (x)	frequency(f)	fx
1.0	2	2
2.0	6	12
3.0	5	15
4.0	4	16
5.0	3	15
	$N = 20$	$\Sigma fx = 60$

$$\therefore \text{Mean}(\bar{x}) = \frac{60}{20} = 3$$

$$\therefore \text{mean length} = 3\text{cm}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}}$$

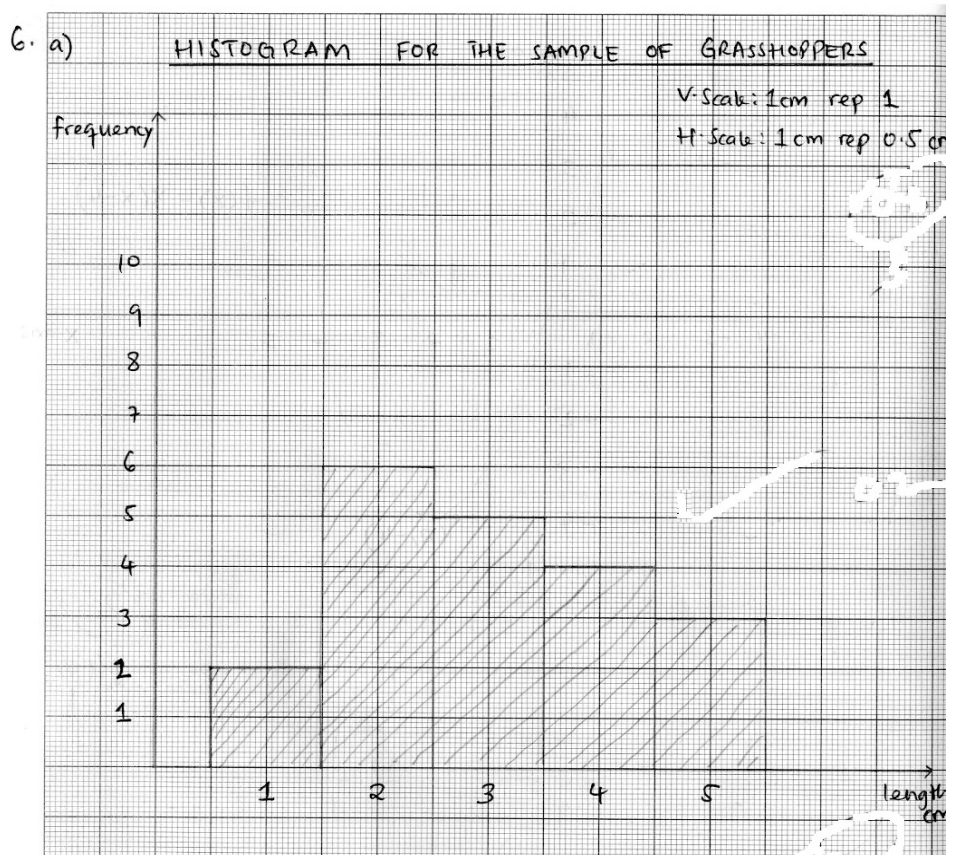
x	f	(x- $\bar{x}$ )	(x- $\bar{x}$ ) <sup>2</sup>	f(x- $\bar{x}$ ) <sup>2</sup>
1.0	2	-2	4	8
2.0	6	-1	1	6
3.0	5	0	0	0
4.0	4	1	1	4
5.0	3	2	4	12
	$N = 20$			$\Sigma f(x-\bar{x})^2 = 30$

$$\text{Standard deviation} = \sqrt{\frac{30}{20}}$$

$$S.D = \sqrt{3/2}$$

$$\therefore S.D = 1.225$$

$\therefore$  The standard deviation is 1.225 cm



Extract 6.1 shows the work of one of the candidates who managed to answer all items in part (a) correctly showing that the candidate had the required knowledge and skills.



## Extract 6.2

6. b)-measures of central tendency:
i) median = 3cm
ii) mean = 3cm
iii) mode = 2cm
- measures of dispersion:
i) Range = 4cm
ii) Standard deviation = 1.225 cm

Extract 6.2 illustrates one of the candidate's good work on the measures of central tendency and measures of dispersion as required.

However, few (12.4%) candidates failed totally to prepare frequency distribution table without grouping the data and hence computed wrongly both the measures of central tendency and dispersion. In addition, they failed to label the axes for histogram while some used the formula for grouped data in their computation. In part (ii) the irrelevance answers were observed when candidates named the graph as a *frequency distribution table*. Some candidates were putting a guessing work and illogical terms indicating that they had very poor communication language in statistics. More confusion was observed when other candidates named a graph as *histogram table for distribution of data* as is indicated by Extract 6.3.

### Extract 6.3

6	frequency distribution table.					
	class interval	frequency (f)	c-mark.			
	1.0 - 2.0	8	1.5	12	2.25	18
	2.0 - 3.0	11	2.5	27.5	6.25	68.75
	3.0 - 4.0	9	3.5	31.5	12.25	110.25
	4.0 - 5.0	7	4.5	31.5	20.25	141.75
						$\Sigma fx^2 = 338.75$
(1c)	Range = 2					
	Mode = $L + \left( \frac{t_1}{t_1 + t_2} \right) L'$					
	$L = 1.5$					
	$t_1 = 11 - 8 = 3$					
	$t_2 = 11 - 9 = 2$					
	Mode = $1.5 + \left( \frac{3}{3+2} \right) 2$					
	$1.5 + 0.36$					
	$= 1.86$					

b. ~~range~~

medium:

$$L + \left( \frac{n - N_b}{2} \right) i$$

$$1.5 + \left( \frac{35 - 8}{2} \right) 2$$

11

$$1.5 + \frac{(9.5)^2}{11} = 9.7$$

$$\text{standard deviation} = \sqrt{\frac{\sum x^2 - (\sum x)^2}{n}}$$

$$\text{But } \bar{x} = \frac{\sum x}{n} = \frac{102.5}{35}$$

$$(\text{Mean}) \quad \bar{x} = 2.93$$

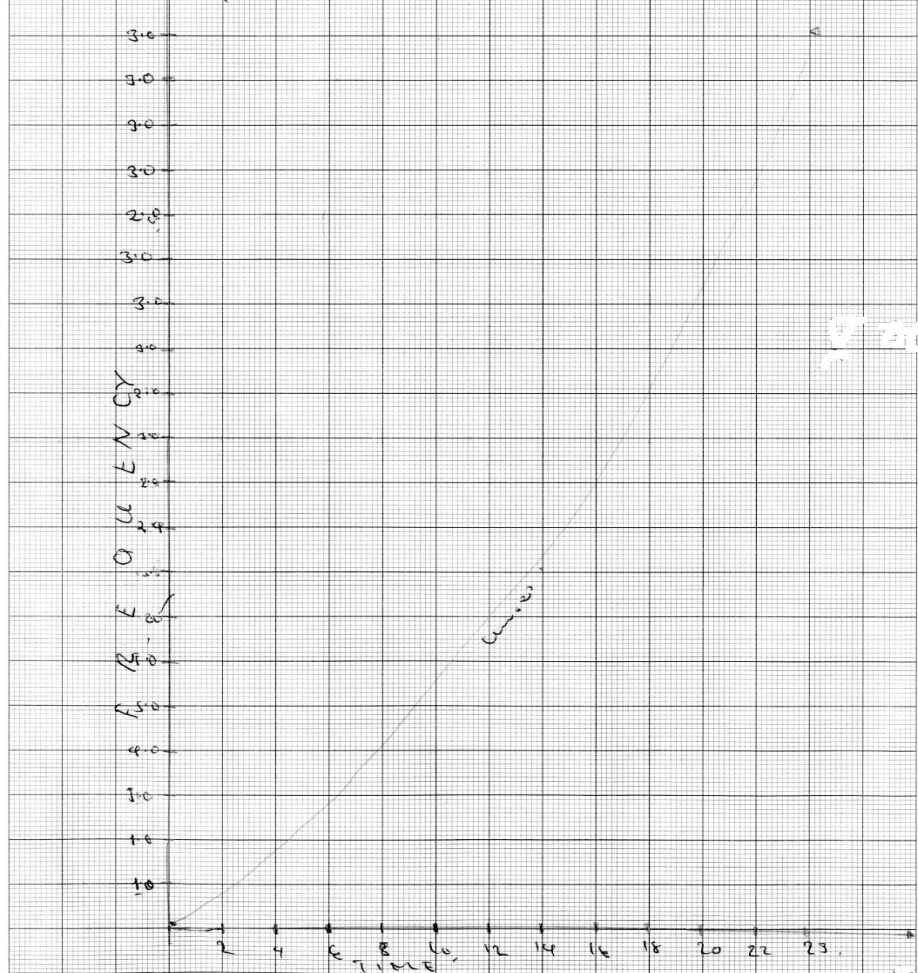
$$\text{standard deviation} = \sqrt{\frac{338.175 - 2.93}{35}}$$

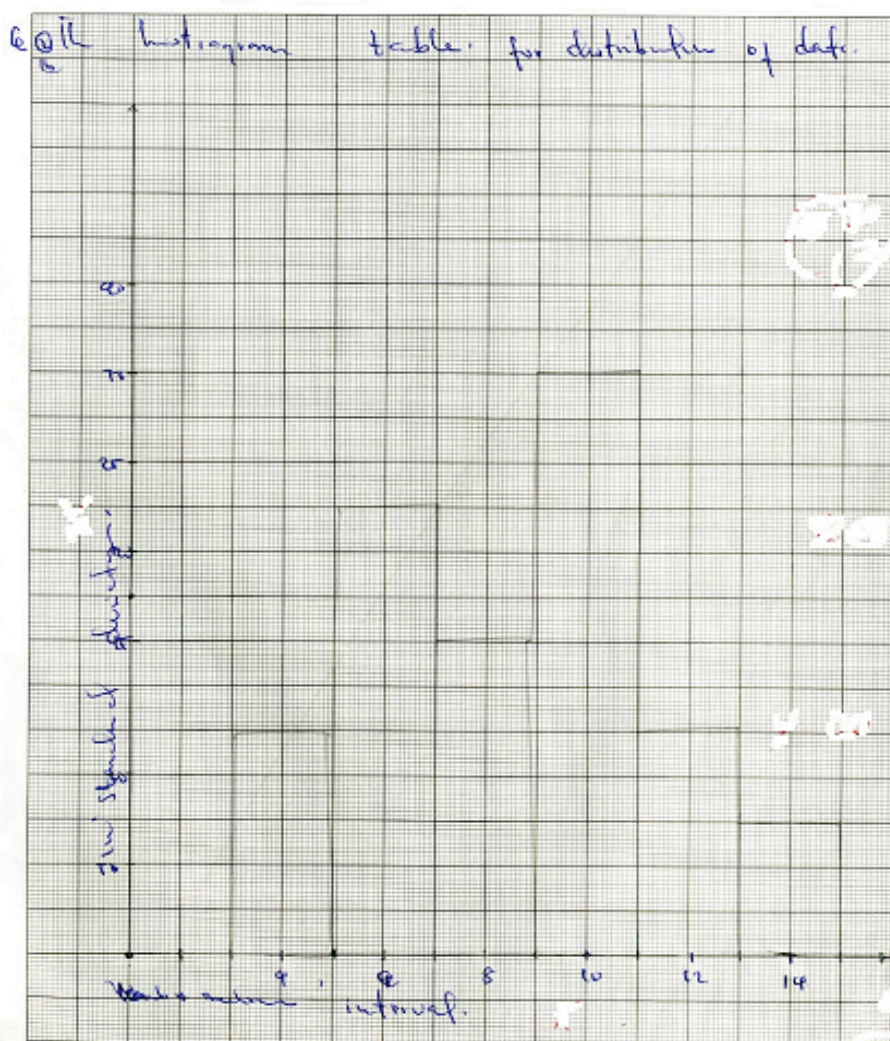
$$\text{standard deviation} = 2.59$$

$$\text{Mean} = \frac{\sum x}{n} = \frac{102.5}{35}$$

$$\text{Mean}(\bar{x}) = 2.93$$

Q. The frequency distribution table





Extract 6.3 shows a poor response from the candidate's work. The candidate failed to label both vertical and horizontal axes, failed to prepare frequency distribution table for ungrouped data and used the formula for grouped data in their computation

## 2.7 Question 7: Probability

The question had four parts; (a), (b), (c) and (d). In part (a) there were two subparts where the candidates were required to evaluate (i)  ${}^9P_4$  and (ii)  ${}^9C_4$ . In part (b) the candidates were required to find number of different ways that the letters in the word *STATISTICS* could be arranged. In part (c)

the candidates were required to find the probability that the next card drawn is a heart, when in a pack of 52 playing cards, two cards which are not hearts were removed and not replaced. In part (d) the candidates were given that  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{8}$  and  $P(C) = \frac{1}{6}$  then required to find; (i)  $P(A \cap B)$  when A and B are independent and (ii)  $P(A \cup C)$  when A and C are mutually exclusive events.

This question was attempted by 99.3 percent of the candidates of which 8,802 candidates equivalent to 59.7 % scored 3 marks or above indicating that the question had a good performance. Also, the analysis shows that 5,940 (40.3%) candidates scored in the range 0 to 2.5 marks, 3,158 (21.4 %) in the range 3.0 to 4.5 marks and 5,644 (38.3%) in the range 5.0 to 10 marks.

The candidates who had high marks in part (a) understood clearly the concepts of permutation and combination. They succeeded to compute the permutation  ${}^9P_4$  and combination  ${}^9C_4$  using the appropriate steps as illustrated by Extract 7.1.

### Extract 7.1

7. a) i)	question
	Required to evaluate
	${}^9P_4$
	from $nPr = \frac{n!}{(n-r)!}$
	${}^9P_4 = \frac{9!}{(9-4)!}$
	$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!}$
	$= 3024$
	$\therefore {}^9P_4 = 3024$

7. a/ii)  ${}^9C_4$

from  ${}^nC_r = \frac{n!}{r!(n-r)!}$

$${}^9C_4 = \frac{9!}{4!(9-4)!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!}$$

$$= 9 \times 7 \times 3$$

$$= 126$$

$\therefore {}^9C_4 = 126$

In Extract 7.1 the candidate demonstrated the ability to evaluate  ${}^9P_4$  and got the correct answer on this part. He/she illustrated also the required procedure to compute the combination when given 9 taking 4. The candidates demonstrated the competence and evaluated the problem correctly.

