

THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**CANDIDATES' ITEM RESPONSE ANALYSIS
REPORT ON THE CERTIFICATE OF SECONDARY
EDUCATION EXAMINATION (CSEE) 2025**

ADDITIONAL MATHEMATICS



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FOREWORD

In response to the 2025 Certificate of Secondary Education Examination (CSEE), the National Examinations Council of Tanzania (NECTA) has produced a Candidates' Item Response Analysis (CIRA) report in 042 Additional Mathematics. This report aims at providing feedback to education stakeholders about candidates' responses to the examination items. Specifically, it clarifies the specific skills and competencies the examination assessed, identifies the strengths and weaknesses candidates demonstrated and pinpoints areas of high and low performance. Consequently, the CIRA report serves as a critical tool for teachers, curriculum developers, and examination officers, helping them understand prevalent errors and improve future learning outcomes.

The analysis revealed that candidates' overall performance was good. The performance was primarily attributed to candidates' ability to interpret questions' requirements accurately, apply appropriate formulae and rules correctly, and employ appropriate problem-solving strategies. Furthermore, many candidates exhibited clear, logically structured solution pathways. Nonetheless, performance varied across specific items, indicating divergent levels of conceptual understanding and application.

This report empowers educational stakeholders to design and implement targeted interventions. Specifically, it provides the evidence base needed to refine pedagogical approaches, structure revision programmes, and allocate resources effectively. These actions are intended to remedy identified areas of weakness and thereby elevate candidates' overall achievement.

The National Examinations Council of Tanzania (NECTA) extends its sincere appreciation to the examinations officers and all contributors involved in preparing this report.



Prof. Said Ally Mohamed

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This Candidates' Item Response Analysis (CIRA) report evaluates candidates' performance in the November 2025 Certificate of Secondary Education Examination (CSEE) in 042 Additional Mathematics. It is designed to provide analytic feedback to teachers, students, and other education stakeholders, thereby supporting instructional improvement and students' learning.

The analysed examination was developed in alignment with the 2022 CSEE format, derived from the 2010 Additional Mathematics Syllabus for Secondary Schools (Form I–IV). The paper comprised fourteen (14) questions divided into two sections, A and B. Section A contained ten (10) short-answer questions worthing six (6) marks each, and Section B contained four (4) extended-response questions valued at ten (10) marks each.

In 2025, a total of 395 candidates sat for the examination, achieving a pass rate of 95.91 percent. This performance represents a 0.77 percentage increase over the 2024 pass rate of 95.14 percent. A comparative analysis of grade distributions between 2024 and 2025 is illustrated in Figure 1.

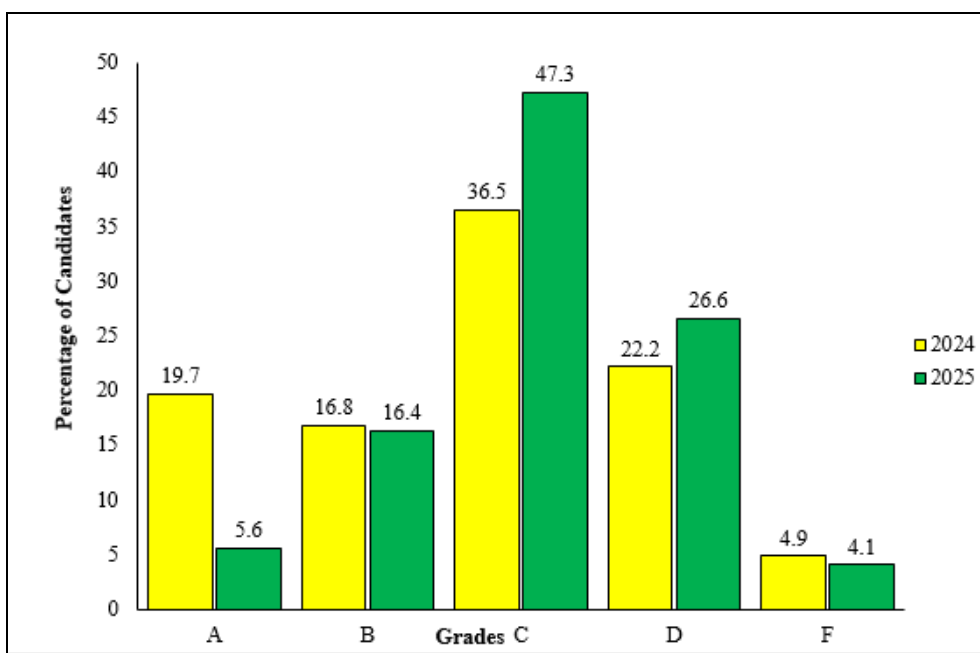


Figure 1: *Distribution of Candidates by Grades in 2024 and 2025*

The grade distribution analysis reveals an increase of pass rate in 2025 compared to 2024 for C and D by 10.8 per cent and 4.4 per cent respectively. However, a decline in pass rate was observed in grades A, B and F by 14.1 per cent, 0.4 per cent and 0.8 per cent respectively, calling for targeted instructional interventions.

The report is structured into four sections, beginning with Section 1.0, the Introduction. Section 2.0 provides a detailed item-by-item analysis. For each question, it covers the assessed skills, item requirements, and a synthesis of candidates' responses, supported by annotated extracts from scripts to exemplify both proficient and deficient performance. This section also examines the factors influencing outcomes. Section 3.0 shifts to a thematic analysis by topic area. Finally, Section 4.0 presents the report's conclusions and targeted recommendations for education stakeholders, designed to guide interventions for improving future examination performance.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH QUESTION

Candidates' performance by question is analysed using three categories: weak (0–29), average (30–64), and good (65–100). In Section A, scores are classified as weak (0–1.5 marks), average (2.0–3.5 marks), and good (4.0–6.0 marks), while in Section B they are classified as weak (0–2.5 marks), average (3.0–6.0 marks), and good (6.5–10 marks). For ease of interpretation, these categories are represented using colours: red for weak, yellow for average, and green for good performance.

2.1 Question 1: Variations

This question was designed to assess candidates' ability to comprehend and apply concepts of direct and joint variation and to employ proportional reasoning in real-life contexts. It comprised parts (a) and (b):

- (a) *It is observed that, the length of the wire, $l^2 + 1$ is directly proportional to the width, w . Given the length, $l = 3$ cm when the width, $w = 2$ cm.*
- (i) *Formulate a mathematical equation to represent this information.*
- (ii) *Calculate the width of the wire when its length, $l = 7$ cm.*
- (b) *Given that 6 students cultivated 45 hectares of land in 15 days, how long will it take for 15 students to cultivate 60 hectares of land?*

The analysis indicates that 2 (0.51%) candidates scored between 0 and 1.5 marks, while 389 (99.49%) scored between 2.0 and 6.0 marks. This distribution suggests that overall candidates' performance on this question was satisfactory. A summary of the performance data is presented in Figure 2.

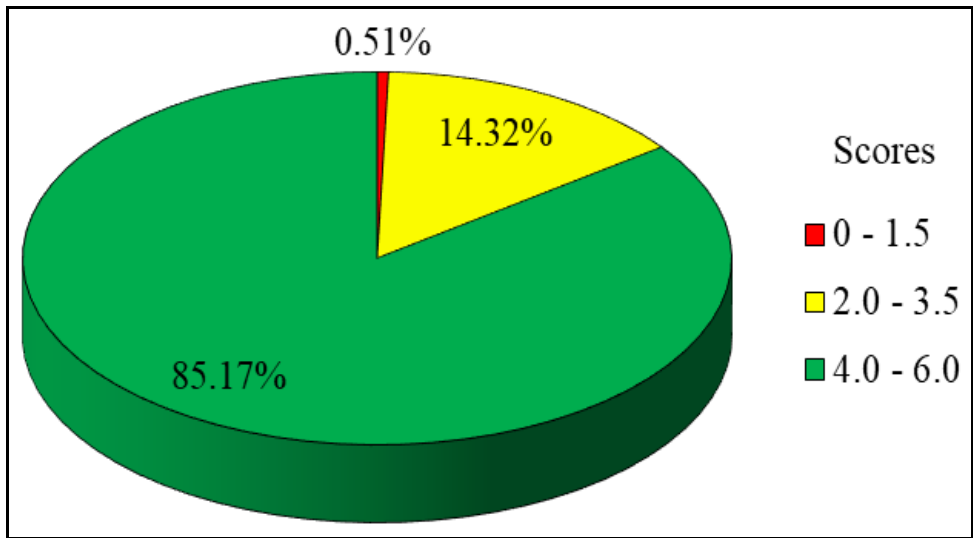


Figure 2: *Candidates' Performance on Question 1*

The candidates who scored all six marks had sufficient knowledge and skills in variations. In part (a) (i), they rewrote the word problem in mathematical model $(l^2 + 1)\alpha w$ and introduced the constant to produce an equation $(l^2 + 1) = kw$. With this equation, they substituted the corresponding values and performed calculations correctly, resulting in $k = 5$ and consequently $(l^2 + 1) = 5w$. Likewise, they responded to part (a) (ii) by substituting $l = 7$ into the equation $(l^2 + 1) = 5w$ and correctly solved the equation, resulting in $w = 10$.

Similarly, in part (b), these candidates correctly interpreted the word problem by first assigning letters to the variables, commonly s for students, h for hectares of land, and d for days. They also realized that the number of students is directly proportional to the number of hectares of cultivated land and inversely proportional to the number of days spent cultivating the land. Thus, they mathematically rewrote the problem as $s \propto \frac{h}{d}$ and consequently $s = k \frac{h}{d}$. With this relation, they substituted the values appropriately and performed necessary calculations, ending up with the correct answer of 8 days, as illustrated in Extract 1.1.

	b)	soln
		Given: $n \propto h/d$
		$n = kh/d$
		When $n=6$; $h=45$ $d=15$.
		$6 = \frac{45k}{15}$
		$3k = 6$.
01.	(b) ...	
		$3k = 6$
		$3 \quad 3$
		$k = 2$
		$n = \frac{2h}{d}$
		When $n=15$; $h=60$ $d=?$
		$15 = \frac{2 \times 60}{d}$
		$15d = 120$
		$15 \quad 15$
		$d = 8$.
		$\therefore$ It will take 8 days

Extract 1.1: A sample of the correct responses to question 1

In Extract 1.1, the candidate correctly formulated the required joint variation and determined the number of days.

In spite of the good performance, some candidates faced challenges when attempting this question. In part (a) (i) some candidates managed to formulate the proportionality equation after introducing the constant k as $(l^2 + 1) = kw$ but failed to express k in terms of the other variables correctly. For example, some candidates wrote $k = \frac{w-1}{l^2}$ instead of

$k = \frac{l^2 + 1}{w}$, obtaining the incorrect answer of k and equation. Other candidates wrote the question wrongly and therefore solved a different question. For instance, some wrote $l^2 + l \propto w$ and introduced the constant k as $l^2 + l = k w$, resulting in an incorrect value of $k = 6$, an incorrect equation $(l^2 + l) = 6w$, as well as an incorrect value of w in part (a) (ii). However, in part (a) (ii), other candidates wrote the equation incorrectly due to carelessness. For instance, some wrote an incorrect equation as $l^2 = 5w + 1$ and obtained an incorrect answer, $w = 9.6$ cm, instead of 10 cm.

In part (b) candidates failed to formulate the relation that defines the given variation. They failed to associate the relationship between number of students, time and number of hectares; that is, when the number of students increases, the number of hectares increases but the number of days will decrease and vice versa. This led to the wrong formulation of the variations relation, like $s \propto \frac{h}{d}$ and $s \propto \frac{1}{hd}$, and then obtained the incorrect value of k and hence the incorrect value of $d = 4.5$ days.

1.	G)	<u>Solution</u>
Q17	$(l^2 + l) \propto W$	
	$(l^2 + l) = kW$	
	$k = \frac{l^2 + l}{W}$	
	$l = 3\text{cm}$ when $W = 2\text{cm}$	
	$k = \frac{3^2 + 3}{2}$	
	$k = \frac{9 + 3}{2}$	
	$k = \frac{12}{2} = \underline{\underline{6}}$	
	$l^2 + l = 6W$	
	∴ The equation is $l^2 + l = 6W$	
Q17	When $l = 7\text{cm}$, $W = ?$	
	$l^2 + l = 6W$	
	$7^2 + 7 = 6W$	
	$49 + 7 = 6W$	
	$56 = 6W$	
	$W = \frac{56}{6} \text{cm} \text{ or } 9.33\text{cm}$	
	∴ The width of the wire is 9.33cm when $l = 7\text{cm}$	

1. (b) invers.
 Student $\propto \frac{1}{\text{hectare and time}}$
 $s \propto \frac{1}{h \cdot d}$
 $s = \frac{k}{h \cdot d}$
 $k = s h d$
 then, $k_1 = k_2$
 $s_1 h_1 d_1 = s_2 h_2 d_2$
 $6 \times 45 \times 15 = 15 \times 60 \times d_2$
 $\frac{4050}{900} = \frac{900}{900} d_2$
 $4.5 = d_2$
 The time is 4 days and 5 hours.

Extract 1.2: A sample of the incorrect responses to question 1

In Extract 1.2, the candidate, in part (a) (i) copied the question incorrectly; hence they got an incorrect value of k and an incorrect value of w . In part (b), the candidate formulated an incorrect equation, hence obtained an incorrect number of days.

2.2 Question 2: Statistics

The question aimed to assess candidates' ability to find the quartiles and to use the assumed mean method to calculate the variance and standard deviation of a given data set. It comprised two parts, (a) and (b), as follows.

The following table shows the distribution of masses of Form One students recorded during registration period:

Masses	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Number of Students	8	7	14	23	26	12	6	4

Use this information to calculate:

- (a) The second quartile correct to four decimal places.
- (b) the variance and standard deviation of the masses given the assumed mean, $A = 54.5$.

The data show that 71 (18.16%) candidates scored 0 to 1.5 marks, 153 (39.13%) scored 2.0 to 3.5, and 167 (42.71%) scored 4.0 to 6.0. The summary of candidates' performances on this question is shown in Figure 3, indicating that it was generally good.

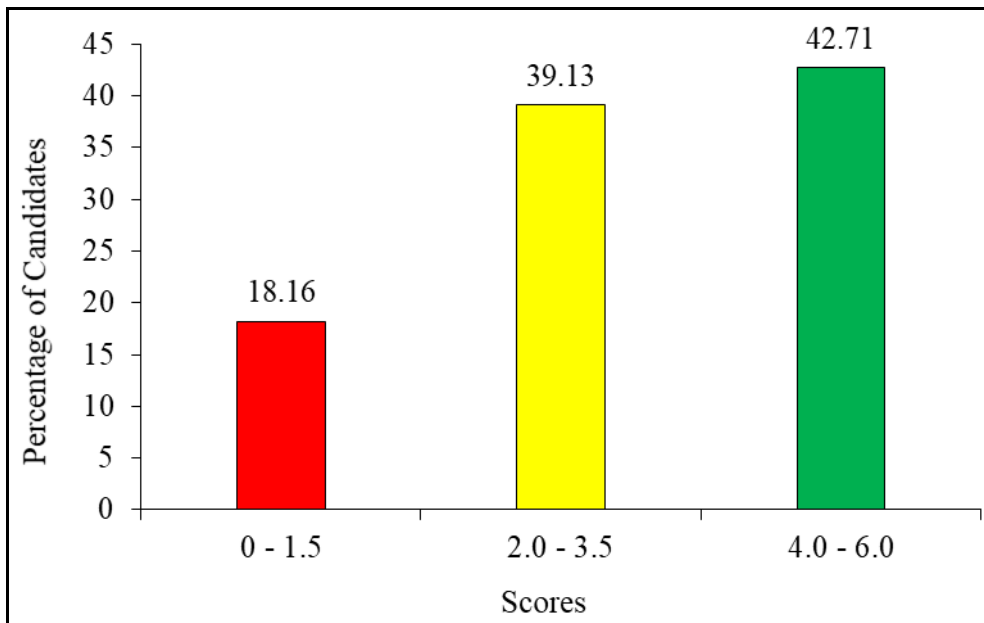


Figure 3: Candidates' Performance on Question 2

In part (a), the candidates correctly determined the position of the second quartile, Q_2 , by calculating $\frac{2N}{4}$, obtaining 50, and thus taking the class whose cumulative frequency is 50 or more, which is 40–50. Thereby, they correctly determined the values of the lower real limit ($L = 39.5$), the total frequency of all classes below the second quartile class ($f_b = 29$), the frequency of the second quartile class ($f_w = 23$) and the class size ($c = 10$). They then substituted these values into the formula

$$Q_2 = L + \left(\frac{\frac{2N}{4} - f_b}{f_w} \right) \times c \text{ and correctly performed calculations, resulting in}$$

$Q_2 = 48.6304$. In part (b), the candidates prepared a frequency distribution table which comprised columns for class marks (x), frequency (f), deviations (d), fd and fd^2 , and employed the formula correctly, as illustrated in Extract 2.1.

2.	(b) consider the table below.						
	$A = 54.5$						
	c.i	x	f	d	d^2	fd^2	fd
	10-19	14.5	8	-40	1600	12800	-320
	20-29	24.5	7	-30	900	6300	-210
	30-39	34.5	14	-20	400	5600	-280
	40-49	44.5	23	-10	100	2300	-230
	50-59	54.5	26	0	0	0	0
	60-69	64.5	12	10	100	1200	120
	70-79	74.5	6	20	400	2400	120
	80-89	84.5	4	30	900	3600	120
	$\Sigma f = 100$ $\Sigma fd = -680$ $\Sigma fd^2 = 34200$						

	$\text{Var}(x) = \frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2$
	$= \frac{34200}{100} - \left(\frac{-680}{100} \right)^2$
	$= 342 - 46.24$
	$\text{Var}(x) = 295.76$
	$\text{Devall } \sigma = \sqrt{\text{var}(x)}$
	$= \sqrt{295.76}$
	$= 17.2$

Extract 2.1: A sample of the correct responses to question 2

In Extract 2.1, the candidate recalled the correct formula for calculating variance, $\delta^2 = \frac{1}{N} \sum_{i=1}^n f_i d_i^2 - \left(\frac{1}{N} \sum_{i=1}^n f_i d_i \right)^2$, and performed calculations correctly, obtaining the correct value of variance, $\delta^2 = 295.76$, as well as the standard deviation, $\delta = 17.2$.

On the other hand, 329 (84.14%) candidates lost marks on this question. In part (a), some candidates failed to determine the correct class containing the second quartile, whereby 50–59 was commonly observed. Thus, they got incorrect values of L , f_b and f_w , and consequently an incorrect value of the second quartile. Other candidates made computational errors, $N=108$ in particular, resulting in incorrect value of the second quartile. In part (b), many candidates tabulated an incomplete frequency distribution table which missed some potential columns such as deviation (d), fd or fd^2 . Moreover, some candidates used an inappropriate formula for variance, $\delta^2 = \sum f(x - \bar{x})^2$, and ended up with an incorrect answer.

D2	(a) Solution					
	FREQUENCY TABLE					
	Masses	f	x	$x - \bar{x}$	$(x - \bar{x})^2$	
	10 - 19	8	14.5	-33.2	1102.24	
	20 - 29	7	24.5	-23.2	538.24	
	30 - 39	14	34.5	-13.2	174.24	
	40 - 49	23	44.5	-3.2	10.24	
	50 - 59	26	54.5	6.8	46.24	
	60 - 69	12	64.5	16.8	282	
	70 - 79	6	74.5	26.8	718.24	
	80 - 89	4	84.5	36.8	1353.2	
		$\Sigma f = 100$				
	Mean = $\frac{A + \Sigma fd}{N}$					
	$= \frac{54.5 + (-680)}{100}$					
	$\bar{x} = 54.5 - 6.8$					
	$\bar{x} = 47.7$					
	b) Solution					
	Variance $s = \frac{\Sigma f(x - \bar{x})^2}{N}$					
	$= \frac{29688.14}{100} = 296.88$					
	$\therefore \text{Variance} = 296.88$					
	Standard deviations = $\sqrt{296.88}$					
	$= 17.23$					
	$\therefore \text{Standard deviation} = 17.23$					

Extract 2.2: A sample of the incorrect responses to question 2

In Extract 2.2 the candidate employed an inappropriate formula for calculating variance and standard deviation.

2.3 Question 3: Coordinate Geometry

This question is intended to assess the candidates' ability to express the general equation of a circle in standard form and to apply it to solve mathematical problems related to circles. It consisted of parts (a) and (b) as follows:

- (a) *Justify that the centre and radius of the equation of a circle given by $\frac{1}{2}x^2 + \frac{1}{2}y^2 + 3x + 4y = 12$ are $(-3, -4)$ and 7 respectively.*
- (b) *Prove that the circles whose equations are $2x^2 + 2y^2 - 8x - 4y - 8 = 0$ and $3x^2 + 3y^2 + 18x - 6y - 18 = 0$ are orthogonal.*

The analysis of the data reveals that 189 (48.34%) candidates scored 0 to 1.5, 154 (39.39%) candidates scored 2.0 to 3.5, and 48 (12.27%) scored 4.0 to 6.0. Therefore, the candidates' overall performance on this question was average. The summary of candidates' performances in this question is shown in Figure 4.

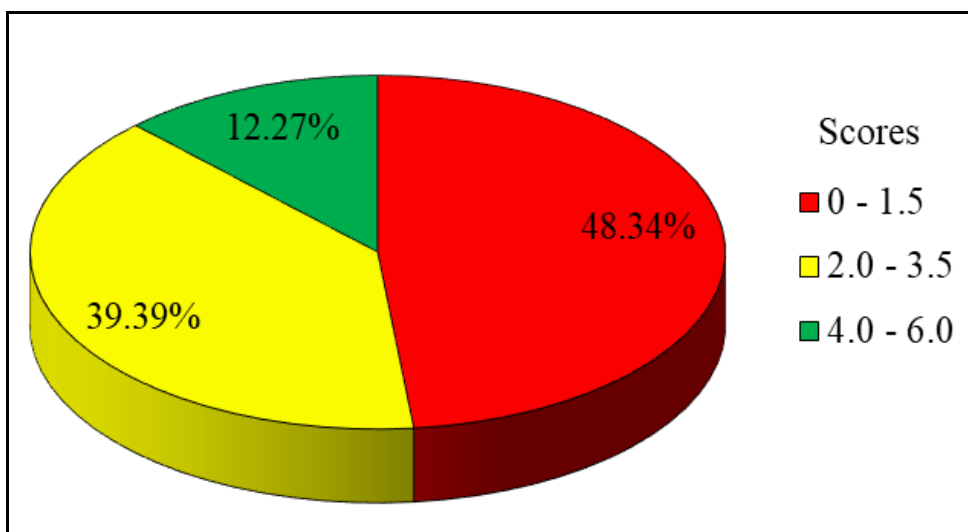


Figure 4: Candidates' Performance on Question 3

In part (a) candidates successfully wrote the equation $\frac{1}{2}x^2 + \frac{1}{2}y^2 + 3x + 4y = 12$ in the standard form, $(x+3)^2 + (y+4)^2 = 49$. Comparing with $(x-a)^2 + (y-b)^2 = r^2$, they obtained $a = -3$, $b = -4$ and $r = 7$, concluding that the centre is $(-3, -4)$ and the radius is 7. Other

candidates rewrote the equation in the general form, $x^2 + y^2 + 6x + 8y - 24 = 0$, and compared it with $x^2 + y^2 + 2gx + 2fy + c = 0$, obtaining $g = 3$, $f = 4$ and $c = -24$. With these values, they recognized that the centre and radius of the circle are given by $(-g, -f)$ and $r = \sqrt{g^2 + f^2 - c}$, respectively. Therefore, they performed appropriate substitutions and computations, obtaining the centre $(a, b) = (-3, -4)$ and radius $r = 7$. In part (b), the candidates rearranged the equations in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, obtaining $C_1 \equiv x^2 + y^2 - 4x - 2y - 4 = 0$ and $C_2 \equiv x^2 + y^2 + 6x - 2y - 6 = 0$, and consequently $g_1 = -2$, $f_1 = -1$ and $c_1 = -4$, as well as $g_2 = 3$, $f_2 = -1$ and $c_2 = -6$. They then substituted the values into the condition for orthogonal circles, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ and obtained -10 on both sides, verifying that the two circles are orthogonal. Other candidates proved the orthogonality of given circles by first rearranging the given equations in the standard form, as illustrated in Extract 3.1.

Qn 3'	(b) sh prove they are orthogonal
	From
	$\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{8x}{2} - \frac{4y}{2} - \frac{8}{2} = 0$
	$x^2 + y^2 - 4x - 2y - 4 = 0$
	$x^2 - 4x + (4x/2)^2 + y^2 - 2y + (2y/2)^2 = 4 + 4 + 1$
	$(x^2 - 4x + 4) + (y^2 - 2y + 1) = 9$
	$(x-2)^2 + (y-1)^2 = 9$
	$r^2 = 9 \therefore r = 3$
	$\therefore (x-2)^2 + (y-1)^2 = 3^2$
	And From Centre $(x_1, y_1) = (2, 1)$
	$\frac{3x^2}{3} + \frac{3y^2}{3} + \frac{18x}{3} - \frac{6y}{3} - \frac{18}{3} = 0$
	$x^2 + y^2 + 6x - 2y - 6 = 0$
	$x^2 + 6x + (6x/2)^2 + y^2 - 2y + (2y/2)^2 = 6 + 9 + 1$
	$(x^2 + 6x + 9) + (y^2 - 2y + 1) = 6 + 9 + 1$
	$(x+3)^2 + (y-1)^2 = 16$
	$r^2 = 16, r = 4$
	$\therefore (x+3)^2 + (y-1)^2 = 4^2$ Centre $(x_1, y_1) = (-3, 1)$
	For Orthogonal, sum of square of radii is equal to distance between centres squared

Qn	(b) .	$r_1^2 + r_2^2 = (\overline{C_1 C_2})^2$, $r_1^2 + r_2^2 = 25$
3		$9 + 16 = (\overline{C_1 C_2})^2$
		$25 = (\overline{C_1 C_2})^2$
		where $(\overline{C_1 C_2})^2 = \sqrt{(-3-2)^2 + (1-1)^2}$
		$(\overline{C_1 C_2}) = \sqrt{25+0}$
		$(\overline{C_1 C_2})^2 = (\sqrt{25})^2$
		$(\overline{C_1 C_2})^2 = 25$
		$\therefore r_1^2 + r_2^2 = (\overline{C_1 C_2})^2 = 25$ hence they
		are orthogonal.

Extract 3.1: A sample of the correct responses to question 3

In Extract 3.1, the candidate rewrote the given equations in standard form and correctly recalled the condition for orthogonal circles, $(\overline{C_1 C_2})^2 = r_1^2 + r_2^2$, and used it to justify the proof.

In contrast, 379 (96.93%) candidates faced challenges while responding to this question. In part (a), they used an inappropriate formula in computing the radius of a circle. For instance, a candidate used $r = \left| \frac{ax_1 + by_1 + c}{\sqrt{a+b}} \right|$, then substituted the given coordinates of the centre $(-3, -4)$ as (x_1, y_1) and the coefficients of the quadratic terms of the given circle as (a, b) and $c = 12$, ending up with an incorrect value of the radius as $\frac{30}{\sqrt{12}}$ (see Extract 3.2).

Other candidates applied $r = \sqrt{g^2 + f^2 - c}$ to find the radius but wrongly substituted $c = 12$ instead of $c = -12$, resulting in the incorrect value of $r = \sqrt{13}$. In part (b), some candidates differentiated the given equations, equated the derivative to zero, and then applied the concept that $m_1 m_2 = -1$. These candidates confused the concept of orthogonal circles with the concept of two perpendicular lines. Other candidates equated the two equations of the circles and simplified to form a single equation. For instance, some formulated an equation $x^2 + y^2 + 26x - 2y - 10 = 0$, used it

to determine the radius $r = 6\sqrt{5}$ and finally concluded that the circles are not orthogonal.

3a)	soluhen.
	$\frac{1}{2}x^2 + \frac{1}{2}y^2 + 3x + 4y = 12$
	$(-3, -4)$
	$r = \frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$
	$= \frac{\frac{1}{2}x - 3 + \frac{1}{2}3x - 4 + 12}{\sqrt{\frac{1}{4} + 1}}$
	$= \frac{6 + 12 + 12}{\sqrt{12}}$
	$= \frac{6 + 24}{\sqrt{12}}$
	$= \frac{30}{\sqrt{12}}$
	$\therefore$ the radius of the circle is $\frac{30}{\sqrt{12}}$.

3	b)
	$\frac{dy}{dx} (2x^2 + 2y^2 - 8x - 4y - 8) = 0$
	$4x + 4y - 8 - 4 = 0$
	$\frac{4y}{4} = \frac{-4x + 12}{4}$
	$y = -x + 3 \quad M_1 = -1$
	$\frac{dy}{dx} (3x^2 + 3y^2 + 18x - 6y - 18) = 0$
	$6x + 6y + 18 - 6 = 0$
	$\frac{6y}{6} = \frac{-12 - 6x}{6}$
	$y = -x - 2 \quad M_2 = -1$
	$M_1 M_2 = 1$
	$-1 \times -1 = 1$
	$1 = 1$
	<u>$\therefore$ the Circles are not Orthogonal.</u>

Extract 3.2: A sample of the incorrect responses to question 3

In Extract 3.2, the candidate, in part (a), used an incorrect formula to find the centre and radius, hence failed to justify the argument, and in part (b), used the formula for perpendicular lines instead of orthogonal circles.

2.4 Question 4: Locus

This question was designed to assess the candidates' ability to describe the locus of a point relative to two fixed points and the area traced by that locus. The question comprised two parts, (a) and (b), as follows:

The locus of point Q moves such that its distance from the point P(3, 2) is two times the distance from the point R(1, 1).

- Compute the locus of point Q.
- Find the area traced by the locus of point Q (Leave your answer in π).

The analysis of the data revealed that 175 (44.76%) candidates scored 0 to 1.5, 129 (32.99%) scored 2.0 to 3.5 and 87 (22.25%) scored 4.0 to 6.0. This indicates that the candidates' overall performance on this question was moderate. A summary of candidates' performances is presented in Figure 5.

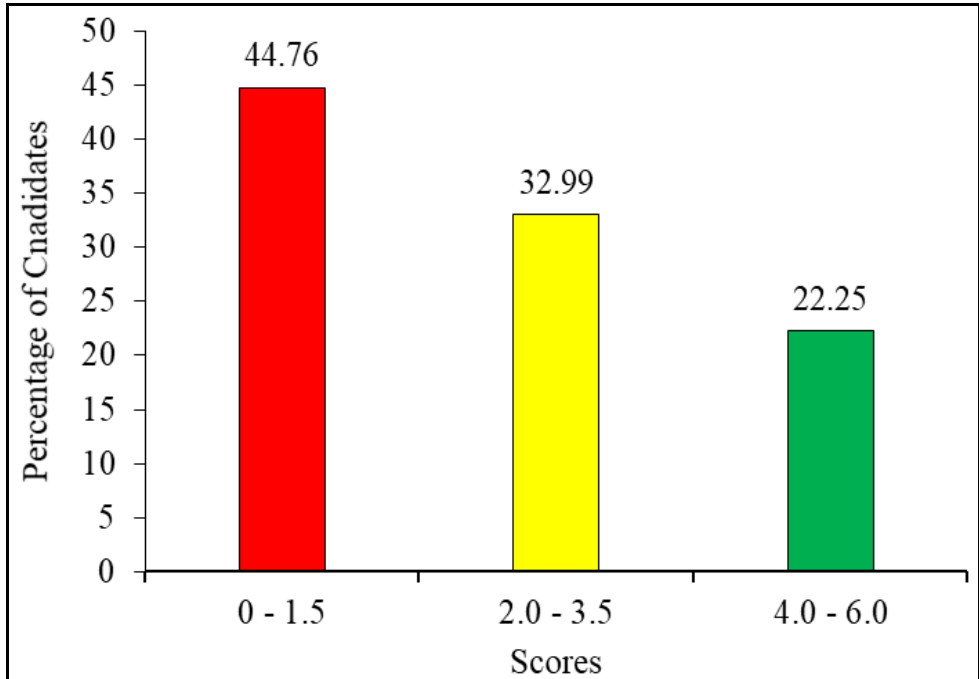


Figure 5: *Candidates' Performance on Question 4*

The good performance was attributed to the adequate knowledge of candidates on the examined competences. In part (a), many candidates interpreted the problem correctly and produced the condition $QP = 2QR$ or $(QP)^2 = (2(QR))^2$. They applied the distance formula $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ to find the distances $(QP)^2$ and $(QR)^2$, substituted in the obtained condition of locus, resulting in $(x-3)^2 + (y-2)^2 = 2^2((x-1)^2 + (y-1)^2)$, and simplified it into $3x^2 + 3y^2 - 2x - 4y - 5 = 0$. In part (b), the candidates recognised that the locus obtained in part (a) was a circle; therefore, they rewrote the equation in the standard form $(x - \frac{1}{3})^2 + (y - \frac{2}{3})^2 = \frac{20}{9}$, from which they obtained the square of the radius of the circle, $r^2 = \frac{20}{9}$. They then recalled and

applied the formula to find the area of a circle as πr^2 and obtained $\frac{20}{9}\pi$ square units.