On the other hand, the candidates who scored low marks in part (a) mixed up the concepts of permutation versus combination in part (a). The misunderstanding cropped up from interchanging the two formulas. It is a fact that the formula for permutation cannot be used interchangeably when one wants to compute the combinations. To demonstrate the case Extract 7.2 is given as an example.

## Extract 7.2

07	To Evaluate
(i)	${}^9P_4$
	soln:
	${}^nP_r$
	$\frac{n!}{n(n-r)!}$
	where $n=9$
	$r=4$
	$\frac{9!}{9(9-4)!}$
	$\frac{9!}{9(5)!}$
	$\frac{9!}{45!} = \frac{362880}{1.196 \times 10^{10}}$
	${}^9P_4 = 3.03355 \times 10^{-5}$
(ii)	${}^9C_4$
	soln
	${}^nC_r$
	$= \frac{n!}{(n-r)!}$ where $n=9$
	$r=4$
	$= \frac{9!}{(9-4)!}$
	$\frac{9!}{5!}$
	$= \frac{9!}{5!}$
	$= \frac{362880}{120}$
	${}^9C_4 = 3024$

The Extract 7.2 shows the factual misconception made by a candidate for part (a) who interchanged the formulas for evaluating the permutation and combination.

The candidates who had good scores in part (b) and (c) showed to understand the concept of arrangement, independent events and mutually exclusive events and showed the required procedures clearly. The Extract 7.3 and 7.4 respectively expresses a good solution of the candidates who performed these parts correctly.



### Extract 7.3

b)	<u>Solution</u>
	Given the word STATISTICS
	Number of letters = 10
	from the question <del>for</del>
	Repeated letters are S = 3
	T = 3
	I = 2
	from
	${}^n P_r = \frac{n!}{(n-r)!}$
	$P = \frac{10!}{3! \times 3! \times 2!}$
	$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3! \times 3! \times 2!}$
	$= 50400$
	$\therefore$ In 50400 ways the word STATISTICS can be arranged.

Extract 7.3 shows the required procedure from one of the candidate's work on how to determine different ways of arranging letters in the word STATISTICS.

### Extract 7.4

7. c)	<u>Given</u>
	Total number of cards = 52
	number of Hearts = 13
	if two cards are removed without replacement
	$n(E) = 13$
	$n(S) = 50$
	from
	$P(E) = \frac{n(E)}{n(S)}$
	$= \frac{13}{50}$
	$\therefore$ The probability that the next card drawn is heart is $\frac{13}{50}$ .

Extract 7.4 shows a good response of one of the candidates who managed to apply the appropriate formula and substituted values accordingly. This indicates that the candidate had sufficient knowledge and skills for the topic.

However, in part (b), the candidates who did not manage to arrange in different ways the letters in the word STATISTICS failed to identify the arrangement rule and the repeating letters. Similarly, the candidates who failed part (c) had inadequate knowledge to identify the number of sample space after removing the two cards and failed to comprehend the required formula. Samples of poor responses from the scripts of candidates are illustrated by Extract 7.5 and 7.6.

### Extract 7.5

7	(b) STATISTICS
	There are two way of arranging this word.
	(i) Number form (n)
	(ii) Rate form (r)
	$n = 10$
	$r_1 = 3$
	$r_2 = 3$
	$\frac{n!}{r_1! r_2!} = \frac{10!}{3! 3!} = \frac{10!}{36}$
	$= 100,800$

Extract 7.5 represents the work of one of the candidates who failed to arrange the letters in different ways indicating that the candidate had no sufficient knowledge of the topic.

## Extract 7.6

	<u>Data:</u>
(c)	Number of sample = 52
	Number of card removed = 4
	Number of Events = 2
	Probability = $\frac{\text{Number of sample } M_1}{\text{Number of Event } M_2}$
	$= \frac{50}{2} = 25$
	$\therefore$ Probability that the next drawn is a heart = 25

Extract 7.6 shows the work of one of the candidates who failed to answer the question due to lacking of the knowledge of the number of sample space after removing the two cards.

The candidates who scored high marks in part (d) managed to synthesize the givens  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{8}$  and  $P(C) = \frac{1}{6}$  to find  $P(A \cap B)$  and  $P(A \cup B)$  and used the required procedure to compute the independent events and mutually exclusive events as required. Extract 7.7 illustrates the case.

## Extract 7.7

d)	<u>Solution</u>
	Given $P(A) = \frac{1}{4}$
	$P(B) = \frac{1}{8}$
	$P(C) = \frac{1}{6}$
	if Required $P(A \cap B)$
	for independent event -
	$P(A \cap B) = P(A) \times P(B)$
	$= \frac{1}{4} \times \frac{1}{8}$
	$= \frac{1}{32}$
	$\therefore P(A \cap B) = \frac{1}{32}$

7.	d/ii/	Required $P(A \cup C)$ for mutually exclusive event
		$P(A \cup C) = P(A) + P(C)$
		$P(A \cup C) = P(A) + P(C)$
		$= \frac{1}{4} + \frac{1}{6}$
		$= \frac{5}{12}$
		$\therefore P(A \cup C) = \frac{5}{12}$

Extract 7.7 shows the work of one of the candidates who used the correct procedure to get the correct answer. This indicates that the candidate had the required knowledge of the topic.

On the other hand, the candidates who failed part (d) made confusion in using the two formulas for independent and mutually exclusive events, finally ended with wrong answers. Extract 7.8 illustrates the case.

## Extract 7.8

(d) Data given
$P(A) = \frac{1}{4}$
$P(B) = \frac{1}{8}$
$P(C) = \frac{1}{6}$
From formula
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$1 = \frac{1}{4} + \frac{1}{8} - P(A \cap B)$
$1 = \frac{3}{8} - P(A \cap B)$
$1 - \frac{3}{8} = P(A \cap B)$
$P(A \cap B) = \frac{5}{8}$

7(d)(iii) $P(A \cup C)$ if A and C are Mutually
From formula
$P(A \cup B) + P(A \cap B) = 1$
$P(A \cup C) + P(A \cap C) = 1$
$P(A)' = 1 - \frac{1}{4} = \frac{3}{4}$
$P(C)' = 1 - \frac{1}{6} = \frac{5}{6}$
$= \frac{19}{12}$
$P(A \cup C) + P(A \cap C) = 1$
$P(A \cup C) + \frac{19}{12} = 1$
$P(A \cup C) + \frac{19}{12} - \frac{19}{12} = 1 - \frac{19}{12}$
$P(A \cup C) = \frac{7}{12}$

In Extract 7.8 the candidate mixed the idea of mutually exclusive events and independent events.

## 2.8 Question 8: Trigonometry

The question had three parts; (a), (b) and (c). In part (a) the candidates were required to find the value of  $\sin(A + B)$  given that  $\sin A = \frac{3}{5}$ ,  $\cos B = \frac{12}{13}$  and that A and B are both acute angles. In part (b) the candidates were required to show that  $\frac{\sec\theta + \operatorname{cosec}\theta}{1 + \tan\theta} = \operatorname{cosec}\theta$ . In part (c) the candidates were required to find the length CA of a triangular flower garden ABC which had an angle  $\hat{A}BC = 110^\circ$ ; given that AB = 50m and BC = 40m.

This question was attempted by 99.3 percent of the candidates of which 4,835 equivalent to 32.8% scored 3 marks or above showing that the question was averagely performed. Furthermore, the analysis shows that 9,907 (67.2%) candidates scored in the range 0 to 2.5 marks, 1,926 (13.1%) in the range 3.0 to 4.5 marks and 2,909 (19.7%) in the range 5.0 to 10 marks.