04) (a) $\overline{QP} = 2\overline{QR}$.

$$\sqrt{(x-3)^2 + (y-2)^2} = 2\sqrt{(x-1)^2 + (y-1)^2}$$

$$(\sqrt{(x-3)^2 + (y-2)^2})^2 = (2\sqrt{(x-1)^2 + (y-1)^2})^2$$

$$(x-3)^2 + (y-2)^2 = 4[(x-1)^2 + (y-1)^2]$$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 4(x^2 - 2x + 1 + y^2 - 2y + 1)$$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 4x^2 - 8x + 4 + 4y^2 - 8y + 4$$

$$4x^2 - x^2 - 8x + 6x + 4y^2 - y^2 - 8y + 4y + 8 - 13 = 0$$

$$3x^2 - 2x + 3y^2 - 4y - 5 = 0$$

$$3x^2 - 2x + 3y^2 - 4y - 5 = 0$$

$$3x^2 + 3y^2 - 2x - 4y - 5 = 0$$

$\therefore$ The locus of Q is $3x^2 + 3y^2 - 2x - 4y - 5 = 0$

04) (b) $3x^2 + 3y^2 - 2x - 4y - 5 = 0$

$$\frac{3x^2}{3} + \frac{3y^2}{3} - \frac{2x}{3} - \frac{4y}{3} - \frac{5}{3} = 0$$

$$x^2 + y^2 - \frac{2}{3}x - \frac{4}{3}y = \frac{5}{3}$$

$$x^2 - \frac{2}{3}x + \left(-\frac{2}{6}\right)^2 + y^2 - \frac{4}{3}y + \left(-\frac{4}{6}\right)^2 = \frac{5}{3} + \frac{4}{36} + \frac{16}{36}$$

$$\left(x - \frac{2}{6}\right)^2 + \left(y - \frac{2}{3}\right)^2 = \frac{60 + 20}{36}$$

$$\left(x - \frac{2}{6}\right)^2 + \left(y - \frac{2}{3}\right)^2 = \frac{80}{36}$$

$$r^2 = \frac{80}{36}$$

$$= \frac{8 \cdot 94}{36}$$

$$= \frac{1 \cdot 49}{36}$$

$$r^2 = \frac{80}{36}$$

From;

$$\text{Area} = \pi r^2$$

$$= \pi \times \frac{80}{36}$$

$$= \frac{20\pi}{9} \text{ square units}$$

$\therefore$ The Area traced by Q is $\frac{20\pi}{9}$ sq. units

Extract 4.1: A sample of the correct responses to question 4

In Extract 4.1, a candidate, in part (a), used the correct distance formula to generate the locus, and in part (b), employed the appropriate formula to calculate the area.

In spite of the good performance, some candidates faced some challenges. In part (a), most candidates formulated an incorrect condition for the locus, commonly $2QP = QR$ instead of $QP = 2QR$. Other candidates used inappropriate formulae in computing the locus. For instance, some employed the gradient formula, that is, $PQ = \frac{y-2}{x-3}$ and $QR = \frac{y-1}{x-1}$, obtaining incorrect loci such as $x^2 + y^2 - 2x - 2y + 5 = 0$. In part (b), some applied inappropriate concepts such as integration. For instance, some wrote $A = \int_0^1 f(x)dx = [x^2 - y^2]_0^1$, which led to the incorrect area, 80π square units in particular. Similarly, some candidates employed a distance formula to calculate the area of a circle that is $A = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$, which eventually led to the wrong value. Moreover, a few used the coordinates (3, 2) and (1, 1) to compute the area of a circle, resulting in the incorrect answer of $\sqrt{5}$ units.

	$2QP = PQ$
	Then For QP
	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
	$d^2 = (3-x)^2 + (2-y)^2$
	$d^2 = 9 - 6x + x^2 + 4 - 4y + y^2$
	$d^2 (QP)^2 = x^2 + y^2 - 6x - 4y + 13$
	$(PQ)^2 = (1-x)^2 + (1-y)^2$
	$(PQ)^2 = 1 - 2x + x^2 + 1 - 2y + y^2$
	$(PQ)^2 = x^2 + y^2 - 2x - 2y + 2$
	but $2(QP)^2 = PQ$
	$2(x^2 + y^2 - 6x - 4y + 13) = x^2 + y^2 - 2x - 2y + 2$
	$2x^2 + 2y^2 = 12x - 8y + 26 = x^2 + y^2 - 2x - 2y + 2$
	$x^2 + y^2 - 10x - 6y + 24 = 0$
	$\therefore$ Locus of Point Q = $x^2 + y^2 - 10x - 6y + 24$

4.	b>.	$r = \sqrt{g^2 + f^2 - c}$
		$g = \frac{4}{2} = 2$
		$f = \frac{2}{2} = 1$
		$c = -11$
		$r = \sqrt{2^2 + 1^2 + 11}$
		$r = \sqrt{4 + 1 + 11}$
		$r = \sqrt{26}$
		$\text{Area} = \pi r^2$
		$= \pi \times (\sqrt{26})^2$
		$= \pi \times 26$
		$\text{Area} = 26\pi \text{ sq. units}$
		$\therefore \text{Area of the locus is } 26\pi \text{ sq. units}$

Extract 4.2: A sample of the incorrect responses to question 4

In Extract 4.2 the candidate, in part (a) made a computational error by omitting a 2 on QR and, in part (b), obtained the wrong value of area due to wrong values obtained from part (a).

2.5 Question 5: Algebra

In this question, the examiner intended to assess the candidates' ability to solve inequalities involving absolute values and to locate the solutions of linear inequalities on the number line. The question comprised parts (a) and (b) as follows:

The absolute value of the difference between twice a certain number and 5 is less than or equal to 7 and greater than or equal to 3.

- (a)
 - (i) Formulate the inequality representing this information.
 - (ii) Determine the solution sets of the obtained inequality.
- (b) On separate number lines represent the solution of the inequality obtained in part (a) (i).

The data reveal that 342 (87.47%) candidates scored 0 to 1.5, 31(7.42%) scored 2.0 to 3.5, and 18 (4.60%) scored 4.0 to 6.0 marks. Figure 6 summarises the candidates' performance on this question, which was generally weak.

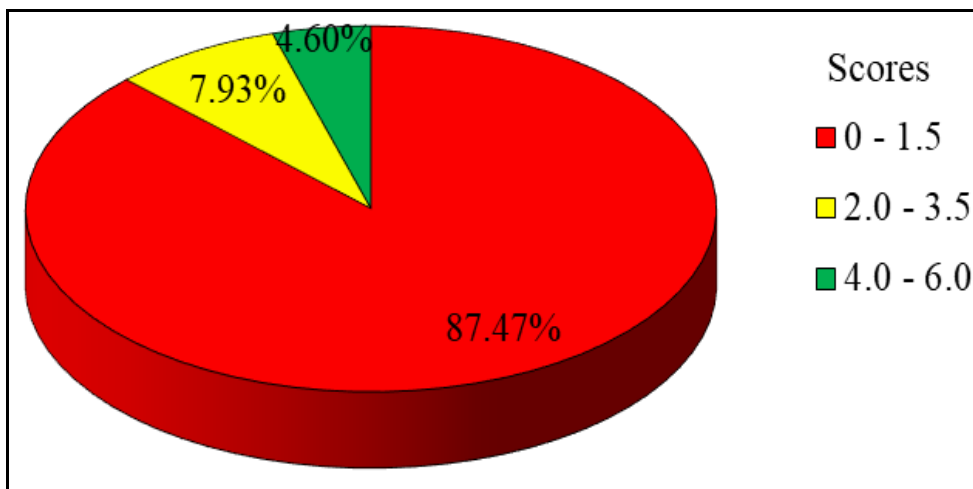


Figure 6: *Candidates' Performance on Question 5*

As shown in Figure 5, the majority had low scores. In part (a) (i), many candidates failed to interpret the word problem; for example, some wrote $|2x-5| \leq 7 \geq 3$ instead of $3 \leq |2x-5| \leq 7$. Other candidates formulated only one part of the inequality. For instance, some wrote $|2x-5| \leq 7$, introduced the $\pm$ symbol and solved it inappropriately, ending up with the incorrect value of $x \leq \frac{13}{2}$. Furthermore, some candidates replaced the inequality sign with an equal sign and solved the resulting equation, obtaining $x=6$. In part (b), the candidates failed to represent the inequalities on a number line, as illustrated in Extract 5.1.

5.	(i) $ 2x-5 \leq 7 \geq 3$
	(ii) $ 2x-5 = 7$
	$-2x+5=7$
	$2x-5=7$
	$-2x=2$
	$2x=12 \quad x=6$
	$x=-1$
	$ 2x-5 \leq 3$
	$2x-5=3$
	$-2x+5=3$
	$2x=8$
	$-2x=-2$
	$x=4$
	$=2 \quad -2x=1$
	$x \leq 6, x \leq -1$
	$x \geq 4, x \geq 1$
	(b)

Extract 5.1: A sample of the incorrect responses to question 5

In Extract 5.1, the candidate, in (a) (i) interpreted the word problem incorrectly, leading to incorrect inequality; in (a) (ii), combined the solution set wrongly; and in part (b), represented incorrect values on the number line.

Despite weak performance, 10 (2.56%) candidates scored full marks on this question. In part (a) (i), the candidates correctly interpreted the given information and rewrote it as $3 \leq |2x-5| \leq 7$. In part (a) (ii), candidates discarded the absolute value sign by introducing the $\pm$ symbol on the term $(2x-5)$, that is, $3 \leq \pm(2x-5) \leq 7$. They further solved the inequalities by first maintaining the positive sign, obtaining $4 \leq x \leq 6$, and second maintaining the negative sign, obtaining $-1 \leq x \leq 1$. In part (b) the candidates represented the solution set obtained in part (a) (ii) on a number line correctly, as shown in Extract 5.2.

Q5 a/ $|2x - 5| \leq 7$
 $|2x - 5| \geq 3$

$3 \leq |2x - 5| \leq 7.$

i/ The inequality is:
 $3 \leq |2x - 5| \leq 7.$

ii/ $3 \leq |2x - 5| \leq 7.$
 $3 \leq \pm(2x - 5) \leq 7$
 $3 \leq 2x - 5 \leq 7$ or $3 \leq -2x + 5 \leq 7.$

Starting with $3 \leq 2x - 5 \leq 7$
 $3 \leq 2x - 5$ and $2x - 5 \leq 7.$
 $3 + 5 \leq 2x$ and $2x \leq 7 + 5$

$\frac{8}{2} \leq \frac{2x}{2}$ and $\frac{2x}{2} \leq \frac{12}{2}.$
 $4 \leq x$ and $x \leq 6.$
 $4 \leq x \leq 6$ ——— First solution.

Following with:
 $3 \leq -2x + 5 \leq 7$
 $3 \leq -2x + 5$ and $-2x + 5 \leq 7$
 $3 - 5 \leq -2x$ and $-2x \leq 7 - 5.$
 $\frac{-2}{-2} \leq \frac{-2x}{-2}$ and $\frac{-2x}{-2} \leq \frac{2}{-2}.$
 $1 \geq x$ and $x \geq -1.$
 $1 \geq x \geq -1.$
 $-1 \leq x \leq 1.$ ——— Second solution.

b/ Number line for first solution, ($4 \leq x \leq 6$)

Number line for second solution, ($-1 \leq x \leq 1$)

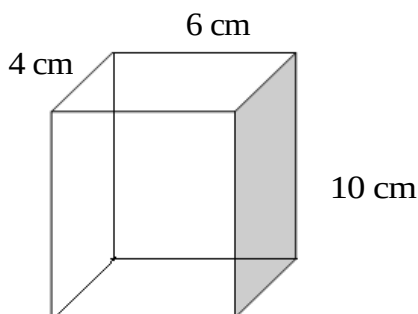
Extract 5.2: A sample of the correct responses to question 5.

In Extract 5.2, the candidate formulated the correct inequality, determined the solution set required and represented the solution on a number line.

2.6 Question 6: Plan and Elevations and Geometrical Constructions

The question aimed to assess candidates' ability to draw plans and elevations of a given figure and to apply the formula for computing the sum of interior and exterior angles for any polygon. It comprised parts (a) and (b) as follows:

(a) Draw the plan, side and front elevation of the following figure:



(b) Each interior angle of a regular polygon is greater than its exterior angle by 120° . Calculate;

- (i) the exterior angle of the polygon.
- (ii) the number of sides of the polygon.
- (iii) the sum of the interior angles of the polygon.

The data reveal that 93 (23.79%) candidates scored 0 to 1.5, 29 (7.42%) scored 2.0 to 3.5, and 269 (68.79%) scored 4.0 to 6.0. The overall performance of the candidates on this question was good, as shown in Figure 7.

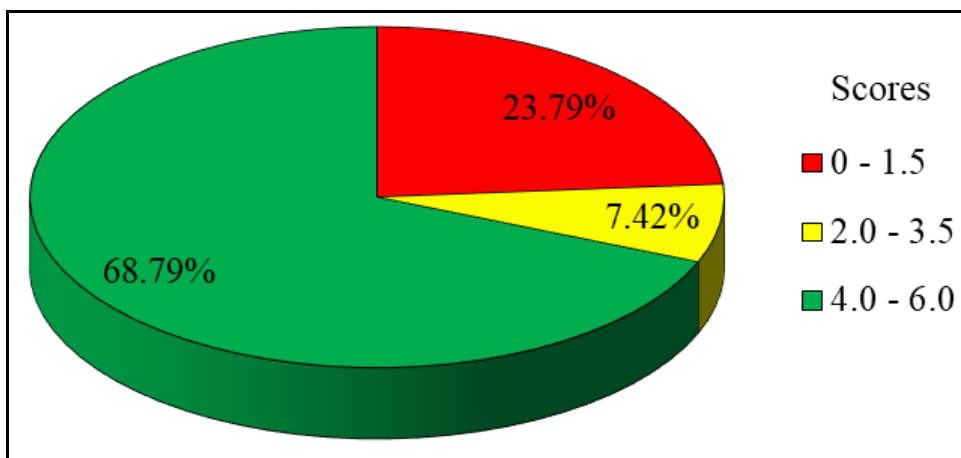
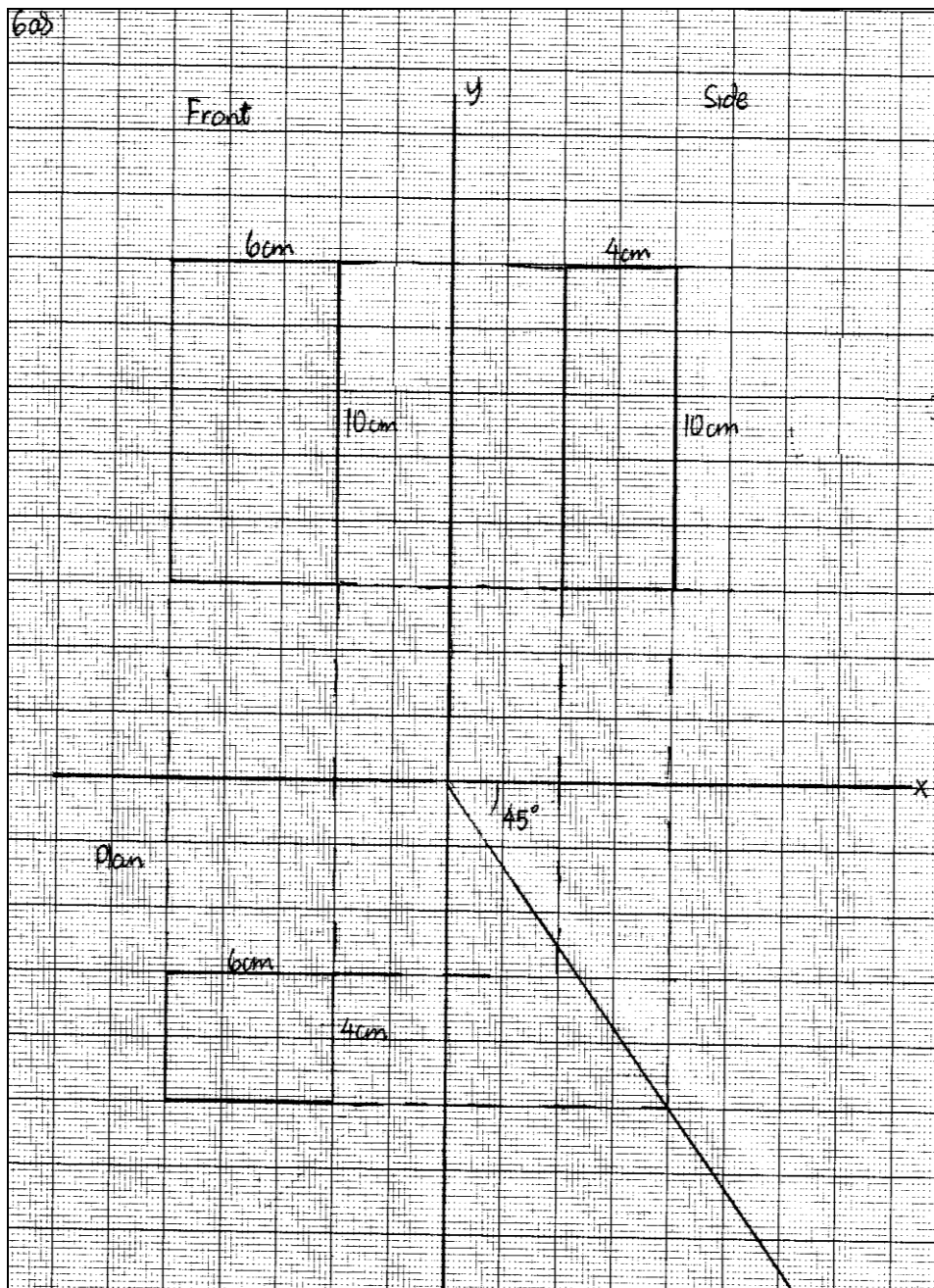


Figure 7: Candidates' Performance on Question 6

The candidates who performed well in this question were well skilled in the examined areas. In part (a), they correctly drew the plan, front and side elevation (Extract 6.1). In part (b) (i), they were conversant with the fact that the sum of interior angles and exterior angles is 180° . Thus, they used the given information of interior angle (i) and exterior angle (e) to formulate two simultaneous equations, $i - e = 120^\circ$ and $i + e = 180^\circ$, and solved them correctly, obtaining the exterior angle as $e = 30^\circ$. In part (b) (ii), they recalled and applied the formula $number\ of\ sides(n) = \frac{360^\circ}{exterior\ angle(e)}$, whereby they substituted the value of e and obtained the number of sides of the polygon, $n = 12$. In part (b) (iii), they applied the formula for finding the sum of interior angles as $(n - 2) \times 180^\circ$, where the value of $n = 12$, and obtained the sum of interior angles correctly as 1800° .



b)	interior angle = i
	exterior angle = e
	$i = 120 + e$
	$i - e = 120^\circ$ — (i)

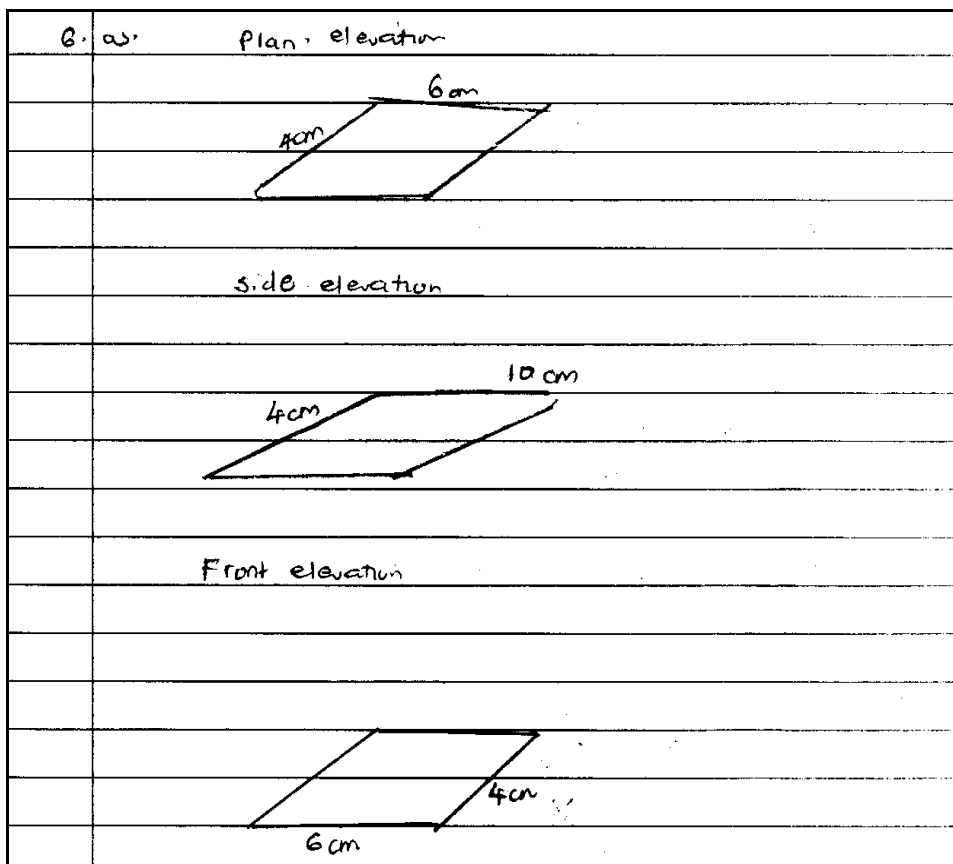
6b)	$i + e = 180^\circ$ (straight line) — (i)
	$i - e = 120^\circ$ }
	$i + e = 180^\circ$ }
	$i = 150^\circ$
	$e = 30^\circ$
	i) Exterior angle is 30°
ii)	$n = \frac{360^\circ}{e}$
	$n = \frac{360^\circ}{30^\circ}$
	$n = 12$
	∴ There are 12 sides in the polygon
iii)	sum of interior angles = $180(n-2)$
	$= 180(12-2)$
	$= 180(10)$
	$= 1800^\circ$
	∴ The sum of interior angles of the polygon is 1800°

Extract 6.1: A sample of the correct responses to question 6.

In Extract 6.1, the candidate correctly drew the plan, side and front elevations of the given figure and calculated the exterior angle, the number of sides and the sum of the interior angles of the polygon.

On the other hand, some candidates were incompetent in plans and elevations as well as geometrical constructions. In part (a), they failed to draw a plan, a front and a side elevation, commonly a parallelogram. Additionally, some drew relevant figures but with wrong dimensions. For example, some drew a rectangle with dimensions 4 cm and 6 cm but by looking at it, the side labelled 4 cm appears longer than the side labelled 6 cm. In part (b) (i), some candidates formulated incorrect equations such as $x + 120 + x = 360^\circ$ instead of $x + 120 + x = 180^\circ$, giving an incorrect exterior angle, $x = 120^\circ$. In part (b) (ii), some candidates made

computational mistakes, whereby some got the incorrect answer of $n=9$ from $n = \frac{360^\circ}{30}$ instead of $n=12$. In part (b) (iii), some candidates applied an incorrect formula to calculate the sum of interior angles where $(n-1) \times 180^\circ$ was commonly observed, obtaining 1960° instead of 1800° .



6	a	<u>Solution -</u>
	i)	The exterior angle of the polygon -
		Interior angle + Exterior angle = 360°
		$x + 120 + x = 360^\circ$
		$x + x + 120 = 360^\circ$
		$2x + 120 = 360$
		$2x = 360 - 120$
		$2x = 240$
		$x = 120^\circ$
		$\therefore$ The exterior angle of the polygon is 120°
6.	b	
	i)	The number of sides of the polygon -
		Exterior angle = $\frac{(n-2)180}{n}$
		$120 = \frac{(n-2)180}{n}$
		$120 = \frac{180n - 360}{n}$
		$120n = 180n - 360$
		$120n - 180n = -360$
		$-60n = -360$
		$-60 \quad -60$
		$n = 6$
		$\therefore$ The number of sides are 6.
	ii)	The sum of Interior angle -
		Interior angle + Exterior angle = 360°
		Interior + $120 = 360$
		Interior = $360 - 120$
		Interior = 240
		$\therefore$ Sum of Interior angle is 240°

Extract 6.2: A sample of the incorrect responses to question 6

In Extract 6.2 the candidate, in part (a), drew parallelograms; in part (b) (i) employed an incorrect formula; in parts (b) (ii) and (b) (iii), used inappropriate formulae.

2.7 Question 7: Trigonometry

The question was intended to assess the candidates' ability to solve trigonometric equations using trigonometric identities and double- and compound-angle formulae. It consisted of the following parts:

- (a) Two waves are defined by the equations $y = 3\sin^2 \theta + \cos 2\theta$ and $y = 1 + \sin 2\theta$. If the waves meet, what will be the values of θ of the combined waves given that $0^\circ \leq \theta \leq 360^\circ$?
- (b) If $2\sin(x - 30^\circ) = \cos(x + 60^\circ)$, what will be the value of $\tan x$? Leave the answer in surd form.

The data reveal that 148 (37.85%) candidates scored 2.0 to 6.0, indicating that the overall candidates' performance was average. The summary of candidates' performances is shown in Figure 8.

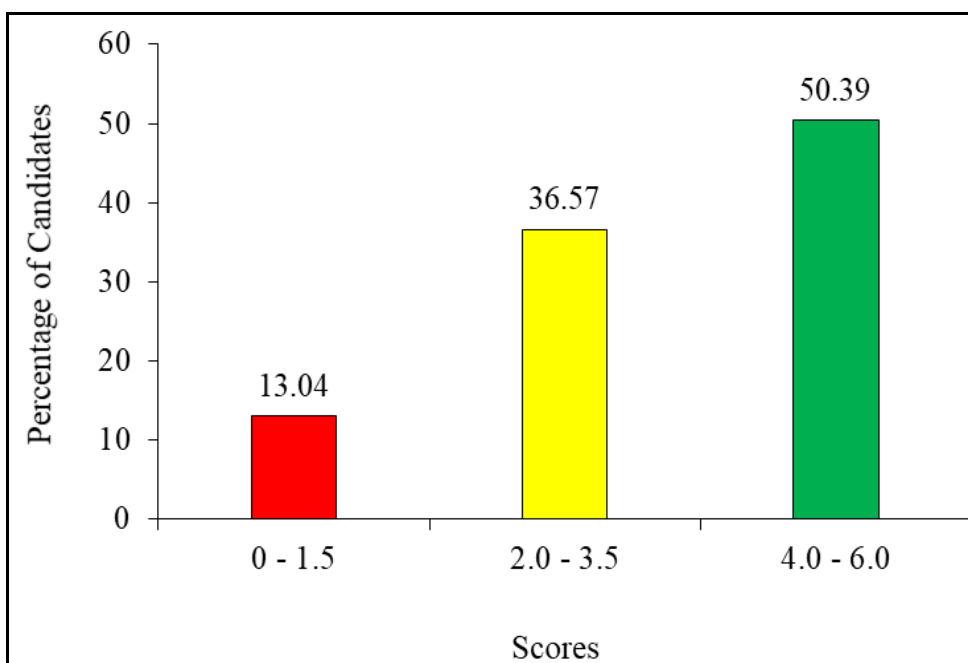


Figure 8: Candidates' Performance on Question 7

In this question, only 4 (1.02%) candidates responded to both parts correctly. In part (a), they realised that they are required to find the intersection points of the two curves, $y = 3\sin^2 \theta + \cos 2\theta$ and $y = 1 + \sin 2\theta$. Thus, they equated the two equations and got $3\sin^2 \theta + \cos 2\theta = 1 + \sin 2\theta$. They then applied the double angle formulae correctly, $\cos 2\theta = 1 - 2\sin^2 \theta$ and $\sin 2\theta = 2\sin \theta \cos \theta$ simplifying into

$\sin^2 \theta - 2 \sin \theta \cos \theta = 0$. With factorisation and further simplifications, they got $\sin \theta = 0$ and $\tan \theta = 2$, and consequently the values of θ are 0° , 63.4° , 180° , 243.4° and 360° . In part (b), many candidates applied the compound angle identities of sine and cosine, resulting in $2(\sin x \cos 30^\circ - \sin 30^\circ \cos x) = \cos x \cos 60^\circ - \sin x \sin 60^\circ$. They further substituted values of special angles for sine and cosine of 30° and 60° into the equation and obtained $2\left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x\right) = \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$, which led to $\tan x = \frac{\sqrt{3}}{3}$.

7b/	Solution.
	$2 \sin(x-30) = \cos(x+60)$
	$2(\sin x \cos 30 - \cos x \sin 30) = \cos x \cos 60 - \sin x \sin 60$
	$2\left(\sin x \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cos x\right) = \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$
	$\frac{\sqrt{3} \sin x - \cos x}{2} = \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$
	$\frac{3\sqrt{3} \sin x}{2} = \frac{3}{2} \cos x$
	divide by $\cos x$ throughout.
	$\frac{3\sqrt{3}}{2} \tan x = \frac{3}{2}$
	$\tan x = \frac{\sqrt{3}}{3}$
	Therefore $\tan x = \frac{\sqrt{3}}{3}$

Extract 7.1: A sample of the correct responses to question 7.

In Extract 7.1, in part (a) the candidate equated two equations, used double-angle formulae to simplify and obtained values of angles. In part (b), the candidate correctly expanded compound angle formulae and obtained $\tan x$.

However, some problems related to the use of trigonometric identities and the expansion of compound angle formulae were observed from the responses of some candidates. In part (a) some of them failed to correctly

formulate a single equation from the given two equations. For instance, a candidate, instead of equating the two equations, found the sum of the equations, then equated it to zero, obtaining $3\sin^2 \theta + \cos 2\theta + 1 + \sin 2\theta = 0$. Thereafter, they substituted the double angle identities and performed manipulation, formulating an irrelevant equation such as $(\sin^2 \theta)^2 + 2\sin^2 \theta - 2\sin^2 \theta (\sin^2 \theta) + 4 = 0$ (see Extract 7.2). Other candidates substituted incorrect trigonometric identities. For example, some substituted $\cos 2\theta = 1 - \sin^2 \theta$ in $3\sin^2 \theta + 1 - 2\sin^2 \theta = 1 + 2\sin \theta \cos \theta$, ending up with incorrect answers, $\theta = 45^\circ$ and 135° . In part (b), they failed to write the compound angle formulae correctly. For instance, some wrote $2\sin(x - 30^\circ) = 2\sin x - 2\sin 30^\circ$, while others illogically divided $2\sin(x - 30^\circ) = \cos(x + 60^\circ)$ by $\cos$ as $\frac{2\sin(x - 30^\circ)}{\cos} = \frac{\cos(x + 60^\circ)}{\cos}$, obtaining $2\tan(x - 30^\circ) = x + 60^\circ$ and consequently an incorrect value of $\tan x = 0.5$.

I. @	$y = 3\sin^2 \theta + \cos 2\theta$ and $y = 1 + \sin 2\theta$
	$3\sin^2 \theta + \cos 2\theta + 1 + \sin 2\theta = 0$
	$3\sin^2 \theta + \cos^2 \theta - \sin^2 \theta + 1 + 2\sin \theta \cos \theta = 0$
	$2\sin^2 \theta + \cos^2 \theta + \sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = 0$

$$3\sin^2\alpha + 2\cos^2\alpha + 2\sin\alpha\cos\alpha = 0$$

$$3\sin^2\alpha + 2(1 - \sin^2\alpha) + 2\sin\alpha(\sqrt{1 - \sin^2\alpha}) = 0$$

$$3\sin^2\alpha + 2 - 2\sin^2\alpha + 2\sin\alpha\sqrt{1 - \sin^2\alpha} = 0$$

$$\sin^2\alpha + 2 + 2\sin\alpha\sqrt{(1 - \sin^2\alpha)^2} = 0$$

$$\sin^2\alpha + 2\sin\alpha\sqrt{1 - \sin^2\alpha} + 2 = 0$$

$$(\sin^2\alpha)^2 + 2\sin\alpha^2(1 - \sin^2\alpha)^{\frac{1}{2}x^2} + 2x^2 = 0$$

$$(\sin^2\alpha)^2 + 2\sin^2\alpha(1 - \sin^2\alpha) + 4 = 0$$

$$(\sin^2\alpha)^2 + 2\sin^2\alpha - 2\sin^2\alpha(\sin^2\alpha) + 4 = 0$$

let $\sin^2\alpha = x$

$$x^2 + 2x - 2x(x) + 4 = 0$$

$$x^2 + 2x - 2x^2 + 4 = 0$$

$$-x^2 + 2x + 4 = 0$$

$$x^2 - 2x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

	$x = \frac{2 + 2\sqrt{5}}{2} \text{ or } x = \frac{2 - 2\sqrt{5}}{2}$
	$x = 1 + \sqrt{5} \text{ or } x = 1 - \sqrt{5}$
	$\sqrt{\sin^2 \alpha} = \sqrt{1 + \sqrt{5}}$
	$\sin \alpha = 1 + 1.5$
	$\sin \alpha = 2.5$
	$\alpha = \sin^{-1}(2.5)$
(b)	from
	$2 \sin(x-30) = \cos(x+60)$
	$2(\sin x \cos 30 - \sin 30 \cos x) =$
	$(\cos x \cos 60 - \sin 60 \sin x)$
	$2 \sin x \frac{\sqrt{3}}{2} - \frac{1}{2} \cos x = \cos x \frac{1}{2} - \frac{\sin x \sqrt{3}}{2}$
	$2 \sin x \left(\frac{\sqrt{3}}{2} \right) - \cos x \frac{\sqrt{3}}{2} = \cos x \frac{1}{2} + \sin x \frac{1}{2}$
	$2 \sin x \frac{\sqrt{3}}{2} - \cos x \frac{\sqrt{3}}{2} = \cos x$

Extract 7.2: A sample of the incorrect responses to question 7

In Extract 7.2, in part (a), the candidate found the sum of the equations and equated it to zero to obtain an incorrect value of θ , and in part (b), the candidate wrote equations that inappropriately led to incorrect simplification.