The candidates who had high scores in part (a) demonstrated their competence in using the given ratios and made proper manipulation to obtain the required value of  $\sin(A+B)$  as illustrated by Extract 8.1.

### Extract 8.1

8.	a)	SOLUTION
		Given: $\sin A = \frac{3}{5}$ .
		$\cos B = \frac{12}{13}$ .
		A, B are acute.
		Required:
		$\sin(A+B)$
		$\sin(A+B) = \sin A \cos B + \sin B \cos A$ .
		From $\sin A = \frac{3}{5}$
		$\cos A = \sqrt{1 - \sin^2 A}$ .
		$\cos A = \frac{4}{5}$ .
		$\cos B = \frac{12}{13}$ .
		$\sin B = \sqrt{1 - \cos^2 B}$ .
		$\sin B = \frac{5}{13}$ .
		$\sin(A+B) = \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$
		$= \frac{36}{65} + \frac{20}{65}$
		$= \frac{56}{65}$ .
		$\therefore \sin(A+B) = \frac{56}{65}$ .

Extract 8.1 shows the work of one of the candidate who got all the marks for part (a). The candidate was able to make the appropriate expansion of  $\sin(A+B)$  and the correct substitution of the ratios.

However, the candidates who failed part (a) started to differentiate the given ratios, the concept which was not applicable to the given problem. Also, they completely lacked the skills on the asked concepts as is illustrated by the extract 8.2.

### Extract 8.2

Handwritten student work for question 8(a) on lined paper. The student has written:

$$08(a) \sin(A+B)$$

$$= A = \frac{3}{5}$$

Hence you differentiate

$$b = \frac{14}{13}$$

$$= A = (10.47)$$

$$B = (-16-11)$$

$$\therefore = -6.36$$

$$\therefore = \sin(A+B) = 6.36$$

Extract 8.2 shows a sample of a poor response to question 8 (a) where the candidate had completely no knowledge of how to solve problems involving trigonometric ratios.

The candidates who did part (b) appropriately managed to express the left hand side as a single fraction. For example, some used the trigonometric identities  $\sec\theta = \frac{1}{\cos\theta}$  and  $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$  to simplify the expression on the left hand side, while others multiplied to both numerator and denominators by  $\sin\theta$ . Ultimately, they did correct manipulation and finally had the correct answer. Extract 8.1(b) portrays one of the cases. This is illustrated by extract 8.3.

### Extract 8.3

8.	b1	<u>Soln</u>
		Given:
		$\frac{\sec \theta + \operatorname{cosec} \theta}{1 + \tan \theta} = \operatorname{cosec} \theta.$
		prove:
		from L.H.S.
		$\frac{\sec \theta + \operatorname{cosec} \theta}{1 + \tan \theta} =$
		$= \frac{\frac{1}{\cos \theta} + \frac{1}{\sin \theta}}{1 + \frac{\sin \theta}{\cos \theta}}$
		$= \frac{\cos \theta + \sin \theta}{\cos \theta \sin \theta} \times \frac{\cos \theta}{\cos \theta + \sin \theta}$
		$= \frac{\cos \theta}{\cos \theta \sin \theta}$
		$= \frac{1}{\sin \theta}.$
		L.H.S. = $\operatorname{cosec} \theta.$
		Since L.H.S. = R.H.S. = $\operatorname{cosec} \theta$

Extract 8.3 shows the work of one of the candidates who managed to show that  $\frac{\sec \theta + \operatorname{cosec} \theta}{1 + \tan \theta} = \operatorname{cosec} \theta$  using the required procedures.



On the other hand, the candidates who provided the incorrect answers failed to write the left side expression as a single fraction. It is evident that the candidates had poor background knowledge on trigonometric identities. For example in one step the candidates put an incorrect simplification to justify that the left side is equal to the right side; that is

$$\frac{\sec\theta + \operatorname{cosec}\theta}{1 + \tan\theta} = \frac{\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right) \times \frac{\cos\theta}{\sin\theta}}{1}.$$

This is illustrated by the following  
Extract 8.4.

#### Extract 8.4

08 b)  $\frac{\cos\theta + 1}{1 + \tan\theta} = \frac{1}{\sin\theta}$

$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right) \times \frac{\cos\theta}{\sin\theta} = 1$

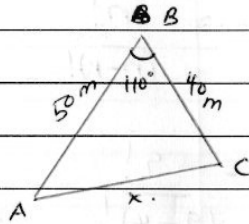
$\therefore \frac{1}{\sin\theta} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$

Hence Proven.

Extract 8.4 indicates how candidates failed to attempt part (b). The candidate could not manage to show that the left side expression is equal to the right side expression.

The candidates who had good scores in part (c) knew the requirement of the question and they had adequate knowledge in the topic of trigonometry as they applied cosine rule to give an appropriate solution. Extract 8.5 displays a good response from one of the candidates.

### Extract 8.5

8	c/	<u>Solution</u>
		Given $\triangle ABC$ .
		$\hat{A}BC = 110^\circ$
		$AB = 50\text{m}$ .
		$BC = 40\text{m}$
		Required: $CA = ?$
		
		from Cosine rule.
		$(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC) \cos B.$
		$(AC)^2 = (50)^2 + (40)^2 - 2(40)(50) \cos 110^\circ$
		$(AC)^2 = 2500 + 1600 - 4000 \cos 110^\circ$
		$(AC)^2 = 4100 - 4000(-0.34202)$
		$(AC)^2 = 4100 + 1368.08.$
		$(AC)^2 = 5468.08.$
		$AC = 73.95\text{m}.$
		$\therefore$ length $AC$ is <u>73.95m.</u>

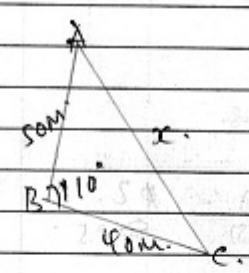
Extract 8.5 demonstrates the work of one of the candidates who answered this part correctly. The candidate was able to use the cosine rule to solve the real life problem.

However, some candidates who failed part (c), confused the use of the cosine rule versus sine rule while others made wrong application of