2.8 Question 8: Numbers

The question was intended to examine the candidates' competence in generating number patterns guided by some rules and their knowledge of the divisibility rule for numbers. The question comprised the following parts:

- (a) *During the football match between teams A and B, the time of scores in minutes of each team were recorded in ascending order as follows:*

Team A: 1, 8, 27, ...

Team B: 1, 2, 6, 24, ...

What will be the subsequent time of the next score obtained by each team?

- (b) *The chairperson of Msufini village reported the amount of harvests in tons of the three crops; maize, beans and tomatoes as 4250, 4257 and 5742 respectively in a village meeting. State and use the divisibility rules to determine the amount of harvests that is divisible by:*

(i) 3

(ii) 6

The data analysis disclosed that 48 (12.28%) scored from 0 to 1.5, 120 (30.69%) scored from 2 to 3.5, and 223 (57.03%) scored from 4 to 6. This means that the general performance of candidates for this question was good.

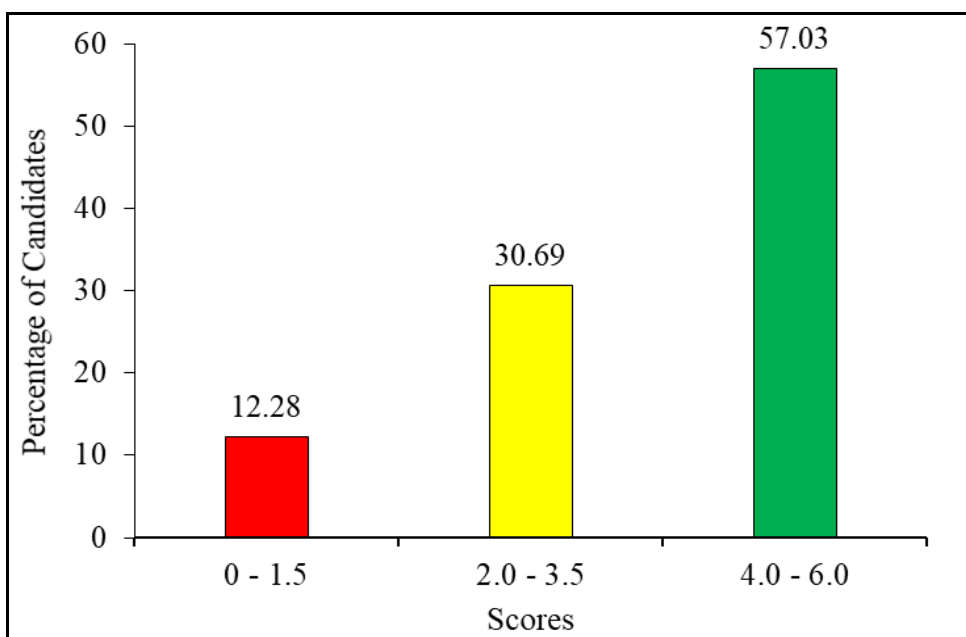


Figure 9: *Candidates' Performance on Question 8*

In this question, 80 (20.46%) scored full marks. In part (a) (i), the candidates recognised that the pattern 1, 8, and 27 are the cubes of the natural numbers 1, 2, and 3, respectively. Therefore, they computed the cube of 4, resulting in 64, and concluded that team *A* will score the next goal in the 64th minutes. Considering the numbers 1, 2, 6, and 24, the candidates recognised that each of the numbers 2, 6, and 24 is the product of the preceding number and the position of the number to be determined. Thus, they correctly got the 5th number after multiplying 24 by 5, obtaining 120. Therefore, they concluded that team *B* will score the next goal in the 120th minute.

In part (b) (i), the candidates recalled the divisibility rule of 3 that ‘a number is divisible by 3 if the sum of its digits is the multiple of 3’. Thus, they correctly inspected 4250, 4257 and 5742 to see whether they are divisible by 3 or not and identified that 4257 and 5742 are divisible by 3. In part (b) (ii), the candidates were also conversant with the rule that the number is divisible by 2 if its digit in ones is either 0, 2, 4, 6 or 8. They also recalled the rule that the number is divisible by 6 if it is divisible by both 2 and 3. Therefore, they further inspected 4250, 4257 and 5742 to see whether they are divisible by both 2 and 3 before concluding about the divisibility by 6, as illustrated in Extract 9.1.

08.	(b) (ii) Divisibility by 6.
	"The number is divisible by 6, when; the number can be divided by both 2 and 3".
	1st Case; 4250 .
	4250; It is divisible by 2, because at ones position there is zero.
	but;
	4250; It is not divisible by 3, because the sum of the numbers is not divided by 3.
	as; $4+2+5+0=11$.
	Therefore;
	4250 cannot be divisible by 6.
	2nd Case; 4257 .
	4257; It is not divisible by 2, because; at ones there is no an even number or zero
	4257; It is divisible by 3, because sum of the numbers gives 18.
	Therefore;
	4257, It is not divisible by 6.
	3rd Case; 5742 .
	5742 it is divisible by 2 because at ones position there is an even number which is 2.
	5742, It is divisible by 3, because the sum of the numbers or digits gives a number which is divisible by 3.

Extract 8.1: A sample of the correct responses to question 8

In Extract 8.1, the candidate applied the divisibility rule of 3 and 6 correctly.

Despite the good performance, 311 (79.54%) lost marks in this question. In part (a), many candidates failed to recognise the correct rule guiding the terms. Some of them guessed the number without any logical reason. For example, some wrote 58 as the next number in the sequence 1, 8, 27, ... while others wrote 56 as the next number for the sequence 2, 6, 24, ...

In part (b) (i), some candidates failed to recall or use correctly the divisibility rules of either 2, 3 or 6. For instance, some candidates wrongly divided the last two digits from the given numbers, that is, $\frac{50}{3}$ for 5250, $\frac{57}{3}$ for 4257 and $\frac{42}{3}$ for 5742. In part (b) (ii), some candidates incorrectly stated that the number is divisible by 6 if the sum of the last two digits is divisible by 3 or 2 (see Extract 8.2).

	n/14 the sum of two last digit is divisible by 3 or 2 then it is divisible by 6.
	beans = $5+7=12$ it is divisible by 2.
	Tomatoes = $4+2=6$ it is divisible by 2 and 3.

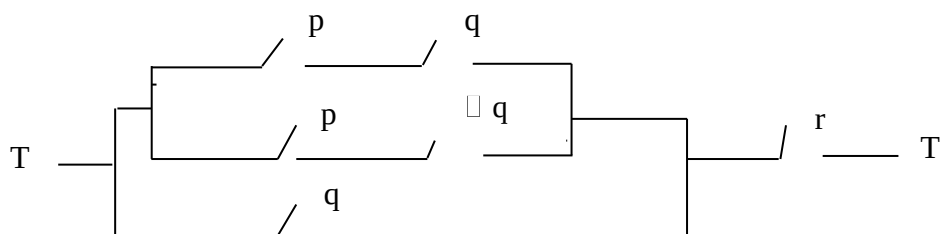
Extract 8.2: A sample of the incorrect responses to question 8

In Extract 8.2, the candidate incorrectly applied the divisibility rule of 2 by dividing the sum of the last two digits by 2.

2.9 Question 9: Logic

The question aimed to assess the candidates' ability to interpret series and parallel switches and to formulate compound statements, use the laws of algebra of propositions, and draw simple circuit diagrams. The question was made up of parts (a) and (b), which were stated as follows:

A school electrician designed an electric circuit for installation in the school laboratory as indicated in the following diagram;



- (a) (i) Use the given diagram to formulate a compound statement representing the circuit.
- (ii) Using the laws of algebra of propositions, simplify the obtained compound statement in (a) (i).
- (b) Draw a circuit of the simplified statement obtained in part (a) (ii).

The data show that 165 (42.20%) candidates scored from 2 to 6 marks, indicating the average overall performance of the candidates. Figure 10 shows the percentage of candidates who obtained low, average and high scores in this question.

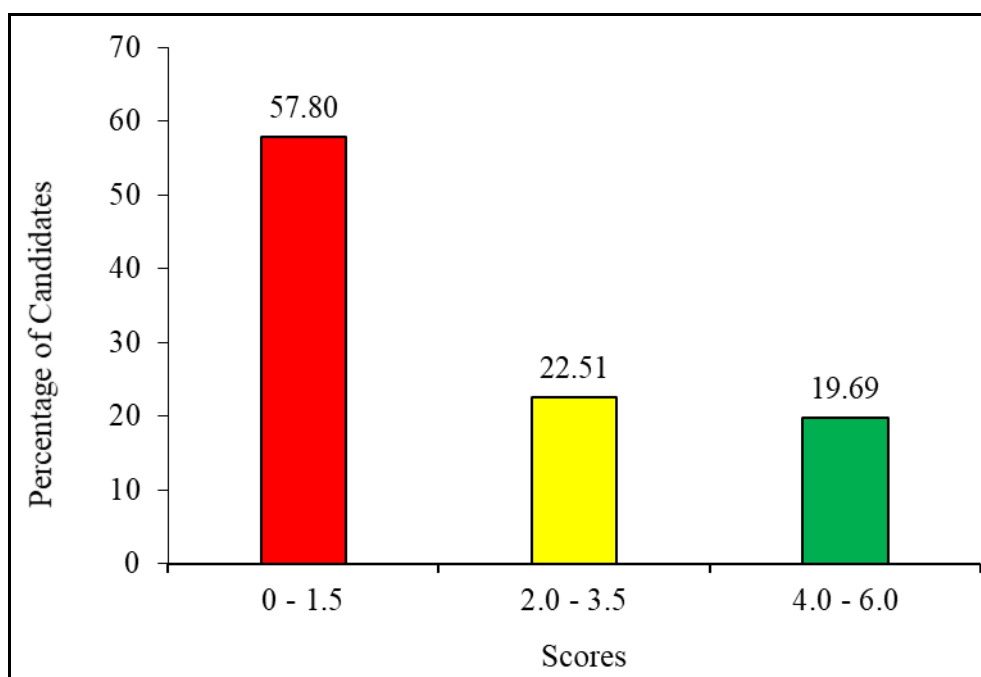


Figure 10: Candidates' Performance on Question 9

Candidates who performed well in this question were knowledgeable about logical reasoning. In part (a) (i), they interpreted correctly the concept of series and parallel connections of switches and formulated the compound

statement that corresponds to the circuit diagram correctly. In part (a) (ii), they used correctly appropriate laws such as distributive, negation and identity laws to simplify the formulated statement and obtained $(p \vee q) \wedge r$. In part (b), they used the knowledge of series and parallel connections of switches to draw the diagram correctly.

9 (a) i)	Soln.	
		$[(p \wedge q) \vee (p \wedge \sim q) \vee q] \wedge r$ compound statement of the given circuit.
(ii)		$[(p \wedge q) \vee (p \wedge \sim q) \vee q] \wedge r$ --- given.
		$(p \wedge (q \vee \sim q) \vee q) \wedge r$ --- distributive law.
		$[(p \wedge T) \vee q] \wedge r$ --- Negation.
		$(p \vee q) \wedge r$ --- Identity.
(b)	Solution	

Extract 9.1: A sample of the correct responses to question 9

In Extract 9.1 the candidate, in part (a) (i) formulated correctly the compound statement; in part (a) (ii), used appropriate laws of algebra to simplify the compound statement; and in part (b), drew the simplified circuit correctly.

Nevertheless, some candidates scored low marks in this question due to various reasons. In part (a) (i), formulated an incorrect compound statement due to lack of knowledge of series and parallel connection of switches. For instance, some wrote $[(p \wedge q) \vee (p \wedge \sim q) \vee q] \rightarrow r$. Since all other parts of the question depend on (a) (i) then it is obvious that the misconception on this part led to incorrect answers to subsequent parts. However, in part (a) (ii), some candidates correctly formulated the compound statement but failed to recall and apply the appropriate laws correctly. For instance, the

candidate correctly wrote $[(p \wedge q) \vee (p \wedge \sim q)] \vee q \wedge r$ but simplified incorrectly to $[p \wedge (p \wedge \sim q) \wedge q \vee (p \wedge \sim q) \vee q] \wedge q$, indicating failure to apply the laws. In part (b), many candidates drew incorrect circuits despite the correct simplification from part (a) (ii). Other candidates made wrong simplifications without stating the names of laws.

9.1(i)	$[(p \wedge q) \vee (p \wedge \sim q)] \vee q \rightarrow p \wedge r$
ii)	$(p \wedge q) \vee (p \wedge \sim q)$ $(p \wedge q) \vee [p \wedge (\sim q \vee q)] \rightarrow p \wedge r$ $(p \wedge q) \vee p \wedge \vee \rightarrow p \wedge r =$ $(p \wedge q) \vee p \rightarrow p \wedge r$ $(q \wedge p) \vee p \rightarrow p \wedge r$ $q \wedge (p \vee p) \rightarrow p \wedge r$ $(q \wedge p) \rightarrow p \wedge r$
b)	

Extract 9.2: A sample of the incorrect responses to question 9

In Extract 9.2 the candidate, in part (a) (i) formulated an incorrect compound statement; in part (a) (ii), simplified wrongly without stating the used laws; and in part (b), interpreted implication as a series connection.

2.10 Question 10: Sets

The question intended to examine candidates' ability in using laws of algebra in sets to simplify set expressions and to solve set problems involving the number of elements in a set. The question consisted of parts (a) and (b) as follows:

- (a) Using the laws of algebra in sets, verify that $(M \cup N)' \cap (M \cap N)' = (M \cup N)'$

(b) Given the universal set $U = \{1, 2, 3, 4, 5 \dots 20\}$ and the subsets $P = \{1, 4, 7, 10, 13, 15\}$ and $Q = \{2, 4, 6, 8, 10\}$, determine;

(i) $n(P' \cup Q')$

(ii) $n(P \cup Q)'$

The data reveal that 214 (54.73%) candidates scored 0 to 1.5, while 177 (45.27%) candidates scored 2.0 to 6.0. Therefore, the candidates' overall performance was moderate. The summary of candidates' performances is illustrated in Figure 11.

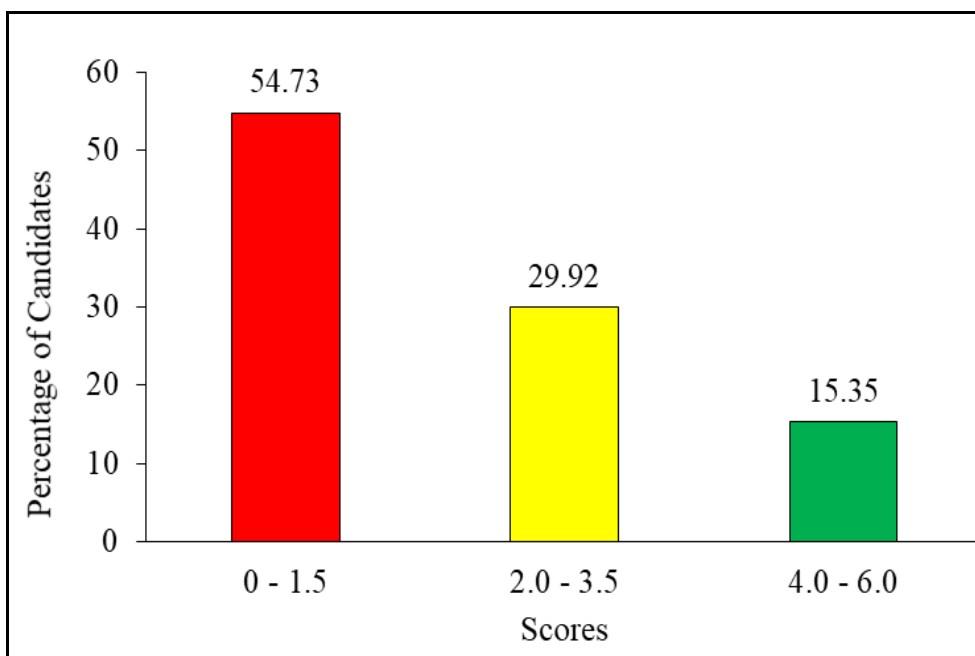


Figure 11: Candidates' Performance on Question 10

The data further show that 15 (3.84%) candidates scored full marks. In part (a), they recalled and applied the appropriate laws correctly, including *De Morgan's*, *Distributive*, *Commutative* and *Idempotent* laws to simplify the given set expression $(M \cup N)' \cap (M \cap N)'$, obtaining $(M \cup N)'$. Alternatively, other candidates applied only three laws appropriately, such as *De Morgan's*, *Associative* and *Absorption* laws, and obtained the same answer. In part (b) (i), the candidates were able to find $n(P' \cup Q')$ by first listing all the elements in the universal set μ but not in set P , that is, P' , and then all the elements in the universal set μ but not in Q , that is, Q' .

They then listed all the elements found in either P' or Q' without repetition, that is $P' \cup Q' = \{1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, and counted the elements and got $n(P' \cup Q') = 18$. In part (b) (ii), they first found all the elements in either P or Q without repetition, that is, $(P \cup Q)$. They then listed all the elements in the universal set μ but not in $(P \cup Q)$, that is $(P \cup Q)' = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$, and consequently $n(P \cup Q)' = 11$. Alternatively, other candidates applied De Morgan's law, $n(P \cup Q)' = n(P' \cap Q')$, and therefore, they correctly found P' and Q' as well as $P' \cap Q' = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$, resulting in $n(P' \cap Q') = n(P \cup Q)' = 11$.

10a)	soln
	$(M \cup N)' \cap (M \cap N)' = (M \cup N)'$
	Consider left hand side
	$(M \cup N)' \cap (M \cap N)'$ De morgans law
	$(M' \cap N') \cap (M' \cup N')$ Distributive law.
	$M' \cap (N' \cup N')$ Idempotent law.
	$M' \cap N'$ De morgans
	$(M \cup N)'$ hence verified
	<u>$(M \cup N)' \cap (M \cap N)' = (M \cup N)'$ verified</u>
10b)	soln
	$U = \{1, 2, 3, 4, 5, \dots, 20\}$
	$P = \{1, 4, 7, 10, 13, 15\}$
	$Q = \{2, 4, 6, 8, 10\}$
	$P' = \{2, 3, 5, 6, 8, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$
	$Q' = \{1, 3, 5, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$
i)	$(P' \cup Q') = \{1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
	<u>$n(P' \cup Q') = 18$</u>
ii)	$n(P \cup Q)'$
	$P \cup Q = \{1, 2, 4, 6, 7, 8, 10, 13, 15\}$
	$(P \cup Q)' = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$
	<u>$n(P \cup Q)' = 11$</u>

Extract 10.1: A sample of the correct responses to question 10

In Extract 10.1, the candidate, in part (a), applied the appropriate laws to verify the statement; in part (b) (i) found the number of elements of the complement of sets correctly; and in part (b) (ii), found the number of complements of the union of the set correctly.

However, the analysis also revealed that some candidates performed poorly in this question due to various challenges. In part (a), some candidates stated inappropriate names of laws. For example, a candidate simplified $(M \cup N)' \cap (M \cap N)'$ into $M' \cap N' \cap M' \cup N'$ and stated it as the idempotent law instead of De Morgan's law. Other candidates simplified $(M' \cap N') \cap (M' \cup N')$ into $(M' \cap M') \cap (N' \cup N')$, stating distributive law instead of commutative law. In part (b) (i), they failed to find the complement of a set. For example, some candidates correctly found correctly $(P \cup Q)$ but incorrectly got $(P \cup Q)' = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19\}$ instead of $(P \cup Q)' = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$, leading to an incorrect number of elements, 10 instead of 11. In part (b) (ii), some failed to distinguish between the set and number of elements of a set, as they wrote $n(P' \cap Q') = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$ instead of $n(P' \cap Q') = 11$.

10.	(a) Soln
	$(M \cup N)' \cap (M \cap N)'$ - Expression
	$M' \cap N' \cap M' \cup N'$ - Idempotent law
	$M' \cap M' \cap N' \cup N'$ - Commutative law..
	$M' \cap N'$ - Idempotent law
	$(M \cup N)'$ - De Morgan's law

10.	(b) <u>Soln</u>
	$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
	$P = \{1, 4, 7, 10, 13, 15\}$
	$Q = \{2, 4, 6, 8, 10\}$
	$i, n(P' \cup Q')$
	$P' = \{2, 3, 5, 6, 8, 9, 11, 12, 14\}$
	$Q' = \{1, 3, 5, 7, 9\}$
	Q'
	$(P' \cup Q') = \{1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 14\}$
	$ii, n(P \cup Q)'$
	$P \cup Q = \{1, 2, 4, 6, 7, 8, 10, 13, 15\}$
	$(P \cup Q)' = \{3, 5, 9, 11, 12, 14, 16, 17, 18, 19, 20\}$

Extract 10.2: A sample of the incorrect responses to question 10

In Extract 10.2 the candidate, in part (a), applied and stated the laws wrongly; in part (b) (i) did not find the number of elements but listed the elements wrongly; and in part (b) (ii), listed the elements instead of finding the number of elements.

2.11 Question 11: Functions, Equations and Remainder Theorem

The question examined the candidates' competences in sketching and stating properties of rational functions. It also intended to observe their knowledge on the application of the remainder theorem. The question consisted of parts (a) and (b) as follows:

- (a) A curve representing the motion of a particle is expressed as $y(x-3) = 2x+6$.
- (i) Find the asymptotes of the curve.
- (ii) Sketch the given curve in the xy -plane.

- (iii) Use the graph to find the value of y when x is numerically very large.
- (b) One of the zeroes of the polynomial function $p(x) = 4x^4 - 4x^3 - mx^2 + 12x - 3$ is $x = \frac{1}{2}$. Compute the value of m .

As shown in Figure 12, the candidates' performance on this question was generally good.

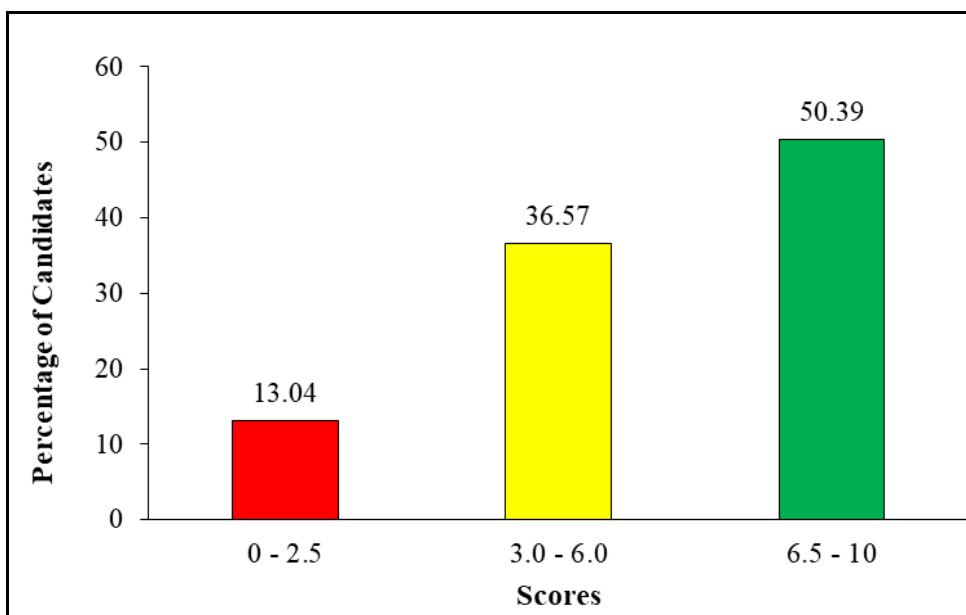


Figure 12: Candidates' Performance on Question 11

The data reveal that 6 (1.53%) candidates responded to this question perfectly. In part (a) (i), the candidates correctly expressed y as the subject of the equation $y(x-3) = 2x+6$ and obtained a rational function $y = \frac{2x+6}{x-3}$. They then correctly determined the vertical asymptote $x=3$ and the horizontal asymptote $y=2$. In part (a) (ii), the candidates determined the intercepts, obtaining $x=-3$ and $y=-2$. They then correctly plotted the graph in the xy -plane, as illustrated in Extract 11.1. In part (a) (iii), the candidates realised that as the value of x increases, the value of y approaches 2. Hence, they concluded that as the value of x becomes numerically very large, the value of y is 2. In part (b), the

candidates were conversant with the fact that if a is a zero of $p(x)$, then $p(a) = 0$. Therefore, they correctly formulated the equation by working on $p\left(\frac{1}{2}\right) = 0$ and solved it, obtaining $m = 11$.

11. (a)

Soln

$$y(x-3) = 2x+6.$$

$$y = \frac{2x+6}{x-3}.$$

(i). Where by

$$\text{H.A. } y = \frac{a}{b} = \frac{2}{1} = 2$$

We N.A = let denominator = 0.

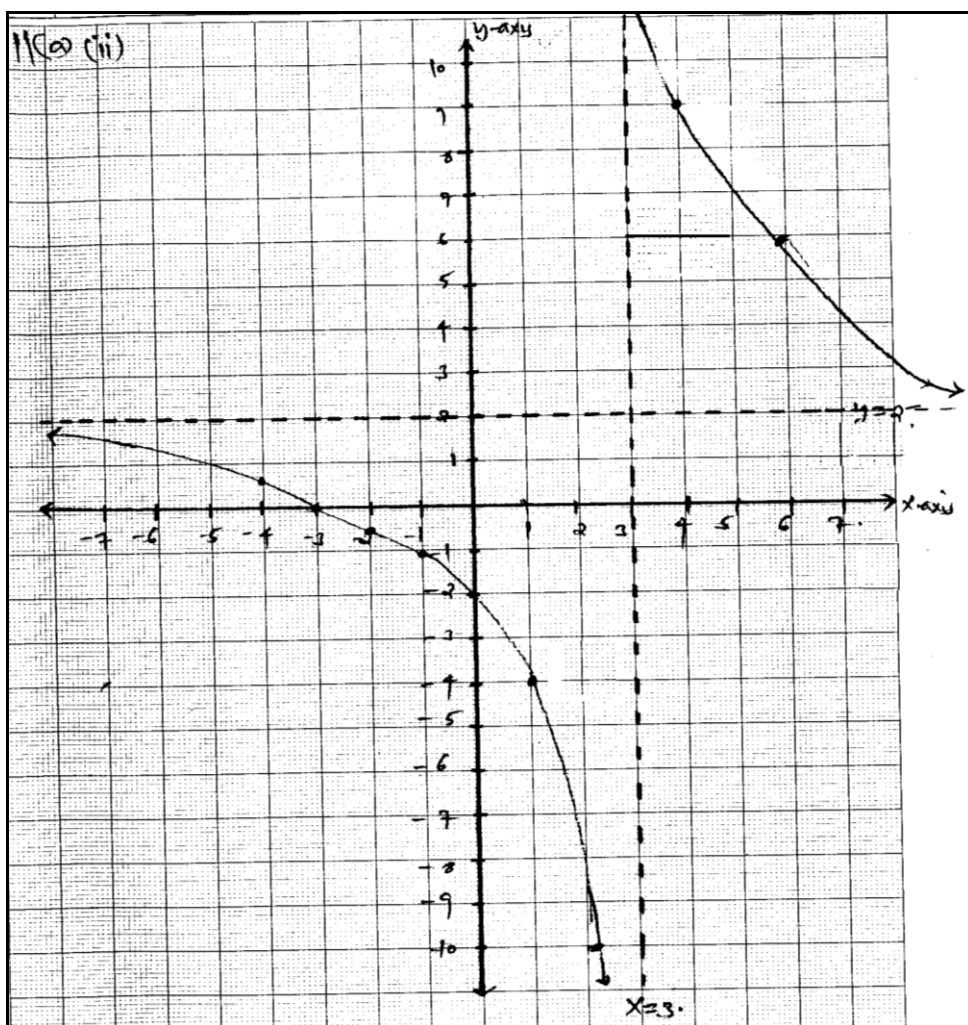
$$x-3=0.$$

$$x=3.$$

$$\text{N.A } x=3$$

Table value.

x	-3	-2	-1	0	1	2	3	4	5	6.
y.	0	-0.4	-1	-2	-4	-10	∞	14	8	6



(ii) graph done in graph provided.

(iii) $y=2$ when x numerical target.

11(b).

Soln

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^3 - m\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right) - 3$$

Where. $p(x) = 0$.

$$0 = \frac{1}{4} - \frac{1}{2} - \frac{1m}{4} + 3$$

$$-\frac{1}{4} + \frac{1}{2} - 3 = \frac{-1m}{4}$$

11	(b).	$-2\frac{3}{4} = -\frac{11}{4}$
		$\frac{-11}{4} = -\frac{m}{4}$
		$m = 11$
		Value of $m = 11$

Extract 11.1: A sample of the correct responses to question 11

In Extract 11.1, the candidate, in part (a) (i) found the asymptotes correctly; in part (a) (ii), sketched correctly the graph indicating all the necessary information; and in part (b), applied the remainder theorem correctly.

In contrast, 385 (98.47%) candidates were incompetent in rational function. In part (a) (i), many candidates failed to express y in terms of x . For example, some incorrectly wrote $y = \frac{x-3}{2x+6}$, resulting in incorrect

asymptotes, $y = \frac{1}{2}$ in particular. In part (a) (ii), some got the correct asymptotes but incorrectly represented asymptotes with continuous lines instead of being dotted. In part (a) (iii), most candidates wrongly interpreted the graph by substituting a particular value of x and obtained an incorrect corresponding value of y . In part (b), they had problems performing algebraic computations, ending up with incorrect answers such as $m = -13$ instead of $m = 11$.

11	(a) (i) soln
	$y(x-3) = 2x+6$
	$yx-3y = 2x+6$
	$y(x-3) = 2x+6$
	$\frac{yx-3y}{x-3} = \frac{2x+6}{x-3}$
	$y = \frac{2x+6}{x-3}$

Q(a)(i) But

$$V.A = X+3 \quad X-3=0$$

$$X-3=0$$

$$X=3$$

H.A

$$y = \frac{2x+6}{x-3}$$

$$= \frac{2x}{x} + \frac{6}{x}$$

$$\frac{x}{x} - \frac{3}{x}$$

$$= 2 + \frac{6}{x}$$

$$1 - \frac{3}{x}$$

$$= 2 + \frac{6}{0}$$

$$1 - \frac{3}{0}$$

$$H.A = 0$$

∴ The V.A is 3 and H.A is 0

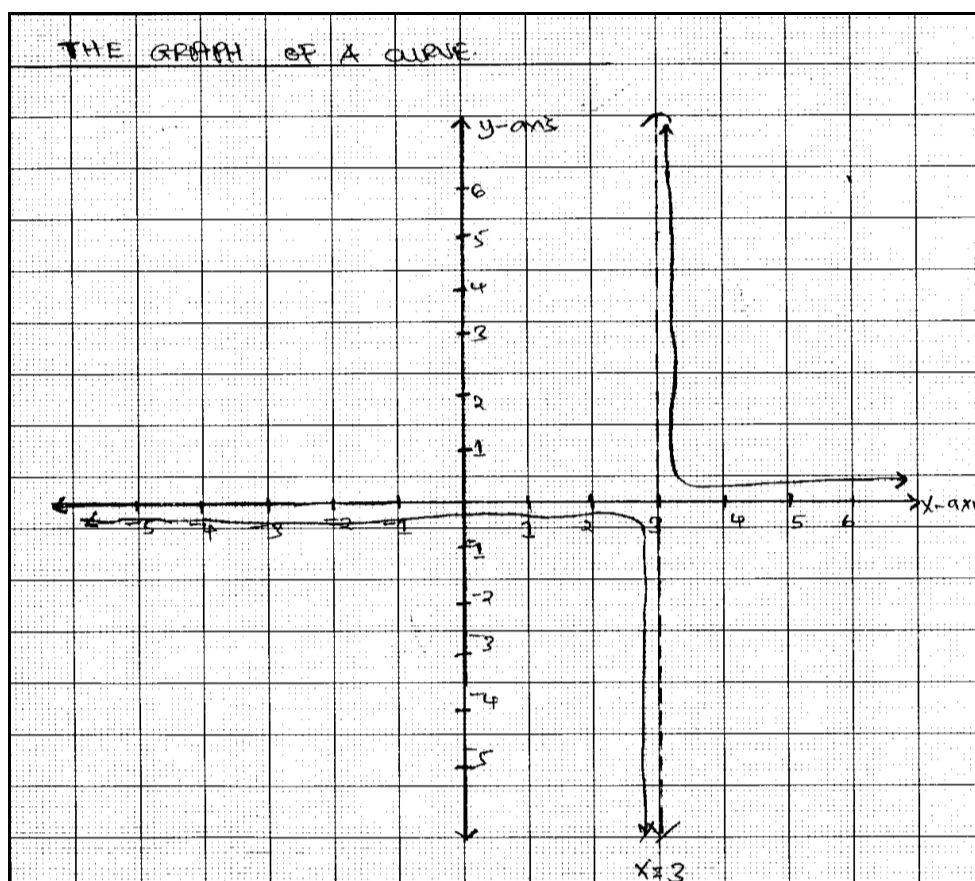
(ii) soln

$$H.A = 0$$

$$V.A = 3$$

Find X Intercept when $y=0$

$$y = \frac{2x+6}{x-3}$$



11(a) iiii)	$y(x-3) = 2x+6.$
	$\div 1$
	$y(x-3) = 2x+6$
	$xy-3y = 2x+6.$
	$(xy-3y)-(2x+6)=0.$
	~~$a=x, b=3$~~
	$y(x-3)+2(x-3)=0$
	$(y+2)(x-3)=0$
	$y+2=0 \text{ or } x-3=0$
	$\therefore y = -2 \text{ or } x=3.$
11. b)	$p(x) = 4x^4 - 4x^3 - mx^2 + 12x - 3$
	$p(1/2) = 4(1/2)^4 + -4(1/2)^3 - m(1/2)^2 + 12(1/2)$
	-3
	$p(1/2) = \frac{1}{4} + -\frac{1}{2} - \frac{1}{4}m + 6 = 3$
	$= -\frac{13}{4} - \frac{1}{4}m$
	$\frac{13}{4} = -\frac{1}{4}m$
	$\frac{-4m}{-4} = \frac{13 \times 4}{-4}$
	$m = -13.$
	$\therefore \text{The value of } m = -13.$

Extract 11.2: A sample of the incorrect responses to question 11.