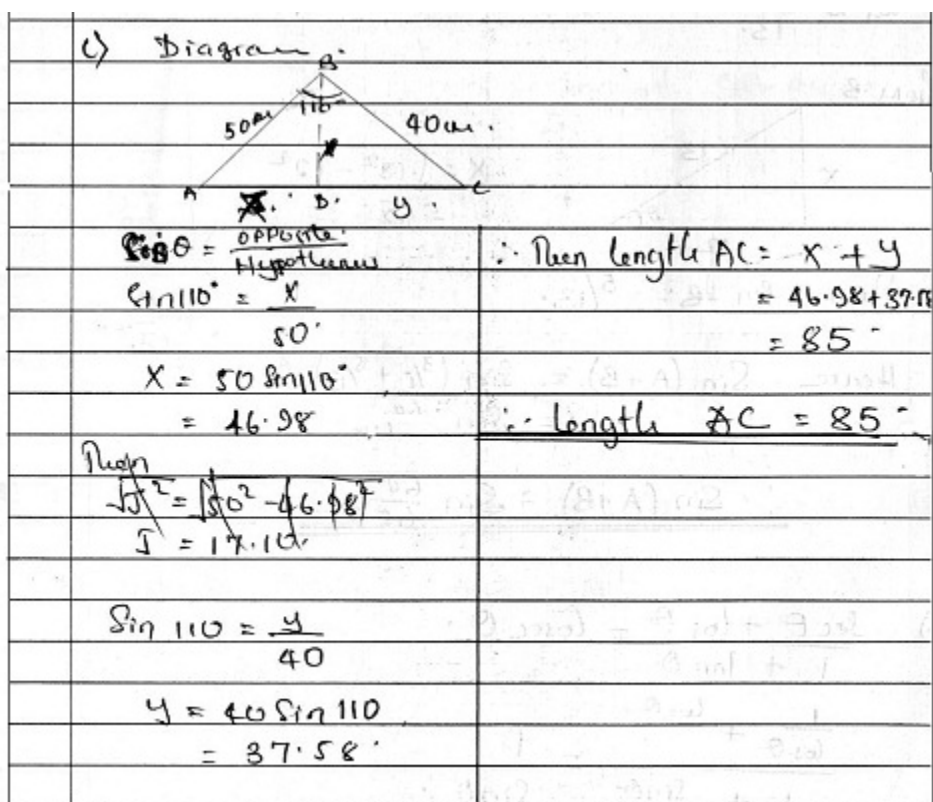
Pythagoras theorem. Extract 8.6 is a sample of poor answer provided by one of the candidate.

### Extract 8.6

(C)



$a^2 + b^2 = c^2$   
 $50^2 + 40^2 = c^2$   
 $2500 + 1600 = c^2$   
 $\sqrt{4100} = \sqrt{c^2}$   
 $\therefore CA = 64.03$   
 $\therefore CA = 64.03 \text{ m.}$



Extract 8.6 shows the poor response which was associated with improper use of the Pythagoras Theorem and the incorrect use of trigonometric ratios.

## 2.9 Question 9: Matrices

This question had two parts; (a) and (b). Part (a) consisted of three subparts which required the candidates to find the matrix multiplication (i) AB, (ii)

BA, (iii) CA and comment on the results given that  $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}$ ,

$B = \begin{pmatrix} 1 & -2 & -1 \\ -3 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$  and  $C = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ . In part (b) the candidates were required

to solve the following system of simultaneous equations by the inverse method:

$$\begin{cases} x + y + z = 3 \\ 5x - y - 3z = 1 \\ x - 3y + 2z = 0 \end{cases}$$

The question was attempted by 99.3 percent of the candidates of which 10,534 candidates equivalent to 71.5% scored 3 marks or above which is a good performance. Additionally, the analysis specified that 4,208 (28.5%) of the candidates scored in the range 0 to 2.5 marks, 4,580 (31.1%) in the range 3.0 to 4.5 marks and 5954 (40.4 %) in the range 5.0 to 10 marks.

The candidates who performed well in parts (a) and (b) had competence in matrix multiplications, as they were able to give the appropriate comment about the commutative property of matrix multiplication and applied the inverse to solve the system of equations. Also, the candidates made a correct computation of cofactors, an adjoint and the determinant. The good responses of part (a) and (b) from one of the candidates are shown in Extract 9.1 and Extract 9.2 respectively.

# Extract 9.1

9(a) (i)  $AB$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & -1 \\ -3 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} (1+3+0) & (-2+2+0) & (-1+1+0) \\ (1+0+1) & (-2+0+0) & (-1+0+1) \\ (1+3+2) & (-2+2+0) & (-1+1+2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

(ii)  $BA$

$$= \begin{bmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} (1+2+1) & (1+0+1) & (0+2+2) \\ (-3+2+1) & (-3+0+1) & (0+2+2) \\ (1+0+1) & (1+0+1) & (0+0+2) \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

(a) (iii) It is impossible to multiply  $(3 \times 1)$  Matrices to  $(3 \times 3)$  Matrices  
 $\therefore$  But on the (i) and (ii) it is possible to multiply  $(3 \times 3)$  Matrices to  $(3 \times 3)$  Matrices.

Extract 9.1 shows the work of one of the candidates who managed to show the required solution indicating that candidates had the required knowledge.

## Extract 9.2

9 (b)  $x + y + z = 3$   
 $5x - y - 2z = 1$   
 $x - 3y + 2z = 0$

$$\begin{bmatrix} 1 & 1 & 1 & | & x & = & 3 \\ 5 & -1 & -2 & | & y & = & 1 \\ -1 & -3 & 2 & | & z & = & 0 \end{bmatrix}$$

Co-factors

$$\begin{bmatrix} \begin{vmatrix} -1 & -2 \\ -3 & 2 \end{vmatrix} & \begin{vmatrix} 5 & -2 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 5 & -1 \\ -1 & -3 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & -3 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 5 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & -3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & -3 \end{vmatrix} \end{bmatrix}$$

$$\begin{bmatrix} (-2-6) & (10-2) & (-15-1) \\ (2-3) & (2-1) & (-3-1) \\ (-3-1) & (-3-5) & (-1-5) \end{bmatrix}$$

9(b)

Co-factor =  $\begin{bmatrix} -11 & 13 & -14 \\ 5 & 1 & -4 \\ -2 & -8 & -6 \end{bmatrix}$

P-transpose  $\begin{bmatrix} -11 & 5 & -2 \\ 13 & 1 & -8 \\ -14 & -4 & -6 \end{bmatrix}$

Adjunt  $\begin{bmatrix} -11 & 5 & -2 \\ 13 & 1 & 8 \\ 14 & 4 & -6 \end{bmatrix}$

Determinant of  $|D|$   $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & -3 \\ 1 & -3 & 2 \end{bmatrix}$

$$1 \begin{vmatrix} -1 & -3 \\ -3 & 2 \end{vmatrix} - 1 \begin{vmatrix} 5 & -3 \\ 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 5 & -1 \\ 1 & -3 \end{vmatrix}$$

$$(-2-9) - (10+3) + (-15+1)$$

$$-11-13-14$$

Determinant,  $|D| = -38$

$$= \frac{1}{-38} \begin{bmatrix} -11 & 5 & -2 \\ -13 & 1 & 8 \\ 14 & 4 & -6 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\frac{1}{-38} \begin{bmatrix} (-33+5+0) \\ (-39+1+0) \\ (-42+4+0) \end{bmatrix}$$



9(b)	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-38} \begin{bmatrix} -38 \\ -38 \\ -38 \end{bmatrix}$
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Extract 9.2 demonstrates one of the good answers from the candidate who was able to compute correctly the cofactors, adjoint and determinant in solving the system of equations.

However, the candidates who could not answer part (b) correctly failed to find cofactors especially the use of  $(-1)^{i+j}$  entry and applied inappropriate method (Cramer's rule). In addition to that, others failed to find the transpose which was important to get the adjoint. The extract 9.3 illustrates the work of one of the candidates' poor responses.