In Extract 11.2, the candidate, in part (a) (i) failed to find a horizontal asymptote; in part (a) (ii), the graph had incorrect intercepts and a horizontal asymptote. In part (a) (iii), he/she did not use a graph to find the value of y as x became very large, and in part (b) they made computational errors.

2.12 Question 12: Integration and Differentiation

The question intended to check the extent to which the candidates were competent in differentiation and integration. The question comprised parts (a), (b), and (c) as follows:

- (a) A parabola is given by the curve $y = \sqrt{x+1}$. Use the first principles of differentiation to find the slope of the curve at any point.
- (b) Differentiate the function $y = \sin \sqrt{3x^2 + 1}$ with respect to x .

- (c) Suppose a curve given by the equation $y = 2x^2$ is bounded from $x=1$ to $x=3$. Calculate the area under the curve along:
- (i) the x -axis.
 - (ii) the y -axis.

In this question, 157 (40.15%) candidates scored from 3 to 10 marks, indicating that the overall performance of the candidates was average.

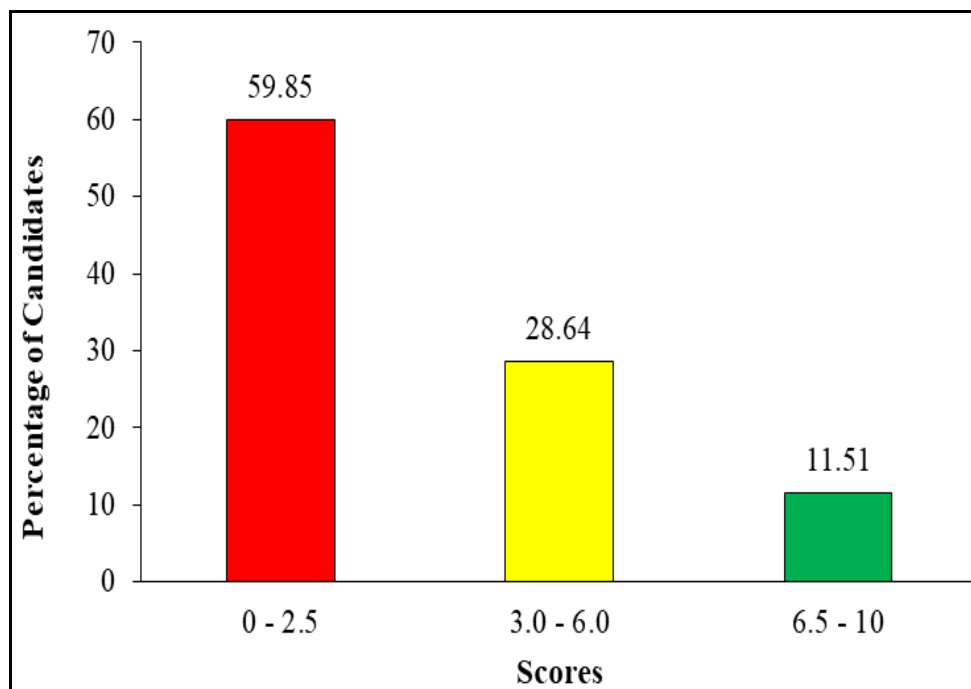


Figure 13: Candidates' Performance on Question 12

In this question, only 2 (0.51%) candidates were well equipped with skills and knowledge in differentiation and integration. In part (a), the candidates recognised that the slope of the curve is obtained by finding the derivative of the curve at the particular point. They correctly employed the formula for differentiating the function from the first principles, $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, as illustrated in Extract 12.1.

In part (b), the candidates correctly applied the chain rule to differentiate $y = \sin \sqrt{3x^2 + 1}$. They let $u = \sqrt{3x^2 + 1}$ making $y = \sin u$; then they found $\frac{du}{dx} = \frac{6x}{2\sqrt{3x^2 + 1}}$ and $\frac{dy}{du} = \cos u$, and employed the chain formula

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, obtaining $\frac{dy}{dx} = \frac{3x \cos \sqrt{3x^2+1}}{\sqrt{3x^2+1}}$. In part (c) (i), the

candidates applied the formula $Area = \int_a^b y dx$ to calculate the area of a

curve along the x – axis. Therefore, they evaluated $\int_1^3 2x^2 dx$ and obtained

$Area = 17\frac{1}{3}$ square units. Similarly, in part (c) (ii), they applied the

formula $Area = \int_a^b x dx$ to calculate the area of a curve along the y – axis.

Therefore, they evaluated $\int_2^{18} \sqrt{\frac{y}{2}} dy$ and obtained $Area = 34\frac{2}{3}$ square units.

12 a) Soln.

$y = \sqrt{x+1}.$

Soln.

$f(x) = \sqrt{x+1}.$

$f(x+h) = \sqrt{x+h+1}.$

$f'(x) = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+1} - \sqrt{x+1})}{h} \cdot \frac{(\sqrt{x+h+1} + \sqrt{x+1})}{(\sqrt{x+h+1} + \sqrt{x+1})}$

$f'(x) = \lim_{h \rightarrow 0} \frac{x+h+1 + \cancel{\sqrt{x+h+1}}(x+1) - \sqrt{x+h+1}(x+1)}{h\sqrt{x+h+1} + h\sqrt{x+1}}$

$f'(x) = \lim_{h \rightarrow 0} \frac{x+h+1 - x-1}{h\sqrt{x+h+1} + h\sqrt{x+1}}$

$f'(x) = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$

$f'(x) = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+1} + \sqrt{x+1}}$

$f'(x) = \lim_{h \rightarrow 0} \frac{1}{2\sqrt{x+1}}$

13) from:

x	1	2	3
$2x^2$	2	8	18

$y_1 = 2.$

$y_2 = 18$

$y = 2x^2.$

$\sqrt{\frac{y}{2}} = x$

$$A = \int_{y_1}^{y_2} x \, dy$$

$$A = \int_2^{18} \sqrt{\frac{y}{2}} \, dy$$

$$A = \int_2^{18} \frac{\sqrt{y}}{\sqrt{2}} \, dy$$

$$A = \int_2^{18} \left[\frac{2 y^{3/2}}{3\sqrt{2}} \right]_2^{18}$$

$$A = \left(\frac{2 (18)^{3/2}}{3\sqrt{2}} - \frac{2 (2)^{3/2}}{3\sqrt{2}} \right)$$

$$A = \frac{104}{3} \text{ square units}$$

Extract 12.1: A sample of the correct responses to question 12.

In Extract 12.1 the candidate, in part (a), used the first principles correctly to find slope, and in part (c), used integration to find the required area by integration.

Nevertheless, some candidates were incompetent in attempting this question. In part (a), they were not able to write the formula of differentiating the function from the first principles. For example, some wrote $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ or $\frac{dy}{dx} = \frac{f(x+\partial x) - f(x)}{\partial x}$ instead of $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. Other candidates failed to rationalise the numerator. Though the candidates made correct substitutions, they could not rationalise the numerator and ended up with an incorrect response,

commonly $\frac{dy}{dx} = -\frac{1}{2}(x+1)$. Moreover, some candidates inappropriately applied the chain rule, contrary to the requirement of the question. In part (b), they failed to apply the chain rule. Some of them started well but could not accomplish the work. For example, some let $u = \sqrt{3x^2+1}$ so that $y = \sin u$ and found $\frac{dy}{du} = -\cos u$ instead of $\frac{dy}{du} = \cos u$, thus ending up with an incorrect response. In part (c) (i), they substituted the limits of $x=1$ and $x=3$ in the function as $\left[2x^2\right]_1^3$ without integrating and obtained an incorrect area of 16 square units. Similarly, in part (c) (ii), they substituted the values of $x=1$ and $x=3$ in the given function $y = 2x^2$ and found the difference of the obtained values.

12 c. $y = 2x^2$.
 given $x = 1$ to $x = 3$.
 $\int_1^3 2x^2 dx$
 The area under the curve along x-axis.
 $\int_1^3 2x^2 dx$
 $= 2(3)^3 - 2(1)^3$
 $= 18 - 2$
 $= 16$
 $\therefore$ The area under the curve along x-axis = 16.

ii) Along y-axis.
 $y = 2x^2$
 $\frac{y}{2} = x^2$
 $x = \sqrt{\frac{y}{2}}$
 Given $x = 1$ to $x = 3$.
 $(1)^2 = \left(\sqrt{\frac{y}{2}}\right)^2$, $(3)^2 = \left(\sqrt{\frac{y}{2}}\right)^2$
 $1^2 = \frac{y}{2}$, $9 = \frac{y}{2}$
 $2 = y_1$, $18 = y_2$
 $y_2 - y_1$
 $18 - 2 = 16$
 $\therefore$ The area under the curve along y-axis = 16.

Extract 12.2: A sample of the incorrect responses to question 12

In Extract 12.2, the candidate calculated the difference of corresponding values of y from the given limits of x.

2.13 Question 13: Permutations and Combinations, and Probability

The question intended to examine candidates on their competences in the choice of r items from n items using the combination formula ${}^nC_r$ and use of probability knowledge to solve problems on combined events. The question consisted of the following parts, stated as follows:

- (a) *A classroom consists of 6 girls and 8 boys. If a discussion group of 4 students is formed, determine the number of ways in which the group will have;*
- (i) *any girl and a boy.*
 - (ii) *one girl and three boys.*
- (b) *The probabilities that YETU and KWETU companies will be producing their products in twenty years to come are 0.65 and 0.7 respectively. Calculate the probability that;*
- (i) *both companies will be producing their products in twenty years to come.*
 - (ii) *either YETU or KWETU company will be producing in twenty years to come.*
 - (iii) *in the twenty years to come YETU company will not be producing given that KWETU company is also not producing.*

The data reveal that 204 (52.17%) candidates scored from 0 to 2.5, 151 (38.62%) scored from 3 to 6 and 36 (9.21%) scored from 6.5 to 10. The overall performance of candidates in this question was average, as Figure 14 shows.

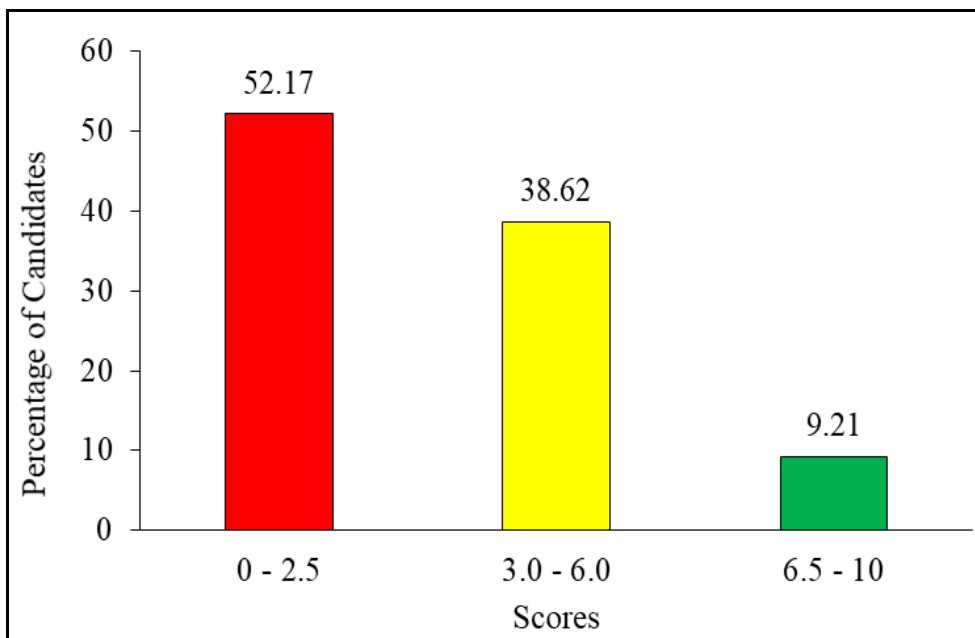


Figure 14: *Candidates' Performance on Question 13*

The data further reveal that only 2 (0.51%) candidates responded correctly to this question. In part (a) (i), they correctly interpreted the given statement and realised that the group could have any girl and a boy in such a way that there were *1 girl and 3 boys or 2 girls and 2 boys or 3 girls and 1 boy*. They then correctly calculated combinations $\binom{6}{1} \times \binom{8}{3} + \binom{6}{2} \times \binom{8}{2} + \binom{6}{3} \times \binom{8}{1}$, resulting in 916 ways. In part (a) (ii), the candidates correctly determined the number of ways in which the group of 1 girl and 3 boys could be selected by computing $\binom{6}{1} \times \binom{8}{3}$, obtaining 336.

In part (b) (i), the candidates correctly rewrote the task symbolically as $P(Y \cap K)$ and correctly defined it as $P(Y \cap K) = P(Y) \times P(K)$. They then substituted $P(Y) = 0.65$ and $P(K) = 0.7$, obtaining $P(Y \cap K) = 0.455$. In part (b) (ii), the candidates translated the task into $P(Y \cup K)$, defined it as $P(Y \cup K) = P(Y) + P(K) - P(Y \cap K)$ and resulted in $P(Y \cup K) = 0.895$. In part (b) (iii), the candidates recognised the existence of conditional probability $P(Y' | K')$ and correctly defined it as $P(Y' | K') = \frac{P(Y' \cap K')}{P(K')}$.

They also recalled the probability equivalences, $P(Y' \cap K') = P(Y \cup K)'$ and $P(Y \cup K)' = 1 - P(Y \cup K)$, and rewrote the formula $P(Y' | K') = \frac{P(Y' \cap K')}{P(K')}$ as $P(Y' | K') = \frac{1 - P(Y \cup K)}{1 - P(K)}$. With appropriate substitutions and calculations, obtaining $P(Y' | K') = 0.35$.

12	i/ $P(Y) \times P(K) = P(Y \cap K)$
	$P(Y \cap K) = 0.65 \times 0.7$
	$= 0.455$
	$\therefore$ The Probability is 0.455.
	ii/ $P(Y \cup K) = P(Y) + P(K) - P(Y \cap K)$
	$= 0.65 + 0.7 - 0.455$
	$= 0.895$
	$\therefore$ The Probability is 0.895
	iii/ $P(Y') = 0.35$, $P(K') = 0.3$
	$P(Y' \cap K') = 0.105$
	$P(Y') / K' = \frac{P(K' \cap Y')}{P(K')}$
	$= \frac{0.105}{0.3}$
	$= 0.35$
	$\therefore$ The Probability is 0.35

Extract 13.1: A sample of the correct responses to question 13

In Extract 13.1, the candidate employed the correct formula for probability of intersection, union and conditional probability.

Contrarily, 389 (99.59%) lost all or some marks in this question. In part (a), many candidates computed permutations instead of combinations. For instance, they wrote ${}^6P_3 \times {}^8P_1$, obtaining 960 instead of 336. Some calculated probability instead of the number of ways of having one girl and three boys, indicating that they did not understand the task correctly. In part (b) (i), some candidates used the inappropriate formula for finding the probability of $P(Y \cap K) = P(Y) + P(K)$ instead of $P(Y \cap K) = P(Y) \times P(K)$, resulting in an incorrect answer of 1.35. In part (b) (ii), some also applied an inappropriate formula, commonly $P(Y \cup K) = P(Y) + P(K)$ instead of $P(Y \cup K) = P(Y) + P(K) - P(Y \cap K)$. In part (b) (iii), some candidates failed to realise that it was conditional probability; instead, they applied the incorrect formula, $P(Y \cap K) = P(Y) + P(K) - P(Y \cap K) + P(Y \cap K)$, in particular.