### Extract 9.3

$$\begin{aligned} 9. (b) \quad & x + y + z = 3 \\ & 5x - y - 3z = 1 \\ & x - 3y + 2z = 0 \end{aligned}$$

sol

To write the equations in the matrix form

$$\begin{pmatrix} 1 & 1 & 2 \\ 5 & -1 & -3 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Let } A = \begin{pmatrix} 1 & 1 & 2 \\ 5 & -1 & -3 \\ 1 & -3 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1(-2-6) - 1(10-(-3)) + 2(-15-(-1)) \\ 5(2-(-6)) - 1(2-2) + 3(-3-1) \\ 1(-7-(-2)) + 3(-3-10) + 2(-1-5) \end{vmatrix}$$

$$|A| = \begin{pmatrix} -8 + 13 + 28 \\ -40 - 0 + 12 \\ -1 - 39 - 12 \end{pmatrix}$$

$$|A| = \begin{pmatrix} 7 \\ -52 \\ -52 \end{pmatrix}$$

$$x = 0.78846$$

$$y = 0.75$$

$$z = 0.7307$$

Extract 9.3 shows the poor response from one of the candidates who lacked the knowledge of matrices. The candidate calculated the column matrix as the determinant which was a wrong procedure to solve the system of equations by the inverse method.

## **2.10 Question 10: Linear Programming**

This question had three parts; (a), (b) and (c). In part (a), the candidates were required to state what a linear programming problem is. In part (b) the candidates were required to write 6 steps one would undertake in solving a linear programming problem graphically. In part (c) the candidates were required to maximize the given objective function  $f(x, y) = 10x + 15y$  subject to constraints;  $3x + 12y \leq 36$ ,  $9x + 6y \leq 30$  and  $x \geq 0$ ,  $y \geq 0$ .

The question was attempted by 99.3 percent of the candidates of which 12,215 equivalent to 82.9 % scored 3 marks or above showing a very good performance. The analysis has also indicated that the question takes the second position among the questions in having very few (9.5 %) zero scores. Further analysis indicates that 2527 (17.1%) candidates scored in the range 0 to 2.5 marks, 2,356 (16.0%) in the range 3.0 to 4.5 marks and 9,859 (66.9%) in the range 5.0 to 10 marks.

The candidates who had good scores in part (a) provided a good constructed definition. The statement given possessed the necessary concepts which were organized to support the required definition. Generally, most candidates had a good knowledge on the topic. Extract 10.1 illustrates the typical case.

### Extract 10.1

10(a) Linear programming is the study which deal with formulation of inequalities, drawing of graph and then using of corner point obtained from graph to maximize or maximize profit by using subject function given. OK In generally linear programming is the study of maximizing or minimizing profit from the given plans or objectives.

Extract 10.1 demonstrates one of the candidate's responses who provided a good definition of the term linear programming problem.

However, the candidates who could not give a clear meaning of the linear programming in problem part (a), some had very poor mathematical communication language while others failed to organize concepts. Therefore, they provided the irrelevant definitions. Extract 10.2 shows a sample of a response from one of the candidates who wrote a statement which does not bring sense.

### Extract 10.2

10 (a) Linear programming problem may be defined as the way of linear programming equation graphically

Extract 10.2 shows inappropriate definition of linear programming problem indicating that the candidate had inadequate knowledge of the topic.

The candidates who had high scores in part (b) managed to write the main six steps required in solving the linear programming problem graphically. They wrote them clearly and in chronological order as is illustrated by Extract 10.3.

### Extract 10.3

10.3	
(i)	Using $X$ and $Y$ to represent given variables
(ii)	Formation of inequalities by using $X$ and $Y$
(iii)	Formation of objective function
(iv)	Drawing graphs of the inequalities formed in (ii) above
(v)	Determining the feasible region and corner points from the graph
(vi)	Finding the optimum point (from the corner points) which satisfies the objective function.

Extract 10.3 represents a sample of one response of a candidate who wrote the main six steps underlying the procedure to solve linear programming problem.

However, the candidates who could not state the required steps in solving a linear programming in part (b) had insufficient knowledge of the topic. For example, some candidates wrote “write the heading, labeling the axis, indicating the value of  $x$  and  $y$  and joining the value of  $x$  and while others mentioned sentences which do not lead into the required steps. Therefore all were irrelevant points in the context of solving linear programming. This is illustrated by Extract 10.4.

### Extract 10.4

(b)	(i) Write the heading.
	(ii) Labelling the Axis.
	(iii) Indicating the value of $x$ and $y$ .
	(iv) Joining the value of $x$ and $y$ .

10(b)	(iv) -- with one straight line
	(v) Shade the upper side of the graph.
	(vi) Shade the lower side of the graph.

Extract 10.4 shows the work of one of the candidates who had a misconception of the required points. The points given do not bring sense and indicates that the candidates had no adequate concepts of the topic.

Likewise, the candidates who had good performance in part (c) made an appropriate sketch of the required graph for the inequalities and from the graph they managed to work out the maximum value. This case is illustrated by Extract 10.5.

#### Extract 10.5

10(c)	<u>Solution.</u>
	given that.
	$f(x, y) = 40x + 15y$ .
	$30x + 12y \leq 36$
	$9x + 6y \leq 30$ ; $x \geq 0, y \geq 0$

10(c)

Then:

$$3x + 12y \leq 36. \quad \text{----- (i)}$$

if x-intercept  $y=0$

$$3x + 12(0) = 36$$

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

$$\hookrightarrow (12, 0)$$

if y-intercept  $x=0$

$$3(0) + 12y \leq 36$$

$$\frac{12y}{12} = \frac{36}{12}$$

$$y = 3$$

$$\hookrightarrow (0, 3)$$

$$9x + 6y \leq 30$$

if x-intercept  $y=0$

$$9x + 6(0) = 30$$

$$\frac{9x}{9} = \frac{30}{9}$$

$$x = 3.33$$

$$\hookrightarrow (3.3, 0)$$

if y-intercept  $x=0$

$$9(0) + 6y = 30$$

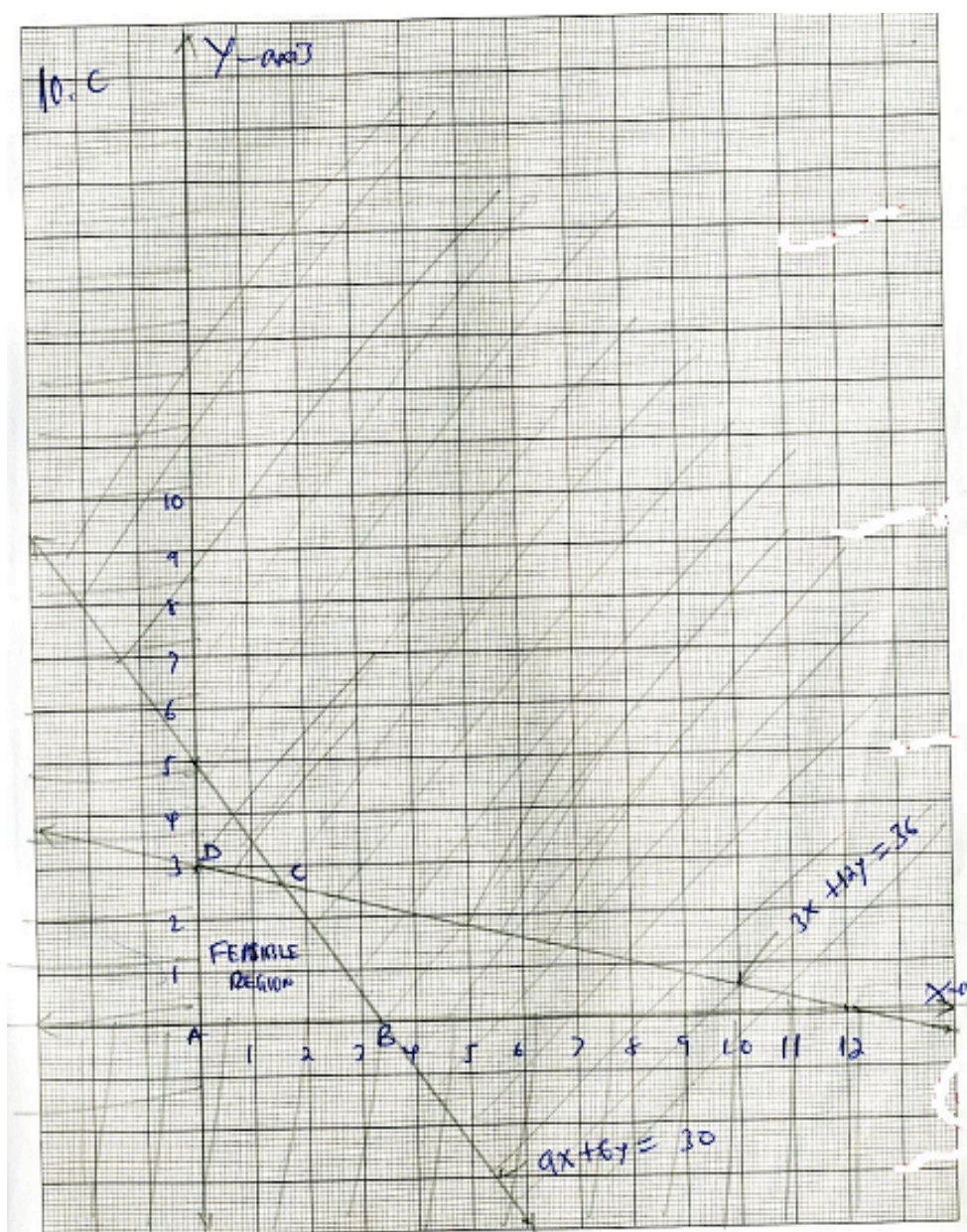
$$\frac{6y}{6} = \frac{30}{6}$$

$$y = 5$$

$$\hookrightarrow (0, 5)$$

$\Rightarrow$  Then Consider the graph drawn in the graph paper question number 10C







100.				
		Corner Point	subject: $f(x,y) = 10x + 15y$	total.
		A (0,3)	$0 + 45$	45
		B (1.6, 2.6)	$16 + 39$	55
		C (3.3, 0)	$33 + 0$	33
		D (0, 0)	$0 + 0$	0
	Therefore from the objective function $f(x,y) = 10x + 15y$ , The maximum objective is 55 when $x = 1.6$ and $y = 2.6$ .			

Extract 10.5 shows the response of one of the candidates who sketched the graph of the system of linear equations, identified the feasible region and the corner points. Then, the candidate was able to work out the maximum value indicating that the candidate had the required skills and knowledge of the topic.

### 3.0 SUMMARY OF THE CANDIDATES' PERFORMANCE

This section provides a summary of the overall performance in tabular and graphical forms where one can compare the candidates' performance more easily.

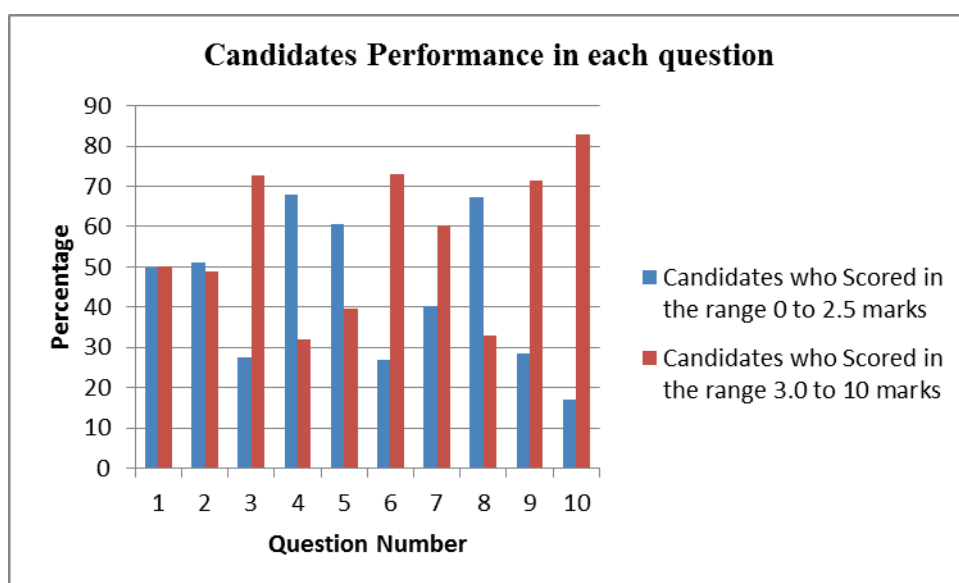
The summary shows the categorization basing on the fact that candidates who scored 30 percent or more of the marks allocated in each question or topic are regarded to have passed the particular question or topic. Further classification revealed that if the percentage of candidates performance falls in the interval 0 to 29 is termed as poor, 30 to 49 is average and it is termed as good performance if the percentage is from 50 to 100.

Basing on these criteria, table 1 and figure 1 were established. However, table 2, figures 2 and 3 were also prepared to show the overall candidates performance in the entire paper.

**Table 1: A Summary of Candidates' Performance in Each Question including the Topic Tested.**

Qn. No.	Topic	The % of Candidates' Performance			Comments
		0 – 2.5	3.0 – 4.5	5 – 10	
1.	Calculating Devices	49.8	28.8	21.4	Good
2.	Functions	51.1	21.2	27.7	Average
3.	Algebra	27.4	15.9	56.7	Good
4.	Differentiation	67.9	15.6	16.5	Average
5.	Integration	60.5	16.3	23.2	Average
6.	Statistics	27	24.4	48.6	Good
7.	Probability	40.3	21.4	38.8	Good
8.	Trigonometry	67.2	13.1	19.7	Average
9.	Matrices	28.5	31.1	40.4	Good
10.	Linear Programming	17.1	16	66.9	Good

**Figure 1: Candidates' Performance in the Range 0 to 2.5 Marks and 3.0 to 10 Marks in Each Question**



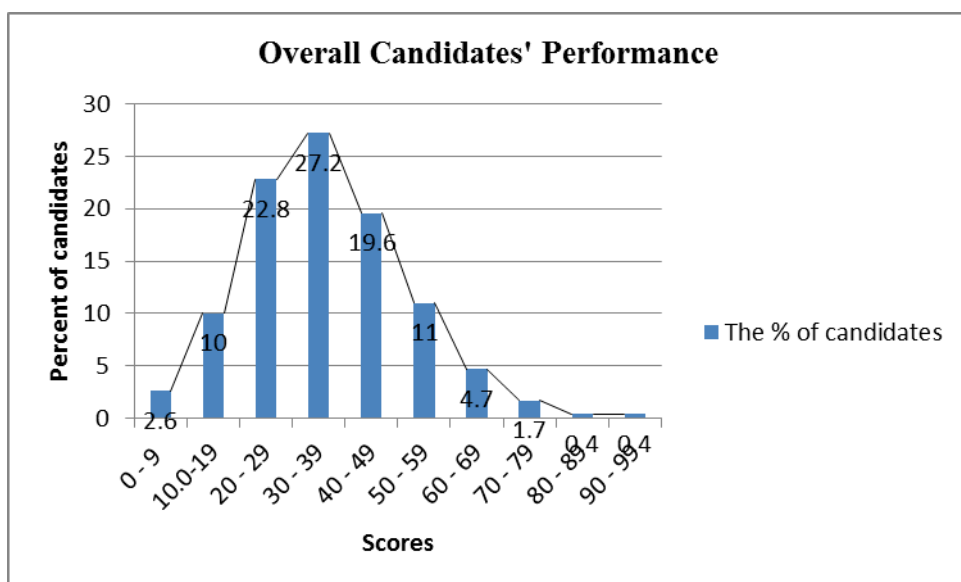
From table 1 and figure 1, we observe that the most well performed question is number 10 on a topic of linear programming. On the other hand, the most poorly performed question is number 4 on a topic of differentiation.