13	b) <u>Solution</u>
	<u>Data provided</u>
	let A represent VETU
	B represent KLUETU
	then; $P(A) = 0.65$
	$P(B) = 0.7$
	i) $P(A \cup B) = P(A) + P(B)$
	$P(A \cup B) = 0.65 + 0.7$
	<u>$P(A \cup B) = 1.35$</u>
	(ii) $P(A \cap B) = P(A) \times P(B)$
	$P(A \cap B) = 0.65 \times 0.7$
	<u>$P(A \cap B) = 0.455$</u>
	(iii) $P(A/B) = \frac{P(A \cap B)}{P(B)}$
	$P(A/B) = \frac{0.455}{0.7} = 0.65$
	<u>$\therefore P(A/B) = 0.65$</u>

Extract 13.2: A sample of the incorrect responses to question 13

In Extract 13.2, the candidate in part (b) (i) incorrectly interpreted the question, in (b) (ii) applied an inappropriate formula and in part (b) (iii) made a wrong computation of conditional probability.

2.14 Question 14: Vectors, Matrices and Linear Transformation

The question examined the candidates' skills in the application of perpendicular vectors to solve related problems and their knowledge of the dot and cross products of vectors. Furthermore, it intended to examine their ability in the transformation of word problems into mathematical models, the use of Cramer's rule and finding the image of a point after rotation. The question comprised three parts, (a), (b) and (c), as follows:

- (a) (i) Given two vectors, $\underline{a} = 6\underline{i} - 3\underline{j} + 3\underline{k}$ and $\underline{b} = 4\underline{i} + c\underline{j} - 2\underline{k}$. Find the value of c such that vector $\underline{b}$ is perpendicular to vector $\underline{a}$.
- (ii) If $\underline{a} = 2\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = 3\underline{i} - \underline{j} + \underline{k}$, what will be the value of $(\underline{a} \times \underline{b}) \cdot \underline{b}$?
- (b) The costs of onions, wheat and rice at three different markets were as follows: In the first market, the total cost for 4 kg of onions, 3 kg of wheat and 2 kg of rice was 60 dollars. In the second market, 2 kg of onions, 4 kg of wheat and 6 kg of rice amounted to 90 dollars. In the third market, 6 kg of onions, 2 kg of wheat and 3 kg of rice had a total cost of 70 dollars. Find the cost of each item per kg by using the Crammer's rule.
- (c) Find the image of the point $(-7, 3)$ after rotation of 45° about the origin in a clockwise direction.

The general candidates' performance was good because 271 (69.31%) scored from 3 to 10 marks. Figure 15 summarises the candidates' performance on this question.

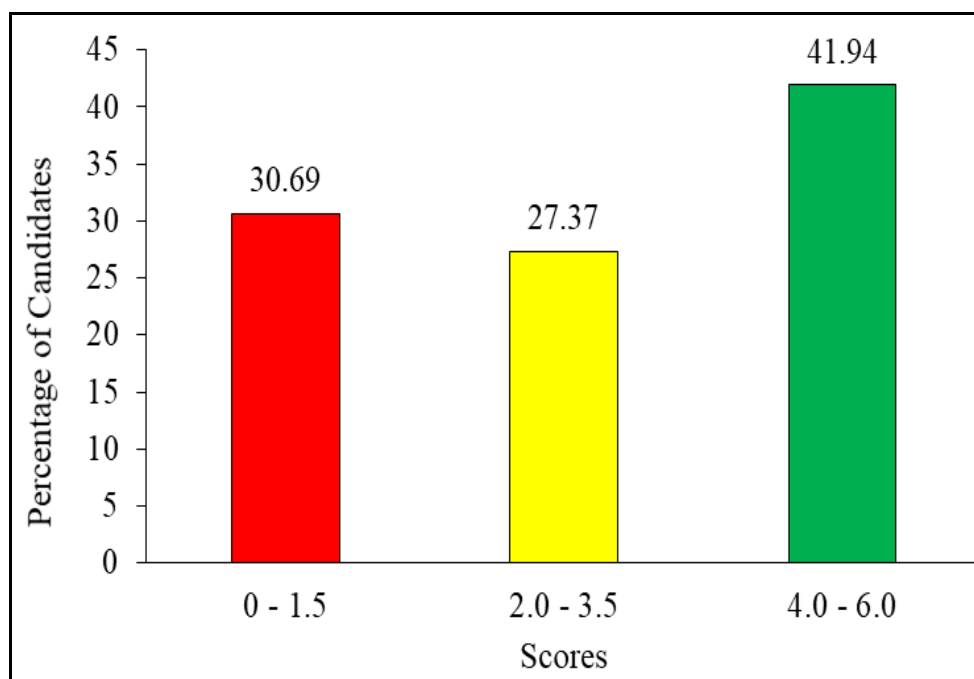


Figure 15: Candidates' Performance on Question 14

In this question, 36 (9.21%) candidates responded to all parts correctly. In part (a) (i), they recognized that if vectors $\underline{a}$ and $\underline{b}$ are perpendicular, then $\underline{a} \cdot \underline{b} = 0$. They used vectors $\underline{a}$ and $\underline{b}$ and this condition to formulate the equation $24 - 3c - 6 = 0$ and correctly solved it obtaining $c = 6$. In part (a) (ii), they correctly computed the value of $(\underline{a} \times \underline{b}) \cdot \underline{b}$, as illustrated in Extract 14.1.

In part (b), they assigned letters to represent the kilograms of onions, wheat and rice, commonly x , y and z , respectively. With these letters, they rewrote the word problem using three simultaneous equations, $4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$ and $6x + 2y + 3z = 70$, and correctly solved them simultaneously using Cramer's rule, obtaining $x = 5$, $y = 8$ and $z = 8$.

In part (c), the candidates recalled the correct formula $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, for finding the image (x', y') of a point (x, y)

after rotation of 45° about the origin. Since the rotation was done in a clockwise direction, they realised that the angle is negative, that is, -45° . Therefore, they substituted the points $(x, y) = (-7, 3)$ and

$$\sin(-45^\circ) = -\frac{\sqrt{2}}{2}, \quad -\sin(-45^\circ) = \frac{\sqrt{2}}{2}, \quad \text{and}$$

$$\cos(-45^\circ) = \frac{\sqrt{2}}{2} \quad -\cos(-45^\circ) = -\frac{\sqrt{2}}{2} \quad \text{into}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(-45^\circ) & -\sin(-45^\circ) \\ \sin(-45^\circ) & \cos(-45^\circ) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and performed the calculations,}$$

correctly obtaining the image $(x', y') = (-2\sqrt{2}, 5\sqrt{2})$.

(ii), Solution,

$$a = 2\hat{i} + \hat{j} - 2\hat{k} \quad b = 3\hat{i} - \hat{j} + \hat{k}.$$

$$(a \times b) \cdot b = ?$$

$$a \times b = .$$

$\hat{i}$	$\hat{j}$	$\hat{k}$
2	1	-2
3	-1	1

$$(1-2)\hat{i} - \hat{j}(2+6) + \hat{k}(-2-3).$$

$$(a \times b) = -\hat{i} - 8\hat{j} - 5\hat{k}.$$

$$(a \times b) = -\hat{i} - 8\hat{j} - 5\hat{k}.$$

$$(-\hat{i} - 8\hat{j} - 5\hat{k}) \cdot (3\hat{i} - \hat{j} + \hat{k}).$$

$$-3 + 8 - 5 = 0.$$

$$(a \times b) \cdot b = 0.$$

14(b). Solution.	
By formulating the equations	
Let $x = \text{onions}$ $y = \text{wheat}$ $z = \text{rice}$.	
$\begin{cases} 4x + 3y + 2z = 60 \\ 2x + 4y + 6z = 90 \\ 6x + 2y + 3z = 70 \end{cases}$	
write in matrix form,	
$A = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix}.$	
$ A = 4(0) - 3(6 - 36) + 2(4 - 24).$	
$ A = (-36 - 30) + (2 \times -20)$	
$90 - 40 = 50$	
$ A = 50.$	
$ A_x = \begin{vmatrix} 60 & 3 & 2 \\ 90 & 4 & 6 \\ 70 & 2 & 3 \end{vmatrix}$	$ A_x = 60(0) - 3(270 - 420) + 2(180$
$ A_x = 450 - 200$	$ A_x = 250.$
$ A_y = \begin{vmatrix} 4 & 60 & 2 \\ 2 & 90 & 6 \\ 6 & 70 & 3 \end{vmatrix}$	$ A_y = 4(-150) - 60(-30) + 2(-400).$
$ A_y = 400.$	$-600 + 1800 - 800$
$ A_z = \begin{vmatrix} 4 & 3 & 60 \\ 2 & 4 & 90 \\ 6 & 2 & 70 \end{vmatrix}$	$ A_z = 4(100) - 3(-400) + 60(-20).$
$ A_z = 400.$	$400 + 1200 - 1200$
$z = \frac{ A_z }{ A } = \frac{400}{50} = 8.$	$y = \frac{ A_y }{ A } = \frac{400}{50} = 8$
$x = \frac{ A_x }{ A } = \frac{250}{50} = 5.$	
The value of $x = 5$ $y = 8$ and $z = 8.$	

Extract 14.1: A sample of the correct responses to question 14

In Extract 14.1, in part (a) (ii), the candidate correctly performed cross and dot products, and in part (b), the candidate rewrote the word problem in three equations and correctly solved them using Cramer's rule.

In spite of good performance, some of them faced some challenges. In part (a) (i), many candidates employed the incorrect condition for two perpendicular vectors, as shown in Extract 14.2. Other candidates performed dot product $\underline{a} \cdot \underline{b} = 0$ incorrectly, whereby the majority multiplied the vectors as if they were performing basic operation on numbers.

In part (a) (ii), some candidates performed the dot product and cross product interchangeably, resulting in the incorrect statement $(\underline{a} \times \underline{b}) \cdot \underline{b} = (3i - j + k)(3i - j + k) \times (2i + j - 2k)$ and consequently an incorrect answer, $22\hat{i} + 11\hat{j} - 21\hat{k}$.

In part (b), most candidates failed to write the equations in the correct matrix form, and thus, they could not use Cramer's rule to solve them.

Also, some candidates failed to generate correct equations from a given word problem including $4x + 3y + 3z = 60$, $2x + 4y + 6z = 90$ and $6x + 2y + 3z = 70$. In addition, though some candidates correctly wrote the

equations in the matrix form
$$\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix},$$
 they failed to

calculate the determinant of the 3×3 matrices.

In part (c), some candidates applied the wrong formula in finding the image of point $(-7, 3)$, especially $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, ending up with

incorrect answer, $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -4 \\ 10 \end{pmatrix}$. Some candidates substituted $\theta = +45^\circ$ into

the formula $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, indicating failure to interpret well the sign for the angle measured in a clockwise direction.

14. a) i.	$\underline{a} = 6\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$
	$\underline{b} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$
	$\underline{a} \bullet \underline{b} = -1$
	$(6 \times 4) + (-3 \times 1) + (3 \times -2) = -1$
	$24 - 30 - 6 = -1$
	$24 - 6 - 30 = -1$
	$24 - 18 - 30 = -1$
	$18 + 1 = 30$
	$19 = 30$
	$c = 19$
	3

Extract 14.2: A sample of the incorrect responses to question 14

In Extract 14.2, the candidate employed the incorrect condition for perpendicular vectors, that is, $\underline{a} \bullet \underline{b} = -1$ instead of $\underline{a} \bullet \underline{b} = 0$.

3.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH TOPIC

In CSEE 2025, the Additional Mathematics paper had sections A and B with a total of fourteen (14) questions with a total of sixteen (16) topics. The analysis shows that 6 questions set from 8 topics had good performance. These topics are *Variations* (99.49%), *Numbers* (87.72%), *Equations and Remainder Theorem* (86.96%), *Statistics* (81.84%), *Symmetry* (76.21%), and *Matrices, Linear Transformation and Vectors* (69.31%). The good performance of candidates in these topics was fortified by their ability to identify the requirement of the questions, recall and use appropriate formulae, facts, laws and theorems, and their ability to translate word problems into mathematical models.

Additionally, the analysis depicted that 7 questions were performed on average. These questions derived from 8 topics of *Locus* (55.24%), *Coordinate Geometry* (51.66%), *Probability and Permutations and Combinations* (47.83%), *Sets* (45.27%), *Logic* (42.20%), *Differentiation and Integration* (40.15%) and *Trigonometry* (37.85%). The average performance was due to the moderate ability in subject matter, attributed to insufficient knowledge and skills on the examined concepts, inability to

apply appropriate formulae, errors in doing basic operations, misinterpretation of some word problems, and failure to adhere to the instructions of a specific question.

Moreover, the candidates had weak performance on one question, which was set from the topic of *Algebra*, attaining an overall performance of 12.53 per cent. This performance was highly attributed to the inability of the candidates to solve inequalities involving absolute values.

Finally, the analysis shows that there is an increase in the performance of the candidates in the topics of *Statistics* (36.4%), *Vectors, Matrices and Linear Transformation* (10.9%), *Symmetry* (2.7%), *Equations and Remainder Theorem* (1.6%) and *Variations* (1.4%) for 2025 as compared to 2024. In contrast, performance on the other topics decreased as compared to 2024. These topics are *Algebra* (79.1%), *Sets* (35.3%), *Trigonometry* (23.0%), *Locus* (22.1%), *Differentiation and Integration* (20.1%), *Logic* (14.8%), *Coordinate Geometry* (10.5%), *Numbers* (9.0%) and *Probability and Permutations and Combinations* (4.1%). Appendix II provides the candidates' performance per topic in 2024 and 2025.

4.0 CONCLUSION AND RECOMMENDATIONS

The analysis of students' performance in CSEE 2025 for Additional Mathematics, with few students failing and limited representation in grade A. Item and competence-level analyses also identify specific areas of difficulty across the syllabus. In response to these findings, the following recommendations are proposed to enhance mathematics learning and teaching in ordinary secondary schools in Tanzania:

- (a) Strengthening Set Theory and Counting Techniques
 - (i) Teachers are encouraged to emphasise the application of set properties and De Morgan's law in simplifying set expressions by providing additional revision exercises to strengthen learners' knowledge and skills.
 - (ii) Teachers should clarify the application of combinations and permutations through additional exercises on a variety of problems, ensuring learners acquire sufficient knowledge and skills and reducing confusion caused by the occasional interchangeable use of these concepts.

(b) Enhancing Geometry, Coordinate Geometry and Trigonometry Competence

- (i) Teachers are advised to teach the topic of Plan and Elevation practically, using physical 3-dimensional objects and simulations, to enable learners to develop competence in drawing plans, side views, and front elevations of different figures. This approach will enhance learners' confidence and improve performance.
- (ii) Through group classwork and quizzes, teachers should guide students to derive the general formula of a circle and apply it in solving mathematical problems.
- (iii) Teachers are recommended to provide additional revision exercises to help students distinguish between the properties of perpendicular lines and orthogonal circles.
- (iv) The derivation of trigonometric identities should be prioritised to enhance students' ability to deduce double-angle formulae from compound-angle formulae, thereby improving their problem-solving skills in trigonometry.

(c) Improving Calculus Application Skills

Greater emphasis should be placed on differentiation and integration, particularly on their applications, by addressing common mistakes made by students. This will help improve performance in these topics.

(d) Mathematical Reasoning, Logic, and Problem Interpretation

Students should develop competence in the following areas:

- (i) Interpreting word problems and translating them into mathematical models by learning important mathematical keywords, such as *sum*, *difference*, *product*, *quotient*, *at least*, and *at most*, as commonly used in word problems.
- (ii) Developing and simplifying compound statements and testing the validity of arguments using truth tables and laws of propositional algebra through frequent application of De Morgan's law and other relevant laws.

- (iii) Applying appropriate formulae in statistics, coordinate geometry, and probability through regular practice to acquire necessary skills and problem-solving techniques.

APPENDIX I

Analysis of Candidates' Performance on Each Topic in 2025

S/N	Topic	Question Number	Percentage of Candidates who Passed	Remarks
1	Variations	1	99.49	Good
2	Numbers	8	87.72	Good
3	Equations and Remainder Theorem	11	86.96	Good
4	Statistics	2	81.84	Good
5	Symmetry	6	76.21	Good
6	Vectors, Matrices and Linear Transformation	14	69.31	Good
7	Locus	4	55.24	Average
8	Coordinate Geometry	3	51.66	Average
9	Permutations and Combinations, and Probability	13	47.83	Average
10	Sets	10	45.27	Average
11	Logic	9	42.20	Average
12	Differentiation and Integration	12	40.15	Average
13	Trigonometry	7	37.85	Average
14	Algebra	5	12.53	Weak

APPENDIX II

Candidates' Performance on Each Topic for the Years 2024 and 2025