In table 2, figure 2 and figure 3 show the overall performance of the candidates. The scores are shown in 10 class intervals together with the corresponding candidates' percent.

**Table 2: Overall Candidates' Performance that includes the Entire Examination.**

S/n	The Candidates' Performance (scores)	The % of Candidates
1.	0 - 9	2.6
2.	10 - 19	10
3.	20 - 29	22.8
4.	30 - 39	27.2
5.	40 - 49	19.6
6.	50 - 59	11
7.	60 - 69	4.7
8.	70 - 79	1.7
9.	80 - 89	0.4
10.	90 - 99	0.4

**Figure 2: Overall Candidates' Performance for the Entire Paper.**



**Figure 3: Overall Candidates Performance Using Scores in Groups.**

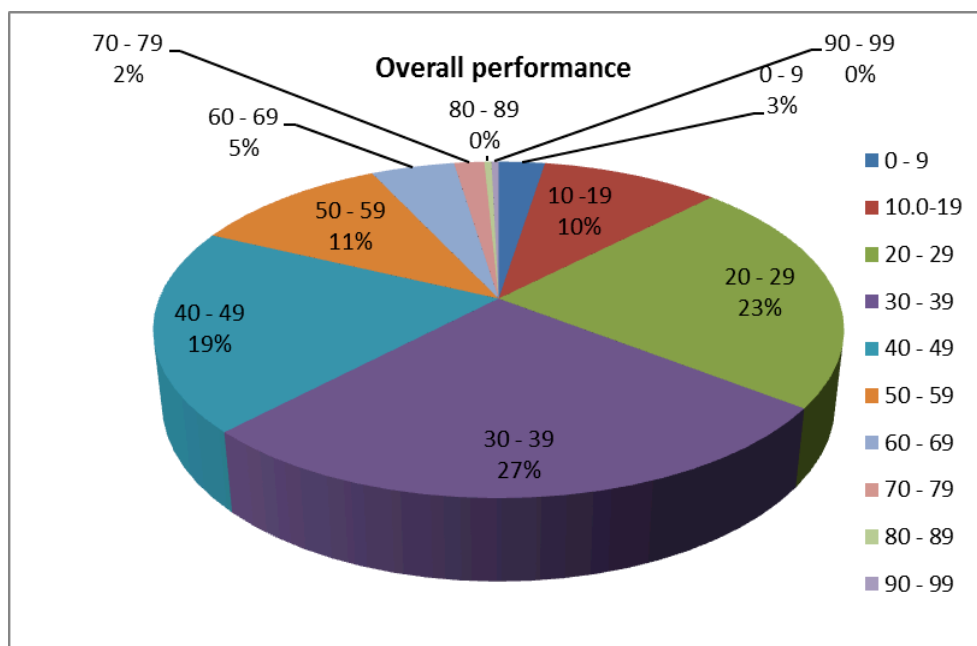


Table 2, figure 2 and figure 3 show a normal distribution of examination scores and the overall performance of the candidates. Majority of the candidates are located in the following classes: 20 – 29, 30 – 39, 40 – 49 and 50 – 59 whereby the class interval 30 – 39 being the modal class with the highest frequency. The trend line is plotted to justify the normal distribution of scores. Hence, it is evident that the performance of the examination was good and the examination was standard.

## 4.0 CONCLUSION AND RECOMMENDATIONS

### 4.1 Conclusion

The question wise analysis shows that the average of 44 percent of the candidates scored between 29 and 59 marks. This shows that the performance was good. The candidates showed strength in all topics except the topics of functions, integration, trigonometry and

differentiation which were averagely performed. The best performed topic was the linear programming of which the candidates had the average of 82.9 percent (refer appendix). More effort therefore should be taken in the topic of differentiation which had the average performance of 32.1 percent.

The following are some of the reasons attributed for the good performance:

- (a) Ability of the candidates to use mathematics skills and notations correctly when manipulating mathematical problems in the examination.
- (b) Ability of the candidates to apply their knowledge and mathematics concepts in solving various questions asked in the examination.
- (c) The competence of the candidates to follow the instructions and procedures of each question so as to obtain the correct solution in the appropriate demand of the question.
- (d) The candidates were able to use the appropriate rules, formula calculators and some principles to comprehend the requirement of the question.
- (e) The candidates were able to make analysis of the question and managed to show the required steps of the problem.

However the reasons that made some of the candidates not to perform well include;

- (a) Lack of knowledge of the topic and poor understanding of the requirements of the question.
- (b) The misuse of some mathematical symbols and use of wrong formulae contrary to the demand of the question.

## 4.2 Recommendation

The Scholars, Teachers and the Ministries responsible for education are all advised to observe the following recommendations to improve the performance of Basic Applied Mathematics in Tanzania:

- (a) Having noted the performance of Basic Applied Mathematics ACSEE 2014, the candidates are advised to;
  - (i) Put more effort in learning mathematics concept so as to get the required knowledge of the topic and minimize the factual misconception.
  - (ii) Do more exercises in order to get different techniques used in solving various questions.
  - (iii) Learn the proper use of symbols, formula and different rules for easy manipulations of mathematical problems.
- (b) Based on the performance of the candidates in this paper teachers should;
  - (i) not only teach mathematics but also teach mathematics concepts so as to help students acquire sufficient knowledge of the topic;
  - (ii) teach communication language in mathematics so that candidates can improve the use of mathematical symbols;
  - (iii) cover the syllabus and make enough practice in solving different problems so as the candidates can acquire and exhaust enough knowledge and techniques in answering questions.
- (c) In order to improve learning and teaching, the Ministries responsible for education should;

- (i) make a comprehensive monitoring and evaluation of the teaching and learning of Basic Applied Mathematics in secondary schools;
- (ii) prepare on job training of Basic Applied Mathematics Teachers to learn how to teach communication language in mathematics and teach mathematics concepts.



## APPENDIX

### The Performance of Candidates in Basic Applied Mathematics Topic- wise

S/N	Topic	Number of Questions	The Percentage of Candidates who Scored the Average of 30% or Above	Comments
1	Linear Programming	1	82.9	Good
2	Statistics	1	73.0	Good
3	Algebra	1	72.6	Good
4	Matrices	1	71.5	Good
5	Probability	1	59.7	Good
6	Calculating Devices	1	50.2	Good
7	Functions	1	48.9	Average
8	Integration	1	39.5	Average
9	Trigonometry	1	32.8	Average
10	Differentiation	1	32.1	Average
Average			56.8	Good

